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Fuzzy Modeling and Evaluation of an IMTS User

Modelación y Valuación Fuzzy del Usuario de un IMTS

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Abstract

A suitable model of an Intelligent Multimedia Tutoring System (IMTS) user is presented in terms of concepts and mastery of abilities. Then it is shown how, via a fuzzy algebra, it is possible to monitor and evaluate user's cognitive and psychological states.

Key words: Fuzzy sets, fuzzy variable, linguistic variable, fuzzy algebraic structure, evaluation, linguistic approximation, intelligent multimedia tutoring system.

Resumen

Un modelo oportuno del usuario de un Intelligent Multimedia Tutoring System (IMTS) es presentado en términos de conceptos y dominio de habilidades. Luego es mostrado como, mediante una algebra fuzzy, es posible controlar y valuar los estados cognitivos y psicològicos del usuario.

Palabras clave: conjunto fuzzy, variable fuzzy, variable linguistica, structura algebrica fuzzy, valuaciòn, aproximaciòn linguistica, sistema tutorial multimedial inteligente

1 Introduction

Beginning from the 1960s, Artificial Intelligence (AI) attempted to build computerized systems that could perform in the same way as human beings. In the 1970s researchers attempted to develop intelligent computer-assisted instruction systems, often said ITSs (acronym for Intelligent Tutoring Systems).

ITSs take into consideration the individual differences of each user and provide dynamic individualization of instruction.

However, even with the unquestionable success of ITSs in certain environments (Sleeman and Brown, 1982, Katz *et al.*, 1992, Ohlsson, 1987)., the overall performance of these systems were not adequate (Self, 1990). IMTSs (Intelligent Multimedia Tutoring System) (Woolf 1996, Di Lascio, Fischetti, Loia, 1998) are the evolutionary successors of ITSs and they heavily rely on the recent dramatic advances in the field of multimedia.

Although the goal of a system capable of autonomous teaching is still perspective, systems can now be developed that offer partial but nevertheless effective solutions and can act as reliable partners for human teachers. The result of bringing together multimedia devices and AI technology represents a significant improvement in the overall performance of the systems that can provide more individualized instruction.

This adaptation to individual students has been made possible through the development of certain modules within an IMTS.

A vital role is played by the *user module* implementing the *user model* and tailoring the system's behavior to the user's needs. User modeling aims at building a model in order to make predictions about user's behavior, so that the system can recognize misconceptions and problems, identify causes and suggest suitable solutions. The user module makes hypotheses about user's conceptions and reasoning strategies employed to achieve the current knowledge state.

It is apparent that the task of obtaining information about the user and inferring conclusions on the basis of what is supplied is very exacting, many interdisciplinary problems are to be tackled, and only partial solutions can be attained.

However, this fact emphasizes the importance of improving knowledge engineering techniques to more effectively capture and portray the user's knowledge. This module plays a crucial role in the overall operation and performance of the entire system, keeps a running "snapshot" of the user and allows that the tutoring strategy be modified "on the fly" to adapt itself to the characteristics of the specific user. A comprehensive list of papers dealing with user modeling problems can be found in (Brusilovsky, Kobsa, Vassileva, 1998).

We note that the learning process is inherently not deterministic and this fact obliges to deal with uncertainty. The uncertainty stems from several reasons, we just quote ambiguity and multiplicity. The ambiguity is related to the fact that user's errors are anyway personal mistakes and thus the same error can be caused, in different users, by different reasons. The multiplicity, in turn, is related to the fact that, in order to solve a specific problem, a wrong behavior can depend on several misconceptions and skill deficiencies.

The use of fuzzy logic represents an interesting solution to the problem of dealing with uncertainty. In fact, in such way one can manage qualitatively different situations that the classical two-valued logic is unable to cope with. The basic principles of fuzzy theory are presented in (Klir, 1995) and several applications to teaching and learning problems are in (Liou and Wang, 1994, Maurice-Baumont and Derognat, 1994, Sawaragi *et al.*, 1991, Hawkes *et al.*, 1990, Biswas, 1995).

It is worth noting that the modeling illustrated in this paper is suitable for both ITSs and IMTSs because the latter is the natural evolution of the first one: multimedia features are present in IMTSs but the problem of dealing adequately with user modeling is basically the same for both systems.

This paper is organized as follows. Section 2 recalls basic results of fuzzy theory. Next Section shows how it is possible to represent user's cognitive states and psychological characteristics through suitable strings. Section 4 presents the basic features of a fuzzy algebraic structure that manages strings that represent users. In Section 5 the concept of fuzzy score is introduced and in Section 6 it is shown that the fuzzy structure can be utilized to monitor the evolution of the learning process. Section 7 tackles the problem of evaluating the learning process and last Section emphasizes the expressive power of the model as compared with other educational taxonomies

2 The Theory of Fuzzy Sets

Given a classical not-empty set U, a fuzzy subset A of U (Zadeh, 1965) is a function A: U —> [0, 1], often denoted by μ_A: U —> [0, 1]. The function $\mu_A(x)$ can be viewed as an extension of the concept of characteristic function. Alternatively, a fuzzy set can be denoted as follows: if $A = \{x_1, x_2, , , x_n\}$ is a finite set and is represented as the union $x_1+x_2+ ... +x_n$ of its elements, then $A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + ... + \mu_A(x_n)/x_n$; if A is infinite then $A = \int_U \mu_A(x)/x$, where the symbol slash links each element of U to its membership grade. In both cases, a fuzzy set A on U is completely characterized by the couples $(x, \mu_A(x))$ where $x \varepsilon U$.

Let U={a, b, c} be a set of persons. Suppose that the individual **a** weighs 85 kilos, **b** weighs 90 kilos and **c** 130 kilos. This is a possible fuzzy set for fat people: A = 0.49/a+0.53/b+0.85/c.

A fuzzy variable is a triple (Vf, U, R(Vf; u)) where Vf is the name of the variable, U is the universe of discourse, u denotes the generic element of U and R(Vf; u) is a fuzzy subset of U. For example "budget" could be the name of a fuzzy variable where $U=[0, \infty]$, and $R(Vf; u) = \{(u, s(u) = 1), u \varepsilon [0, 100]\} \cup \{(u, s(u) = [1 + ((u-100)/200)^2]^{-1}), u \varepsilon [100, \infty]\}$.

A linguistic variable is, in turn, a quintuple (Vl, T, U, g, m) where V1 is the name of a variable, T is the set of values or linguistic terms of V1, U is the universe of discourse, g is a syntactic rule (a grammar) capable of generating such linguistic values and m is a semantic rule that attaches to each element t of T its meaning, m(t), namely a fuzzy set on U. For example, the values of the linguistic variable "age" could be: young, rather young, old, very old, and so on. Each value is the name of a fuzzy variable on the universe of discourse U = [0, 100].

We note that adverbs such as "very", "rather" are called linguistic modifiers (Zadeh, 1972) because modify the meaning of terms such as "old" and "young" which are generators of the set T. The fuzzy sets associated with the generic element t of T are functions of types S and π, as defined by (Zadeh 75). By linearizing these functions one gets the triangular functions, i.e. the triangular symmetric numbers that are the fuzzy sets used to mathematize the terms of a linguistic variable [Pedrycz, 1994]. Lakoff has shown (Lakoff, 1973) that the terms of a linguistic variable can be defined beginning from the generator terms. The following function is capable of generating the term set of a linguistic variable:

$\mu(x) = 2 * (x-a)/(b-a) * I_{[a, (a+b)/2]}(x) +$
$+2 * (x-a)/(a-b) * I_{[(a+b)/2, b]}(x)$,
$x \varepsilon U \subseteq R$, a, b ε U,
where $I[x_1, x_2](x)=1$ if $x \varepsilon [x_1, x_2]$ otherwise 0.

Some terms of the linguistic variable "evaluation" are: good, fairly good, not good, very good; according to what suggested by (Schwartz, 1989), in the following Sections we use for this variable a sequence of values adjacent and uniformly distributed, rather than an infinite sequence of values. If one wishes to use n terms for evaluation, the i-th term is the fuzzy set with membership function different from zero on the interval [Max/n*i, Max/n*(i+1))/2], where $x \varepsilon$ [0, Max]$\subseteq$R and Max=highest possible rating. Each element of the variable "evaluation" can also be denoted by the triple [a, c, b] where $c=(a+b)/2$ and $\mu(x)=0$, if $x\leq a$, $\mu(c)=1$, $\mu(x)=0$ if $x\geq b$.

3 Fuzzy Modeling of an IMTS User

The fuzzy sets can be used to model the knowledge of a concept grasped by a generic user. In fact, by introducing the notions of linguistic variable and fuzzy variable one can:

–model the inherent vagueness about the knowledge of a concept,

– dynamically introduce new terms (fuzzy labels) of the linguistic variable, using suitable modifiers,

–achieve a rich descriptive structure of the user's cognitive, psychological and mental states,

–attain these results with limited computational efforts.

In fact, we propose that the user's cognitive state be represented by a string of the type:

$$a_n^{\alpha n}\ a_{n-1}^{\alpha n-1} \ldots a_1^{\alpha 1}$$

where the elements of the sets $\{a_i\}$ are the didactic goals related to a specific navigation within the IMTS, whereas the elements $\{\alpha_i\}$ are fuzzy variables, namely values of a linguistic variable.

Let $\{C_i\}$ be the set of concepts related to a part of the knowledge domain present in the IMTS and let $\{O_i\}$ be the set of goals. We say that mastery of the concept Ci is very good if the system, interacting with the user, generates the string $[O_{i1}, O_{i2}, , ,O_{in}]^{vg}$ where the label "vg" denotes the value "very good". In general, a string such as the following represents and singles out different learning levels for each goal related to a specific concept:

$$[O_{i1}, . . O_{ik}]^{\alpha n}\ [O_{i(k+1)}, \ldots O_{i(k+2)}, , .O_{ih}]^{\alpha n-1} \ldots [O_{i(h+1)}, \ldots, O_{in}]^{\alpha 1}$$

where $\{\alpha_n\}$ are the values of the variable evaluation.

Of course, the user representation can be enriched by taking into account other states, such as the psychological state. This state could be evaluated using the linguistic variables "attention" and "interest" associated with each node n_i visited by the user during the navigation within the IMTS. In this case the elements $\{a_i\}$ are the hypermedia nodes and the labels $\{\alpha_i\}$ are values of the linguistic variables. For example, a couple such as (node k, very interested) denotes that the user, during the visit of the node k, has been deeply involved.

User's navigation within the hypermedia network takes place by means of actions such as answers to questions, choice of a specific hypertextual link, deepening on a concept, request of an example, submission of a topic already examined, access to the data base. These actions can involve one or more nodes in the hypertext network and can be linked to a specific semantics, i.e. each action can be interpreted in order to get information about user's cognitive state. In such way the system can adapt its behavior to user's needs.

The following table summarizes the possible actions. To each element describing the model, are "attached" the functions and variables therein defined.

Cognitive state:
- -Right translation
- -Left transation
- -Comparison
- -Change in learning states
- -Distance between cognitive states

Psychological state:
- -Attention
- —Interest

Data base orientation:
- -Peak
- -Peak difference
- - Right translation percentage
- - Left translation percentage

Learning styles:
- -Average length of deepening path
- -Average duration of node visits
- -Average duration of a tutoring session

Once chosen the linguistic variable for evaluating user states, the table shows the translations that activate changes in the values of the linguistic variable. Each architectural element is linked with suitable functions and linguistic variables. Each linguistic variable and each function can be updated depending upon the type and strength of the evidence that appears in the user's action. It is worth emphasizing that several linguistic variables could be taken into account. However, for the sake of simplicity, we limit our investigation and computation to the variable "evaluation"

4 The Fuzzy Algebraic Structure

Let U = {a, b, c, d, ...} be the finite universe of discourse and let At = {A, B, C, D, ...} be the set of attributes used to classify the elements of U. We denote with $\alpha_1 \leq \alpha_2 \leq, \ldots, \leq \alpha_n$ the fuzzy numbers that represent the elements of the variable evaluation Vt. $a_{ij} = A_i^{-1}(\alpha_j)$ is the set of evaluated elements α_j with respect to the attribute A_i belonging to At. The string $a_n^{\alpha n}\ a_{n-1}^{\alpha n-1} \ldots a_1^{\alpha 1}$ is the classification of U with respect to the attribute A_i (Gisolfi, 1992, Gisolfi and Loia, 1995, Gisolfi and Cicalese, 1996, Gisolfi and Nunez, 1993, Di Lascio, Fischetti, Gisolfi 1999). The symbols a_i denote the *first parts* of the string, whereas α_n are the *second parts*.

Given the strings:

$A = a_n^{\alpha n}\, a_{n-1}^{\alpha n-1} \ldots a_1^{\alpha 1}$

and $B = b_m^{\beta m}\, b_{m-1}^{\beta m-1} \ldots b_1^{\beta 1}$,

with $n \geq m$, the operation Π, commutative and associative, associates the string C with A and B:

$$(a_n^{\alpha n}\, a_{n-1}^{\alpha n-1} \ldots a_1^{\alpha 1})\,\Pi$$

$$(b_m^{\beta m}\, b_{n-1}^{\beta m-1} \ldots b_1^{\beta 1}) =$$

$$= c_{m+n-1}^{\gamma m+n-1}\, c_{m+n-2}^{\gamma m+n-2} \ldots c_1^{\gamma 1}$$

The operation Π splits in two operations: the first is denoted by * and acts upon the first parts, the second with $\bullet$ and acts on the second ones. The traditional subsets c_i and the fuzzy numbers γ_i are obtained by applying the operators * and $\bullet$, respectively.

The operation for the first parts is defined as follows:

$$c_i = \begin{cases} \cup_{j=1..i}\ a_{i-j+1} \cap b_j, & \text{if } 1 \leq i \leq m-1 \\ \cup_{j=1...m}\ a_{i-j+1} \cap b_j, & \text{if } m \leq i \leq n-1 \\ \cup_{j=i-n+1...i}\ a_{i-j+1} \cap b_j, & \text{if } n \leq i \leq n+m-1 \end{cases}$$

where $\cup$ and $\cap$ are the usual set-theoretic operations defined on the power set P(U).

This operation has the following properties:

1. Closure
2. Commutativity
3. Associativity
4. Existence of the zero element

The operation for the second parts, in turn, is defined as follows:

$$(\alpha_n \alpha_{n-1} \ldots \alpha_1) \bullet (\beta_m \beta_{m-1} \ldots \beta_1) =$$
$$= \gamma_{m+n-1}\, \gamma_{m+n-2} \ldots \gamma_1$$

where:

$$\gamma_i = \begin{cases} 1/(k_1+k_2)\, d_i * \Sigma_{j=1...i}\ d2_j * d1_{i-j+1} * (k_1 * \alpha_{i-j+1} + k_2 * \beta_j), & 1 \leq i \leq m-1 \\ 1/(k_1+k_2)\, d_i * \Sigma_{j=1...m}\ d2_j * d1_{i-j+1} * (k_1 * \alpha_{i-j+1} + k_2 * \beta_j), & m \leq i \leq n-1 \\ 1/(k_1+k_2)\, d_i * \Sigma_{j=i-n+1...m.}\ d2_j * d1_{i-j+1} * (k_1 * \alpha_{i-j+1} + k_2 * \beta_j), & n \leq i \leq m+n-1 \end{cases}$$

and the values d_i are obtained as follows: $\Sigma_{j=1...i}\ d1_j * d2_{i-j+1}$.

In this operation the indices k_1 and k_2, respectively for A and B, and the arrays $d1_i$(i=1..n) and $d2_j$ (j=1..m), induce the distributivity of the operation $\cap$ with respect to $\cup$. The indices $d1_i$ and $d2_i$ represent, respectively for the first and the second string, the number of sets for which the union has been carried out in order to get the i-th class; the indices k_1 and k_2 denote the number of sets whose intersection has produced these sets.

The operation $\bullet$ is closed on the set of fuzzy numbers defined in [0,1], is associative, commutative and preserves the ordering among fuzzy numbers.

Example

Consider the following matrix (concepts, goals) that can be viewed as representative of the cognitive state of an IMTS user:

	C1	C2	C3
O1	g	i	q
O2	i	g	g
O3	i	q	q
O4	s	s	q
O5	s	s	s

We can write:

$C_1 = [O_1]^g[-]^q[O_4, O_5]^s[O_2,O_3]^i$,
$C_2 = [O_2]^g[O_3]^q[O_4, O_5]^s[O_1]^i$;
$C_3 = [O_2]^g[O_1,O_3,O_4]^q[O_5]^s[-]^i$.

Let U = [0, 1] and suppose that
g(ood) = [0.8, 1, 1],
q(uite good) = [0.5, 0.7, 0.9],
s(ufficient) = [0.2, 0.4, 0.6],
i(nsufficient) = [0, 0, 0.2].

Multiplying the first parts:

i) $C_1 * C_2$

			$[O_1]$	[-]	$[O_4,O_5]$	$[O_2,O_3]$
			$[O_2]$	$[O_3]$	$[O_4,O_5]$	$[O_1]$
			$[O_1]$	[-]	[-]	[-]
		[-]	[-]	$[O_4,O_5]$	[-]	
	[-]	[-]	[-]	$[O_3]$		
[-]	[-]	[-]	$[O_2]$			
[-]	[-]	[-]	$[O_1,O_2]$	$[O_3,O_4,O_5]$	[-]	[-]

ii) $C_1 * C_2 * C_3$

			[-]	[-]	[-]	[O_1,O_2]	[O_3,O_4,O_5]	[-]	[-]
						[O_2]	[O_1,O_3,O_4]	[O_5]	[-]
			[-]	[-]	[-]	[-]	[-]	[-]	[-]
		[-]	[-]	[-]	[-]	[O_5]	[-]	[-]	
	[-]	[-]	[-]	[O_1]	[O_3,O_4]	[-]	[-]		
[-]	[-]	[-]	[O_2]	[-]	[-]	[-]			
[-]	[-]	[-]	[O_2]	[O_1]	[O_3,O_4]	[O_5]	[-]	[-]	[-]

Thus $C_1 * C_2 * C_3 =$
$[-][-][-][[O_2][O_1][O_3,O_4][O_5][-][-][-]$

Multiplying the second parts:
i) $H = C_1 \bullet C_2 = (g, q, s, i) \bullet (g, q, s, i) =$
$(h_7, h_6, h_5, h_4, h_3, h_2, h_1)$
We note that: $d_{C1} = d_{C2} = (1, 1, 1, 1, 1, 1, 1)$;
$K_{C1} = K_{C2} = 1$;
$d_{H1} = 1, d_{H2} = 2, d_{H3} = 3; d_{H4} = 4;$
$d_{H5} = 3; d_{H6} = 2; d_{H7} = 1$
$K_H = 2$, because the string H represents the product of two strings:
$h_1 = 1/2\,(i+i) = [0, 0, 0.2]$
$h_2 = 1/4\,(i+s+i+s) = [0.1, 0.2, 0.4]$
$h_3 = 1/6\,(i+d+s+s+d+i) = [0.233, 0.366, 0.5]$
$h_4 = 1/8\,(i+b+s+d+d+s+b+i) =$
$= [0.375, 0.525, 0.625]$
$h_5 = 1/6\,(s+b+d+d+b+s) = [0.5, 0.7, 0.833]$
$h_6 = 1/4\,(d+b+b+d) = [0.65, 0.85, 0.95]$
$h_7 = 1/2\,(b+b) = [0.8, 1, 1]$.

Now we carry out the multiplication

$H \bullet (b, d, s, i) = (\delta_{10}, \delta_9, \delta_8, \delta_7, \delta_6, \delta_5, \delta_4, \delta_3, \delta_2, \delta_1)$ and considering the not empty first parts we get:

$(\delta_7, \delta_6, \delta_5, \delta_4) = ([0.510, 0.690, 0.810], [0.417, 0.583, 0.733]$
$[0.325, 0.475, 0.642]\,[0.240, 0.360, 0.540])$.

By applying the procedure for linguistic approximation described in [Gisolfi and Nunez, 1994], the fuzzy numbers can be replaced by the following labels:

[0.510, 0.690, 0.810] —> q
[0.417, 0.583, 0.733] —> IB(s,q), namely included between "s" and "q"
[0.325, 0.475, 0.642] —> NT(s), namely next to "s"
[0.240, 0.360, 0.540] —> s

The final result is $C_1 * C_2 * C_3 = = [O_2]^q [O_1]^{IB(s,q)} [O_3,O_4]^{NT(s)} [O_5]^s$.

5 From Fuzzy Scores to Linguistic Terms

Let $t[a_1, c, a_2]$ and $t_1[b_1, d, b_2]$ be two fuzzy sets whose membership functions are either type s or π. In both cases two parameters are sufficient to single out univocally the fuzzy set. The *center* of the shell of a fuzzy set t is the real number $m_t = 0.5*(a+b)$.
The function

$$\Omega(t[a_1, c, a_2], t_1[b_1, d, b_2]) =$$
$$= ((a_1 + a_2 - (b_1 + b_2)) / (a_2 - a_1 + b_2 - b_1)$$

is said *superposition grade* of two fuzzy sets t and t_1 [Hellendoorn, 1992]. It is worth noting that: if $\Omega = 0$, then the center of the shell of the fuzzy sets is identical; if $\Omega > 0$, then $m_t > m_{t1}$; if $\Omega < 0$, then $m_t < m_{t1}$. Moreover, if $\Omega \geq 1$, then we can adfirm that $m_t >> m_{t1}$, whereas if $\Omega \leq -1$, then $m_t << m_{t1}$. If t and t_1 are linguistic elements of the set Vt, then the previous relations can be translated into more or less favorable assessments. For example, $\Omega \geq 1$ means that the evaluation expressed by t is more favorable than that expressed by t_1.
Let U=[0, Max] be the interval of possible scores. A *fuzzy score* is a fuzzy number FS: U —> [0, 1] defined as follows:

$$FS(x) = \begin{cases} l(x) & \text{if } x \,\varepsilon\, [0, a] \\ 1 & \text{if } x \,\varepsilon\, [a, b] \\ r(x) & \text{if } x \,\varepsilon\, [b, Max] \end{cases}$$

where l(x) is a function defined in U and whose range is [0, 1], increasing, left continuous and such that l(x)=0 for $x \,\varepsilon\, [0, \lambda_1] \subseteq [0, a]$ whereas r(x) is also defined in U and ranges in [0, 1], but is decreasing, left continuous and such that r(x) = 0 for $x \,\varepsilon\, [\lambda_2, Max] \subseteq [b, Max]$. A class of functions satisfying these conditions are just the above mentioned functions S and π. In this way each element of Vt is the linguistic translation of a fuzzy score.
It is easy, given a fuzzy score, to find the corresponding term of Vt. Let p be a FS, if $|\Omega(p, t_0)| = \min_{t \varepsilon Vt} |\Omega(p, t)|$,

then t_0 is the term of the linguistic variable Vt corresponding to the fuzzy score.

Example

U = [0, 10];
Vt={insufficient, sufficient, quite good, good, very good};
insufficient = [0, 0, 10/5]
sufficient = [10/5, 0.5*(10/5+10/5*2), 10/5*2
quite good = [10/5*2, 0.5*(10/5*2+10/5*3), 10/5*3]
good = [10/5*3, 0.5*(10/5*3+10/5*4), 10/5*4]
very good = [10/5*4, 10, 10]

Given FS = p(u; 4; 6), one gets:
$|\Omega$(p, insufficient)$| = |$ (10-2)/(2+2) $|$
$|\Omega$(p, sufficient)$| = |$ (10-6)/(2+2) $|$
$|\Omega$(p, quite good)$| = |$ (10-10)/(2+2) $|$
$|\Omega$(p, good)$| = |$ (10-14)/(2+2) $|$
$|\Omega$(p, very good)$| = |$ (10-18)/(2+2) $|$
In conclusion, FS = quite good

If FS = p(u; 2; 3), one has:
$|\Omega$(p, insufficient)$| = |$ (5-2)/(1+2) $|$
$|\Omega$(p, sufficient)$| = |$ (5-6)/(1+2) $|$
$|\Omega$(p, quite good)$| = |$ (5-10)/(1+2) $|$
$|\Omega$(p, good)$| = |$ (5-14)/(1+2) $|$
$|\Omega$(p, very good)$| = |$ (5-18)/(1+2) $|$
and we get FS = sufficient

For the sake of simplicity, without loss of generality, we use for the fuzzy score just the functions S and π, in such way a fuzzy score can be expressed either giving directly S (or π) - fuzzy number or considering a set of couples (x, FS(x)). We denote this choice by PF_Z. For example, given the fuzzy set 0/0+0/20+0.1/40+0.5/50+0.6/60+0.9/80+1/100, if we take seven points and Max = 100, then we have an example of discretized fuzzy set. In the first case, in order to translate the fuzzy score FS_Z into a term of the variable Vt, it is sufficient to compute the minimum of the function Ω in Vt. In the second case, the couples of values can be interpolated, and the problem can be solved by applying the least squares method first to l(x) and then to r(x). These second grade polynomial functions allow to single out the Zadeh's function represented by them. In such way we are led to the first case. We can also improve the linguistic approximation by adding suitable linguistic modifiers to the set Vt.

6 The Assessment

The first column of the following table contains the goals related to a specific concept C:

							Evaluation
O_1	y_{11}	y_{12}	...	...	y_{1n}		E_1
...	...	...	...	...	...		
O_i	y_{i1}	...	...	...	y_{1in}		E_i
...	...	...	...	...	...		
O_n	y_{n1}	...	...	...	y_{mn}		E_m
							E

The IMTS expresses for each O_i the fuzzy score, where y_{ij} j belongs to the interval [0,1]. $s(x_i) = y_{ij}$ means that the assessment, which evaluates equal to x_i% the achievement of the goal, is assigned a credibility level equal to y_{ij} [3]. A possible range of values for x_i is the following: 0%, 20%, 40%, 50%, 60%, 80%, 100%. For the i-th row the set of couples (x_i, $\mu_{[0,1]}(x_i)=y_{ij}$) represents a fuzzy score (FS_Z) and thus this set complies with the above definition of fuzzy score (FS_Z). In turn, if the score is expressed by the fuzzy number [a,c,b] it means that we are sure that the goal cannot be achieved with a confidence level higher than b% and lower that a%, whereas this conviction for other values is expressed by the function l(x) and r(x) introduced in Section 5.

Let us consider the i-th row of the previous table. The values $\{y_{ij}\}$ single out either a fuzzy set t or a number S or π according to the choice made. It is possible to locate two values t_1 and t_2 of the linguistic variable Vt such that $t_1 \le t \le t_2$. They can be computed by means of the Hellendoorn's function Ω:
$t_2 = \min |\Omega(t, Vt)|$
$t_1 = \min |\Omega(t, V')|$, where $V' = Vt \backslash \{t_2\}$.

However, in general t is different from t_2. In this case the above mentioned procedure of linguistic approximation [12] allows to single out the linguistic modifier and consequently the value of the variable Vt related to each row O_i in the table. These values are reported in the last column as E_i. By using a larger subset of Vt the problem of finding the element of Vt which best represents the linguistic translation of the values y_{ij} can be tackled. The overall evaluation can be computed as follows:
$E = E_1 \bullet E_2 \bullet \ldots \bullet E_i \bullet \ldots \bullet E_m$.

A linguistic term gets associated with each fuzzy number E. However it is also possible to state an overall assessment about the user. In fact, let us denote a generic user with A_k. For each table k we get the following string:
$A_k = [O_{i1}, \ldots, O_{ih}]^{Ej1} \ldots [O_{i1}, \ldots, O_{ih}]^{Ejm}$.
The product $A = \Pi A_k$ gives an overall evaluation of the user described by a table of the kind reported above.

7 Monitoring Learning Process

Let {C1, C2, ..., Cn} be the set of concepts that can be grasped by the user during a tutoring session and suppose that the system builds for each concept C_j a string whose first parts satisfy the conditions $O_i = Cj^{-1}(\alpha_i)$, where the quantities α_i are the values of the variable "evaluation" as defined in Section 3. In such way we have in different moments different matrices (concept, goal). The set of such matrices describes the temporal evolution of the user's learning process. At time $t = t_k$ the representation of the cognitive state is given by the k-th matrix:

	O_1	O_2	.....	O_h
C_1	α_{11}	α_{12}	.....	α_{1h}
C_2	α_{21}	α_{22}	.....	α_{2h}
.....			.....	
C_n	α_{n1}	α_{n2}	.....	α_{nh}

For each matrix it is possible to carry out the products among rows:

$C(t=t_1)=C_1(t=t_1)\,\Pi\,....\Pi\, C_n(t=t_1)$, matrix 1;
$C(t=t_2)=C_1(t=t_2)\,\Pi\, ...\,\Pi\, C_n(t=t_2)$, matrix 2;

$C(t=t_k)=C_1(t=t_k)\,\Pi\, ...\,\Pi\, C_n(t=t_k)$, matrix k;

Each product gives a classification, i.e. an overall evaluation referred to the instant the product is carried out. In turn, the product

$$C = \Pi_i\, C(t=t_i)$$

gives rise to a string that represents the overall cognitive state of the user.

By multiplying rows belonging to different matrices, one can get information about cognitive levels for each concept, "on the fly" during the tutoring session or at the end of several sessions:

$C^1 = C_1(t=t_1)\,\Pi\, ...\,\Pi\, C_1(t=t_k)$;
$C^2 = C_2(t=t_1)\,\Pi\, ...\,\Pi\, C_2(t=t_k)$;

$C^n = C_n(t=t_1)\,\Pi\, ...\,\Pi\, C_n(t=t_k)$;

Every C^i represents the user's learning level of the concept C_i. Recalling that each string is built at different instants, the product represents the final state of the cognitive evolution for a specific concept. While the previous string C gives the final evaluation for each didactic goal beginning from the representation of the conceptualization level achieved, the new product:

$$C^+ = \Pi_i\, C^i$$

emphasizes the temporal evolution of the learning process.

It is also possible to build strings relating goals and concepts. We note that in mastery learning the tutoring strategy (Block, 1971) aims at giving students mastery about the basic elements of knowledge. By linking knowledge with concepts and abilities with goals we have a string in which $C_i = Oj^{-1}(\alpha_i)$, evaluates the mastery level of each ability related to the concepts. Thus a string such as the following:

$$O = [C_{i1}, ..., C_{ik}]^{\alpha n}[C_{i(k+1)}, ..., C_{i(k+2)}, ..., C_{ih}]^{\alpha n-1} ... [C_{i(h+1)}, ..., C_{in}]^{\alpha 1}$$

adfirms that the concepts $[C_{i1},,, C_{ik}]$, expressed by mastery O, have been mastered with grade α_n. These string gives us a computable schema to tackle the problem. For example, the ability to solve a polynomial equation is related both to the concept of zero of a polynomial and to the notion of algebraic structure on which depends the solubility of an equation. A string such as the previous one offers information about the way learning levels of these concepts contribute to the overall ability. Thus by multiplying the columns belonging to the same matrix one gets the following string:

$O(t=t_1)=O_1(t=t_1)\,\Pi\, ...\,\Pi\, O_n(t=t_1)$, matrix 1;
$O(t=t_2)=O_1(t=t_2)\,\Pi\, ...\,\Pi\, O_n(t=t_2)$, matrix 2;

$O(t=t_k)=O_1(t=t_k)\,\Pi\, ...\,\Pi\, O_n(t=t_k)$, matrix k;

and thus $O = \Pi_i\, O(t=t_i)$, and a final label expressing the contribution of each concept to the overall mastery is attached to each concept.

Moreover the product of columns belonging to different matrices:

$O^1 = O_1(t=t_1)\,\Pi\, ...\,\Pi\, O_1(t=t_k)$;
$O^2 = O_2(t=t_1)\,\Pi\, ...\,\Pi\, O_2(t=t_k)$;

$O^n = O_n(t=t_k)\,\Pi\, ...\,\Pi\, O_n(t=t_k)$;

expresses the overall contribution, in the interval $[t_1, t_1]$, of each concept to the goal taken into account. Thus in different strings the same concept can appear with different labels. The final product:

$$O^+ = \Pi_i\, O^i$$

permits to attach, at the end of the tutoring sessions, one assessment label to each concept.

It is worth noting that we have obtained a suitable representation of the learning process during the interaction user-system. In fact the temporal evolution of the process is described by the set of matrices, the strings represent the overall cognitive state of the user, information about cognitive levels for each concept are obtainable by multiplying rows, the strings can offer information about the way learning levels of the concepts contribute to the overall ability.

8 Concluding Remarks

Our approach to user modeling for IMTSs allows to represent cognitive states, learning styles, psychological states, mastery levels. The fuzzy strings that are our basic elements are able to express concepts and goals presented in the taxonomies well known in the literature [De Landesheere, 1990]. The algebraic structure dealing with these strings allows to monitor and evaluate all the elements present in the learning process of an IMTS user and gives us a fuzzy calculus on a set of words.

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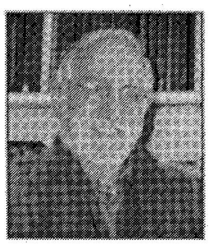

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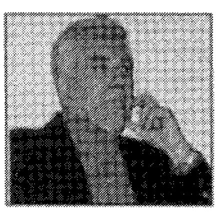

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