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Centro de Ciencias Aplicadas y Desarrollo Tecnológico
Distrito Federal, México

Available in: http://www.redalyc.org/articulo.oa?id=47413026001
Automatic synthesis of 2D-n-scrolls chaotic systems by behavioral modeling

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ABSTRACT
This paper introduces the guidelines to synthesize 2D chaotic systems by means of high-level descriptions. The aim of this investigation is to synthesize 2D-n-scrolls chaotic systems based on saturated functions with multisegments. The new methodology of circuit synthesis is performed by three hierarchical levels. First, the 2D chaotic oscillator is numerically simulated at the electronic system level by applying state variables and piecewise-linear approximation. Second, the excursion levels of the chaotic signals are scaled to control the breaking points and slopes of the saturated functions within practical values. Additionally, the frequency scaling of 2D-n-scrolls chaotic attractors is performed. Finally, current and voltage saturated functions are synthesized using Verilog-A models for the operational amplifiers and in this manner a 2D chaotic system is synthesized using operational amplifiers to generate 2D-n-scrolls attractors. Numerical results are confirmed by H-SPICE simulations to show the usefulness of the proposed synthesis approach.

RESUMEN
El presente artículo introduce una guía para sintetizar sistemas caóticos en 2D a través de descripciones de alto nivel. El objetivo de esta investigación es sintetizar sistemas caóticos de 2D-n-enrollamientos basados en funciones saturadas con multisegments. La nueva metodología para síntesis de circuitos se desarrolla en tres niveles jerárquicos. Primero, el oscilador caótico en 2D se simula numéricamente a nivel de sistema electrónico aplicando variables de estado y aproximación lineal a tramos. Segundo, los niveles de excursión de las señales caóticas se escalan para controlar los puntos de rompimiento y las pendientes de las funciones saturadas dentro de valores prácticos. Adicionalmente, se desarrolla un escalamiento en frecuencia de los atractores caóticos de 2D-n-enrollamientos. Finalmente, las funciones saturadas de corriente y voltaje se sintetizan usando modelos en Verilog-A de amplificadores operacionales para que de esta manera se sintetice un sistema caótico en 2D usando amplificadores operacionales para generar atractores de 2D-n-enrollamientos. Los resultados numéricos se confirman con simulaciones en H-SPICE para mostrar la utilidad del enfoque de síntesis propuesta.

Keywords: Chaos, 2D-n-scroll attractors, behavioral modeling, circuit synthesis, opamp.

1. Introduction
Nonlinear science has had quite a triumph in all conceivable applications in science and technology [1]. For instance, chaotic systems have been an attractive field for research in various areas, among them physics, communications and electronics [1]-[25]. Every new chaotic system [2], [9], [21] is a candidate to improve applications in engineering [11], [13], [14], [15]. The circuit implementation of reliable nonlinear circuits [7], [15], [18], [22], [23], for generating various complex chaotic signals is a key issue for future applications of chaos-based information systems [3], [11], [13], [14], [15], [17], [19], [20]. For instance, in [25] it is introduced an automatic system to design multiscroll chaotic attractors [3], [4], [5], [6], [8], [14], which are good candidates in communication applications because they present more complex behaviors than the ones based in Chua’s circuit [7], [18], [22], [23]. In particular, creating various complex multidirectional multiscroll chaotic attractors by using some simple electronic devices is a topic of both theoretical and practical interests [10], [11], [26], [30]. Research on generation of multiscroll chaotic attractors has been developing for more than a decade and there are many approaches reported
in [3]. All these multiscrolls have been verified by numerical simulations and theoretical proofs [1], [4], [12], [30]. However, it has been identified that it is quite difficult to synthesize multidirectional multiscrolls by analog electronic circuits [5], [6], [7], [8], [10]. To cope with this problem, the electronic design automation (EDA) industry is developing design tools with a high degree of abstraction (behavioral modeling) [26], [27], [28]. Nevertheless, automated synthesis of analog systems from a description of its desired behavior, such as continuous chaotic systems, has not progressed as it is done for the digital domain [16]. Furthermore, behavioral modeling can be exploited to give a solution on the synthesis of chaotic systems as shown in [32] because it offers one possible way to abstract the features of interest in a circuit block or a system [26], [27], [28], [29].

In this manner, this work introduces an extended version of the synthesis approach presented in [25] and [32]. This new version is focused on the design of 2D-n-scrolls chaotic attractors [10], [30], beginning with Electronic System Level (ESL) simulations [7], [27], [28], [30], [31], and ending with the synthesis of each individual block using operational amplifiers (opamps) [33]. To speed-up time simulation, the chaotic oscillator is modeled by applying state variables and piecewise-linear (PWL) approximation [25]. Two saturated functions (SFs) of voltage and current are needed to generate a 2D-mesh of n-scrolls, in contrast to those in [25] and [32]. Besides, the position of the scrolls on a 2D-mesh is evaluated by matrix representations and it depends on the value of the saturated plateaus in SFs. When SFs have been computed, they can be synthesized using high-level Verilog or SPICE opamp models [26], [27], [29], [33]. Therefore, our proposed approach is oriented to synthesize 2D chaotic oscillators based on current and voltage SFs using opamps. In section II, the high-level synthesis methodology to design 2D chaotic systems is shown. In section III, the numerical simulations and the synthesis of 2D-n-scrolls chaotic systems using opamps are shown. In section IV, the SPICE results for 2D-3-scrolls and 2D-4-scrolls chaotic attractors are shown. Finally, the conclusions are given in section V.

## 2. Synthesis Methodology

The proposed synthesis methodology for 2D chaotic systems is performed by three hierarchical levels in a similar fashion as shown in [32, Fig. 1]. The high-level descriptions capture the behavior of the 2D chaotic attractor and include the number of scrolls on the 2D-mesh, position of the scrolls on X-axis and Y-axis, voltage or current level of the chaotic signals and frequency of the attractor. As is well known, it is much more difficult to physically realize a nonlinear resistor that has an appropriate characteristic with many segments [3], [8]. However, the realization of a nonlinear characteristic with multisegments is the basis for implementing chaotic attractors with multidirectional orientation and a large number of scrolls.

### A. Behavioral modeling

The chaotic system in [25] is modified here to generate chaotic behavior on a 2D-mesh. The 2D chaotic system is modeled by applying state variables approach as shown in (1), where $x, y, z$ are state variables, and $a, b, c, d_1, d_2$ are positive real constants. Two saturated function series $f(x)$ and $f(y)$ in (1) are needed to generate 2D-n-scrolls attractors and are defined by (2), where $k > 0$ is the slope and plateau of the saturated function series, $h > 2$ is the saturated delay time of the saturated function series, $p_1, p_2, q_1$ and $q_2$ are positive integers [8]. Therefore, the chaotic system has the potential to create a 2D $(p_1 + q_1 + 2)\times(p_2 + q_2 + 2)$-even-scrolls mesh and a 2D $(p_1 + q_1 + 1)\times(p_2 + q_2 + 1)$-odd-scrolls mesh. Besides, the saturated plateau in saturated

function series is as follows: plateau = ±nk for 2D-even-scrolls and plateau = ±mk for 2D-odd-scrolls. The saturated delays for the slopes centers are defined by $h_i = ±mk$ for 2D-even-scrolls and $h_i = ±nk$ for 2D-odd-scrolls as shown in Fig. 1. The multiplier factors for the above-mentioned expressions are defined by $n = 1, 3, \ldots (p_2 + q_2 + 1)$ for 2D-even-scrolls and $n = 1, 3, \ldots (p_2 + q_2 - 1)$ for 2D-odd-scrolls; and $m = 2, 4, \ldots (p_1 + q_1)$ for both types of scrolls.

$$\begin{align*}
\dot{x} &= y - \frac{d_2}{b} f(y; k_2, h_2, p_2, q_2) \\
\dot{y} &= z \\
\dot{z} &= -ax - by - cz + d_1 f(x; k_1, h_1, p_1, q_1) + d_2 f(y; k_2, h_2, p_2, q_2) \\
\end{align*}$$

(1)

$$f(x; k, h, p, q) =
\begin{align*}
(2q + 1)k & \quad x > qh + 1 \\
(kx - ih) + 2ik & \quad |x - ih| \leq 1, p \leq i \leq q \\
(2i + 1)k & \quad ih + 1 < x < (i + 1)h - 1, p \leq i \leq q - 1 \\
-(2p + 1)k & \quad x < -ph - 1 \\
\end{align*}$$

(2)

Additionally, the centers of scrolls and connections among neighbors-scrolls in a 2D-scrolls mesh depend on the value of $k$ and they are evaluated by the matrix representations shown in (3) to (7). The matrixes are filled in a $(x, y)$ form, where $x$ and $y$ are the values on the X-axis and Y-axis, respectively. All scrolls have a radius of $k$. In (4) and (7), the operation $(*)$ means an interchange in the axis as shown here, $(x, y) \rightarrow (*) = (y, x)$. Also, one needs to evaluate all quadrants in (3) to (7), this is $+(x, y), -(x, +y), -(x, -y), +(x, -y)$. Consequently, centers of the 2D scrolls are defined by $C$ matrix in (3); the connections are defined by $U_x, U_y, U$, and $U'$ matrixes in (4) and (7). For 2D odd-scrolls, centers of scrolls are defined by $C'$ matrix in (5) and the connections are defined by $U'_x, U'_y, U'$, and $U'$ matrixes in (6) and (7). Similarly, one can design 2D-n-scrolls attractors in $(x, z)$ or $(y, z)$ directions.

**B. Scaling of excursion levels and frequency**

Equation (2) cannot be synthesized because it cannot have small excursion levels (ELS) as shown in [30, section 3]. Therefore, to implement 2D-n-scrolls attractors using practical opamps one needs to redefine (2) by (8), where $\alpha$ allows that ELS of the attractors be within the ELS of real opamps [32]. The frequency scaling consists in multiplying the state variables system in (1) by a required factor of scaling as described in [25, section 4].

![Figure 1. PWL description of an SF to generate 2D-odd and 2D-even-scrolls](image-url)
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\[ C = \begin{bmatrix} (k,k) & \ldots & (nk,k) \\ \vdots & \ddots & \vdots \\ (k,nk) & \ldots & (nk,nk) \end{bmatrix} \]  

\[ U_j = \begin{bmatrix} (k,2k) & \ldots & (nk,2k) \\ \vdots & \ddots & \vdots \\ (k,nk) & \ldots & (nk,nk) \end{bmatrix}, \quad U_j^T = \begin{bmatrix} (2k,k) & \ldots & (mk,k) \\ \vdots & \ddots & \vdots \\ (2k,nk) & \ldots & (mk,nk) \end{bmatrix} \]  

\[ C' = \begin{bmatrix} (0,0) & \ldots & (mk,0) \\ \vdots & \ddots & \vdots \\ (0,mk) & \ldots & (mk,mk) \end{bmatrix} \]  

\[ U'_j = U_j^T = \begin{bmatrix} (2k,k) & \ldots & (2k,nk) \\ \vdots & \ddots & \vdots \\ (mk,k) & \ldots & (mk,nk) \end{bmatrix}, \quad U'_j = U_j^T = \begin{bmatrix} (k,2k) & \ldots & (k,mk) \\ \vdots & \ddots & \vdots \\ (nk,2k) & \ldots & (nk,mk) \end{bmatrix} \]  

\[ U = [k,0] \ldots (nk,0] \quad U' = U^*[0] \ldots (0,nk)] \]  

\[ f(x;\alpha,k,h,p,q) = \begin{cases} (2q+1)k & x > qh + \alpha \\ k/\alpha(x - ph) + 2hk & |x - ph| \leq \alpha, -p \leq i \leq q \\ (2i+1)k & ih + \alpha < x < (i+1)h - \alpha, -p \leq i \leq q - 1 \\ -(2p+1)k & x < -ph - \alpha \end{cases} \]  

of real opamps are considered using the Verilog–A model given in [32, section IV-B].

\[ k = \text{RixSat} \quad \text{Isat} = V_{\text{sat}}/R_c \quad \alpha = \text{Ri}/V_{\text{sat}}/R_f \]  

\[ s = k/\alpha \quad h = Ei(1 + (R_i/R_f)) \]  

C. Synthesis of voltage and current saturated functions

A voltage SF can be described by the opamp finite-gain model, so that if a shift-voltage (±E) is added, one gets the shifted-voltage SFs for positive and negative shifts [25]. The basic cell (BC) shown in [25, Fig. 4(b)] is used herein to synthesize voltage and current SFs. In the following, the general connection of BCs to implement \( f(x) \) and \( f(y) \) in (1) is shown in Fig. 2. \( E \) takes different values in (9) to synthesize the required plateaus and slopes. The value of plateaus \( k \) in voltage and current, breakpoints \( \alpha \), slope \( s \) and \( h \) are evaluated by (9). The gain, bandwidth, slew rate and saturation

\[ \text{Figure 2. Synthesis of voltage and current saturated functions} \]
3. Synthesis of 2D-n-Scroll Attractors

The ESL speeds up time simulation since it allows the use of behavioral models for nonlinear systems [27],[28]. A 2D-3-scrolls chaotic attractor is generated by setting \( a=b=c=d_1=d_2=0.7 \), \( k=250\times10^{-3} \), \( \alpha=2.5\times10^{-3} \), \( h=250\times10^{-3} \), \( p_1=q_1=p_2=q_2=1 \) to evaluate (1) and (8) as shown in Fig. 3, and a 2D-4-scrolls attractor is generated with \( a=b=c=d_1=d_2=0.7 \), \( k=250\times10^{-3} \), \( \alpha=2.5\times10^{-3} \), \( h=500\times10^{-3} \), \( p_1=q_1=p_2=q_2=1 \), as shown in a scaled version in [30, Fig. 7]. The position of scrolls on a 2D-mesh, the centers of scrolls and the connections among neighbors scrolls are given by evaluating (3) to (7). For 2D-even-scrolls, it results in 16 scrolls with a radius of 250e-3 and 24 connections as shown in (10). Similarly, the evaluation for 2D-odd-scrolls is given in (11) and it results in 9 scrolls with a radius of 250e-3 and 12 connections.

\[
\begin{bmatrix}
(±0.25,±0.25) \\
(±0.75,±0.75)
\end{bmatrix}
\] \[
\begin{bmatrix}
(±0.25,±0.25) \\
(±0.75,±0.75)
\end{bmatrix}
\]

\[
C' = \begin{bmatrix}
(0,0) \\
(0,±0.5)
\end{bmatrix}
\]

\[
\begin{bmatrix}
±0.5,±0.5
\end{bmatrix}
\]

\[
U_x = \begin{bmatrix}
±0.25,±0.5
\end{bmatrix}
\]

\[
U_y = \begin{bmatrix}
±0.5,±0.5
\end{bmatrix}
\]

\[
U' = \begin{bmatrix}
0,±0.25
\end{bmatrix}
\]

\[
\begin{bmatrix}
±0.25,±0.5
\end{bmatrix}
\]

\[
\begin{bmatrix}
±0.25,±0.5
\end{bmatrix}
\]

\[
U'' = \begin{bmatrix}
0,±0.25
\end{bmatrix}
\]

\[
\begin{bmatrix}
±0.25,±0.5
\end{bmatrix}
\]

\[
\begin{bmatrix}
±0.25,±0.5
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\begin{bmatrix}
±0.25,±0.5
\end{bmatrix}
\]

\[
\begin{bmatrix}
±0.25,±0.5
\end{bmatrix}
\]

On the other hand, the 2D chaotic system in (1) can be synthesized with opamps as shown in Fig. 4 and 5. From Fig. 5 one obtains the system given in (12), and its parameters are determined by (13). By selecting \( R_{ix}=10K\Omega \), one obtains \( C=143uF \), \( R=7K\Omega \), \( R_x=R_y=R_z=10K\Omega \), \( R_{fs}=10K\Omega \) and \( R_{is}=10K\Omega \).
The general connection shown in Fig. 2 can synthesize the current saturated function $SF(x)$ and voltage and current saturated function $SF(y)$ in Fig. 5. Therefore, the circuit level synthesis of the saturated functions is shown in Fig. 6 for 2D-3-scrolls and 2D-4-scrolls, respectively.

$$\frac{dx}{dt} = \frac{y}{RC} \frac{v(y)}{RC}$$

$$\frac{dy}{dt} = \frac{z}{RC}$$

$$\frac{dz}{dt} = -\frac{x}{RxC} \frac{y}{RyC} \frac{z}{RzC} \frac{[i(x)Rix]}{RixC} \frac{[i(y)Riy]}{RiyC}$$

(12)

$$C = 1/0.7Rix \quad Rx = Ry = Rz = 1/0.7C \quad R = 1/C$$

(13)
4. H-SPICE Simulation results

By selecting $V_{sat}=\pm 2.5V$, $R_i=10\Omega$, $R_c=100K\Omega$, $R=1K\Omega$ and $R_f=1M\Omega$, in (9) one gets $k=250mV$, $I_{sat}=25\mu A$, $\alpha=2.5mV$, $s=100$ and $h=E_1=250mV$ for 2D-3-scrolls and $h=E_1=500mV$ for 2D-4-scrolls. Furthermore, the H-SPICE simulations for 2D-3-scrolls attractors and 2D-4-scrolls attractors are shown in Fig. 7.

5. Conclusion

The synthesis of 2D-3-scrolls attractors and 2D-4-scrolls attractors by behavioral modeling has been shown. The 2D-n-scrolls chaotic oscillator was modeled by state variables and PWL approximations, and the synthesis process was focused on the implementation of PWL approximations by scaling ELs of the chaotic signals to implement those using practical opamps. In this manner, it was shown that voltage and current saturated functions can be synthesized with opamps by controlling the breaking points and slopes. Finally, since SPICE simulations are in good agreement with the ESL numerical simulations, one can conclude on the usefulness of high-level behavioral modeling to synthesize 2D-n-scrolls attractors.

References


Acknowledgment

J.M. Muñoz-Pacheco acknowledges CONACyT for support granted through Ph. D. scholarship #204409. This research is supported by CONACyT/MEXICO under project number 48396-Y.
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