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A New Analytical Method to Calculate the Characteristic Impedance $Z_C$ of Uniform Transmission Lines

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Abstract. A new analytical method to calculate the characteristic impedance of transmission lines embedded in identical, symmetrical and reciprocal connectors is herein presented. To calculate the characteristic impedance of transmission lines, the proposed method uses S-parameter measurements performed on two uniform transmission lines having the same characteristic impedance and propagation constant but different lengths. The method was successfully applied to characterize microstrip lines printed on an FR4 substrate in the 0.45-4GHz frequency range.

Keywords. Characteristic impedance, propagation constant, microstrip line, symmetrical-reciprocal connectors.

1 Introduction

Nowadays, microprocessor chips run at frequencies higher than 2GHz. At these frequencies, the interconnects that link the multichip modules (MCMs) on the circuit boards start to behave like high-frequency transmission lines, and problems related to mismatch between lines and chips (receiver and transmitters), and line losses cannot be ignored by the designer of high-speed circuits [6]. For instance, in digital circuits, line losses and mismatch may distort the clock signal and produce high order harmonics [4, 6]. Therefore, when using microstrip lines printed on circuit boards to interconnect MCMs, the line’s characteristic impedance and propagation constant are crucial parameters that have to be considered in the system design stage at these clock frequencies [6].

A uniform transmission line is characterized by its propagation constant $\gamma$ and characteristic impedance $Z_C$. In addition, the line’s characteristic impedance and propagation constant are useful parameters to determine the distributed R, L, C and G components of the transmission line.
model. Usually, the propagation constant of a uniform transmission line, embedded within arbitrary connectors, is determined from scattering parameter measurements performed on two lines (L-L method [3, 8]) having the same characteristic impedance but different lengths. The L-L method for $\gamma$ determination can be implemented using either the ABCD matrix [8] or wave cascading matrix [3, 12]. With respect to $Z_c$, this parameter is not easily determined when the transmissions line is embedded between arbitrary connectors [9, 10]. Indirect methods dealing with $Z_c$ determination, based on calibration independent methods, have been reported in [15, 16]. Indirect methods for $Z_c$ determination use measurements of both the line propagation constant and the line capacitance. Other works, focused on characteristic impedance determination, use two-tier calibration methods [9, 10] and measurements of the line propagation constant. Examples of those methods are the ones proposed by Mangan et al. [9] and Narita et al. [10]. In [9], the transition in which the line is embedded is modeled with a shunt admittance and ignores the pad losses and pad phase delay. In the case of lines embedded between symmetrical and reciprocal connectors, Narita et al. [10] proposed a new analytical method to calculate $Z_c$. However, in [10] the solutions of the equations, which provide $Z_c$ and the elements of the transition matrix $E_L$ used to model the connectors between which the line is embedded, are not sufficiently justified. For this reason, in this paper we present in detail and in a comprehensive manner a new and straightforward analytical method to determine $Z_c$, first introduced in [17]. The method uses S-parameter measurements performed on two uniform transmission lines having the same characteristic impedance and propagation constant, but having different length. The method assumes, as already reported by Narita [10], that the transitions (connectors) in which the line is embedded are identical, symmetrical, and reciprocal. To demonstrate the usefulness of the proposed method, experimental data of $Z_c$ for microstrip lines printed on FR4 and embedded in female connectors are reported.

2 Method to Calculate the Line’s Characteristic Impedance

When a uniform transmission line of arbitrary length and arbitrary characteristic impedance is embedded between connectors, as shown in Fig. 1, it is difficult—if not impossible—[4, 13], to determine its characteristic impedance directly from a single set of S-parameter measurements. According to [4], the structure shown in Fig.1 consists of a uniform transmission line and two connectors referred to as $A$ and $B$, represented using the formalism of the ABCD transmission matrix as $T_{li}$, $T_s$ and $T_{in}$ matrices, respectively. Using the ABCD matrix formalism, the equivalent matrix $M$ of the structure shown in Fig. 1 is equal to the product of the three individual matrices expressed as:

$$M = [T_{li}] [T_s] [T_{in}] \quad (1)$$

with

$$\begin{bmatrix} \gamma \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (2)$$

and

$$\begin{bmatrix} \gamma \end{bmatrix} = \begin{bmatrix} a_{22} & a_{12} \\ a_{11} & a_{21} \end{bmatrix} \quad (3)$$

where $T_{li}$ is the ABCD matrix for the uniform transmission line, $\gamma$ is the line propagation constant, $Z_c$ is the line characteristic impedance, and $l_i$ is the line length.

Under the assumption that the connectors $A$ and $B$ are identical ($B = A$), symmetrical ($a_{22} = a_{11}$) and reciprocal ($a_{11}a_{22} - a_{12}a_{21} = 1$) as shown in Fig. 2, the equivalent matrix $M$ of the structure can be expressed as:
A new analytical method for calculating the characteristic impedance $Z_C$ of uniform transmission lines

The method proposed in this work to calculate $Z_C$ requires $S$-parameter measurements performed on two uniform transmission lines having the same characteristic impedance and propagation constant, but different lengths as shown in Fig. 3. The ABCD matrices $M_1$ and $M_2$, resulting from the measurements of lines $l_1$ and $l_2$ are expressed as

$$[M] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \cosh(\gamma l) & Z_C \sinh(\gamma l) \\ \frac{1}{Z_C} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(5)

The method proposed in this work to calculate $Z_C$ requires $S$-parameter measurements performed on two uniform transmission lines having the same characteristic impedance and propagation constant, but different lengths as shown in Fig. 3. The ABCD matrices $M_1$ and $M_2$, resulting from the measurements of lines $l_1$ and $l_2$ are expressed as

$$[M_1] = [T_\lambda][T_{l1}][T_\lambda]$$

(6)

$$[M_2] = [T_\lambda][T_{l2}][T_\lambda]$$

(7)

In order to find an analytical expression to determine the characteristic impedance of the line, equations (6) and (7) are written as:

$$[M_1][T_\lambda]^{-1} = [T_\lambda][T_{l1}]$$

(10)

$$[M_2][T_\lambda]^{-1} = [T_\lambda][T_{l2}]$$

(11)

Then, taking advantage of the symmetry property of the structure shown in Fig. 2 ($[T_{l1}] = [T_{l2}]$), it is easy to conclude from (6) and (7) that $m_{11} = m_{22}$ and $p_{11} = p_{22}$. Using these results and writing out (10) it follows that:

$$m_{11} - \cosh(\gamma l_1) = m_{12} \frac{a_{12}}{a_{11}} + \frac{1}{Z_C} \sinh(\gamma l_1) \frac{a_{12}}{a_{11}}$$

(12)
\[ m_{i1} - \cosh(\gamma l) = m_{2i} \frac{a_{i2}}{a_{i1}} + Z_c \sinh(\gamma l) \frac{a_{2i}}{a_{1i}} \]  (13)

\[ m_{i2} = (m_{i1} + \cosh(\gamma l)) \frac{a_{i2}}{a_{i1}} + Z_c \sinh(\gamma l) \]  (14)

\[ m_{2i} = (m_{i1} + \cosh(\gamma l)) \frac{a_{2i}}{a_{1i}} + \frac{1}{Z_c} \sinh(\gamma l) \]  (15)

In the same way, but now writing out (11), one obtains

\[ p_{i1} - \cosh(\gamma l_z) = p_{2i} \frac{a_{i2}}{a_{i1}} + \frac{1}{Z_c} \sinh(\gamma l_z) \frac{a_{2i}}{a_{1i}} \]  (16)

\[ p_{i2} = (p_{i1} + \cosh(\gamma l_z)) \frac{a_{i2}}{a_{i1}} + Z_c \sinh(\gamma l_z) \]  (17)

\[ p_{2i} = (p_{i1} + \cosh(\gamma l_z)) \frac{a_{2i}}{a_{1i}} + \frac{1}{Z_c} \sinh(\gamma l_z) \]  (18)

Then, a set of two simultaneous equations is obtained from (10) and (11). On the one hand, the first set of simultaneous equations, (14) and (18), can be expressed in matrix form as:

\[
\begin{bmatrix}
    m_{i2} \\
    p_{i2}
\end{bmatrix} = \begin{bmatrix}
    a_{i2} \\
    a_{i1}
\end{bmatrix} K \begin{bmatrix}
    Z_c
\end{bmatrix}
\]  (20)

On the other hand, the second set of simultaneous equations (15) and (19) can be expressed in matrix form as:

\[
\begin{bmatrix}
    m_{2i} \\
    p_{2i}
\end{bmatrix} = \begin{bmatrix}
    a_{2i} \\
    a_{1i}
\end{bmatrix} K \begin{bmatrix}
    \frac{1}{Z_c}
\end{bmatrix}
\]  (21)

where

\[
K = \begin{bmatrix}
    m_{i1} + \cosh(\gamma l) & \sinh(\gamma l) \\
    p_{i1} + \cosh(\gamma l_z) & \sinh(\gamma l_z)
\end{bmatrix}
\]  (22)

Since (20) and (21) have the same \( K \) matrix, they can be grouped as:

\[
M_X = [K][B]
\]  (23)

with

\[
M_X = \begin{bmatrix}
    m_{2i} & m_{i2} \\
    p_{2i} & p_{i2}
\end{bmatrix}
\]  (24)

and

\[
B = \begin{bmatrix}
    a_{2i} & a_{i2} \\
    a_{1i} & a_{i1} \\
    \frac{1}{Z_c} & Z_c
\end{bmatrix}
\]  (25)

Solving equation (23) for \( B \), one has:

\[
B = [K^{-1}][M_X]
\]  (26)

Finally, an analytical expression to calculate \( Z_c \) is obtained from (26) as:

\[
Z_c = \frac{p_{i2}(\cosh(\gamma l_z) + m_{i1}) - m_{2i}(\cosh(\gamma l_z) + p_{i1})}{\det(K)}
\]  (27)

where \( \det(K) \) is the value of the determinant of matrix \( K \), expressed as

\[
\det(K) = (m_{i1} + \cosh(\gamma l)) \sinh(\gamma l_z) - (p_{i1} + \cosh(\gamma l_z)) \sinh(\gamma l) \]  (28)

It is important to comment that the analytical expression to calculate \( Z_c \) depends only on \( l \), \( l_z \) and the line propagation constant. This expression is different from the one reported in [10, 14]. The proposed method to calculate the line's characteristic impedance requires the knowledge of the propagation constant, which will be determined in this work using the method proposed in [12].
3 The Test Structures

Figure 3 shows the test structures used for this work. These consist of two SMA female connectors and a microstrip line. Each microstrip line is printed on an FR4 substrate (dielectric constant $\varepsilon_r$ 4.8 and thickness (h) of 0.16cm). Commercially available software (LinCal included in ADS software [2]) was used to determine the width (w) of the line in order to achieve a 50 Ω characteristic impedance. The lengths of the lines are 2.5 and 4 cm, referred to as $l_1$ and $l_2$, respectively, and are shown in Fig.3b.

4 Experimental Results

The S-parameters measurements of the test structures were performed using an HP8510C vector network analyzer (VNA) in the frequency range from 0.045 to 4 GHz. Since the two SMA connectors are of the same sex, the SOLT (Short-Open-Load-Thru) calibration technique with adapter removal was performed to calibrate the VNA to the boundary of the coaxial cables.

The implementation of the proposed method to determine the line’s characteristic impedance, as indicated by equation (27), requires the knowledge of both the line propagation constant and the physical lengths of the lines. For this work, the line propagation constant was determined using the two-lines method proposed in [12]. The propagation constant was determined from S-parameter measurements performed on two lines presenting the same characteristic impedance and propagation constant, but having different lengths [12]. Moreover, the two-lines method [12] does not require of the knowledge of the connector matrix, and uses the raw S-parameter measurements. However, the method in [12] can also be implemented using S-parameter measurements performed with a vector network analyzer calibrated to the boundaries of the connectors of the test structures. Therefore, to determine the line propagation constant and the line impedance we used the same S-parameter data set of the test structures.

The calculated propagation constant is directly related to the attenuation constant, $\alpha$, and the phase constant, $\beta$, as $\gamma = \alpha + j\beta$; $\beta = \frac{2\pi f \sqrt{\varepsilon_{\parallel}}}{c}$, where $f$ is the frequency and $c$ is the speed of light. For the microstrip lines used in this work, the attenuation constant $\alpha$ per unit length and the effective dielectric constant $\varepsilon_{\parallel}$ versus frequency

![Fig. 4. a) Effective dielectric constant versus frequency and b) attenuation per physical length versus frequency](image-url)
are reported in Fig. 4. Some atypical peaks in the $\alpha$ and $\varepsilon_{\|}$ traces can be observed around 2 GHz, but since they do not appear periodically, these peaks are attributed to the non-ideal coaxial to microstrip transition.

Once the propagation constant has been determined, the characteristic impedance can be calculated from (27). The characteristic impedance extracted using the new method ($l_1$ = 2.5 cm, and $l_2$ = 4 cm) is shown in Fig. 5. Notice that the real part of the characteristic impedance agrees with the nominal value calculated using ADS’s Lin-Cal; unfortunately, this program does not provide information on the imaginary part of $Z_c$. It is important to comment that the characteristic impedance of the line is expected to be slightly different from the nominal value due to the standard etching resolution limits.

On the other hand, once the propagation constant $\gamma$ and the characteristic impedance $Z_c$ were calculated, the distributed transmission line parameters; resistance $R$, inductance $L$, conductance $G$, and capacitance $C$ per unit length, were determined using the following equations:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_c = \frac{R + j\omega L}{\sqrt{(G + j\omega C)}}$$

Solving these for $R$, $L$, $G$, $C$, one obtains:

$$R = \text{Re} \left( \frac{\gamma Z_c}{\gamma Z_c} \right)$$

$$L = \text{Im} \left( \frac{\gamma Z_c}{\gamma Z_c} \right)$$

$$G = \text{Re} \left( \frac{\gamma}{Z_c} \right)$$

$$C = \text{Im} \left( \frac{\gamma}{Z_c} \right)$$

The extracted parameters $R$, $L$, $G$, $C$ and their frequency dependence are shown in figures 6a and 6b. It should be noted that $L$ and $C$ remain almost constant in the whole frequency range, while the frequency dependence of $R$ and $G$ show the expected behavior with frequency. It is important to comment that these $R$, $L$, $G$, $C$ values agree with those reported in [14] for an FR4 substrate ($C(f = 1\text{GHz}) \approx 1.6 \text{pF/cm}$, $L(f = 1\text{GHz}) \approx 3.0 \text{nH/cm}$ $G(f = 1\text{GHz}) \approx 0.1 \text{mS/cm}$, $R(f = 1\text{GHz}) \approx 0.3 \Omega/\text{cm}$).

5 Conclusions

We have presented an analytical method to calculate the characteristic impedance $Z_c$ of microstrip transmission lines embedded between symmetrical and reciprocal connectors. The method uses scattering parameter measurements performed on two uniform transmission lines having the same characteristic impedance and
propagation constant but different lengths. The method was experimentally validated, in the frequency range of 0.45 to 4GHz, using microstrip lines printed on FR4 and embedded within female-female connectors. The main advantage of this method over already published methods [15, 16] is that knowledge of line capacitance is not necessary to calculate $Z_C$. From the knowledge of $\gamma$ and $Z_C$, the distributed transmission line parameters $R$, $L$, $G$, and $C$ have also been determined and agree with those provided in [10].

References


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A new analytical method for calculating the characteristic impedance $Z_C$ of uniform transmission lines

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