Confluent Rewriting Systems in Non-Monotonic Reasoning

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Abstract

We introduce the general notion of a Confluent LP-System, which is a rewriting system on the set of all logic programs over a signature $\mathcal{L}$. Such a system is based on certain transformation rules and induces a semantics SEM in a natural way. We show that most of the well-known semantics for normal logic programs are induced by confluent LP-systems. Moreover, we show by introducing several new transformation rules that the corresponding LP-systems induce interesting semantics which are polynomial time computable and extend WFS. Moreover we use our approach to define new semantics for disjunctive programs.

Keywords:

Well-founded semantics, stable semantics, logic programming, non-monotonic reasoning, rewriting systems, negation as failure.

1 Introduction

In this paper we try to combine methods from rewriting with logic programming technology to get a framework for considering semantics of logic programs.

The main idea (already introduced in [Brass and Dix, 1998] for disjunctive logic programs) is to determine a set of rewriting rules that is confluent. These rules transform programs into simpler programs. Confluence and termination guarantees that every program is associated a normal form. This normal form then induces a semantics and a simple and efficient method to answer queries with respect to this semantics.

In this paper we do not stick to a particular semantics but we develop the beginnings of a general theory of confluent LP systems and investigate which semantics can be represented as such systems.

We also extend the confluent system which corresponds to the wellfounded semantics WFS by new rewriting rules. These rules are motivated by the use of aggregates in logic programming and the problems of modelling aggregates in semantics such as WFS (see [Osorio and Jayaraman, 1997; Dix and Osorio, 1997]).

An advantage of our approach is its declarative nature. The transformation rules express natural conditions and different applications might ask for different selections of such rules. We show that the naturally induced semantics of a confluent LP-system is in fact the weakest semantics satisfying all our transformations.

The first proposal to provide a convincing declarative semantics to NAF was given in [Clark, 1978] and it is called the Clark's completion. The main idea is that, to deduce negative information from a normal program, we could "complete" the program by adding the only-if halves of the definitions of the predicate symbols$^1$. Clark's completion is obtained as our weakest LP-system $CS_0$.

$^1$For the details we refer to [Lloyd, 1987].
An abstract rewriting system is a pair $\langle S, \rightarrow \rangle$ where $\rightarrow$ is a binary relation on $S$. Let $\rightarrow^*$ be the reflexive, and transitive closure of $\rightarrow$. When $x \rightarrow^* y$ we say that $x$ reduces to $y$. An irreducible element is said to be in normal form.

We say that a rewriting system is

noetherian: if there is no infinite chain $x_1 \rightarrow x_2 \rightarrow \ldots \rightarrow x_i \rightarrow x_{i+1} \rightarrow \ldots$

confluent: if whenever $u \rightarrow^* x$ and $u \rightarrow^* y$ then there is a $z$ such that $x \rightarrow^* z$ and $y \rightarrow^* z$.

locally confluent: if whenever $u \rightarrow x$ and $u \rightarrow y$ then there is a $z$ such that $x \rightarrow^* z$ and $y \rightarrow^* z$.

In a noetherian and confluent rewriting system, every element $x$ reduces to a unique normal form that we denote by $\text{norm}(x)$.

A signature $\mathcal{L}$ is a finite set of elements that we call atoms. A literal is an atom or the negation of an atom that we denote by $\neg a$. A given set of atoms $\{a_1, \ldots, a_n\}$, we write $\neg \{a_1, \ldots, a_n\}$ to denote $\{\neg a_1, \ldots, \neg a_n\}$. We may denote a normal clause $C$ as usual [Lloyd, 1987]: $a :- l_1, \ldots, l_n$, where $a$ is an atom and each $l_i$ is a literal; or by $a \leftarrow B^+, \neg B^-$, where $B^+$ contains all the positive body atoms and $B^-$ contains all the negative body atoms. We also use $\text{body}(C)$ to denote $B^+ \cup \neg B^-$. A program is a finite set of clauses. Sometimes we will consider the logical constants $t$ and $f$ with their intended interpretation. Let $\text{Prog}_{\mathcal{L}}$ be the set of all normal propositional programs with atoms from $\mathcal{L}$. By $\mathcal{L}_P$ we understand the signature of $P$, i.e. the set of atoms that occur in $P$. A (partial) interpretation based on a signature $\mathcal{L}$ is a disjoint pair of sets $\langle I_1, I_2 \rangle$ such that $I_1 \cup I_2 \subseteq \mathcal{L}$. A partial interpretation is total if $I_1 \cup I_2 = \mathcal{L}$. Given two interpretations $I = \langle I_1, I_2 \rangle$, $J = \langle J_1, J_2 \rangle$, we define $I \leq_k J$ iff $I_i \subseteq J_i$, $i = 1, 2$. Clearly $\leq_k$ is a partial order. We may also see an interpretation $\langle I_1, I_2 \rangle$ as the set of literals $I_1 \cup \neg I_2$. When we look at interpretations as sets of literals then $\leq_k$ corresponds to $\subseteq$.

A general semantics SEM is a function on $\text{Prog}_{\mathcal{L}}$ which associates to every program a partial interpretation.

What are the minimal requirements we want to impose on a semantics? Certainly we want that facts, i.e. rules with empty bodies are true. Dually, if an atom does not occur in any head, then its negation should be true. This gives rise to the following definition (given by Brass and Dix), which will play an important role later.

**Definition 2.2 (SEMmin)**

For any program $P$ we define $\text{HEAD}(P) = \{a \; | \; a \leftarrow B^+, \neg B^- \in P\}$ — the set of all head-atoms of $P$. We also define $\text{SEMmin}(P) = \langle p_{\text{true}}, p_{\text{false}} \rangle$, where $p_{\text{true}} := \{p \; | \; p \leftarrow \in P\}$, and
\[ p^{\text{false}} := \{ p \mid p \in L_P \setminus \text{HEAD}(P) \} \]

3 Non-monotonic confluent LP-Systems

The main concept we use is that of a LP system is based upon, is the concept of a transformation rule, see [Brass and Dix, 1997].

Definition 3.1 (Basic Transformation Rules)
A transformation rule is a binary relation on \( \text{Prog}_C \). The following transformation rules are called basic. Let a program \( P \in \text{Prog}_C \) be given.

\[ \text{RED}^+ : \text{This transformation can be applied to } P, \text{ if there is an atom } a \text{ which does not occur in } \text{HEAD}(P). \text{ RED}^+ \text{ transforms } P \text{ to the program where all occurrences of } \neg a \text{ are removed.} \]

\[ \text{RED}^- : \text{This transformation can be applied to } P, \text{ if there is a rule } a \leftarrow \text{ in } P. \text{ RED}^- \text{ transforms } P \text{ to the program where all clauses that contain } \neg a \text{ in their bodies are deleted.} \]

\[ \text{SUB}: \text{This transformation can be applied to } P, \text{ if } P \text{ contains two clauses } a \leftarrow \text{body}_1 \text{, and } a \leftarrow \text{body}_2 \text{, where } \text{body}_1 \subseteq \text{body}_2. \text{ SUB transforms } P \text{ to the program where the clause } a \leftarrow \text{body}_2 \text{ has been removed.} \]

Although these rules are not really functions on \( \text{Prog}_C \) (e.g. \( \text{RED}^- \) is only determined if an occurrence of a certain rule is distinguished), we usually write them as operators on \( \text{Prog}_C \) when it is understood by context how they are uniquely determined.

Obviously, the just mentioned transformations are among the minimal requirements a well-behaved semantics should have (see [Dix, 1995b]). From now on, when we speak of a semantics \( \text{SEM} \), we understand that \( \text{SEM} \) is invariant under the transformations \( \text{RED}^+, \text{RED}^- \) and SUB.

Given a program \( P \) and a list of operators \( \text{ops} \), we define the application of \( \text{ops} \) to \( P \), denoted as \( P^{\text{ops}} \), as follows:

\[ P^[] := P \]

\[ P^{op[\text{ops}]} := (P^{op})^{\text{ops}} \]

We define the size of \( \text{ops} \) as the number of elements of \( \text{ops} \). We are now ready to introduce our main notion:

Definition 3.2 (Confluent LP-system \( \text{CS} \))
A non-monotonic confluent LP-system \( \text{CS} \) over the signature \( \mathcal{L} \) is a pair

\[ \{(op_i \mid i = 1, \ldots, n), \rightarrow\} \]

that satisfies the following conditions:

1. \( \{op_i \mid i = 1, \ldots, n\} \) is a finite set of transformation rules on \( \text{Prog}_C \). By abuse of language we often view them as operators and we write \( P^{op} \) to denote the result of the application of the given operator. We say that the operator is executed in this case.

2. \( \langle \text{Prog}_C, \rightarrow \rangle \), where \( \rightarrow \) is the union of all the transformation rules in \( \text{CS} \), is a noetherian and confluent rewriting system.

3. If \( P \rightarrow P_1 \) then \( \text{SEM}_{\text{min}}(P) \leq_k \text{SEM}_{\text{min}}(P_1) \).

We denote the uniquely determined normal form of a program \( P \) with \( \text{norm}_{\text{CS}}(P) \).

Every LP-system \( \text{CS} \) induces a semantics \( \text{SEM}_{\text{CS}} \) as follows:

\[ \text{SEM}_{\text{CS}}(P) := \text{SEM}_{\text{min}}(\text{norm}_{\text{CS}}(P)) \]

When there is no ambiguity about the given confluent LP-system we drop the subscripts.

Some points are worth mentioning:

1. Thanks to confluence and termination, our transformation rules have both a declarative and an operational meaning. From the declarative point of view, they tell us that our semantics is closed under the given transformation rule. From an operational point of view the transformations are computable functions that can be applied to simplify the program. When we arrive to the normal form, then computing its semantics is immediate.

2. Rationality (introduced later) has been argued as a desirable property of semantics in logic programming. Confluence plus a simple property that we call partial distribution implies rationality. It is straightforward to see that all (but one) of our semantics satisfy partial distribution. Therefore we know that these semantics are rational.

3. The nature of simple transformations, confluence and termination allow us to prove many properties by induction on the number of steps that we need to compute the normal form.

3.1 Basic Results

We omit the proofs of most of our results but they can be found in the research report [Dix et al., 1997].

Lemma 3.1 For every confluent LP-system \( \text{CS} \) and every program \( P \) such that every step in \( P^{\text{ops}} \) is executed we have: \( \text{SEM}_{\text{min}}(P) \leq_k \text{SEM}_{\text{min}}(P^{\text{ops}}) \).

Corollary 3.1

For every program \( P \) of a confluent LP-system \( \text{CS} \):

\[ \text{SEM}_{\text{min}}(P) \leq_k \text{SEM}_{\text{CS}}(P) \]
The following condition is also desirable in logic programming.

**Definition 3.3 (Three-valued Based)**
A semantics $SEM$ is called a 3-valued based if for every program $P$ the partial interpretation $SEM(P)$ is a 3-valued model of $P$.

In 3-valued based semantics, it can not happen that the body of a rule evaluates to true in $SEM(P)$ while the head of this rule evaluates to false or to undefined. For the details of 3-valued based semantics see [Dix, 1995a; Brass and Dix, 1997].

### 3.2 Rationality

A semantics $SEM$ is called rational, if for every atom $a$ and program $P$ the following holds:

$\neg a \not\in SEM(P)$ implies $SEM(P) \leq_k SEM(P \cup \{a \leftarrow \})$.

**Definition 3.4 (Partial distribution)**

A confluent LP-system $CS$ satisfies partial distribution if for every op, $P$ and a such that $a \in HEAD(P^{op})$ and $P^{op}$ is executed, the following holds:

1. $(P \cup \{a\})^{op}$ is executed,
2. $(P \cup \{a\})^{op} = P^{op} \cup \{a\}$.

**Lemma 3.2** For every partial distributive confluent LP-system $CS$ the following is true. Let $a \in HEAD(P^{ops})$, where ops is any list of operators such that every step in $P^{ops}$ is executed. Then:

1. every step in $(P \cup \{a\})^{ops}$ is executed,
2. $(P \cup \{a\})^{ops} = P^{ops} \cup \{a\}$.

Rationality is a nice property for a semantics SEM, especially if we plan to use the system in logic programming. The following result is important because it shows that a relatively simple condition, namely partial distribution on $CS$ suffices to ensure rationality of the induced semantics.

**Theorem 3.1 (Rational semantics)**

Every partial distributive confluent LP-system $CS$ induces a rational semantics $SEM_{CS}$.

### 4 Clark’s completion, WFS and WFS$^+$

In this section we show that the 3-valued version of Clark’s completion semantics (introduced by Fitting in [Fitting, 1985, and 1986]), WFS ([Gelder et al., 1988, and 1991]) and an extension of WFS (introduced independently by [Dix, 1992] and [Schlipf, 1992]) are induced by confluent LP-system.

To this end, we introduce the following transformation rules:

**GPPE** (Generalized Principle of Partial Evaluation)
Suppose $P$ contains $a \leftarrow B^+, \neg B^-$ and we fix an occurrence of an atom $g \in B^+$ different from $a$. Then we can replace $a \leftarrow B^+, \neg B^-$ by the $n$ clauses ($i = 1, \ldots, n$)

$a \leftarrow (B^+ \setminus \{g\}) \cup B_i^+, \neg B_i^- \cup \neg B_i^-$

where $g \leftarrow B_i^+, \neg B_i^- \in P$, (for $i = 1, \ldots, n$) are all clauses with $g$ in their heads. If no such clauses exist, we simply delete the former clause.

**TAUT** (Tautology) Suppose $P$ contains a rule $C$ which has the same atom in its head and in its body. Then we can remove this rule.

**LC** (Logical Consequence) Suppose $P \models a$ for an atom $a$. Then we can add the rule $a \leftarrow P$.

Let $CS_0$ be the LP-system which contains, besides the basic transformation rules, the rule GPPE. Let $CS_1$ be $CS_0$ enlarged by TAUT. Finally, let $CS_2$ be $CS_1$ plus the rule LC. By using results of [Dix, 1995a; Brass and Dix, 1998] we get

**Theorem 4.1 (Classifying comp$_3$, WFS)**

1. Fitting’s semantics comp$_3$ is the weakest 3-valued based semantics satisfying GPPE. It is induced by the confluent LP-system $CS_0$.
2. The wellfounded semantics is the weakest 3-valued based semantics satisfying GPPE and TAUT. It is induced by the confluent LP-system $CS_1$.

Let us note that although the $CS_1$ system has the nice property of confluence (and termination), its computational properties are not that efficient. In fact, computing the normal form is exponential, whereas it is known that the WFS can be computed in quadratic time. It turned out that recently an equivalent confluent LP-system was defined, where the normal form can be computed in quadratic time (see [Brass et al., 1997]). The idea is to restrict the GPPE rule and to replace it by two simple instances plus an additional check for an unfounded set (Loop detection):

**Success (S):** Suppose that $P$ includes a fact $a$ and a clause $q \leftarrow Body$ such that $q \in Body$. Then we replace the clause $q \leftarrow Body$ by $q \leftarrow Body \setminus \{a\}$.

**Failure (F):** Suppose that $P$ includes an atom $a$ and a clause $q \leftarrow Body$ such that $a \notin HEAD(P)$ and $a$ is a positive literal in $Body$. Then we erase the given clause.

**Loop Detection (Loop):** We say that $P_2$ results from $P_1$ by Loop$_A$ iff there is a set $A$ of atoms such that
1. for each rule \( a \leftarrow \text{Body} \in P_1 \), if \( a \in A \), then \( \text{Body} \cap A \neq \emptyset \),
2. \( P_2 \) := \( \{ a : \neg \text{Body} \in P_1 : \text{Body} \cap A = \emptyset \} \),
3. \( P_1 \neq P_2 \).

We call the resulting system \( \mathcal{CS}_1 \), i.e. \( \text{RED}^+ + \text{S} + \text{F} + \text{Loop} \). We now show that \( \text{F} \) is redundant, for that we need the following lemma.

**Lemma 4.1 (Zepeda, 1997)**

Let \( \Rightarrow_X \) denotes the reduction relation of the \( \mathcal{CS}_1 \) system. Let \( \Rightarrow X = F \) be as \( \Rightarrow_X \) without \( \text{F} \). For all programs \( P, P', P_1, P_2 \), where,

\[
P \Rightarrow^*_{X-F} P' \Rightarrow^*_F P_1 \Rightarrow^* X P_2
\]

there is a program \( P'' \) such that

\[
P \Rightarrow^*_{X-F} P' \Rightarrow^*_F P'' \Rightarrow^*_L P'' \Rightarrow^* X P_2
\]

**Proof:** If \( P' \Rightarrow^* X P_1 \), where \( a \) is the atom resposable of this failure reduction. Let \( A = \{a\} \). Suppose that there are exactly \( k \) clauses in \( P' \) such that \( a \) occurs positively in them. Starting with \( P'' \) we can apply a sequence of \( k \) Failure steps (over the same positive literal) leading to some program \( P'' \) where \( a \) no longer occurs positively. And by confluence we get that,

\[
P \Rightarrow^*_{X-F} P' \Rightarrow^*_F \cdots \Rightarrow^*_F P'' \Rightarrow^*_X P_2
\]

Moreover: There is no rule \( A \leftarrow B \) in \( P' \) such that \( A \notin A \). \( P'' \) := \( \{ A \leftarrow B \in P' : \text{B} \cap A = \emptyset \} \), and \( P' \neq P'' \). For hence \( P'' \) also results from \( P'' \) by applying the Loop Detection, so

\[
P \Rightarrow^*_{X-F} P' \Rightarrow^*_F \cdots \Rightarrow^*_F P'' \Rightarrow^*_X P_2
\]

as desired.

The following theorem is a direct consequence of the above lemma.

**Theorem 4.2 (Elimination of Failure, Zepeda, 97)**

Let \( P \) be a program. Then \( P'' \) the normal form of \( P \) w.r.t. the \( \mathcal{CS}_1 \) system, iff \( P'' \) is the normal form of \( P \) w.r.t. the same system but without \( \text{F} \).

**Theorem 4.3 (Classifying WFS\(^+\))**

The WFS\(^+\) semantics is the weakest 3-valued based semantics satisfying S, F, Loop, TAUT and LC. It is induced by the confluent LP-system consisting of exactly these transformations (plus the basic ones).

The reason we gave up GPPE is not only for computational purposes. As has been shown in [Dix and Müller, 1994] GPPE does not hold for WFS\(^+\). Let us consider the programs:

\[
P : \quad a \leftarrow \neg a \quad P' : \quad a \leftarrow \neg b
\]

\[
b \leftarrow \neg a \quad b \leftarrow \neg a
\]

\[
x \leftarrow a \quad x \leftarrow \neg a
\]

\[
x \leftarrow b \quad x \leftarrow \neg b
\]

By applying GPPE twice, we get \( P' \). But WFS\(^+\) \( \{x\} \) while WFS\(^+\) \( \{P'\} \) = \emptyset. We point out that the WFS for both \( P \) and \( P' \) is \emptyset, considered as a shortcoming of WFS.

5 New Semantics extending WFS

In this section we show how our transformation rules can be extended by new rules that still define confluent LP-systems.

The motivation of these rules is the overly sceptical approach of WFS. In particular for aggregation predicates, rules of the form \( a \leftarrow \neg a \) should be considered as being equivalent to \( a \). However, incorporating this into a well-behaved semantics is in general very difficult.

Of course the WFS\(^+\) semantics has this property. But again it can be shown that WFS\(^+\) is on the second level of the polynomial hierarchy (due to the LC-rule) and thus most probably not polynomial. We therefore introduce a weaker form of LC:

**Definition 5.1 (Local Logical Consequence)**

By the application of LLC (local logical consequence) to a program \( P \) that only contains one clause with head \( a \), namely a LC-clause \( a \leftarrow \neg a \in P \), we mean the transformation of \( P \) which simply removes every occurrence of \( \neg a \) in \( P \).

We define \( \mathcal{CS}_3 \) as LP-system \( \mathcal{CS}_1 \) plus the rule LLC. Here is one of our main theorems.

**Theorem 5.1 (\( \mathcal{CS}_3 \) is a confluent LP-System)**

\( \mathcal{CS}_3 \) is a confluent LP-System. Its induced semantics, called WFS\(^9\), is 3-valued based, but not rational.

Here are stronger versions of LLC.

**Definition 5.2 (LLC\(^*\))**

Let \( B \) be an ordered set \( \{a_0, \ldots, a_n\} \), where \( n \geq 0 \). Suppose in addition that the set of clauses \( \text{LLC}_B := \{a_i \leftarrow a_0 \} \cup \{a_i \leftarrow a : 1 \leq i \leq n - 1 \} \cup \{a_0 \leftarrow a_n\} \) is a subset of a program \( P \). If \( n = 0 \) we assume that the set reduces to \( a_0 \leftarrow a_0 \). Then the transformation (LLC\(^*\), \( B \)) substitutes the clause \( a_0 \leftarrow a_n \) by the fact \( a_0 \).

Note that by transitivity of \( \leftarrow \) we get \( a_0 \leftarrow a_0 \). Moreover \( \neg a \leftarrow a \) is a theorem in propositional classical logic and by Modus Ponens we can infer \( a \). So, this rule is sound in propositional classical logic. In [Osorio et al., 1995] it is defined a language that allows to model aggregation in a natural way via POL programs. In [Osorio and Jayaraman, 1997] we showed that this
POL programs can be translated to normal programs such that the declarative semantics of a POL program corresponds to the WFS of the translated normal program. But we have recently noted, [Dix and Osorio, 1997], that not always we needed the full power of WFS but a restricted form that corresponds to LLC.

Let $\mathcal{CS}_4$ be the LP-system $\mathcal{CS}_1^+$ plus the rule LLC.

**Theorem 5.2 (WFS + LLC*)**

$\mathcal{CS}_4$ is a confluent $\text{LP}$-System. Its induced semantics is 3-valued based and rational.

**Definition 5.3 (Contra)**

Let $C$ a clause of the form $a \leftarrow \text{Body}$, where both $b \in \text{Body}$ and $\neg b \in \text{Body}$. We define the transformation Contra as the one that deletes the clause $C$ from the program $P$.

At this point we are not really sure about the potential uses of Contra. The motivation for Contra comes from the following observations. Clearly the rule is a theorem in intuitionistic propositional logic and so is in classical logic. The power of a theory should not be affected if we add or delete theorems from the proper axioms of the theory (the proper axioms correspond to our programs).

**Theorem 5.3 (WFS + Contra)**

The $\text{LP}$-system given by $\mathcal{CS}_1^+$ plus the rule Contra is confluent. In addition its induced semantics is rational but not 3-valued based.

It has been criticized the WFS for its inability to do reasoning by cases and this is one reason to prefer in several cases $\text{STABLE}$ over WFS, see for instance [Baral and Gelfond, 1994]. The following rule allows to add a weak form of this kind of reasoning to WFS, still preserving the polynomial-time computability property of the semantics.

**Definition 5.4 (Weak-Cases)**

Let $C_1$ be of the form $a \leftarrow b$ and $C_2$ be of the form $a \leftarrow \neg b$, where $a \neq b$ are any pair of atoms of the signature. We define the transformation (Weak-Cases, $C$) as the one that substitutes the pair of clauses $C_1, C_2$ by the fact $a$ in the program $P$.

**Theorem 5.4 (WFS + Weak-Cases)**

The $\text{LP}$-system given by $\mathcal{CS}_1^+$ plus the rule Weak-Cases is confluent. In addition its induced semantics is rational and 3-valued based.

6 Disjunctive programs

We may denote a (general) clause $C$ as: $a_1 \lor \ldots \lor a_m \leftarrow l_1, \ldots, l_n$, where $m > 0$, $n \geq 0$, each $a_i$ is an atom, and each $l_i$ is a literal. When $n = 0$ the clause is considered as $a_1 \lor \ldots \lor a_m \leftarrow \text{true}$, where true is a constant atom with its intended interpretation. Sometimes, is better to denote a clause by $A \leftarrow B^+, \neg B^-$, where $A$ contains all the head atoms, $B^+$ contains all the positive body atoms and $B^-$ contains all the negative body atoms. We also use body($C$) to denote $B^+ \cup \neg B^-$. When $A$ is a singleton set, the clause reduces to a normal clause. A definite clause ([Lloyd, 1987]) is a normal clause lacking of negative literals, that is $B^- = \emptyset$. A pure disjunction is a disjunction consisting solely of positive or solely of negative literals. A (general) program is a finite set of clauses. As in normal programs, $\text{HEAD}(P)$ to denote the set of atoms occurring in heads in $P$. We use $\models$ to denote the consequence relation for classical first-order logic. It will be useful to map a program to a normal program. Given a clause $C := A \leftarrow B^+, \neg B^-$, we write $\text{dis-nor}(C)$ to denote the set of normal clauses:

$$\{a \leftarrow B^+, \neg(B^+ \cup \{A \setminus \{a\}\}) \mid a \in A\}.$$

We extend this definition to programs as follows. If $P$ is a program, let $\text{dis-nor}(P)$ denotes the normal program: $\bigcup_{C \in P} \text{dis-nor}(C)$. Given a normal program $P$, we write $\text{Definite}(P)$ to denote the Definite program that we obtain from $P$ just by removing every negative literal in $P$. Given a Definite program, by $\text{MM}(P)$ we mean the unique minimal model of $P$ (that always exists for definite programs, see [Lloyd, 1987]).

**Definition 6.1 (Definition of $\text{def}$ and $\text{sup}$)**

Let $P$ be a normal program. Let $a$ be an atom in a given signature $\mathcal{L}$ (such that $\mathcal{L}_P \subseteq \mathcal{L}$), by the definition of $a$ in $P$, we mean the set of clauses: $\{a \leftarrow \text{body} \in P\}$, that we denote by $\text{def}(a)$. We define

$$\text{sup}(a) := \begin{cases} \text{false} & \text{if } \text{def}(a) = \emptyset \\ \text{body}_1 \lor \ldots \lor \text{body}_n & \text{otherwise} \end{cases}$$

where $\text{def}(a) = \{a \leftarrow \text{body}_1, \ldots, a \leftarrow \text{body}_n\}$.

**Definition 6.2 (comp(P) (Clark,1978))**

For any normal program $P$, we define $\text{comp}(P)$ over a given signature $\mathcal{L}$ (where $\mathcal{L}_P \subseteq \mathcal{L}$) as the classical theory $\{a \leftrightarrow \text{sup}(a) : a \in \mathcal{L}\}$.

We use an example to illustrate the above definitions. Let $P$ be the program:

$$p \lor q \leftarrow \neg r$$
$$p \leftarrow s, \neg t$$

Then $\text{HEAD}(P) = \{p, q\}$, and $\text{dis-nor}(P)$ consists of the clauses:

$$p \leftarrow \neg r, \neg q$$
$$q \leftarrow \neg r, \neg p$$
$$p \leftarrow s, \neg t$$

Definite($\text{dis-nor}(P)$) consists on the clauses:

$$p \leftarrow \text{true}$$
$$q \leftarrow \text{true}$$
$$p \leftarrow s$$

$\text{In the standard definition } \mathcal{L} \text{ is } \mathcal{L}_P \text{. Our paper requires this more general definition}$
**Example 6.1 (Transformation)**

Let $P$ be the program:

\[
\begin{align*}
    a \lor b & \leftarrow c, \neg c, \neg d \\
    a \lor c & \leftarrow b \\
    c \lor d & \leftarrow e \\
    b & \leftarrow \neg c, \neg d, \neg e \\
\end{align*}
\]

then $\text{HEAD}(P) = \{a, b, c, d\}$, and $\text{SEM}_{\text{min}}(P, \mathcal{L}_P) = \emptyset$.

We can apply $\text{RED}^+$ to get the program $P_1$:

\[
\begin{align*}
    a \lor b & \leftarrow c, \neg c, \neg d \\
    a \lor c & \leftarrow b \\
    c \lor d & \leftarrow \text{true} \\
    b & \leftarrow \neg c, \neg d, \neg e \\
\end{align*}
\]

If we apply $\text{RED}^+$ again, we get program $P_2$:

\[
\begin{align*}
    a \lor b & \leftarrow c, \neg c, \neg d \\
    a \lor c & \leftarrow b \\
    c \lor d & \leftarrow \text{true} \\
    b & \leftarrow \neg c, \neg d \\
\end{align*}
\]

Clearly $\{c, d, a\} \in \text{SEM}_{\text{min}}(P_2, \mathcal{L}_P)$ means that $c \lor d \lor a$ is a consequence in $\text{SEM}_{\text{min}}(P_2, \mathcal{L}_P)$. Now, we can apply $\text{SUB}$ to get program $P_3$:

\[
\begin{align*}
    a \lor c & \leftarrow b \\
    c \lor d & \leftarrow \text{true} \\
    b & \leftarrow \neg c, \neg d \\
\end{align*}
\]

We will refer to this example again, that we will start calling our basic example.

Obviously, the just mentioned transformations are among the minimal requirements a well-behaved semantics should have (see [Dix, 1995b]). For this reason, every semantics presented in this paper will be invariant under the transformations $\text{RED}^+$, $\text{RED}^-$ and $\text{SUB}$.

The following transformations are defined in [Brass and Dix, 1997; Brewka and Dix, 1996].

**GPPE:** (Generalized Principle of Partial Evaluation)

Suppose $P$ contains $A \leftarrow B^+, \neg B^-$ and we fix an occurrence of an atom $g \in B^+$. Then we replace $A \leftarrow B^+, \neg B^-$ by the $n$ clauses $(i = 1, \ldots, n)$

\[
A \cup (A_t \setminus \{g\}) \leftarrow (B^+ \setminus \{g\}) \cup B_i^+, \neg B^- \cup \neg B_i^-
\]

where $A_t \leftarrow B_i^+, \neg B_i^- \in P$, for $(i = 1, \ldots, n)$ are all clauses with $g \in A_t$. If no such clauses exist, we simply delete the former clause.

**TAUT:** (Tautology) same as for normal programs.

Let $CS_2$ be the rewriting system which contains, besides the basic transformation rules, the rules GPPE and TAUT. This system is introduced in [Brass and Dix, 1997] and is confluent and terminating as shown in [Brass and Dix, 1998].
Definition 6.5 (D-WFS)
The disjunctive wellfounded semantics D-WFS is defined as the weakest semantics satisfying SUB, RED+, RED−, GPPE and TAUT.

Let us note that although the CS5 system has the nice property of confluence (and termination), its computational properties are not that efficient. In fact, computing the normal form of a program is exponential (even for normal programs, whereas it is known that the WFS can be computed in quadratic time).

We introduce our proposed semantics D1-WFS and D1-WFS-COMP and give some important results about them. Unless stated otherwise we assume that every program is a disjunctive program.

Definition 6.6 (Dloop)
For a program P, let unf(P) := P \ M(M(Definite(dis = nor(P))).
The transformation Dloop reduces a program P to P1 := \{A ← B+, \neg B− | B+ \cap \text{unf}(P) = \emptyset\}. We assume that the given transformation takes place only if P \neq P1.

Let Dsuc be the natural generalization of suc to disjunctive programs, formally:

Definition 6.7 (Dsuc)
Suppose that P is a program that includes a fact a ← true and a clause Q ← Body such that a ∈ Body. Then we replace this clause by the clause Q ← Body \ {a}.

Definition 6.8 (CS6)
Given a relation R, we define R' as follows:

\[
ar'^{b} \text{ iff } \begin{cases}a^{R^{n}b} & \text{if } \exists c|a^{R^{n+1}c}\text{ a.o.w.} \\
a = b & \text{otherwise}
\end{cases}
\]

Given two relations R1 and R2, we define R2 \circ R1 as:

R2 \circ R1 \cup \{(x,y)|(x,y) \in R1, \neg \exists(z,w) \in R2\}, where \circ denotes the standard composition of relations.

Let REDUCE be the binary relation on programs defined by:

Dloop \circ (Dsuc' \circ (SUB \cup RED^+ \cup RED^-)) \setminus I, where I denotes the identity relation.

Finally, let CS6 be the rewriting system based on the basic transformation REDUCE.

Theorem 6.1 (Confl. and termination of CS6)
The system CS6 is confluent and terminating. It induces a semantics that we call D1-WFS. If we consider only normal programs then its induced semantics corresponds to the well-founded semantics.

Proof: Confluence is immediate since REDUCE behaves as a partial function. Moreover, REDUCES always deletes something in the program and so this relation is terminating. For normal programs, the system is clearly equivalent to RED^+ + RED^- + S + Loop + SUB (that is, both systems define the same normal form) which in turns defines the well-founded semantics.

Consider again P from our basic example introduced before. As we noticed before, program P reduces to P3. But P3 still reduces (by RED−) to P4, which is as P3 but the third clause is removed. From P4 we can apply a Dloop reduction to get P5: the single clause c \lor d ← true. So, REDUCES (which can be seen as a macro reduction) transforms P (in one step) to P5. Since REDUCES can not be applied again, P5 is the normal form of the CS6 system.

For this example it turns out that D-WFS is equivalent to D1-WFS, but this is false in general. However for normal programs both systems are equivalent since they define WFS, but note that the normal forms for CS5 and CS6 are not necessarily the same. An advantage of CS6 over CS5 (again for normal programs) is that the normal form of CS6 is polynomial-time computable, while computing the normal form of CS5 is in general exponential as it is shown in [Brass et al., 1997].

We now define a very strong semantics that includes the power of comp.

Definition 6.9 (D1-WFS-COMP)
For every program P, we define DCOMP(P) := comp(dis − nor(normalCS6,P))) over Lp. We define D1-WFS-COMP(P) as the set of pure disjunctions that are logical consequences of DCOMP(P).

It is immediate to see that D1-WFS-COMP is more powerful than D1-WFS. Take for instance the program P:

p \lor q ← true
r ← \neg p
r ← \neg q

Then D-WFS(P) = \{(p,q),\neg(p,q,r)\} = D1-WFS(P), however, D1-WFS-COMP(P) at least derives r. In this case D1-WFS-COMP corresponds to STABLE, but this is not always true. Sometimes STABLE is inconsistent, while D1-WFS-COMP is not. Consider P as:

d \lor e
\neg c
b ← a
a ← \neg b, \neg c

Note that STABLE is inconsistent while D1-WFS-COMP is not. This is because DCOMP(P) is:

d ← \neg e
e ← \neg d
a ← (b \lor \neg b)
c ← \text{false}

Due to its construction, we see that D1-WFS-COMP is similar to STABLE. However, STABLE is inconsistent more often than D1-WFS-COMP (at least for normal programs).

Our current research suggests that logic programming can be extended by adding both disjunctions and
7 Conclusion

We introduced the general notion of a confluent LP-system. Such a system is based on certain transformation rules and induces a semantics SEM in a natural way. Thanks to confluence and termination, our transformation rules have both a declarative and an operational meaning. From the declarative point of view, they tell us that our semantics is closed under the given transformation rule. From an operational point of view the transformations are computable functions that can be applied to simplify the program.

When we arrive to the normalform, then computing its semantics is immediate. Rationality is a desirable property of semantics in logic programming. Confluence plus a simple property that we call partial distribution implies rationality. It is straightforward to see that all (but one) of our semantics for normal programs satisfy partial distribution. Therefore we know that all but one of these semantics are rational.

We showed that most of the well-known semantics for normal logic programs are induced by confluent LP-systems. Moreover, we showed by introducing several new transformation rules that the corresponding LP-systems induce interesting semantics which are polynomial time computable and extend WFS. We also showed how to apply our approach to disjunctive programs.

It is quite surprising that the simple notion of a confluent LP-system that we introduced here, is so flexible that it allows us to extend recently defined calculi for WFS by new transformations in such a way, that the new system still is confluent. We have therefore shown that rewriting methods can be successfully applied in the realm of logic programming semantics.

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