Novel Two-Stage Comb Decimator

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Abstract. A simple method for the design of a multiplier-less comb-based decimation filter is presented. The filter compensates the comb passband droop, and has the desired attenuation in the first folding band. The proposed structure has two decimation stages, both with simpler comb filters. The desired attenuation in the first folding band is achieved by an appropriate number of cascaded combs in the second stage. A simple compensation filter, working at a low output rate, provides the desired compensation in the pass band. The choice of the design parameters is presented along with a comparison of the proposed method to some other existing design methods.

Keyword. Comb filter, decimation, folding bands, compensation.

1 Introduction

Currently, the design of the most popular analog-to-digital (A/D) converter is based on oversampling and delta-sigma ($\Delta\Sigma$) modulation techniques that results in a simpler analog filter, while requiring a more complex digital structure [1]. The key part of this structure is a decimation filter, which is needed to decrease the sampling rate of the oversampled signal. The most popular decimation filter is the comb filter due its computational efficiency and simple implementation requiring only additions/subtractions.

The transfer function of a Kth-order decimation comb filter can be expressed either in a recursive form or a non-recursive form as

$$H_{\text{COMB}}(z) = \left[ \frac{1}{M} \left( \frac{1-\z^{-M}}{1-z^{-1}} \right) \right]^K = \left( \frac{1}{M} \sum_{n=0}^{K-1} z^{-n} \right)^K,$$  \hspace{1cm} (1)

where $M$ is the decimation factor and $K$ is the number of the comb stages. Some authors call the filter of Eq. 1 CIC (cascaded-integrator-comb) filter, although the CIC filter is referred to a structure proposed by Hogenauer [4].

The magnitude response of the filter of Eq. 1 is

$$|H_{\text{COMB}}(e^{j\omega})| = \left| \frac{1}{M} \sin(\omega M / 2) \right|^K$$  \hspace{1cm} (2)

Various methods have been proposed to model the power consumption of the recursive and non-recursive structures of Eq. 1 ([1], [2], [3],

Nuevo decimador comb de dos etapas

Resumen. Se presenta un método simple para el diseño de un filtro de diezmado basado en filtro comb sin multiplicadores. El filtro compensa la caída en la banda de paso del filtro comb y tiene la atenuación deseada en la primera banda del plegado. La estructura propuesta tiene dos etapas de decimación, ambas con filtros comb simples. La atenuación deseada en la primera banda del plegado se alcanza por un número apropiado de combs en cascada en la segunda etapa. Un filtro simple de compensación trabajando en la baja razón de muestreo de salida provee la compensación deseada en la banda de paso. La elección de los parámetros de diseño es presentada junto con una comparación del método propuesto con algunos otros métodos de diseño existentes.

Palabras Clave. Filtro comb, diezmado, bandas del plegado, compensación.
[12]). Abbas et al. extend previous models for the estimation of power and complexity of recursive and non-recursive comb filters and provides a guide-line of the power and implementation complexity by taking into account several factors [1].

The comb filter is usually used at the first decimation stage, whereas a second decimator block with a decimation factor \( R \), which is smaller than that of the comb filter, determines the passband edge frequency \( \omega_p \)

\[
\frac{\omega_p}{\pi} = \frac{1}{RM},
\]

(3)

where the worst-case passband distortion occurs [10]. Similarly in [10], the worst-case aliasing is at the frequency

\[
\frac{\omega_A}{\pi} = \frac{2}{M} - \frac{1}{RM} = \frac{2R-1}{RM}.
\]

(4)

In this paper we consider the minimum value for \( R \), i.e., \( R = 2 \).

The comb filter must have a high alias rejection around its zeros (the folding bands) and a low passband droop in the pass band defined by Eq. 3 in order to avoid the distortion of the decimated signal. However, the comb filter has a high passband droop and low folding band attenuations. The first folding band is the most critical, because it has less attenuation than the sub sequential folding bands.

Various methods have been advanced to solve the above two problems. In order to improve the attenuation around the folding bands, the zero-rotation terms have been introduced in [11] and [13]. Fairly simple compensation filters have been proposed in [6, 7] and [9] for improving the comb passband. The sharpening based methods improve the passband and the stopband at the expense of the increased complexity in [5] and [10].

Recently, in [8], a simple two-stage structure with a good aliasing rejection characteristic and a low passband droop for \( R = 8 \) have been advanced.

The main goal of this work is to generalize this result so as to provide a low passband droop and high stopband attenuation in the first folding band for a minimum value of \( R \).

The paper is organized as follows. Section 2 describes the two-stage structure and the choice of the design parameters. The design of the compensation filter is given in Section 3. Finally, Section 4 provides a discussion of the results.

2 Two-Stage Structure

We consider here the case where the decimation factor \( M \) can be expressed as a product of two integers as \( M = M_1M_2 \).

The transfer function of the corresponding comb-based decimation filter is given by

\[
H(z) = H_1(z)H_2(z) = \left[H_1(z)\right]^{K_1}\left[H_2(z)\right]^{K_2},
\]

(5)

where

\[
H_1(z) = \frac{1}{M_1}\left(\frac{1 - z^{-M_1}}{1 - z^{-1}}\right),
\]

\[
H_2(z^{M_1}) = \frac{1}{M_2}\left(\frac{1 - z^{-M_1M_2}}{1 - z^{-M_1}}\right).
\]

(6)

The corresponding magnitude response is given as

\[
|H(e^{j\omega})| = \left|\frac{1}{M_1}\frac{\sin(\omega M_1/2)}{\sin(\omega/2)}\right|^{K_1}\left|\frac{1}{M_2}\frac{\sin(\omega M_2/2)}{\sin(\omega/2)}\right|^{K_2}.
\]

(7)

2.2 Choice of Design Parameters

2.2.1 Choice of \( M_1 \) and \( M_2 \)

The choice of \( M_1 \) is a matter of compromise between two factors: having less complex polyphase components in the first stage and making the filter in the second stage working as much as possible at a lower rate [8]. To this end, we propose that the factors \( M_1 \) and \( M_2 \) be close in
values as much as possible to each other with $M_1 \leq M_2$.

### 2.2.1.2 Choice of $K_1$ and $K_2$

Next we discuss the benefit of choosing different values for $K_1$ and $K_2$ and how to relate them. From Eq. 6 it can be seen that the filter $H_2(\zeta^{M_1})$ has the same zeros as the corresponding comb filter with a decimation factor $M = M_1M_2$. It follows that for $K = K_2$, both filters will have approximately the same first folding band characteristics. However, a decrease in the attenuation in the lateral folding bands may appear, because $K_1$ is not equal to $K_2$.

As an example, Fig. 1 shows the gain responses of the filter of Eq. 5 with $M_1 = 4$, $M_2 = 7$, $K_1 = 4$ and $K_2 = 5$ along with that of comb filter of Eq. 1 with $M = 28$, $K = 4$ and $K = 5$. Note that the filter (5) has the first folding band width almost the same as the comb filter (1) with $K = 5$. However, the filter (5) has 4 cascaded less complex comb filters working at the high input rate, while the comb filter (1) with $K = 5$ has 5 cascaded more complex comb filters working at the high input rate.

Consequently, we can increase $K_2$ to obtain more attenuation in the first folding band and decrease $K_1$ to arrive at a less complex overall filter, considering that all subsequent folding bands should have at least the minimum attenuation as the first folding band.

As another example, Fig. 2 illustrates the case where $K_1 = 4$ and $K_2 = 10$. Note that the first folding band has the same attenuation as that of the comb filter with $K = 10$. However, the seventh folding band has less attenuation than the first one.

Therefore the appropriate relation between $K_1$ and $K_2$ will prevent that the lateral folding bands have less attenuation than the first folding band.

Using MATLAB simulations we arrive at:

$$K_1 \geq \left\lfloor \frac{K_2}{2} \right\rfloor + 1. \quad (8)$$

For example, for $K_2 = 10$ from Eq. (8) it follows that the minimum value for $K_1$ is equal to 6. Fig. 3 shows the gain responses of the proposed filter with $M_1 = M_2 = 4$, $K_1 = 6$, $K_2 = 10$, and comb filter with $M = 16$, $K = 6$ and 10.

Note the following:

- The proposed filter (5) with $K = 6$, and $K_2 = 10$ has almost the same first folding band attenuation, as the original comb filter (1) with $K = 10$.

- However, the comb filter with $K = 10$ has the cascade of 10 filter at high input rate, as a difference to the proposed filter, which has only 6.

- Both, the proposed and comb filters with $K = 6$, have the cascade of 6 filters at high input rate. However, the attenuation in the first folding band of the proposed filter is much better.
2.2.1.3. Worst-case aliasing attenuation

Next we investigate the worst case aliasing attenuation for the comb filters. Consider the worst-case frequency \( \omega_A \), given in (4), for \( R=2 \),

\[
\omega_A = \frac{3\pi}{2M}.
\] (9)

Placing (9) into (2), we obtain the worst case aliasing attenuation for the comb filter:

\[
|H_{\text{comb}}(e^{j\omega_A})| = \frac{\sin(\omega_A M/2)}{M \sin(\omega_A/2)} \left( \frac{\sin(3\pi/4)}{M \sin(3\pi/4M)} \right)^K.
\] (10)

Fig. 4a presents the worst-case aliasing attenuations (10) in dB, for \( M=8, \ldots, 64 \), and for different values of \( K \), \( K = 1, \ldots, 6 \). The zoom for \( K=6 \) is shown in Fig.4.b.

Note that the worst case aliasing attenuation practically does not depend on \( M \). The same observation is confirmed mathematically, as explained in the following.

Considering \( 3\pi/4M \) being small, we can rewrite Eq. 10 as

\[
|H_{\text{comb}}(e^{j\omega_A})| \approx \frac{\sin(3\pi/4)}{M (3\pi/4M)} \left( \frac{\sin(3\pi/4)}{3\pi/4} \right)^K.
\] (11)

For higher values of \( M \), the expression (11) becomes more correct, as shown also in Fig.4.b.

As given in Sub-section 2.2.1.2, the parameter \( K_2 \) of the proposed filter should be equal to the parameter \( K \) of the comb filter, in order to have approximately equal worst-case attenuation in the first folding band.

Table 1 shows tentative value of the minimum worst-case aliasing attenuations from Fig.4 and the corresponding values of \( K_2 \). The minimum values of \( K_1 \), obtained from (8) are also shown. The method is illustrated by the following example.

**Example 1:** Let \( M = 15 \). We choose \( M_1 = 3 \) and \( M_2 = 5 \), and we want the minimum worst case aliasing attenuation of -60 dB. From Table 1, we choose \( K_2 = 6 \), and \( K_1 = 4 \). The corresponding
gain responses are given in Fig. 5, along with that of the comb filter with $M = 15$ and $K = 6$.

Next issue is the improvement of the passband characteristics.

### 3 Compensation Filter

For small values of $\omega$, sinusoidal functions in the first term of (7) can be approximated as $\sin(\alpha) \approx \alpha$ resulting in:

$$G(z^M) = -2^{-M} - (2^4 + 2)z^{-2M} + z^{-3M}$$  \hspace{1cm} (15)

$$|H(e^{j\omega})|^K_2 \approx \left| \frac{\sin(\omega M / 2)}{M_1 \omega / 2} \right|^K_2 = \left| \frac{\sin(\omega M / 2)}{M_2 \sin(\omega M_1 / 2)} \right|^K_2$$  \hspace{1cm} (12)

$$\Rightarrow \left| H_2(e^{j\omega}) \right|^K_2 = \left| \frac{\sin(\omega M / 2)}{M_2 \sin(\omega M_1 / 2)} \right|^K_2 = \left| H_2(e^{j\omega}) \right|^K_2.$$  \hspace{1cm} (13)

Similarly, the magnitude characteristic of the comb filter (1) can be approximated for small values of $\omega$ as:

$$H_z(e^{j\omega}) \approx \left| \frac{\sin(\omega M / 2)}{M_2 \omega / 2} \right|^K_2 = \left| \frac{\sin(\omega M / 2)}{M_2 \omega / 2} \right|^K_2.$$  \hspace{1cm} (13)

The filter is multiplierless, and its coefficients are independent of $M$ and $K$. However, the filter must be cascaded $K_3$ times, where $K_3$ depends on $K$.

Now, the filter can be moved to the lower sampling rate to become the filter $G(z)$. Using analysis from this section and results from [7] we have

### Table 1. Choice of parameters $K_1$ and $K_2$ for the desired minimum worst-case aliasing attenuation

<table>
<thead>
<tr>
<th>Minimum attenuation in $\omega_{2\sin}$ dB</th>
<th>$K_2$</th>
<th>Minimum value for $K_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>-20</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>-30</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>-40</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>-50</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>-60</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

![Fig. 5. Illustration of Example 1](image-url)
The transfer function of the compensated proposed filter is given as
\[ H_p(z) = \left[ H_1(z) \right]^{K_1} \left[ H_2(z^M) \right]^{K_2} \left[ G(z^M) \right]^{K_3}. \]  \( (17) \)

The corresponding structure is shown in Fig. 7.

Consider the design of comb decimation filter for \( M=20 \) with the minimum alias attenuation of -50dB. From Table 1, we have the following parameters for the proposed method: \( K_1 = 3, K_2 = 5 \), and from (16), \( K_3=4 \). The parameters in method [8] are: \( K_1 = 4, K_2 = 4, b = 0 \). In both methods \( M_1=4 \) and \( M_2=5 \).

The gain responses along with the passband and the first folding band zooms are shown in Figures 8.a and b, respectively.

Note that the proposed method and the method in [8] provide the same attenuation in the first folding band. However, the method in [8] has 4 combs at high input rate, compared with the proposed filter which has only 3 combs.

In the proposed method, the side lobes are increased. However, the minimum desired attenuations in all folding bands are satisfied. Additionally, the pass band droop is less than in [8].

**Comparison to the method in [8]**

4 Discussion and Results

In this section, we compare the performance of the proposed structure with those of some other methods of similar complexity.

**Fig. 6.** Passband droops of filters comb and \( H_2 \)

\[ K_3 = \begin{cases} K_2 & \text{for } K_2 \leq 3, \\ K_2 - 1 & \text{for } K_2 > 3. \end{cases} \]  \( (16) \)

The corresponding structure is shown in Fig. 7.

**Fig. 7.** Proposed structure

Note that the structure includes two decimation stages consisting of the simpler comb decimation filters. The filters can be implemented in either recursive or non-recursive form as explained in [1].

**Fig. 8.** Comparison with the method in [8]

4a. Overall gain responses

4b. Passband and the first folding band zooms
Comparison to the Sharpening method in [10]

The sharpening method in [10] uses the sharpening polynomial $3H^2 - 2H^3$, where $H$ is the comb filter (1), to simultaneously improve both, the passband and the stopband of the comb filter. In this example we consider $M=25$ and $K=2$. In the proposed method, $M_1=M_2=5$, and $K_1=4$, $K_2=6$ and $K_3=5$.

The overall gain responses are shown in Fig. 9a, while the pass band and the first folding band zooms are given in Fig.9b.

Note that the proposed method exhibits much better magnitude characteristic in both, the pass band as well as in the stop band. Additionally, the sharpening structure is more complex than the proposed one.

Comparison to the zero-rotation method in [13]

In order to improve the folding band attenuations in [13], the zero-rotation filter has been introduced as

$$H_r(z) = \frac{1 - 2\cos(\alpha M)z^{-M} + z^{-2M}}{1 - 2\cos(\alpha)z^{-1} + z^{-2}}$$

(18)

where $\alpha$ is the zero rotation.

The filter (18) is cascaded with the comb filter, and this cascade is referred to as Rotated Sinc (RS) filter:

$$H_{RS}(z) = H_{COMB}(z)H_r(z)$$

(19)

Consider the design of RS filter for $M=16$, and $K=3$ and $\alpha=0.0184$. In the proposed method, $M_1=M_2=4$, $K_1=5$, and $K_2=6$, $K_3=5$.

The overall gain responses for the proposed and RS filter are shown in Fig.10a; the passband and the first folding band zooms are shown in Fig. 10b. Observe that the first folding band aliasing rejections are similar, as well as the attenuations in the sub sequential folding bands. However, the RS method requires two multipliers, one working at high input rate.

5 Conclusion

We have presented a simple method to improve alias rejection of the first comb folding band. In contrast with the methods, that make use of the zero-rotation term, the proposed method is
less complex because it does not require multipliers, while having similar folding band attenuations. Also, the method has a less complexity comparing to the Sharpening and other two-stage methods.

The method is based only on comb filters. Consequently, there is flexibility in the use of a recursive or non-recursive comb structure to achieve different goals regarding to the power consumption and complexity as explained in [1].

The compensation filter \( G(z) \) remains the same for all values of \( M \) and \( K_2 \). However, the number of the cascaded filters \( K_3 \) is related to the parameter \( K_2 \). If we wish to avoid multiple cascades of the compensator, we may use the compensator from [9]. However, its parameters are different for different values of \( K_2 \).

References


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*Article received on 13/06/2011; accepted on 29/08/2011.*