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Chiral phase transition in relativistic heavy-ion collisions revisited: toward a description of weak magnetic fields effects

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It has recently been pointed out that in peripheral collisions of heavy nuclei at high energies, a sizable magnetic field is produced in the interaction region. This fact opens up the possibility to study the chiral phase transition in a quark gluon plasma in the presence of a magnetic field. Here we work in the linear sigma model to compute the finite-temperature effective potential. We give an account only up to one-loop order where we show that in the weak field case, the effects of the magnetic field are subdominant. We argue that the magnetic field effects will show up when including the next order correction, the so called ring diagrams.

Keywords: Finite temperature field theory; chiral phase transition; quark gluon plasma; magnetic fields.

En fechas recientes se ha mostrado que se pueden producir campos magnéticos intensos en la región de interacción en colisiones periféricas de iones pesados a altas energías. Este hecho abre la posibilidad de estudiar la transición de fase quiral en un plasma de quarks y gluones en presencia de un campo magnético. En este trabajo usamos el modelo sigma lineal para calcular el potencial efectivo a temperatura finita. Damos una descripción solo hasta el orden de un lazo y mostramos que en el caso de que el campo es débil, sus efectos son subdominantes. Finalmente argumentamos que los efectos del campo magnético se encontrarán solo al considerar la siguiente corrección cuántica al potencial efectivo, la que proviene de los llamados diagramas de anillo.

Descriptores: Magnetic fields; relativistic heavy ion collisions; quiral phase transition; linear sigma model.

PACS: 11.10.Wx; 11.30.Rd; 12.38.Mh; 25.75.Nq

1. Introduction

The experiments at the Relativistic Heavy Ion Collider (RHIC) have found convincing signals that reveal the production of deconfined matter where the degrees of freedom involved are the quarks and gluons of QCD [1]. While the majority of these signals come from the most central collisions, it has also been realized that a host of new phenomena can also happen for reactions with large impact parameter. Among these, it has recently been pointed out that a magnetic field of a non-negligible strength is generated [2]. The origin of this field is two-fold: On one hand, in peripheral collisions, there is a local imbalance in the momentum carried by the colliding nucleons in the target and projectile that generates a non-vanishing local angular momentum [3,4] which in turn produces a magnetic field, given the net positive charge present in the collision. On the other hand, the spectator nucleons can be thought of as currents of net positive charge moving in opposite, off center, directions which in turn produce a magnetic field that adds up in the interaction region.

An interesting question that emerges from this scenario is whether the presence of such magnetic field can influence the phase transitions that may occur during the reaction, in particular the chiral phase transition. There are known examples where magnetic fields are able to change the nature of a phase transition. Most notably is the Meissner effect where the phase transition of superconductors of type I changes from second to first order in the presence of a magnetic field. Magnetic catalysis is another phenomenon whereby the presence of a magnetic field is able to dynamically generate masses in QED, regardless of the strength of the field [5]. More recently, it has been shown that in the presence of primordial magnetic fields, the electroweak phase transition, that took place in the early universe for temperatures of order 100 GeV, gets also strengthened [6].

Calculations of the intensity of the field produced in this kind of collisions show that for very early proper times after the reaction ($\tau \lesssim 0.1$ fm) the field reaches values $eB \simeq 6m_\pi^2$, where m_π is the pion mass, even for mid-peripheral collisions. The intensity decreases with proper time as $eB \propto 1/\tau^3$ in such a way that for $\tau \simeq 1$ fm, namely, for times when the standard picture of a heavy-ion reaction places the existence of the equilibrated QGP, $eB \lesssim 0.1\ m_\pi^2$ [2], that is, already two orders of magnitude smaller than at the very early stages of the collision.

In a recent work, the chiral phase transition in relativistic heavy-ion collisions has been examined in the presence of strong magnetic fields using the linear sigma model [7]. The authors conclude that the effect is to change the nature of the phase transition from second to a crossover. Nevertheless, as mentioned above, a more realistic scenario should be to consider that for the times when the initial chromoelectric fields decohere in the aftermath of the collision and give rise to partons –which in turn are the appropriate degrees of freedom to describe the chiral phase transition– the magnetic field in the interaction region might not be that strong. At these times the hierarchy of scales is such that the magnetic fields strength is the smallest of all and the deconfinement/chiral phase transition temperature is the largest one, while the mass of the Goldstone boson of the symmetry breaking, the pion mass, occupies an intermediate place. Furthermore, the analysis of Ref. 7 neglects the contribution from the so called ring diagrams which are known to be important at high temperatures to account for the infrared properties of the plasma [8, 9] as well as the renormalization of the effective potential which is important to have a perturbative control of the analysis.

In this work we lay down the basis for the calculation of the effective potential at finite temperature to describe the chiral phase transition in relativistic heavy-ion collisions, when this happens in the presence of a weak magnetic field. We use the linear sigma model as the working tool. We work explicitly with the hierarchy of energy scales where $eB \ll m^2 \ll T^2$, with m the generic mass and T the temperature around the phase transition, considering that the interaction region is subject to an external magnetic field directed along the positive $\hat{z}$ axis. We restrict the present analysis to give a somewhat detailed account of the renormalization procedure for the effective potential only up to one-loop. A more detailed discussion including the effects of the ring diagrams and the full effect of a weak magnetic field is given in Ref. 10.

2. The sigma model

The Lagrangian for the linear sigma model is given by

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi)^2 + \frac{\mu^2}{2}(\sigma^2+\pi^2) \\ &- \frac{\lambda}{4}(\sigma^2+\pi^2)^2 + i\bar{\psi}\gamma^\mu\partial_\mu\psi \\ &- ig\bar{\psi}\tau\gamma_5\psi\cdot\pi - g\bar{\psi}\psi\sigma, \end{aligned} \tag{1}$$

where ψ is a SU(2) isospin doublet of massless quarks, $\pi = (\pi_1, \pi_2, \pi_3)$ is an isospin triplet representing the pions and σ is an isospin singlet.

When the mass parameter μ^2 is positive, the Lagrangian admits a broken symmetry vacuum solution given by the minimum of the classical potential

$$V^{(cl)} = -\frac{\mu^2}{2}(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2. \tag{2}$$

Choosing this minimum along the σ direction, the *vacuum expectation values* for the sigma and pion fields are given by

$$\begin{aligned}\langle\sigma\rangle &= \mu/\sqrt{\lambda} \equiv v_0 \\ \langle\pi\rangle &= 0. \end{aligned} \tag{3}$$

v_0 is also called the classical vacuum, that is, the value that, for uniform field configurations, minimizes the classical action. To study the quantum properties of the system, we define the shifted field σ' by

$$\sigma = v + \sigma', \tag{4}$$

where v is taken as a variable. When $v = v_0$, σ' represents the field configuration around the classical vacuum.

As a function of the shifted field, after symmetry breaking, the Lagrangian of Eq. (1) becomes a theory describing a massive σ' field, three pion fields and a massive quark-doublet field with masses $m_q(v) = gv$

$$\begin{aligned} m^2_{\sigma'}(v) &\equiv m^2_{\sigma'} = 3\lambda^2 v^2 - \mu^2 \\ m^2_{\pi}(v) &\equiv m^2_{\pi} = \lambda^2 v^2 - \mu^2 \\ m_q(v) &\equiv m_q = gv, \end{aligned} \tag{5}$$

respectively.

3. Effective potential

The tree level potential is given by

$$V^{(tree)} = -\frac{\mu^2}{2}v^2 + \frac{\lambda^2}{4}v^4. \tag{6}$$

We consider that the σ' field is very heavy and thus treat it only classically. To one-loop order, the contribution to the effective potential in the imaginary-time formulation of thermal field theory is given, for bosons, by

$$V_b^{(1)} = \mathrm{s}_b T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln(D^{-1})^{1/2}, \tag{7}$$

whereas for fermions, by

$$V_f^{(1)} = \mathrm{s}_f T \sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}\ln(S^{-1}), \tag{8}$$

where $\mathrm{s}_{b,f}$ are the degeneracy factors accounting for the internal degrees of freedom for bosons (isospin) and fermions (isospin and color), respectively, n is the index for the Matsubara frequency and D and S represent the boson and fermion Matsubara propagators, respectively. For charged particles, these propagators should include the effect of the external magnetic field. Nevertheless, it has been shown [6] that for the hierarchy of energy scales considered, at the one-loop level, the terms containing the effects of the magnetic field are subdominant.

Equations (7) and (8) contain both a vacuum and a finite temperature pieces. The vacuum piece exhibits the usual ultraviolet divergence that needs to be regulated. In the present context, this means that, up to additive constants, the effective potential should contain only finite v-dependent terms. This is accomplished, for instance, by introducing counter-terms to absorb the infinities. The usual physical conditions implemented to fix the counter-terms require that the position of the minimum of the effective potential, as well as the mass of the σ' field maintain their classical values [11]. However, for theories with massless modes –such as the pions in the present case, whose mass vanishes at $v = v_0$– this procedure breaks down. The problem is that the σ' field, being the inertia along the σ-axis, requires computing the second derivative of the effective potential evaluated at $v = v_0$. This derivative turns out to be not defined at such value. Thus, this second condition needs to be replaced by another appropriate one in a manner that we proceed to explain.

First, let us introduce the general expression for the one-loop renormalized effective potential, where we add the counterterms to absorb the v-dependent infinities

$$V_{ren}^{(1)} = -\frac{\mu^2}{2}v^2 + \frac{\lambda^2}{4}v^4 + \left(\frac{a(\Lambda) - \delta\mu^2}{2}\right)v^2 + \left(\frac{b(\Lambda) + \delta\lambda}{4}\right)v^4 + 3I(m_\pi, \Lambda) - 24I(m_f, \Lambda). \tag{9}$$

The last two terms account for the pion and fermion contributions to the vacuum effective potential, respectively, which from Eqs. (7) and (8) involve the ultraviolet cutoff (Λ) dependent function $I(m, \Lambda)$ defined as

$$I(m, \Lambda) = \frac{1}{2\pi^2}\int_0^\Lambda dk k^2 \sqrt{k^2 + m^2}. \tag{10}$$

The counter-terms in Eq. (9), $a(\Lambda)$ and $b(\Lambda)$, are introduced to take care of the v-dependent infinities, whereas the counter-terms $\delta\mu^2$ and $\delta\lambda$, account for finite terms that might shift the coefficients of the v^2 and v^4 terms, respectively.

After a bit of algebra where the v-dependent infinities are absorbed, the one-loop renormalized effective potential becomes

$$V_{ren}^{(1)} = -\left(\frac{1}{2}\mu^2 + 3\frac{\lambda}{64\pi^2}\mu^2 + \frac{1}{2}\delta\mu^2\right)v^2 + \left(\frac{1}{4}\lambda + 3\frac{\lambda^2}{128\pi^2} - 6\frac{g^4}{32\pi^2} + \frac{1}{4}\delta\lambda\right)v^4 - 24\frac{m_q^4}{64\pi^2}\ln\left(\frac{m_q^2}{4}\right) + 3\frac{m_\pi^4}{64\pi^2}\ln\left(\frac{m_\pi^2}{4}\right). \tag{11}$$

To fix one of the counter-terms, either $\delta\mu^2$ or $\delta\lambda$, we impose the condition that the minimum of the renormalized effective potential remains at its classical value, namely,

$$\frac{1}{2v}\frac{\partial}{\partial v}V_{ren}^{(1)}\bigg|_{v=v_0} = 0, \tag{12}$$

which implies that

$$\delta\lambda = 6\frac{g^4}{4\pi^2}\left[1 + \ln\left(\frac{g^2\mu^2}{4\lambda}\right)\right] + \frac{\lambda}{\mu^2}\delta\mu^2. \tag{13}$$

Inserting Eq. (13) into Eq. (11), we get

$$V_{ren}^{(1)} = -\left(\frac{1}{2}\mu^2 + 3\frac{\lambda}{64\pi^2}\mu^2 + \frac{1}{2}\delta\mu^2\right)v^2 + \left(\frac{1}{4}\lambda + 3\frac{\lambda^2}{128\pi^2} + 6\frac{g^4}{32\pi^2} + \frac{\lambda}{4\mu^2}\delta\mu^2\right)v^4 - 6\frac{m_q^4}{16\pi^2}\ln\left(\frac{m_q^2}{m_q^2(v_0)}\right) + 3\frac{m_\pi^4}{64\pi^2}\ln\left(\frac{m_\pi^2}{4}\right). \tag{14}$$

To fix the second counter-term, $\delta\mu^2$, notice that the argument of the last logarithmic function is dimensionfull, though the whole term is well defined as m_π goes to zero. However, if we choose

$$\delta\mu^2 = -3\frac{\lambda\mu^2}{16\pi^2}\ln\left(\frac{\mu^2}{4}\right), \tag{15}$$

we obtain

$$V_{ren}^{(1)} = -\left(\frac{1}{2}\mu^2 + 3\frac{\lambda}{64\pi^2}\mu^2\right)v^2 + \left(\frac{1}{4}\lambda + 3\frac{\lambda^2}{128\pi^2} + 6\frac{g^4}{32\pi^2}\right)v^4 - 6\frac{m_q^4}{16\pi^2}\ln\left(\frac{m_q^2}{m_q^2(v_0)}\right) + 3\frac{m_\pi^4}{64\pi^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right), \tag{16}$$

which does not contain any longer dimensionfull arguments of the logarithmic functions. The choice of Eq. (15) although seemingly arbitrary, has the advantage of producing a well defined renormalized effective potential which preserves the properties of the three level one, namely, that the pions are massless at v_0 which keeps being the minimum of the potential. Recall that the effective potential is not in itself an observable; only physical properties extracted from it, such as the position of the minimum and the critical temperature are. The choice that produces Eq. (16) is also extensively used in Standard Model calculations [8].

We now proceed to include the finite temperature contribution at one-loop. The finite temperature pieces of Eqs. (7) and (8) are given by

$$V_f^{(1)T\neq 0} = 6\left[-\frac{7\pi^2}{180}T^4 + \frac{m_q^2}{12}T^2 + \frac{m_q^4}{16\pi^2}\ln\left(\frac{m_q^2}{T^2}\right)\right]$$
$$V_\pi^{(1)T\neq 0} = 3\left[-\frac{\pi^2}{90}T^4 + \frac{m_\pi^2}{24}T^2 - \frac{m_\pi^3}{12\pi}T - \frac{m_\pi^4}{64\pi^2}\ln\left(\frac{m_\pi^2}{(4\pi T)^2}\right)\right]. \tag{17}$$

Adding the renormalized effective potential to the finite temperature contributions, we get the full one-loop finite temperature effective potential

$$V_{ren}^{(1)T\neq 0} = -48\frac{\pi^2T^2}{180} - \left(1+3\frac{\lambda}{32\pi^2}\right)\frac{\mu^2}{2}v^2 + \left(\lambda + 3\frac{\lambda^2}{32\pi^2} + 6\frac{g^4}{8\pi^2}\right)\frac{v^4}{4} + \left(12m_q^2 + 3m_\pi^2\right)\frac{T^2}{24} - 3m_\pi^3\frac{T}{12\pi} - 6\frac{m_q^4}{16\pi^2}\ln\left(\frac{T^2}{m_q^2(v_0)}\right) + 3\frac{m_\pi^4}{64\pi^2}\ln\left(\frac{(4\pi T)^2}{\mu^2}\right). \quad (18)$$

A few words about the properties of Eq. (18) are in order: First, notice that the dependence on the pion mass in the argument of the logarithmic functions has canceled upon addition of the vacuum and finite-temperature pieces of the renormalized effective potential. This is an important property for otherwise, this function can develop an imaginary part when the pion mass is negative, namely for $v < v_0$. Second, notice the appearance of a cubic pion mass term. This is also a dangerous term since it gives rise to an imaginary piece when the pion mass is negative. We can show that this term is exactly canceled when considering the contribution from the ring diagrams [10].

4. Thermal analysis

In order to closely examine the behavior of the renormalized finite-temperature effective potential with the temperature, let us reexpress Eq. (18) expanding it in powers of v

$$V_{ren}^{(1)T\neq 0} = -48\frac{\pi^2T^2}{180} - 3\frac{\mu^2}{24}T^2 + 3\frac{\mu^4}{64\pi^2}\ln\left(\frac{(4\pi T)^2}{\mu^2}\right) - \left[1 + 3\frac{\lambda}{32\pi^2} - \frac{2g^2+\lambda/2}{2\mu^2}T^2 + 3\frac{\lambda}{16\pi^2}\ln\left(\frac{(4\pi T)^2}{\mu^2}\right)\right]\frac{\mu^2 v^2}{2} + \left[\lambda + 3\frac{\lambda^2}{32\pi^2} + 6\frac{g^4}{8\pi^2} - 6\frac{g^4}{4\pi^2}\ln\left(\frac{T^2}{m_q^2(v_0)}\right) + 3\frac{\lambda^2}{16\pi^2}\ln\left(\frac{(4\pi T)^2}{\mu^2}\right)\right]\frac{v^4}{4} - 3\frac{m_\pi^3}{12\pi}T. \quad (19)$$

Notice that the critical temperature T_c for the phase transition is determined by the coefficient of the term v^2. When this coefficient is negative, the effective potential at $v = 0$ is concave and the minimum of the potential happens for a finite value of v, that is the broken symmetry phase. However, when this coefficient is positive, the curvature at $v = 0$ is convex and the minimum of the effective potential is located at $v = 0$, that is the symmetric phase. The above takes place provided the coefficient of the term v^4 is positive for otherwise the effective potential for large v becomes concave and thus unstable. This can happen for very high temperatures larger than a given temperature T_{max}. Therefore, the condition for the analysis to be valid is that the critical temperature is smaller than this last temperature, namely, $T_c < T_{max}$.

To test whether the analysis is consistent, we proceed to compute these temperatures. For this purpose, we use the standard values for the parameters $\lambda = 20$, $\mu = 398$ MeV and $g = 3.3$ [7]. The values of these parameters are determined from the vacuum σ mass, $m_\sigma^{vac} = 600$ MeV, the pion vacuum decay constant $f_\pi = 93$ MeV and a constituent quark mass $m_q^{vac} = 300$ MeV. This gives

$$T_c = 154.7 \text{ MeV}$$
$$T_{max} = 7,196.5 \text{ MeV}, \quad (20)$$

which confirms that $T_c < T_{max}$. The phase transition is second order.

5. Conclusions and outlook

In this work we have computed the finite-temperature effective potential up to one-loop order in the sigma model in the presence of an external magnetic field. We have treated the σ field as classical since it corresponds to a very heavy particle. To this order, the magnetic field contribution is subdominant for the hierarchy of energy scales considered. The phase transition is second order and happens for a critical temperature $T_c \simeq 150$ MeV.

The magnetic field contribution comes only when considering the next order correction to the effective potential which arises from the so called ring diagrams. These represent a resummation of the leading infrared divergences in theories containing massless fields, which in the present context are the pions. The leading contribution comes from the $n = 0$ mode in the expression

$$V^{ring} = \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi)^3}\ln[1 + \Pi_1^B D^B(k)], \quad (21)$$

where Π_1^B and D^B are the one-loop pion self-energy and pion propagator, respectively, in the presence of a magnetic field. Since the σ is treated as a classical field, the scalar contribution to Π_1 is obtained from the computation of the pion tadpole diagram. The fermion contribution to this self-energy is obtained from the diagram with a fermion bubble. The magnetic field dependence of these diagrams is obtained, for the hierarchy of energy scales considered here, computed by means of the method described in Ref. 6.

The computation up to the inclusion of ring diagrams is reported in Ref. 10 where however, it is argued that for a reliable conclusion, the pion self-energy needs to be computed non-perturbatively. The question remains as whether the second order nature of the phase transition can be modified even when the magnetic field is weak.

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