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UN ENFOQUE COGNITIVO DEL FHA PARA CORREGIR SESGOS EN EL JUICIO DE EXPERTOS: APLICACIÓN A UN CASO[1]

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Resumen

El propósito de este trabajo es presentar un enfoque metodológico integrador que sirva de apoyo a la toma de decisiones.

Hasta ahora la literatura científica ha producido principalmente trabajos de dos clases: análisis descriptivo, que se refiere a los procesos reales que caracterizan la valoración y selección de los individuos, y el análisis normativo, que analiza los procesos de selección realizados por individuos racionales idealizados.

Entonces, cuando hablamos de un enfoque integrador, pretendemos desarrollar una metodología, que aun partiendo de instrumentos cuantitativos típicos del análisis normativo, tome también en consideración las implicancias cognitivo-comportamentales obtenidas por los especialistas en toma de decisión. Hemos desarrollado un modelo aplicativo basado en el análisis jerárquico fuzzy (FHA), en el que a las capacidades del proceso de jerarquía analítico (AHP) de racionalizar el proceso de decisión sin prescindir de las valoraciones, se añaden elementos de la teoría de conjuntos borrosos que permiten al decisor expresar la ambigüedad de su propia valoración.

Este método corrige los juicios tomando en consideración los llamados sesgos cognitivos, es decir, distorsiones subjetivas relacionadas con la percepción de la utilidad y la incertidumbre.

Por último, se ha llevado a cabo una experiencia para verificar el valor del modelo propuesto, los límites de su aplicabilidad y los posibles desarrollos futuros.

Palabras clave: análisis jerárquico fuzzy, toma de decisión, enfoque cognitivo.

[1] Presentado en XII Congreso Internacional de la Sociedad de Gestión y Economía Fuzzy (SIGEF). 26-28 de Octubre 2005, Bahía Blanca, Argentina.

A COGNITIVE APPROACH FOR FHA TO CORRECT BIASES IN EXPERTS' JUDGEMENT: EMPIRICAL EVIDENCE IN A DECISION MAKING SITUATION[2]

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Abstract

This paper aims to propose a methodological integrated approach to support experts in decision making situation.

Until now scientific literature has mainly produced works of two types: descriptive analysis, that talks about the real processes that characterize the evaluation and selection of individuals, and the normative analysis, that analyses the process of selection made by rational individuals.

When we talk about an integrated approach, we try to develop a methodology, that even starting off of typical quantitative instruments of the normative analysis, also takes in consideration the cognitive side obtained by the specialists in decision making. We have developed an empirical model based on the fuzzy hierarchical analysis (FHA). In addition the capacities of Analytical Hierarchy Process (AHP) to rationalize the process of decision considering the evaluations, elements of the fuzzy set theory are considered in order to allow experts to express the ambiguity of their own evaluations.

The proposed methodology corrects the judgments taking in consideration the so called cognitive biases, that is to say, subjective distortions related to the perception of utility and uncertainty. Finally, an experience has been carried out to verify the value of the proposed model, the limits of its applicability and possible future developments.

Keywords: *Fuzzy hierarchical analysis, Decision making, Cognitive approach.*

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[2] Presented in XII Congreso Internacional de la Sociedad de Gestión y Economía Fuzzy (SIGEF). 26-28 October 2005, Bahía Blanca, Argentina.

1. Introduction

In this paper we show the first empirical results obtained using some models that cognitive psychology built up into the so-called "descriptive analysis", with regard to the multi-criteria decision making support technique known as "fuzzy hierarchical analysis" (FHA). In particular we adopt "prospect theory" and "ambiguity model" to correct biases in experts' judgements corresponding to the pairwise comparisons in the FHA.

We show that using the proposed method we are able to improve the accuracy of the priority vectors assessed by the solutions of the FHA technique.

2. Cognitive Biases influence in decision making: literature review

In literature we find three kind of approaches to decision making, all of them having different subjects' backgrounds.

The first one is called *normative analysis* (Savage 1954; Luce and Raiffa 1957; Fishburn and Kochenberger, 1979; Fishburn, 1982), and takes its origins from the game theory by von Neumann and Morgenstern (1944), that lays the grounds of Utility Theory (UT). Through the assessment of order, independence and continuity axioms, UT implies the existence of a real valued function $u(x)$ defined on the subset X of possible outcomes (or states of the world) that gives a numerical structure to the choice options (see Fig.1). In particular let $p,q,\ldots$, be probability distributions defined on a set X of outcomes. Each $p \in P$ can be viewed as a risky alternative that yields outcome $x \in X$ with probability $p(x)$, with the $p(x)$ summing to unity. The overall utility of an alternative p is therefore

$$u(p) = \sum_{x \in X} p(x)u(x).$$

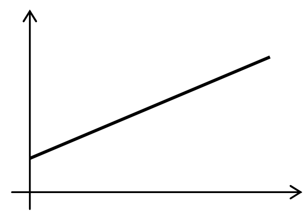

Figure 1. Von Neumann – Morgerstern utility function (1944)

In general normative analysis has to do with how idealized, rational people should think and should act. Such analyses abstract away known cognitive concerns of real people, their internal disorders, their shifting values, their anxieties and post-decisional regrets, their rejection for ambiguity, their inabilities to do intricate calculations, and their limited attention span. The hallmarks of such normative analyses are coherence and rationality, usually captured in axioms of the form: if the decision maker believes so and so, he should do such and such. Axioms and basic principles are motivated by what some investigator thinks is logical, rational, intelligent behaviour, and yield conditions of optimality for choice.

The second approach is known as *descriptive analysis* (Allais 1953; Simon 1955, 1956; McNeil et al. 1962; Slovic and Tversky 1974; Kahneman and Tversky 1979; Einhorn and Hogarth 1978), a highly empirical activity that lies in the social sciences concerned with individual behaviour, which poses questions like: how do real people think and behave? How do they perceive uncertainties, accumulate evidence, learn and update perceptions? What are their hang-ups, biases, internal conflicts? Which are the processes that bring them to make a choice? How can their behaviour be (approximately) described?

In other words, descriptive analysts are concerned with how and why people think and act the way they do. They try answering that questions without any interest in trying to modify, influence or moralize individual behaviour (see Fig.2).

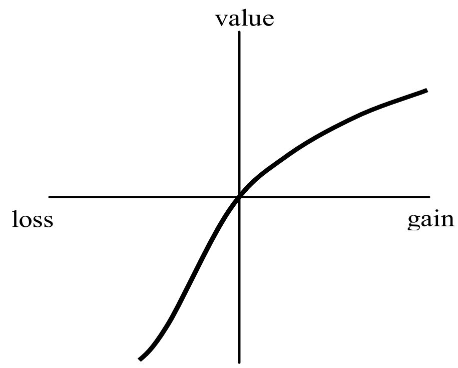

Figure 2. Value function in the prospect theory (Kahneman and Tversky, 1979)

Two among the most noteworthy descriptive models of human behaviour are the *prospect theory* (Kahneman and Tversky 1979) and the *ambiguity model* (Einhorn and Hogarth; see ref. in Hogarth 1986). The first one shows through empirical evidence how perception of value (=utility) in assessing preferences and decisions commonly deviates from the original linear shape assumed by von Neumann-Morgerstern utility function. The second highlights that people are ambiguous concerning the probabilities of events that can affect outcomes. In Einhorn-Hogarth model people are assumed to assess ambiguous probabilities by first anchoring on some value of the probability and then adjusting this figure by mentally simulating or imagining other values the probability could take. This process basically depends on the decision maker's attitude toward uncertainty, which reflects his personal tendencies such as *optimism* or *pessimism*.

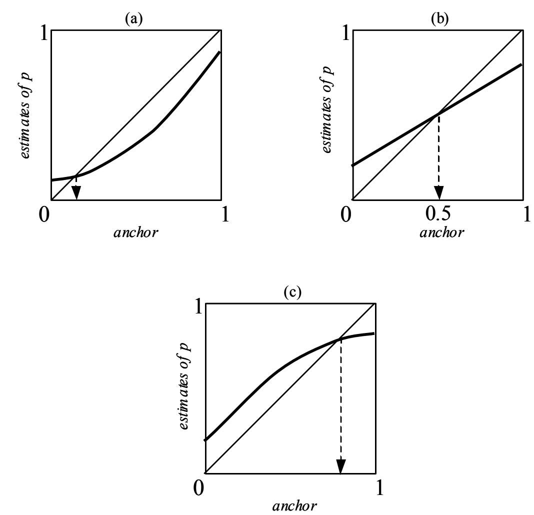

Figure 3. Ambiguity model (Einhorn and Hogarth, 1986)

In panel (a), for example, values of the probability below the diagonal (that represents the anchor value) are weighted in imagination more heavily than those above; in case of determining the chances of obtaining a positive outcome this reflects a pessimistic attitude. In panel (c) values above the diagonal are weighted more heavily than those below; in case of determining the chances of obtaining a positive outcome this reflects an optimistic attitude. Panel (b) reflects what we may call *neutral* attitude.

In the third and last approach to decision making, researchers (often called "methodologist") are concerned with the bottom line: how to improve the quality of decisions in practice? It is one thing to talk of axioms and proofs (normative side) and of cognitive limitations and biases (descriptive side), but how can we really help people making better decisions? They therefore try to devise methods that incorporate the insights gained from normative theories, but in a way that recognizes the cognitive limitations of the decision maker. To this approach belong several methods supporting decision making among which there is the Fuzzy Hierarchical Analysis (FHA). On FHA we implement our model as described in section 4.

We believe this paper also pertains to the last kind of approach to decision making, even though we use in the FHA some cognitive models, that makes our perspective new and maybe wider. But we still adapt that descriptive theories in a method.

Our aim is still posed in the question: how can we help people to make better decisions?

3. THE FHA AS MULTI-ATTRIBUTE DECISION MAKING SUPPORT TECHNIQUE

The FHA develops Saaty's original hierarchical analysis (Saaty 1977, 1978, 1980), when the experts (judges,...) are allowed to use fuzzy ratios in place of exact ratios. In Saaty's hierarchical analysis a person (expert, judge,...) is asked to supply ratios a_{ij} for each pairwise comparison between issues (alternatives, candidates,...) $A_1, A_2, \ldots, A_n$ for each criterion in a hierarchy, and also between the criteria. For some specific criterion, if a person considers A_1 more important than A_5, then a_{15} might equal $3/1$, or $5/1$, or $7/1$. The numbers of the ratios are usually taken from the set $\{1,2,\ldots,9\}$ so a_{15} could be s_1/s_5 for $s_1, s_5 \in \{1,2,\ldots,9\}$ and $s_1 > s_5$. The ratios a_{ij} indicate, for this expert, the strenght with which A_i dominates A_j. If a_{15} is equal to $5/1$, then a_{51} is taken as $1/5$. That is, $a_{ji} = a_{ij}^{-1}$ and $a_{ii} = 1$ for all *i*. Let **A** be the $n \times n$ matrix whose entries are the ratios a_{ij}. **A** is called a positive reciprocal matrix. Saaty's procedure uses the pairwise comparison matrices **A** for each criterion, and also the pairwise comparison matrix for the criteria, to compute a final set of weights w_i $(w_i > 0, w_1 + w_2 + \ldots + w_n = 1)$ for the alternatives which can be used to rank the issues from highest to lowest.

We easily can recognize that is difficult for people to always assign exact ratios when comparing two alternatives. When comparing A_1 and A_5 a person might feel that A_1 is much more important than A_5. Does this mean that a_{15} equals $5/1$, $7/1$ or $9/1$? Using fuzzy numbers (Zadeh 1965; Zimmermann 1993; Dubois and Prade 1980) an expert can respond that a_{15} is between 7 and 9. Also, a person could feel that A_1 is little more important than A_5. An appropriate fuzzy ratio in this case might be approximately 3. Fuzzy numbers automatically incorporate the vagueness of these replies.

There are several methods to compute the fuzzy final weights representing the priority vector. They consist in an extension of Saaty's procedure to fuzzy reciprocal matrices, and was first introduced by van Laarhoven and Pedricz (1983). Other researchers developed more accurate methods (Buckley 1985, Buckley *et al*, 2001; Boender *et al* 1989, Gogus and Boucher, 1997). Anyway, choosing one method rather than another does not change or invalidate the model we introduce in section 4 to improve the accuracy of the FHA technique.

4. Methodological proposal

We made clear the FHA is an expert based technique, that an individual is asked to supply judgements for each pairwise comparison, in particular we will represent these judgements through triangular fuzzy numbers[*]. If we now imagine the support to decision making (in this case the FHA) being an opened system as shown in Fig. 4, we recognize expert's judgements correspond to the input of such a system. Similarly we may define its output as the solution of the decision making problem yielded by the technique.

Figure 4. Decision making support technique in expert's judgements

In this scenario it is easy to realize that if expert's judgements are affected by some cognitive biases, and we know they really are (according to descriptive models), there are no reasons for us to think the output will not keep these distortions no matter how good the support technique is. Hence our focus will be on the "input" of the above-mentioned system, proposing a model that tries to correct expert's judgements and their biases.

For our purposes we first define triangular fuzzy numbers representing expert's judgements as functions of two variables: the modal value v_m and the uncertainty i that contains all the elements that univocally determine the fuzzy number spread. We then perform two independent transformations (see Fig. 5) for the fuzzy number $\tilde{a}(v_m, i)$ thereby defined; one operating on v_m through the value function of prospect theory and another operating on i through the ambiguity function of ambiguity model.

[*] The method can easily be extended to an FHA that uses other types of fuzzy numbers. Using triangular shaped fuzzy numbers does not therefore mean a lost in generality for the concepts of our proposal.

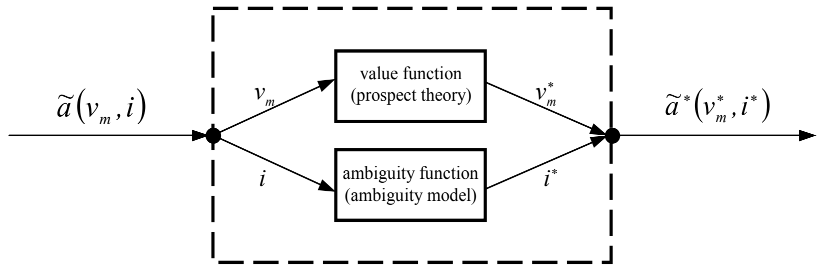

Figure 5. Transformation methodology proposed for the fuzzy number

When a person gives an evaluation of a pairwise comparison that we set as v_m, he is actually supplying a *perceived value*, which fits the shape of prospect theory value function. If we want to obtain something closer to an "objective value" we shall find the point $P(x, y)$ of the value function such that $y = v_m$ (see Fig.6), and then perform the substitution:

$$v_m = y \qquad \rightarrow \qquad v_m^* = x \, .$$

For example, if the expert's assessment is $v_m = 6$, we will find his unbiased evaluation $v_m^* = 7.7$ in the following way.

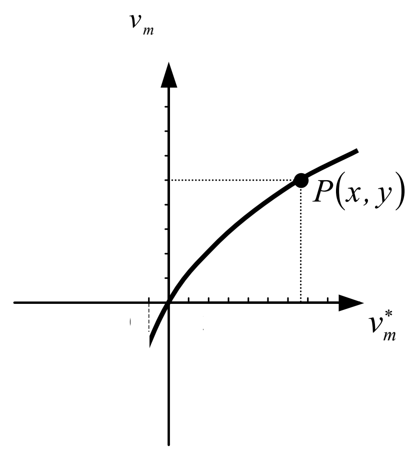

Figure 6. Objectification of the proposed value

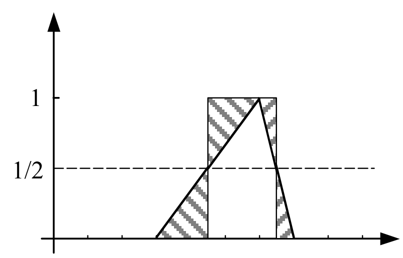

Figure 7. Measure of the fuzziness representing the expert's judgements

For the second transformation we shall first assign to the decision maker (or to the group of decision makers) his proper curve among those shown in Fig.3. We can further work out a *measure of fuzziness* of the fuzzy number representing the expert's judgement (see Fig.7). This can be made through the "index of fuzziness" by Kaufmann (1975) defined on a fuzzy set $\widetilde{A}$ as follows:

$$d(\widetilde{A}) = \sum_i \left| \mu_{\widetilde{A}}(x_i) - \mu_{\widetilde{A}_{1/2}}(x_i) \right| ,$$

where $\widetilde{A}_{1/2}$ is the 1/2-cut of $\widetilde{A}$, remembering from fuzzy set theory that

$$\mu_{\widetilde{A}_{1/2}}(x) = \begin{cases} 1 & \text{if} \quad \mu_{\widetilde{A}}(x) \geq 1/2 \\ 0 & \text{otherwise} \end{cases} .$$

Since we deal with particular kinds of fuzzy set having continuous membership function (fuzzy numbers), it is legitimate to use a slightly different index of fuzziness:

$$D(\widetilde{A}) = \int_X \left| \mu_{\widetilde{A}}(x) - \mu_{\widetilde{A}_{1/2}}(x) \right| \mathrm{d}x ,$$

which represents for the fuzzy number $\widetilde{a} = (3,6,7)$ taken as example, the marked area in Fig. 7.

If we set $D_{max} = \max_{\widetilde{A}} D(\widetilde{A})$ as the index of fuzziness of the most ambiguous fuzzy set, this quantity only depends on the evaluation scale chosen. As a matter of fact the ambiguity of a fuzzy set is higher if its membership function gets closer to the value $1/2$, in which case it is obviously more difficult to determine if the element x belongs or not to the set. If, for example, we use Saaty's scale (from the set $\{1/9, \ldots, 1/2, 1, 2, \ldots, 9\}$), we will have

$$D_{max} \cong \frac{9}{2} = 4.5 ,$$

while the set with highest ambiguity will be defined by a membership function $\mu(x) = 1/2$ for all $0 < x \leq 9$. At this point we can name the ratio $D(\widetilde{a})/D_{max}$ as *relative fuzziness*, and further consider its complement to unity:

$$c(\tilde{a}) = 1 - \frac{D(\tilde{a})}{D_{max}},$$

defining it as a *confidence rate evaluation*. We can interpret this rate as a reliance degree that the decision maker assesses for its own judgement. Hence when a person establishes the shape of the fuzzy number fixing its spread, he is actually assessing the amount of uncertainty of his statement, and consequently a probability estimate of his judgement accuracy.

If we want to remove the biases connected with the decision maker's attitudes toward ambiguity, and obtain a more objective value we shall enter with the value of $c(\tilde{a})$ the individual's ambiguity function from the y-axis (that represents perceived probability) and take out the correspondent value on the x-axis (that represent the objective probability i.e. the anchor). For our example we used the fuzzy number $\tilde{a} = (3,6,7)$ whose confidence rate is $c(\tilde{a}) = 0.78$. Assuming expert's ambiguity curve is the one shown in Figure 8, we shall perform the substitution:

$$c(\tilde{a}) = 0.78 \quad \rightarrow \quad c(\tilde{a})^* = 0.88 .$$

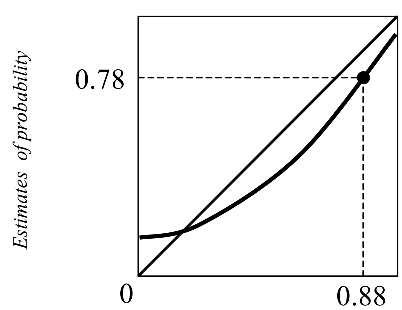

Figure 8. Expert's ambiguity curve

The new value for the confidence rate means we have to change the shape of the fuzzy number, modifying its spread in a way that its fuzziness would yield a confidence rate of 0.88. It is important to highlight that any transformation we perform on the spread will be coherent with the original shape of the fuzzy number, that is they will keep the proportion between the distance from upper to modal value and the distance from lower to modal value.
In our particular case, the spread of the fuzzy number will have to be smaller to increase the confidence rate from 0.78 to 0.88. The resulting fuzzy number after the spread transformation presented in Fig.9.

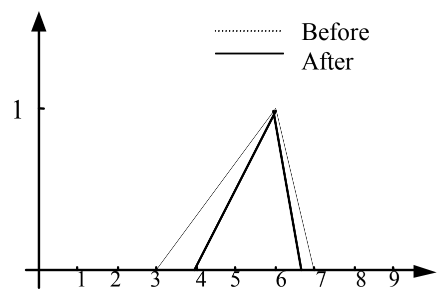

Figure 9. Fuzzy number after spread transformation

For further information about the connection between optimism pessimism and assessing probabilities see also Gibson and Sanbonmatsu (2004).

5. TEST OF THE MODEL: THE EXPERIMENT

The validation of the consistency and reliability of the model has been tested in an experiment compounded by two slightly different perception based tests. In the first one the test subjects have been asked to compare pairs of different size circles (Fig 10a), while in the second one comparisons were concerned with figures having different grey-degree colour (Fig 10b).

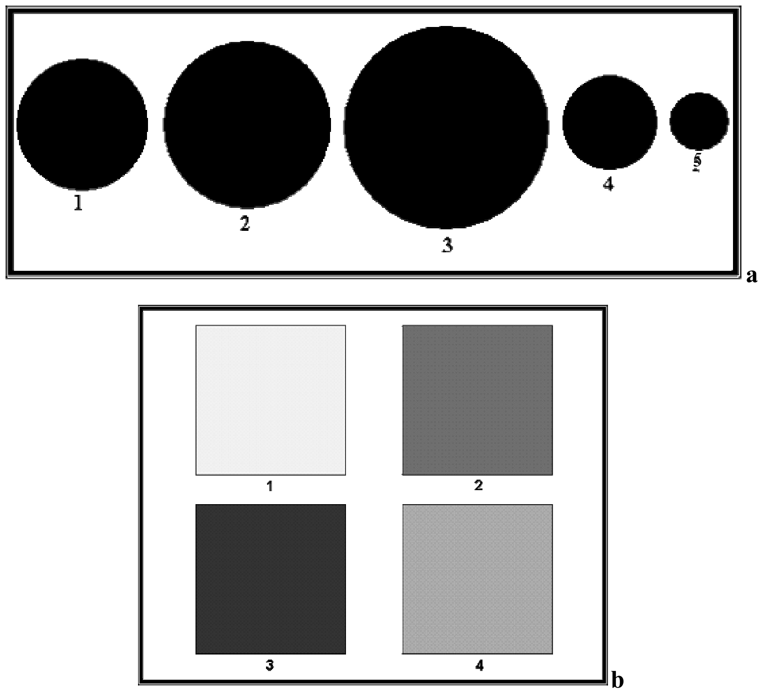

Figure 10. Box of the figure used in the perception test

In both perception based test, the sample was asked to evaluate, following their perception, the difference between a pair of figures. (e.g: showing a box with only circles one and five, we asked individuals: how many times circle one is bigger than circle number 5). Every person had to answer using the evaluation scale proposed in Figure 11. Whenever the evaluator was certain about his/her perception, he could sign with an **X** the exact point to indicate the difference between the two figures. If the evaluator was uncertain, he/she could sign an **X** with an interval of fuzziness (as shown in Figure 11).

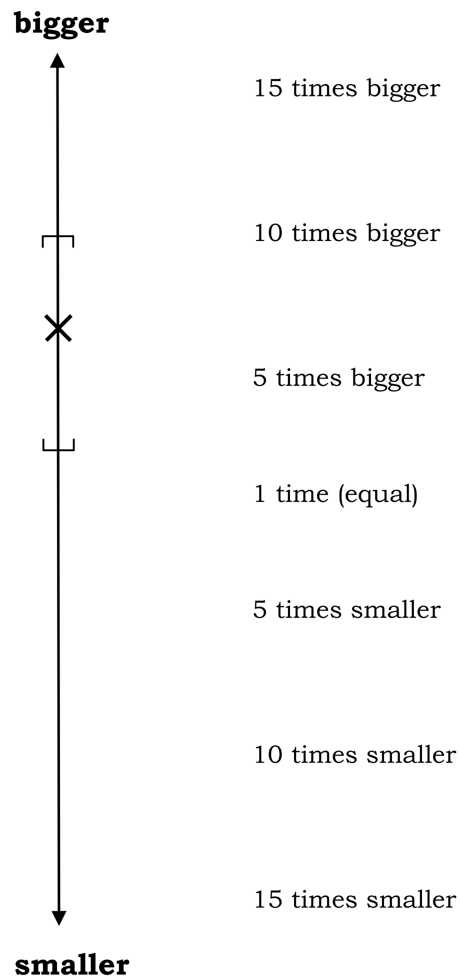

Figure 11. Evaluation scale

From these tests, we wish to evaluate different cognitive perception of individuals. In fact, by asking people to indicate a fuzzy interval, we hoped to measure fuzziness in individual perception of their cognitive bias.

The experiment was submitted to a sample of 60 persons randomly chosen among undergraduate students of the University of Napoli Federico II, and all the responses of the two tests were collected respectively in 10×60 and 6×60 matrices (see Fig.10), whose rows stand for the fuzzy pair wise comparisons and whose columns represent the individual decision maker. Each element $\tilde{a}_{ij}$ of our matrices is a triangular fuzzy number denoting the *i*-th pairwise comparison judgement assessed by the *j*-th decision maker.

First of all we should clarify that we will test just the modal value transformation that uses the value function of prospect theory. The spread modification cannot be examined because the two decision problems used, for their perceptive nature and simplicity, brought up a low amount of uncertainty in judgements. We shall note that simplicity is after all a consequence of the fact that for our purposes we need decision problems whose exact solutions are known to us, if we want to express perceived value in function of objective value and we want to observe such relation.

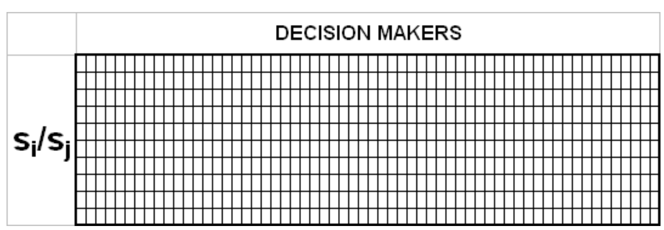

Figure 12. Matrix of the sample

It therefore happens the average spread of our fuzzy numbers does not reach a minimum size under which it is not possible a mathematical transformation (see Fig.13: the functions meet the vertical axis $x = 1$ for ordinate values less than unity; the figure highlights the *no-transformation area*).

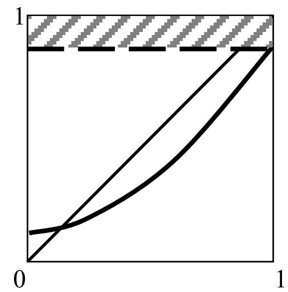

Figure 13. Ambiguity function

We can now proceed to investigate the modal values of the fuzzy judgement, that we arrange in what from now on we will call *data set*. This testing has been done through the statistic method known as *k-fold cross validation* (Stone 1974, 1977).

The data set is divided into *k* subsets. One of the *k* subsets is chosen to be the *testing set* (see Fig.12) and the other $k-1$ subsets are put together to form the *training set*. We considered this two subsets as decision making groups, hence for each of them we worked out the geometric mean of the modal values across all the experts belonging to it (see i.e. Saaty, 1978; Buckley, 1985; Boender *et al*, 1989; Gogus and Boucher, 1997; for group decision making). In this way we obtain two vectors, one from training set and the other from testing set, both of which map the perceived values s_i/s_j on the known solutions w_i/w_j .

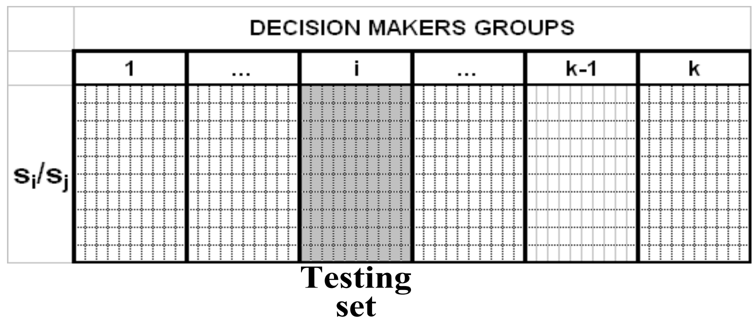

Figure 14. The testing set

We then build a perception of value function through a least square regression method using the training set vector only. The function we thereby fit is assumed to be of the form $y = ax^b + c$ with $0 < b < 1$, which is the closest parameterisation of value function shape of prospect theory. Such function is asked to predict the output values for the data in the testing set. We therefore plot the testing set vector dots on that graph to evaluate the root mean square error (RMSE) of the testing set, which is used to assess the goodness of the model. The described procedure has to be run k times, till each subset has been chosen as testing set exactly once. Then the average RMSE across all k trials is computed.

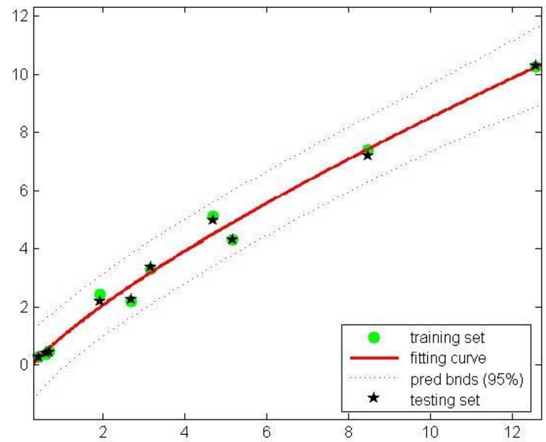

Figure 15. RMSE of the testing set

6. RESULTS

Let's consider first each of the two tests individually. In both cases it happens that the regression functions we fit on training sets represent a very good prediction of testing set data. We have

<u>I test:</u> $RMSE_{fit} = 0.35$, $RMSE_{test} = 0.68$

<u>II test:</u> $RMSE_{fit} = 0.61$, $RMSE_{test} = 0.71$

The quantities $RMSE_{fit}$ and $RMSE_{test}$ respectively represent the goodness of the regression and the extent to which the fitted curve can predict testing set values. We can ascribe the difference between them to the lower variability of training vector that has been built on a larger set of individuals than the testing one. The normal distribution of the errors around a zero mean pushes us still further to accept this explanation. Moreover we applied the modal value transformation to testing vectors using training fitted curves. The thereby modified judgements of the decision makers belonging to testing sets, were yielding more accurate FHA[*] solutions, i.e. closer to the correct solution of 74% for the first test and of 27% for the second.

These facts bring us to the first important conclusion: biases connected with assessing judgement can be recognized and measured. Also they *do not depend* from specific characteristics of individuals who form the experts' group, and once we fix the decision making situation we are able to correct that biases without concern about the features of the people expressing the judgements.

At this point we want to study the consequences of a situational shift for the decision framing. To do this we considered the first and second test data as training and testing set and vice versa.

This time the dimensions of the sets on which we built training and testing vectors are the same, so we might not be able to explain the first case RMSE difference as we previously did. But at a deeper look of the second case, we find the regression curve predicting testing value with less error than the regression itself (see Fig.14).

FIRST CASE	**SECOND CASE**
training set: test n.1 **testing set**: test n.2	**training set**: test n.1 **testing set**: test n.2
$RMSE_{test} = 0.69$ $RMSE_{fit} = 0.41$	$RMSE_{test} = 0.62$ $RMSE_{fit} = 0.86$

Figure 16. Consequences of a situational shift for the decision making

[*] It has to be noticed that considering the modal values only, as stated in our premise, the FHA is reduced to a normal AHP. In the computation of its solution we therefore used the original Saaty's eigenvector method.

This strange phenomenon induces us to believe the second test yields more variable responses than the first one, maybe due to an increased situational ambiguity or to the way the same test has been submitted to people. As we could guess, the application of the modal value transformation yields positive shifts in the FHA solution in the second case only, where it becomes closer of around 20% to “real priorities”. These results leave us more than a suspicion about the independence of decisional frame from the judgements shape.

6. CONCLUSIONS AND FUTURE DEVELOPMENTS OF THE RESEARCH

We can assume that cognitive biases *do not depend* even from the specific decision making problem proposed to the experts’ group, and that they can be recognized, measured and corrected whatever situation has to be faced.

The change of some variables in place of test planning as much as the model application to more complex situations has not been here investigated. We believe this could be of interest for future research either to give more consistency to our assumptions or to propose some others.

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