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Projective computer tomography and image reconstruction of discrete structure of materials

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Computer tomography is efficiently used in different applied areas, particularly for investigation of the material's structure. Traditional approach requires scanning the object with a lot of angles and reconstruction of images, using sufficiently difficult mathematical operations as, for example, the inversion of the Radon transform in X-rays tomography. It requires sufficiently large volume of calculations, but it is necessary for object with difficult structures. We consider in this paper the object with the simple "discrete" structure, *i.e.*, the structure that consists of some separate elements inside of the homogeneous (or quasi homogeneous) substance in the considered domain. These types of structures appear at investigation of defectoscopy of materials, particularly at production of semiconductor's plate and composite materials, so as in other applied areas. We propose for these types of structures the "Projective Computer Tomography" (PCT), which consists in localization of intersections of prolongation of some known projections, obtained as images by some concrete tomography. To realize numerically PCT we develop here the three dimensional generalization of the proposed before the plane scheme of Rotating Projection Algorithm (RPA). It simplifies the image reconstruction of discrete structures, because RPA does not require numerical realization of complicate formulas, such as, for example, the inverse Radon transform in X-rays tomography. Constructed algorithms are realized as program package in MATLAB system, which quality is demonstrated on simulated numerical examples.

Keywords: Computer tomography; discrete structure; rotating projection algorithm.

La tomografía computacional es usada de manera eficiente en diferentes áreas de aplicaciones, especialmente para la investigación de la estructura de los materiales. El enfoque tradicional requiere escanear el objeto con muchos ángulos y reconstrucción de imágenes, usando operaciones matemáticas suficientemente difíciles como, por ejemplo, la transformada inversa de Radon en tomografía de rayos X. Esto requiere suficientemente grande volumen de cálculos, pero es necesario para estructuras complejas. Consideramos en este trabajo los objetos con estructura "discreta" simple, *i.e.*, estructura que consiste de algunos elementos separados dentro de la homogénea (o cuasi homogénea) substancia en el dominio considerado. Tales tipos de estructuras aparecen en investigación de defectoscopia de materiales, particularmente en producción de composites y platas de semiconductores, como en otras áreas aplicadas. Proponemos para este tipo de estructuras la "Tomografía Computacional Proyectiva" (TCP), la cual consiste en la localización de intersecciones de prolongaciones de algunas proyecciones conocidas. A realizar numéricamente TCP desarrollamos aquí la generalización tres dimensional del esquema plano del Algoritmo Rotatorio Proyectivo (ARP), propuesto antes. Esto simplifica reconstrucción de las imágenes de las estructuras discretas, porque ARP no requiere realización numérica de las formulas complicadas, como, por ejemplo, la transformada inversa de Radon en la tomografía de rayos X. Los algoritmos construidos fueron realizados como un paquete de programas en el sistema MATLAB, cual calidad es demostrado en ejemplos numéricos simulados.

Descriptores: Tomografía computacional; estructuras discretas; algoritmo rotatorio proyectivo.

PACS: 02.60.Cb; 42.30.d; 61.18.-j

1. Introduction

We consider the problem of the image reconstruction of the structure, consisting of the component with different characteristics. This problem can be resolved by tomography that corresponds to some external physical field that we use to obtain indirect boundary observations as projections. There are some tomographies such as electrical, infrared, acoustic, X-rays (radiography), etc. [1,2]. The traditional approaches require scanning the object and resolve some mathematical model that corresponds to the type of tomography. For example, in X-rays tomography it is necessary calculating the inversion of the Radon transform. It seems necessary for difficult structures and can be realized in sufficiently fast manner. But sometimes the investigating object has the simple discrete structure, so its reconstruction consists only in localization of some elements with different characteristic inside of the homogeneous (or quasi homogeneous) region. In Ref. 3 it was proposed the simplified variant of scanning algorithm without application of the inverse Radon transform. We call it as Rotating Projection algorithm. In Ref. 4 this algorithm was applied for electric tomography. Here we use this idea and develop the approach for abstract Projective Computer Tomography.

2. Plane rotating projection algorithm

We explain the idea of this algorithm in the simplest plane case for one element to be localized. Suppose that we know values of function $\nu(p, \varphi)$ (projections), which characterizes the intensiveness of passed through the object rays for some fixed angles ϕ and all linear coordinates p of scanning. By another words, we know for some fixed angles $\phi_i, i = 1, \ldots, n$ corresponding number of one-dimensional images (projections) as functions of one variable p.

We need the next steps.

1. Prolongation on a plane of calculated projections $\nu(p, \phi_i)$ for every p along the direction, corresponding to angle $\phi_{i,}$ to obtain the extended two dimensional image.

2. Rotation, *i.e.*, changing number i (scanning for different ϕ) of prolonged projections and localization of the areas of intersection of projections with the same values.

Let us suggest that we have homogeneous substance in the region and only one element with another density. If we do not interesting in the exact geometrical form of the element and want to localize it as a rectangle, it is sufficient [5] to use n=2.

We suppose also that $\phi_1{=}0, \phi_2{=}90$grad, that means we have two orthogonal projections. The intensiveness v_i=v(p,ϕ_i), i=1,2 of rays, passed through the homogeneous substance, is equal to 0, and for rays, passed through the element, is equal to 1. So, our variable p is x for i=1, or y for i=2, and we can localize one dimensional images at axes X and Y.

The graphical illustration of the above supposition is presented at Fig. 1: one-dimensional images as little squares

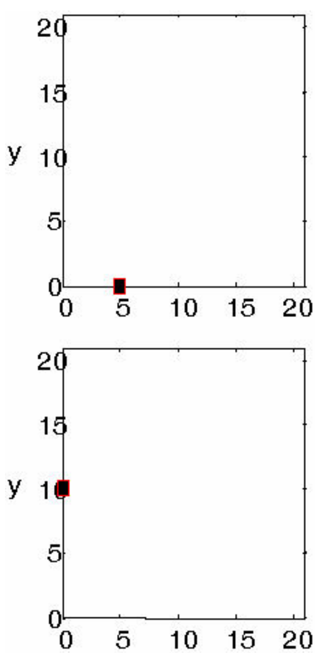

FIGURE 1. One-dimensional images.

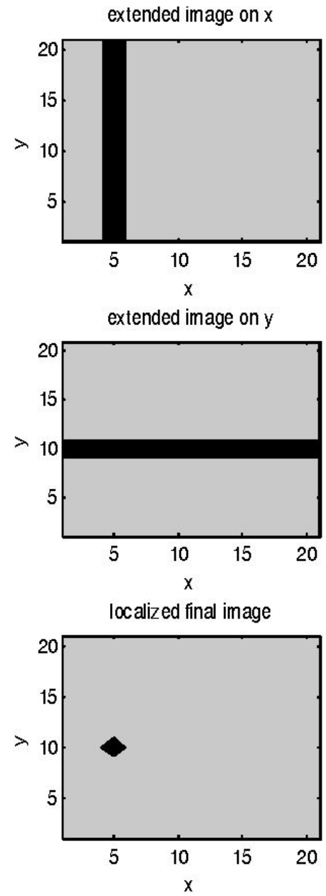

FIGURE 2. Bidimensional extensions and final image of localized element.

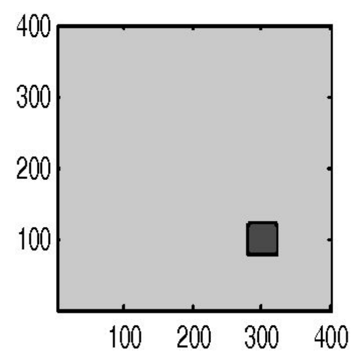

FIGURE 3. Original image.

at axes x and y, which correspond to the value 1 of the projection function. Illustration of the projection algorithm is presented at Fig. 2: two bidimensional extensions and final image of localized element.

3. Numerical experiments on rapidness of RPA in comparing with simulated X-rays tomography

Simulation of the X-rays tomography is based on construction the input data as the direct Radon transform for test image, presented by given function $u(x, y)$, for different number n of values of angle φ and then application of the inverse Radon transform [6]. We present some numerical experiments to illustrate comparison of the quality and rapidness of proposed Projection Tomography and simulated X-rays tomography. The exact image to be recuperated is presented at Fig. 3.

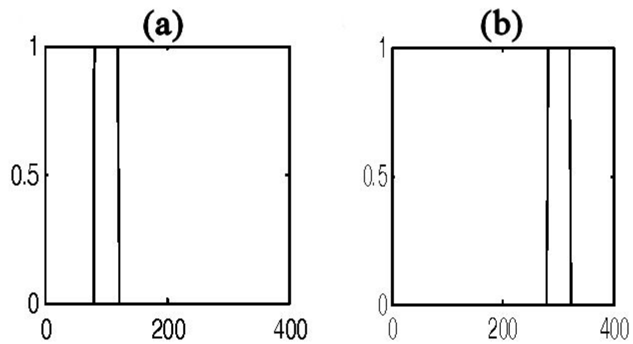

FIGURE 4. Radon transform of original image for angle of: a) 0 degrees and b) 90 degrees.

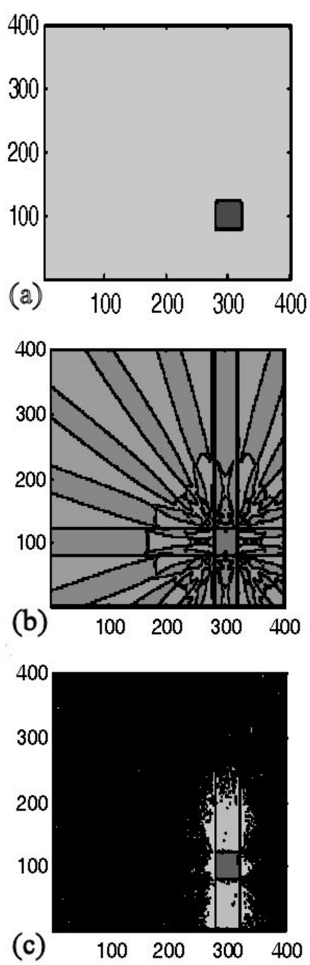

FIGURE 5. Recuperation of image by: a) RPA, b) simulated X-rays method for n=10, and c) simulated X-rays method for n=180.

We calculated Radon transforms of this image for n=2 angles of 0 degrees and 90 degrees that are explained at Figs. 4a and 4b as normalized density in dependence of the variable p. Then we used corresponding one-dimensional images as input data for the RPA. Result of the reconstruction is presented at the graphic of Fig. 5a. Time of reconstruction by RPA is equal to 1.0630 sec. Results of recuperations of image by simulated X-rays method are presented at Figs. 5b and 5c for n= 10, 180 correspondently. Recuperations are not as perfect as at Fig. 5a and require more calculation time, which is equal to: 3.6410 sec for n=10; 163.3440 sec for n=180. Many another numerical experiments also confirm advantages of RPA.

4. Space case of rotating projection algorithm

We developed the explained scheme for the space (three dimensional) case with scaning by beams, which are parallel to plane XY in cartizian coordinates XYZ [7]. We suppose that there are some elements with the same characteristic (density) inside of quasi homogeneous substance in the region. We suppose that the density of these elements is such that their projections have igual intensiveness a strictly greater than intensiveness of the image of the quasi homogeneous fund of the region. Hence, we include in the scheme of the algorithm the procedure of "clearing" of the images as the first step. It consists in normalization of the range of the image in a grey scale and changing intensiveness for 0 in the pixels with initial intensiveness less then 1. The description of the next steps is similar to description of the plane scheme. Constructed algorithm was realized as a package of computer programs in MATLAB system and justified by a lot of numerical model experiments [7].

5. Affine rotating projection algorithm

We propose for considered type of structures to use three plane projections, two of which correspond to the XZ and YZ planes in rectangular coordinates; the third projection corresponds to some affine to XY plane. Proposed scheme is three dimensional affine generalization of the Rotating Projection algorithm. We explain this variant on the simulated example of the image recuperation of the rectangle, displaced at the plane that is parallel to XY plane, as we can see at Fig. 6. Graphics at Figs. 7a, 7b and 7c correspond to projections 00, 60 and 90 degree calculated with parallel to XY plane beam. The graphic at Fig. 7d corresponds to projection onto the plane XZ with beam, which is inclined at angle of 20 degree to the XY plane. At Fig. 8a we can see results of reconstruction of the image by 3 parallel to XY plane projections, at Fig. 8b reconstruction by 2 parallel to XY plane projections and 1 affine projection. From our point of view, such approach can be realized technically at most of consisting equipment for X-rays, electrical, infrared and others tomography [1,2].

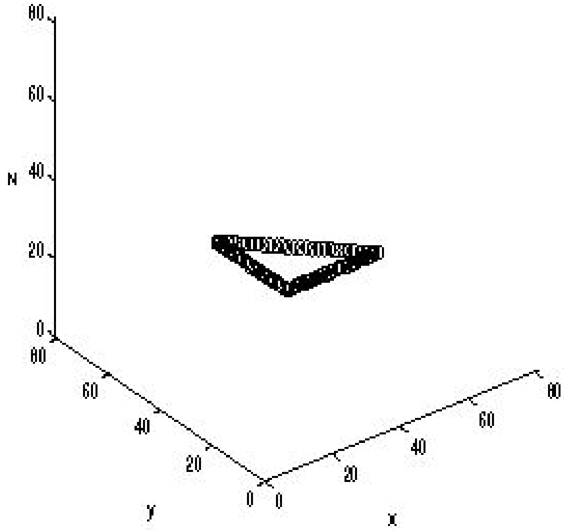

FIGURE 6. Original image.

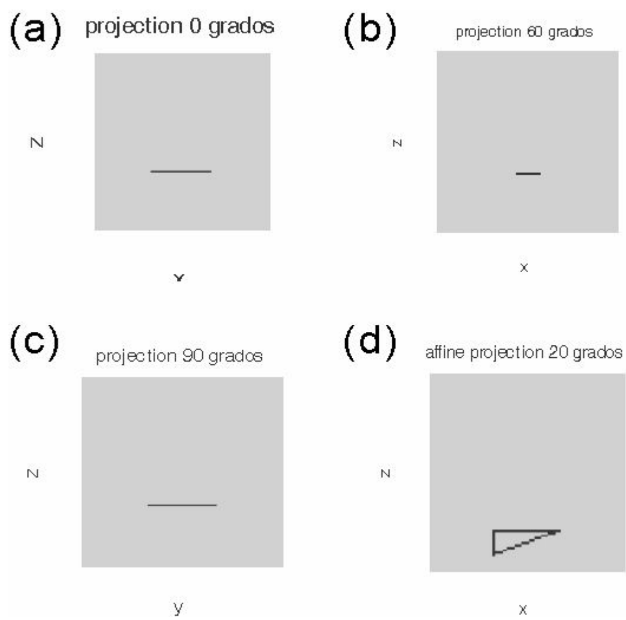

FIGURE 7. a) Projections 00 degree calculated with parallel to XY plane beam, b) Projections 60 degree calculated with parallel to XY plane beam, c) Projections 90 degree calculated with parallel to XY plane beam, and d) Projection onto the plane XZ with beam inclined at angle of 20 degree to the XY plane.

6. Possible applications

Considering types of structures appear at investigation of defectoscopy of materials, particularly at production of semiconductor's plate and composite materials, so as in other applied areas. In papers [3,7] some physical experiments related with electrical and optic tomographies are presented, that confirms possible applications in these areas.

One of the important perspectives in applications of constructed algorithm and programs is related with the medicine diagnostics of cancer of the female bosom using X-rays tomography. This application is under the investigations of some groups of specialists [8]. In this case it is important to reduce the time of the radiation treatment in the tomography process, by other words – to reduce the number of angles of scanning. It seems possible to use the proposed approach for this applied problem.

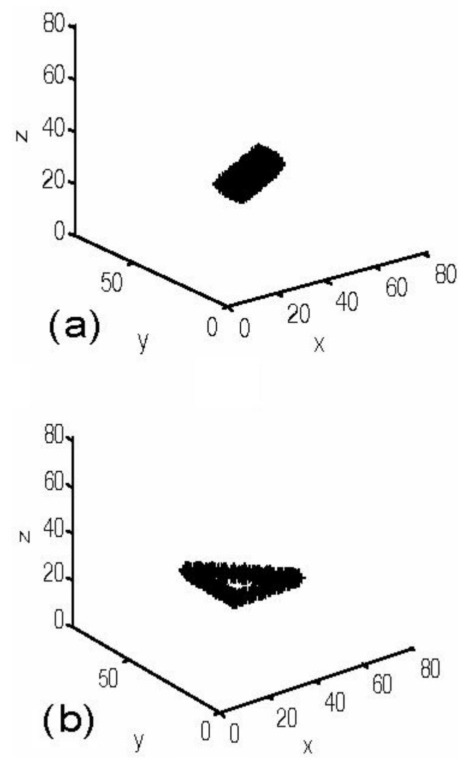

FIGURE 8. a) Reconstruction by 3 parallel to XY plane projections, and b) Reconstruction by 2 parallel to XY plane projections and 1 affine projection.

A seismic tomography for recuperation of the underground structure by seismic methods can be formulated mathematically as a problem, which is analogue of X-rays tomography [9]. For detailed recognition of investigating structures it is necessary to use a lot of measured data and to reconstruct images by the inversion of the Radon transform. In some simplified cases, for example at preliminary recognition, it is possible to use made above supposition about the discrete structure of the investigating object. Under these suppositions it is possible to use the plane version of RPA. It can simplify the image reconstruction for the considering case, because it does not require complicated calculation of the regularized inverse Radon transform or solving ill-conditioned systems of linear algebraic equations, as it is proposed in Ref. 9. This algorithm has also some perspectives in electromagnetic geophysics methods [10].

7. Conclusion

Rotating Projection Algorithm is developed for the reconstruction of images, obtained in computer tomography of objects that have the discrete structure, when its image reconstruction consists only in localization of some elements with different characteristic inside of the homogeneous (or quasi homogeneous) substance in the region. Good properties of the developed algorithm are demonstrated on numerical examples with simulated and experimental data. Performed investigations open possibility to use proposed Projective Computer Tomography, which operates with projections, obtained as images by some concrete tomography. It is important that mathematical, algorithmic and program support developed for PTC does not depend of type of physical field, which is used to realize tomography.

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