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# Asymptotic behavior of the daily increment distribution of the IPC, the mexican stock market index

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In this work, a statistical analysis of the distribution of daily fluctuations of the IPC, the Mexican Stock Market Index is presented. A sample of the IPC covering the 13-year period 04/19/1990 - 08/21/2003 was analyzed and the cumulative probability distribution of its daily logarithmic variations studied. Results show that the cumulative distribution function for extreme variations, can be described by a Pareto-Levý model with shape parameters $\alpha = 3.634 \pm 0.272$ and $\alpha = 3.540 \pm 0.278$ for its positive and negative tails, respectively. This result is consistent with previous studies, where it has been found that $2.5 < \alpha < 4$ for other financial markets worldwide.

*Keywords:* Econophysics; stock market; Power-Law; stable distribution; Levý regime.

Presentamos un análisis estadístico de la distribución de fluctuaciones diarias del índice de la Bolsa Mexicana de Valores, el llamado IPC (Índice de Precios y Cotizaciones). Estudiamos la función de distribución acumulativa de las diferencias logarítmicas diarias calculadas a partir de una muestra del IPC que cubre un periodo de 13 años, que empieza el 19/04/1990 y finaliza el 21/08/2003. Hallamos que esta función de distribución acumulativa puede describirse para los valores extremos de estas diferencias mediante una distribución de Pareto-Levý (ley potencia) con exponentes $\alpha = 3.634 \pm 0.272$ y $\alpha = 3.540 \pm 0.278$ en sus colas positiva y negativa respectivamente. Este resultado es consistente con estudios previos que muestran que $2.5 < \alpha < 4$ para los mercados financieros de diferentes partes del mundo.

*Descriptores:* Econofísica; bolsa de valores; ley potencia; distribución estable; régimen de Levý.

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## 1. Introduction

The behavior of extreme variations of economic indices, stock prices or even currencies, has been a topic of interest in finance and economics, and its study has become relevant in the context of risk management and Financial Risk Theory. However, these analyses are usually difficult to perform due to the small number of extreme observations in the tails of the distributions of financial time series variations. Recently, the interest of the physics community in the behavior of financial markets,has strongly increased, boosted by the availability of worldwide, electronical recorded financial data, giving rise to different aproach to confront problems that arise in the study of economics. The collection of methods and techniques originally developed in the area of physics, which are currently applied in the study of financial complex systems, is now called Econophysics and is becoming an emergent branch of physics by itself [1–6].

In order to describe the behavior of distribution of financial time series variations, several models have been proposed. Some of them are:

- Gaussian distribution [7].
- Log-gaussian distribution (Geometric Brownian Motion) [8].
- Stable Levý distribution [9–11].
- Truncated Levý Distribution [12–14].
- Poisson like distribution [15, 16].
- Power law Distribution with $\alpha \simeq 3$ [17] (Asymptotically).

In this paper, a statistical analysis of the distribution of daily variations of the IPC [18] is presented. It is organized as follows: in the remainder of this section, we briefly review the Pareto-Levý distribution, and some of the phenomena it describes are mentioned. In the next section, a very short introduction to the variables commonly used in the study of financial index and prices variations is given. In Sec. 3, we introduce the data sample analyzed in this work, and some important statistical properties of financial time series variations (fat tails, clustering volatility, etc.) [19, 20] are discussed, all of the above in the context of the IPC data. In Sec. 4, we explain and justify the procedure to estimate the Pareto-Levý exponent from the data and we show the results concerning the fit on the tails of the cumulative distribution of the IPC daily logarithmic variations. Finally, the last section is devoted to the comparison of our results to other related studies previously reported of different international stock markets.

### 1.1. Pareto-Levý Distribution. Stable Distributions

At this point, it is convenient to make a review of the definition of the Pareto-Levý distribution:

An absolutely continuous random variable $Y$, is said to follow a Pareto-Levý distribution with parameters $\alpha$ and $\gamma$, if its cumulative distribution function $F$ has the form:

$$F(y) := P\{Y \leq y_i\} = 1 - \left(\frac{y_0}{y_i}\right)^{\alpha} = 1 - \frac{\gamma}{y_i^{\alpha}} \qquad (1)$$

with $y_i \geq y_0$, $y_0^\alpha = \gamma$ and $\alpha > 0$. When the condition $\alpha > 2$ holds, the mean and variance of Y are both finite and by the central limit theorem, the sum of independent Pareto-Levý distributed random variables, converges in probability to a gaussian law. On the other hand, when $\alpha < 2$, the Pareto-Levý distribution has infinite variance, and it is said that the distribution is stable. For a mathematical treatment of these topics, consult Refs. 19 and 20. For a review from an econophysics point of view, Refs. 6 and 21 are recommended.

The Pareto-Levý distribution is often known as the *Power-law* distribution, and its role in Physics and other areas seems ubiquitous. In particular, we can illustrate this point by the following examples taken from finance:

- $N_{\Delta t}$, the number of trades in a given interval of time $\Delta t$, follows a power-law distribution with exponent $\alpha \simeq 3/2$ [24].
- Pareto-Levý tails, with $\alpha \simeq 3$ for extreme variations of individual stocks prices [25], and also for indices of different leading stocks markets [17,26,27].
- Decay of volatility correlations follows a power law distribution [28,30].
- The tail behavior of the cumulative distribution function of volatility is consistent with a power law distribution with exponent $\simeq 3$ [31].

All the above suggests universality in financial complex systems, and in order to explain the above facts, new models and even theories are currently being proposed Refs. 26 and 29 to 31.

## 2. Study of variations of financial time series

In the study of price variations of financial assets, many observables can be analyzed [6]. If $Y(t)$ is the value of the index at time $t$, some commonly used observables are:

- Prices or indices change themselves, for some interval of time $\Delta t$ :

$$Z(t) := Y(t + \Delta t) - Y(t) \tag{2}$$

- Deflated prices and index changes:

$$Z_D(t) := Z(t) \times D(t) \tag{3}$$

Where $D(t)$ is a statistical factor or index called a discount or a deflation factor,and is used to adjust the time value of money, enabling the comparison of prices while accounting for inflation, devaluation, etc. in different time periods.

- Returns, defined as:

$$R(t) := \frac{Y(t + \Delta t) - Y(t)}{Y(t)} \tag{4}$$

- Differences of the natural logarithm of prices [32], defined for some interval of time $\Delta t$ as:

$$S(t) := \ln Y(t + \Delta t) - \ln Y(t) \tag{5}$$

Each one has its own merits and disadvantages [6]. In this analysis, we have used the former variable $S(t)$.

## 3. Data sample and IPC variations

The database containing the IPC series analyzed here is available at Ref. 32 and covers the 13-year period 04/19/1990 - 08/21/2003. Figure 1 shows the IPC evolution for this time period. We have used in our analysis the daily closure values of the IPC, that is, its recorded value at the end of each trading day

In this work, our observable is $S(t)$ as defined in Eq. 5, where we studied the tail behavior of $P(S(t)) = 1 - F(S(t))$, for $t = 1, \ldots, N$, where $N = 3337$ is our sample size and $\Delta t = 1$ day.

Figure 2a shows the histogram of $Z(t)$, the IPC daily changes. It is interesting to point out that this strongly symmetric and leptokurtic (fat tailed) distribution does not follow any well known model, which could appropriately describe the probability of events in its central region and in its tails at once.

Figures 2b and 2c show the distribution of our observable $S(t)$. Figure 2b shows that the distribution of $S(t)$ appears to follow a gaussian. This is discarded after observing Fig. 2c, the same distribution with a vertical logarithmic scale. There are too many extreme events visible almost as far as ten standard deviations from its mean. To compare easily, Fig. 2c broken line shows a gaussian scaled to the amplitude


FIGURE 1. IPC development for the 13 year period 04/19/1990-08/21/2003.

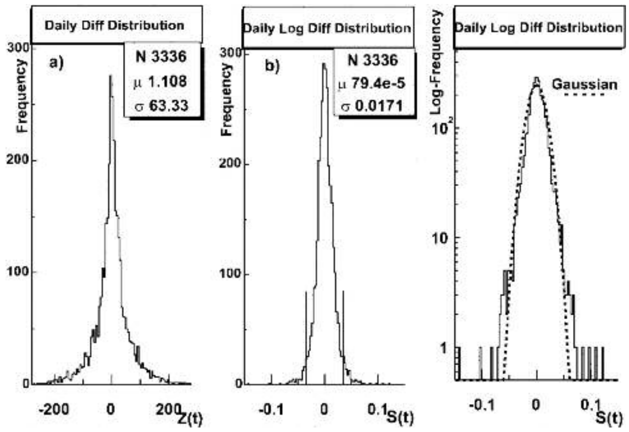


FIGURE 2. IPC variations. a) Histogram of $Z(t)$, the daily changes of the IPC index. The distribution shows symmetry and fat tails. b) Histogram of S(t). Regions studied in this paper, $S(t) > 0.035$ and $S(t) < -0.035$, are indicated by two vertical lines. c) Same as above, but with a vertical logarithmic scale. The broken line corresponds to a gaussian density with the same mean and standard deviation than the $S(t)$ distribution.

of $S(t)$, and with the same mean and standard deviation than those of $S(t)$ series.

Figure 2 shows how IPC variations are distributed; however it does not give any insight about the dynamics of the stochastic process that governs them. Evolution of the IPC daily log-differences is displayed in Fig. 3a. We observe that large variations are not uniformly distributed over time, and that is possible to distinguish phases of higher volatility [37] alternated with phases of a relative financial calm. This is a characteristic of financial time series called clustered volatility.

The clustering phenomenon has not been completely understood, however some successful models, such as the GARCH and Stochastic Volatility models have been proposed [38–40].

In order to appreciate the magnitude of these strong fluctuations, gaussian values with the same mean and standard deviation than those of the $S(t)$ distribution were simulated. In the simulation, nearly no clustering is appreciated, as it is shown in Fig. 3b.

## 4. Parameter estimation from IPC empirical data

The procedure to estimate $\alpha$, the Pareto-Levý exponent from empirical data is straightforward: Pareto-Levý tails behave as straight lines in a logarithmic plot. Then, after performing a linear fit of $\log P(S(t))$ on $\log S(t)$, the obtained slope gives us an estimate of the exponent $\alpha$ of the Pareto-Levý Distribution. Clearly, a distribution whose tails do not behave linearly in a log-log plot can not be properly described by the Pareto-Levý model.

The fit was carried out using data available in the tails regions $|S(t)| > 0.035$. Both tails are marked with vertical lines in Fig. 2b. Those regions were chosen simply by examining the set of points for which the corresponding log-log plot behaves linearly. $P(S(t))$ was then reduced to 79 and 93 events in its negative and positive tails, respectively. Note that, as is showed in Fig. 4, and in order to deal with the undefined logarithmic scale for the left tail of $P(S(t))$, we have used $-S(t)$ in our analysis.

It was found that for these regions the tails of $P(S(t))$ decay following a power law model. A straight line provides a good fit for them in a logarithmic plot. Both fits are shown in Fig. 4.

Table I shows the estimated parameters and the 95% confidence intervals obtained from the lineal regression fit for the negative and positive tails of $P(S(t))$.

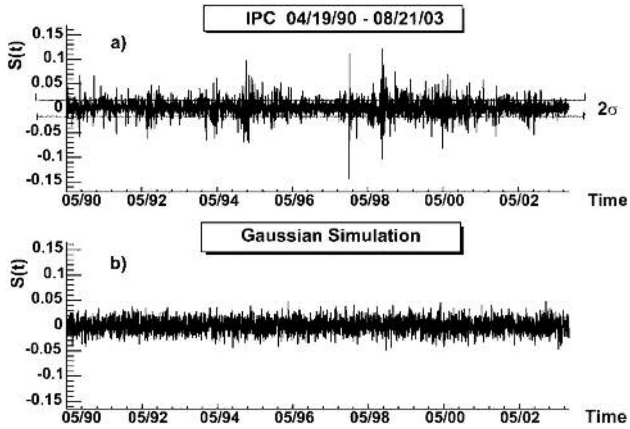


FIGURE 3. a) $S(t)$ behavior for the period of time under study. Large variations in $S(t)$, some of them as far as eight standard deviations from its mean, can be appreciated. It can also be seen that large variations tend to form clusters in time; this phenomena is called clustered volatility. b) Gaussian simulation already shown as a broken line in Fig. 2c. Clustering is virtually not present.

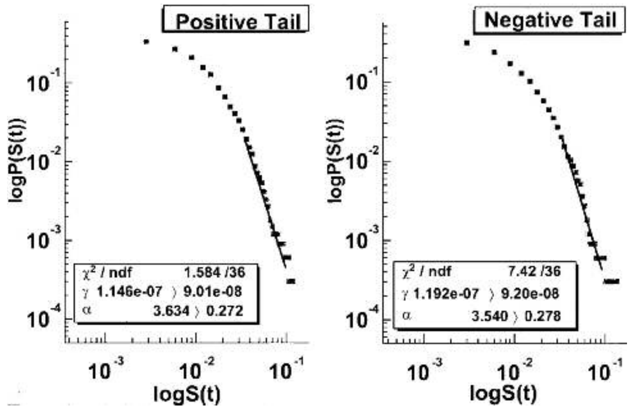


FIGURE 4. Linear fitted tails in a log-log plot of the cumulative distribution function $P(S(t))$ on $S(t)$. Right image positive tail. Left image negative tail. Fitted parameters are shown.

TABLE I. Fitted Parameters plus minus twice the standard error of the estimates, for positive and negative tails.

| Fitted region | $\alpha$ |
|---|---|
| $S(t) > 0.035$ | 3.634 $\pm$ 0.272 |
| $S(t) < -0.035$ | 3.540 $\pm$ 0.278 |

TABLE II. Pareto exponent for some international Stock Markets. Daily data (d). High Frequency data (*).

| Market | $\alpha$ (Right tail) | $\alpha$ (Left tail) | Data Period |
|---|---|---|---|
| AMEX NASDAQ NYSE (Combined) (USA) [17] | $2.84 \pm 0.12$ | $2.73 \pm 0.14$ | 01/94-12/95$^*$ |
| S& P 500 (USA) [26] | $3.66 \pm 0.011$ | $3.61 \pm 0.11$ | 1962-1996$^d$ |
| | $3.39 \pm 0.05$ | $3.37 \pm 0.07$ | 1984-1996$^*$ |
| DAX (Germany) [27,41] | 2.4 (minutely) to 3.5 (hourly) | | 10/97-12/99$^*$ 1959-2001$^d$ |
| NIKKEI (Japan) [26] | $3.05 \pm 0.16$ | | 1984-1997$^d$ |
| Hang-Seng (Hong Kong) [26] | $3.03 \pm 0.16$ | | 1986-1997$^d$ |

## 5. Discussion

Results shown in Table I are consistent with similar studies, where the Pareto-Levý model with $2.5 < \alpha < 4.0$, has been found to be useful to describe the behavior of extreme variations of diverse financial markets. Table II summarizes results of some of these studies.

High frequency studies of price variations, most of them performed for stock markets belonging to developed countries, show that the distribution of returns follows a Pareto-Levý form with exponent converging to $\alpha \simeq 3$ as $\Delta t$ decreases to time intervals of about one minute. For the case of stock markets of emergent economies, it seems that they may belong to a different universality class, some studies [42, 43] show that the return distributions from emergent markets have fatter tails than the observed in developed markets.

In summary, it has been shown that the cumulative probability of daily extreme logarithmic changes of the Mexican IPC index can be approximated by the Pareto-Levý model, with exponents $\alpha = 3.634 \pm 0.272$ and $\alpha = 3.540 \pm 0.278$ for its positive and negative tails, respectively. As a consequence of these values, we can affirm that the stochastic process that governs the time series $S(t)$ is well outside the Levý stable regime $(0 < \alpha < 2)$.

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