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ARTÍCULO

THE JOINT DISTRIBUTION OF INCOME AND WEALTH IN URUGUAY

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Sanroman, G., & Santos, G. (2021). The joint distribution of income and wealth in Uruguay. *Cuadernos de Economía, 40*(83), 609-642.

We analyse the joint distribution of income and wealth in Uruguay and compare it to that of Chile, Spain, and the U.S., using data from Surveys of Household Finances. We analyse income and wealth separately and find that wealth is more concentrated and asymmetric than income. We provide a non-parametric estimation of copulas for income and wealth. It reveals that high-income households are among the wealthiest while low-income households are at the bottom of the wealth distribution. When assessing the sources of income and wealth heterogeneity for Uruguay, we found that education strongly influences income, wealth, and joint distribution.

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Keywords: Income; wealth; inequality; copula; non-parametric estimation.
JEL: C4, C31, D31.

Sanroman, G., & Santos, G. (2021). La distribución conjunta de la renta y riqueza en Uruguay. *Cuadernos de Economía, 40*(83), 609-642.

Este artículo analiza la distribución conjunta del ingreso-riqueza en Uruguay y la compara con Chile, España y Estados Unidos, por medio de encuestas financieras de hogares. Se analizan las distribuciones univariantes de ingreso y riqueza, y se encuentra que la última es más concentrada y asimétrica que la primera. Se estiman cópulas no paramétricas para la distribución conjunta que revelan que los hogares de altos ingresos se encuentran entre los de mayor riqueza y que los de menores ingresos entre los de menor riqueza. Se descubre que, en Uruguay, la educación es la principal fuente de heterogeneidad del ingreso, la riqueza y la distribución conjunta.

Palabras clave: ingreso; riqueza; desigualdad; cópula; estimación no paramétrica.
JEL: C4, C31, D31.

Sanroman, G., & Santos, G. (2021). A distribuição conjunta de renda e riqueza no Uruguai. *Cuadernos de Economía, 40*(83), 609-642.

Este artigo analisa a distribuição conjunta de renda-riqueza no Uruguai e a compara com Chile, Espanha e Estados Unidos, por meio de pesquisas financeiras domiciliares. As distribuições univariadas de renda e riqueza são analisadas, e a última se mostra mais concentrada e assimétrica do que a primeira. Cópulas não paramétricas são estimadas para a distribuição conjunta que revelam que as famílias de alta renda estão entre aquelas com maior riqueza e que aquelas com renda mais baixa estão entre aquelas com menos riqueza. Descobriu-se que, no Uruguai, a educação é a principal fonte de heterogeneidade de renda, riqueza e distribuição conjunta.

Palavras-chave: renda; fortuna; desigualdade; cópula; estimativa não paramétrica.
JEL: C4, C31, D31.

INTRODUCTION

Measuring wealth and income distribution while disentangling the dependence between both variables has not proven to be an easy task. A growing literature concerned with income inequality has pointed out the importance of measuring wealth distribution, not only to address welfare, but also to find explanations for the recent surge of income inequality observed in many economies (Piketty & Zucman, 2014; Saez & Zucman, 2016). While the dependence between income and wealth has been addressed in developed economies, in Cowell et al. (2017) and Jäntti et al. (2015), the evidence for developing countries is scarce. In this paper, we aim to fill this gap by analysing the joint distribution of income and wealth in Uruguay, a small Latin American economy currently ranking as one of the most equal in the continent according to income distribution indicators (CEPAL, 2019). We use survey data from the first wave of Encuesta Financiera de los Hogares Uruguayos (EFHU), which collects detailed, household financial, and economic information. We focus on Uruguay, but throughout most of the paper we compare Uruguayan data with that from Chile, Spain, and the U.S. The reason behind this is because these countries conduct surveys which have served as a framework for designing EFHU in several aspects such as sample design, questionnaire, treatment of non-response, among others. For Chile, we use the 2014 wave of the Encuesta Financiera de los Hogares (EFH). For Spain we use the 2011 wave of the Encuesta Financiera de las Familias (EFF). Finally, for the U.S. we use data from the 2013 Survey of Consumer Finances (SCF).

We start by analysing the univariate distribution of income and wealth. In Uruguay, wealth is much more concentrated than income, as seen in other countries (Jäntti et al., 2015; Kennickell, 2009). At the same time, income is less concentrated than in Chile and the U.S. but more concentrated than in Spain. Wealth, in turn, is less concentrated than in the U.S. and its distribution is similar to the one for Chile. To analyse the joint distribution of income and wealth, we construct empirical smoothed kernel copulas, as in Kennickell (2009) or Jäntti et al. (2015). However, instead of estimating parametric copulas, we estimate copulas which are model free. Results for Uruguay are in line with Kennickell (2009) and Jäntti et al. (2015); top income households are among the wealthiest, while low income households are likely to be more frequent among the poorest. Such dependence is stronger at the top of the joint distribution. To assess the Uruguayan copula, we apply a non-parametric test of equality between copulas proposed by Rémillard and Scaillet (2009). The test allows the full dependence of both variables to be compared across countries without relying on a parameter like the Pearson or Spearman correlation. Data only supports the hypothesis of equality between copulas for the Spain-Uruguay pair.

Finally, we analyse the main household determinants of the joint distribution of income and wealth in Uruguay. We firstly provide mean regression estimates for income and wealth using education, family structure, age structure, inheritances, and region of residence as covariates. Education is the main source of heterogeneity for income and, although it also influences wealth, inheritances have the highest explanatory power for the latter. Family and age structure and region of residence also play a role on both variables. To address to which extent those covariates shape the joint distribution of income and wealth, we build copulas using the residuals from previously estimated mean regressions for the marginal distributions. Then, we test whether this copula is statistically different from the original copula built using observed variables. We find that education is the most relevant variable shaping the joint distribution of income and wealth. However, most of the variation in wealth and its dependence with income is not explained by the previous set of covariates.

Previous evidence for Uruguay is scarce. As part of a project aimed at measuring global wealth, Davies et al. (2017) estimate Uruguayan wealth distribution using EFHU. However, this work is not focused on the Uruguayan case. De Rosa (2019) estimates wealth distribution for Uruguay leveraging the literature on the capitalization method, such as Atkinson and Harrison (1978) or Saez and Zucman (2016) who analyse the UK and the U.S., respectively. In this paper, we extend the works studying wealth distribution in Uruguay by analysing its joint distribution with income. We do not apply the capitalization method; instead, we rely on survey data. Despite the literature having recognized the limitation of surveys to capture the wealthiest households as in Saez and Zucman (2016) or Vermeulen (2018), EFHU allows us to link income and wealth with socio-demographic characteristics, otherwise unavailable for Uruguay at this stage of research.

In searching for income and wealth determinants, Arrondel et al. (2014) analyse the relationship between household characteristics and the joint distribution of wealth and income for EU countries. In a related work, Martinez and Uribe (2017) perform a similar analysis but using data for Chile. Both find that family structure, inheritances, and household income increase the probability of being in higher wealth deciles.

We contribute to the literature by measuring the joint distribution of wealth and income in Uruguay, a small developing economy, using unique dataset. We provide estimations of non-parametric smoothed kernel copulas for income and wealth and identify the household characteristics that shape the dependence between both variables. We extend the literature by analysing the link between income and wealth in developed countries and developing economies (Cowell et al., 2017; Jäntti et al, 2015; among others). Emerging economies show limited access to credit, higher income volatility, and levels of income inequality; all these factors may contribute differently to the link between income and wealth.

DATA

We use data from household finance and wealth surveys for each country. For Uruguay we use data from the Encuesta Financiera de los Hogares Uruguayos

(EFHU) collected during 2013 and 2014 (Table 1). For Spain, we use data from the Encuesta Financiera de las Familias (EFF) collected in 2011, while for the U.S. we use the 2013 wave of the Survey of Consumer Finances (SCF). For Chile, data comes from the 2014 wave of the Encuesta Financiera de los Hogares (EFH).

The SCF was conducted by the University of Chicago as a cross-sectional survey on a triennial basis since the 1980s.[1] The EFF has been conducted by Banco de España every three years since 2002. The EFH of Banco de Chile is a panel dataset, its first wave was collected in 2007. Finally, the Uruguayan EFHU (conducted by dECON-UDELAR and sponsored by Banco Central del Uruguay and Ministerio de Economía) is from 2013-2014 and has only one wave. Both the EFH and the EFHU are similar to the SCF and the EFF. They were designed using the same technical features including questionnaire, sample design, type of interview, selection of the family member to be interviewed and the methods used to deal with non-response.

Consequently, there are important parallels among these surveys that allow us to use them as a framework for analysing Uruguayan data. Firstly, these surveys collect similar information on household assets, liabilities, income, expenditure and socio-economic data on household members. Secondly, they all use stochastic multiple imputation methods to deal with the non-response bias, a major characteristic of household financial surveys.

All these surveys oversample high income/wealth households, which accounts for the fact that some assets are held by a small fraction of the population (Kennickell, 2005, 2007). The SCF, EFH, and EFF follow a dual sample design based on tax records to oversample wealthy households. The EFHU, in turn, oversamples households at the top 20% of income distribution according to the ECH (Encuesta Continua de Hogares), an annually based cross-sectional survey capturing main household information such as living conditions, employment status and income profile.[2] Concerning sample sizes, EFF and SCF survey around 6,000 households, the size of the EFH is 4,500, and EFHU surveys 3,500 (Table 1).

Despite these similarities, each survey has its own unique characteristics. On the one hand, in Uruguay and Chile income data is collected after taxes, while in the U.S. and Spain, it is collected before taxes. For the U.S., data on payroll and federal taxes were computed and removed using TAXSIM programme.[3] [4] This was not possible for Spain, which posed a considerable restriction for the analysis. EFH does not collect information on wealth from business activity, which imposed a limitation since firm ownership has been recognized as a major determinant of the

[1] The SCF is sponsored by the U.S. Federal Reserve Board in cooperation with the Treasury Department.

[2] See Ferre et al. (2016) for a detailed description of EFHU.

[3] TAXSIM is a NBER program which computes households federal, state, and payroll taxes in different surveys. State taxes cannot be deduced because geographical information in the SCF is not publicly disclosed.

[4] See http://users.nber.org/taxsim/

Table 1.
Survey Description

	EFF	EFHU	EFH	SCF
Country	Spain	Uruguay	Chile	U.S.
Year	2011	2013-2014	2014	2013
Observations	6,106	3,490	4,502	6,015
Unit of analysis	Household	Household	Household	Household
Number of imputed datasets	5	10	30	5

right tail of the wealth distribution (Cagetti & De Nardi, 2008). Therefore, to compare data between Uruguay and Chile, we provide some indicators on Uruguayan wealth with and without business.

HOUSEHOLD BALANCE SHEETS

We focus on Uruguayan data, but also provide some cross-country similarities/ differences. We perform a detailed comparison between Uruguayan and the U.S., Spanish, and Chilean case. We also use information from Badarinza et al. (2016) as well as data contained in the Household Finances Survey and Consumption Network (HFCS) collected by the European Central Bank (ECB, 2013). The former analyses household balance sheets for 13 developed economies. The latter is a dataset including financial household-level information for the Eurozone.

We define assets as the sum of financial and non-financial assets. Financial assets include deposits, transactions accounts, bonds, stocks, retirement funds, and mutual funds. For Uruguay, we do not include wealth from pensions. A large portion of pensions are provided by the State as a pay-as-you-go system with compulsory taxes and are not collected by the survey. Despite the survey collecting data on private pension funds, non-response rates were high enough to prevent reliable estimations from being obtained. Arguably, we include pensions for the U.S. because households voluntarily choose a pension plan, and we aim for a wide measure of household wealth. Non-financial assets comprise the main residence, other real estate, business ownership, vehicles, and other valuables such as jewellery or art. Liabilities include all outstanding debt owned by households, such as debts for purchasing the main residence and other real estate (mortgage and non-mortgage credit, debt arising from the purchase of durable and non-durable goods, financial loans, and credit card outstanding balances.

Table 2 reports participation rates for assets and liabilities computed as the percentage of households owning each asset/debt. The participation rate for the main residence in Uruguay is akin to one for Chile and the U.S. Meanwhile, the participation rate for the main residence in Spain is much higher, around 83%.

Table 2.
Participation Rates for Household Assets and Debts (% of Households)

	Uruguay	Chile	Spain	U.S.
Financial Assets	**48.9**	**82.2**	**95.8**	**94.5**
Non-Financial Assets	**85.2**	**79.0**	**96.2**	**91.3**
Main residence	61.7	61.9	83.1	65.2
Other real estate	12.7	13.3	40.2	13.3
Own business	20.9	-.-	12.3	11.7
Vehicles	56.9	50.3	78.4	86.3
Art, jewellery, other	3.6	1.4	22.6	7.3
Liabilities	**44.5**	**72.6**	**49.3**	**74.5**
Main residence	8.0	17.0	26.6	44.5
Other real estate	1.2	3.5	9.5	5.3
Credit card	9.0	54.4	5.9	38.1
Consumption, vehicles	36.5	33.7	21.8	38.1
Education loans	-.-	8.2	-.-	20.0
Other debt	-.-	4.8	3.8	6.6

Notes: Participation rate is computed as percentage of households owning each item. All the imputation sets for each survey as well as sample weights were used.

Data for Uruguay in Table 3 indicates that the main residence accounts for 55% of total assets, a figure close to the estimated for Chile. According to Badarinza et al. (2016), the main residence is the most valuable asset held by households, accounting for at least half of total household assets in countries such as Greece, Italy, Slovakia, Slovenia, and Spain. On average, real assets represent 85% of total gross assets in Europe (Cowell & Van Kerm, 2015), a figure close to that of Uruguay and Chile.

The participation rate for business wealth is substantially larger in Uruguay than in Spain and the U.S., probably due to the inclusion of self-employees and small-business owners in the definition of business. As the informal economy is larger in Uruguay than in developed economies, self-employment may account for a larger fraction of the workforce. Unfortunately, the EFH does not collect information on this item. According to the HFCS, on average 11% of EU households own a business. However, in Italy and Spain these figures were 15% (2010) and 18.4%, respectively, closer to what is found in Uruguay.

Financial assets show a substantially lower participation in Uruguay (49%) than in Chile (82%) and the developed countries analysed. Financial markets in emerging economies, such as Uruguay or Chile, are in an early stage of development.

In Uruguay there are almost no equity markets and the banking system is a stable oligopoly where active interest rates are comparatively very high and passive rates are very low, especially for family loans.

In terms of liabilities, almost 50% of Uruguayan households are indebted, a similar figure to that of Spain and lower than in Chile and the U.S. (75%). The proportion of indebted households in Uruguay is close to the average of the Euro Area, 43% in 2009-2010 (HFCS). Uruguay lacks an 'education' credit market like the ones in the U.S. or Chile, where school debt is significant. Nearly one third of Uruguayan households have consumer debt; this figure is similar to Chile and the U.S., and higher than Spain.

The participation rate for mortgage debt is substantially lower in Uruguay (8%) than in the other countries analysed, while housing tenure ranks in the middle. This fact poses a question regarding alternative funding mechanisms to acquire the main residence. For instance, nearly 19% of Uruguayan households inherit the asset, while 3% received it as a gift.

Table 3 reports the allocation of household liabilities. Debt for purchasing the main residence is the most valuable debt, followed by debt for consumption purposes. Another interesting finding is that consumer debt amounts to almost 40% of total

Table 3.
Allocation of Household Assets and Liabilities (% of Total Assets/Liabilities)

	Uruguay	Chile	Spain	U.S.
Financial Assets	**4.5**	**9.2**	**15.1**	**40.8**
Non-Financial Assets	**95.5**	**90.7**	**84.8**	**59.2**
Main residence	55.2	63.4	49.6	27.6
Other real estate	23.4	19.7	24.2	6.8
Own business	12.2	-.-	7.8	20.8
Vehicles	4.5	6.8	2.4	3.1
Art, jewellery, other	0.2	0.8	0.8	0.7
Liabilities				
Main residence	52.3	58.4	62.4	73.7
Other real estate	9.2	15.2	24.3	8.9
Credit card	0.7	8.7	0.2	2.4
Consumption, vehicles	37.6	12.8	11.3	7.5
Education loans	-.-	3.6	-.-	6.3
Other debt	-.-	1.3	1.6	1.0

Note: Each item share is computed as the proportion of each item in total assets/debt value. All the imputation sets for each survey as well as sample weights were used.

liabilities in Uruguay, while this figure sits at around 10% in the rest of the countries analysed.

INCOME AND WEALTH: A UNIVARIATE ANALYSIS

In this section, we separately analyse the univariate distribution of income and wealth for Uruguay and compare them to those of Spain, Chile and the U.S.. We report descriptive statistics for the distribution of both variables and provide commonly used indicators to assess wealth and income inequality.

We consider household net worth as a measure for net wealth. The variable is constructed as the difference between the value of total assets and liabilities defined in the previous section. Since we aim to measure the joint distribution of income and wealth with no explicit theoretical link between them, we define the variables as broadly as possible.[5] For income, we capture income from all concepts. For wealth we include different items such as real estate, financial wealth, wealth from business, vehicles, jewellery, art, etc. We use household as a unit of analysis, since the survey does not collect personal data no data on the 'legal ownership' of assets and debts. To measure household income, we consider the sum of all revenues retrieved by the household. Income is considered after taxes in all cases, except for Spain for which we discuss the potential effects of taxes when referring to inequality measures. EFH and EFHU collect information about after-tax income directly, while the SCF and the EFF collect income before taxes. For the U.S., we obtain a measure for after tax income by using TAXSIM program.

Table 4 reports the main descriptive statistics for wealth distribution. In Uruguay, about 80% of households have positive net wealth. The figure is similar to Chile and lower than Spain or the U.S., where it is close to 90%. The proportion of households holding negative net wealth is similar in Uruguay, Chile and the U.S., but it almost doubles in Spain. The proportion of "hand to mouth" consumers is larger in Uruguay and Chile (the developing economies). Panel b of Table 4 shows that in Uruguay mean wealth is approximately USD 90,000, median wealth is close to USD 35,000, the 10th percentile is USD -357, and the 75th and 90th percentiles are USD 88,704 and USD 186,332, respectively. Uruguayan figures are closer to Chile than to the other countries.[6] In panel c of Table 4, we compute the

[5] The definition for income and wealth can be different depending on the objectives of the analysis. For instance, if one attempts to address consumption smoothing, measuring 'liquid and non-liquid assets' could be more effective. Other definitions of wealth and income could be also led by different objectives, such as measuring the joint distribution over the life-cycle or precautionary savings motives.

[6] Since EFH does not collect information on business, we also compute the statistics for Uruguay removing wealth from business. In that case, the mean wealth in Uruguay is USD 78,615, while the 75th and the 90th percentiles are USD 86,258 and USD 177,168 respectively. Those figures are even closer to Chilean data.

Table 4.
Net Wealth -Main Descriptive Statistics

	Uruguay	Chile	Spain	U.S.
W>0	0.79	0.78	0.94	0.87
W=0	0.10	0.07	0.01	0.01
W<0	0.12	0.15	0.05	0.12
Mean	90,417	75,104	369,093	536,876
10th percentile	-357	-559	8,549	-2,099
25th percentile	857	2,498	93,205	8,924
50th percentile	35,534	31,298	215,011	82,759
75th percentile	88,704	75,993	414,357	320,763
90th percentile	186,332	172,532	732,717	958,754
p75th/p25th	103.00	30.4	4.45	35.9
Mean/Median	2.54	2.39	1.72	6.5

Note: The first panel corresponds to the number of households in each category as percentage of the total. In the second panel, figures are in 2014 U.S.D. All the imputation sets for each survey as well as sample weights were used.

"mean/median" and the 75^{th}/25th ratios as measures of dispersion for wealth distribution. The first indicator for Uruguay is approximately 2.5 similar to Chile (2.4), lower than the U.S. (6.5) and higher than Spain (1.7). The 75^{th}/25th ratio is remarkably higher in Uruguay (close to 100) than in the other countries. This might be due to the 25th percentile being considerably lower in Uruguay than in the other economies.

Annual mean income in Uruguay is close to USD 19,000, the lowest value among the analysed countries (Table 5). Despite income dispersion being lower in Uruguay, income for Spain is before taxes. To the extent that taxes have an equalizer effect on income distribution, inequality measures considering after tax income for Spain may indeed indicate a less unequal income distribution.

In Figure 1, we provide estimations for the marginal distribution of income (grey) and wealth (light grey). The variables are scaled by an inverse hyperbolic sine, a transformation that deals relatively well with negative and zero values while preserving data properties (Burbidge et al., 1988). We observe the same patterns for Uruguay that have been described before for developed economies (Cowell et al., 2017); Jäntti et al., 2015). Wealth distribution is asymmetric and bimodal, with a first mode at zero, and its dispersion is remarkably larger than for income. Kernel density estimates show that wealth inequality is substantially larger than income inequality in all analysed countries, particularly in the U.S.

Table 5.
Income - Main Descriptive Statistics

	Uruguay	Chile	Spain	U.S.
Mean	18,703	27,031	46,527	69,825
10th percentile	5,004	4,841	11,572	13,577
25th percentile	8,400	8,734	18,758	23,845
50th percentile	14,400	16,041	34,161	40,976
75th percentile	24,000	30,084	57,698	72,692
90th percentile	36,078	56,098	88,078	120,881
p75th/p25th	2.86	3.4	3.08	3.1
Mean/Median	1.30	1.69	1.36	1.7

Note: Figures are in 2014 U.S.D. After tax income is considered in all cases expect for Spain. All the imputation sets for each survey as well as sample weights were used.

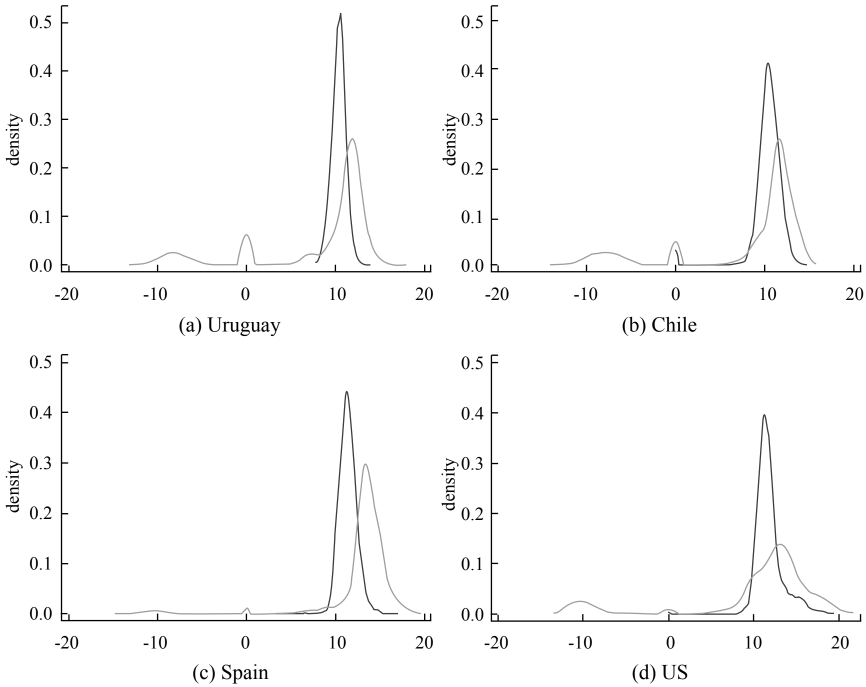
To further analyse wealth and income inequality we compute the Gini coefficient, shown in Table 6, and construct Lorenz curves as depicted in Figure 2. We first consider total household income and wealth and in a second step per capita income and wealth. The latter considers household size, which is important when measuring inequality in developing economies because numerous households could be more frequent among the poorest. While we compute the Gini coefficient in two ways, Lorenz curves are constructed considering total household income only to simplify the comparison with previous literature. In Uruguay, the Gini coefficient for income is 0.42 when taking the household as a unit of measure and 0.46 when income is considered per capita. Figures for Uruguay are similar to Spain and much lower than for Chile and the U.S. As previously mentioned, income for Spain is collected before taxes. When considering the Spanish scheme for taxes

Table 6.
Gini Coefficient: Income and Wealth

	Uruguay	Chile	Spain	U.S.	Uruguay	Chile	Spain	U.S.
	Wealth				Income			
Gini	0.75	0.74	0.60	0.85	0.42	0.53	0.44	0.53
Gini (capita)	0.77	0.79	0.62	0.86	0.46	0.56	0.42	0.53

Note: After tax income is considered in all cases except for Spain. All the imputation sets for each survey as well as sample weights were used

Figure 1.
Kernel Densities For Income and Wealth

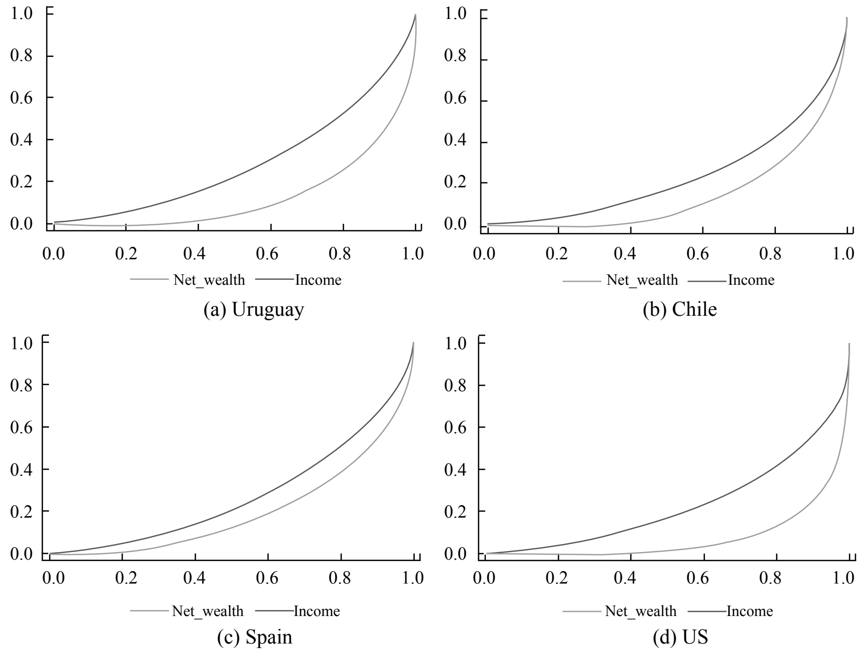


Note: x-scale is scaled by an inverse hyperbolic sine transformation.

and transfers, the literature found lower values for the Gini coefficient. Indeed, Anghel et al. (2018) compare the Gini coefficient for household income before and after personal taxes using data from the 2008 and 2014 EFF waves and conclude that the tax scheme in Spain reduces income inequality. Similar results have been obtained using other datasets and inequality indicators (see OECD, 2012). While the latter poses some obstacles to perform cross-country comparisons, the focus of the paper is to assess the joint distribution of income and wealth in Uruguay, and we provide data for other countries as a framework for assessing Uruguayan data.

There are some differences in the estimation of the Gini coefficients for the U.S. and Spain with respect to other datasets. For the U.S., the index is higher than the one computed using Census or OECD data, which may be due to the ability of the SCF to capture the top 1% of income distribution and revenues from capital (Guner et al., 2014). Census data is top censored, which may lead to lower estimates of the Gini coefficient in comparison to data from the SCF (Burkhauser et al. 2011; Guner et al., 2014). For Spain, the Gini coefficient is higher when using data from the EFF

Figure 2.
Lorenz Curves For Income and Wealth



Note: All the imputation sets for each survey as well as sample weights were used.

in comparison to other data such as the 'Encuesta de Condiciones de Vida' (ECV), which was most commonly used to measure income distribution. In that survey, the Gini coefficient was 0.33 in 2011. Similarly to the U.S., differences could be due to survey design, mainly because the EFF is designed to capture the wealthiest households, which are also higher income households. This characteristic is absent in the ECV.

When considering wealth, the Gini coefficient is larger than income in all countries analysed. For Uruguay, the shape of the figure remains between that of the U.S. and Spain.[7] When analysing Lorenz curves, Uruguay exhibits the same pattern as all other countries, income seems to be much less concentrated than wealth. This result is also observed in 13 of the 15 European countries analysed in Cowell and Van Kerm (2015).

Finally, we compute the concentration ratios, which are defined as the proportion of income and wealth held by percentiles of the population (Table 7). Wealth distribution in Uruguay is similar to Chile, lower than the U.S. and higher than

[7] The Gini coefficient is well defined despite wealth taking negative values. (Chen et al., 1982)

Table 7.
Concentration Ratios (% of Income and Wealth)

	Uruguay	Chile	Spain	U.S.	Uruguay	Chile	Spain	U.S.
Percentiles	Wealth				Income			
Bottom 50	3.6	3.5	12.0	1.1	21.9	15.9	20.6	16.9
Middle 40	37.3	38.8	42.4	23.9	47.3	42.4	46.3	39.1
Top 10	59.1	57.6	45.5	75.0	30.7	41.6	33.1	44.0

Note: After tax income is considered in all cases except for Spain. All the imputation sets for each survey as well as sample weights were used.

Spain. In Uruguay, the 10^{th} percentile at the top of the distribution holds nearly 59% of total wealth. The bottom 50% holds less than 10% in all countries except in Spain. In Uruguay, that figure is similar to Chile, but marginally higher than the U.S. The EFHU sample size (3,490 observations) prevents us providing concentration ratios for the top 1% or 0.1%. Unlike the other surveys which oversample according to estate tax records, EFHU oversamples the top 20% of income distribution, which may not be necessary the wealthiest due to the presence of "wealthy hand to mouth" households. This phenomenon is, partially, what we are trying to address.

De Rosa (2019) computes concentration ratios for Uruguay using the capitalization method, indicating that 63% of total wealth is owned by the top 10%, 31% by the middle 40%, and 5% by the bottom 50%. These results are surprisingly aligned with the survey, if one accepts that surveys are generally unable to capture wealthiest households.[8] [9] According to De Rosa (2019), results are similar in the survey and the capitalization method when considering different categories of assets, such as business, financial, and real estate wealth.[10]

THE JOINT DISTRIBUTION OF INCOME AND WEALTH: METHODS

This section presents the different approaches used in the paper to address the joint distribution of income and wealth. We first review single indices to measure the correlation between both variables, such as the Pearson and Spearman indices. Then we describe the estimations of empirical copulas and the non-parametric test by Rémillard and Scaillet (2009) to test the hypothesis of equality between copulas.

[8] Davies et al. (2017) also computes wealth concentration ratios for Uruguay using EFHU; ratios for income are not computed.

[9] See Vermeulen (2018) for a discussion

[10] De Rosa (2019) performs a detailed comparison between the two methods.

The Pearson index measures the linear correlation between both variables using cardinal data, while the Spearman index exploits ordinal information and evaluates the association between individual rankings within the distributions of income and wealth. As Jäntti et al. (2015) point out, an advantage of the Spearman index is that it is less sensitive to outliers, which may exert a strong influence on Pearson correlations.

Although single indices are useful to summarize wealth and income correlations, they may be unable to capture the complexity of the relationship between the two. Aiming to explore the full dependence between both variables, we construct copulas for the joint distribution of income and wealth, similarly to Kennickell (2009) or Jäntti et al. (2015). We estimate kernel smoothing copulas, an alternative to the purely empirical approach of Kennickell (2009), and to the fully parametric approach of Jäntti et al (2015). Subsequently, we perform the non-parametric test proposed by Rémillard and Scaillet (2009) for the hypothesis of equality between copulas of each pair of countries.

NON-PARAMETRIC ESTIMATION OF COPULAS

A copula is a joint distribution with uniform margins that allows the full dependence structure between random vectors (Chen & Huang, 2007) to be observed. The estimation of copulas is not the only way to assess the joint distribution of two variables and their degree of dependence. However, they enable the analysis of the full dependence structure, which may not be adequately captured by single summary statistics (Jäntti et al. 2015). This could be the case of income and wealth, as their marginal distribution has its own specificities.

In a different approach to Jäntti et al. (2015), who estimate parametric Plackett copulas for the joint distribution of income and wealth, we estimate non-parametric copulas. The Plackett copula is a single-parameter specification and has a one-to-one telation with Spearman's index. A non-parametric copula is model free. Thus, we do not assume any parametric model for the marginal distributions of income and wealth or for the copula itself.

We first obtain purely empirical copulas, which is what Kennickell (2009) did. Considering X = (X1, X2) a random vector and F a distribution function with marginal distributions F1 and F2; a copula can be defined as a bivariate distribution function C on $[0; 1]^2$ such that:

$$F(x_1, x_2) = C\{F_1(x_1), F_2(x_2)\} \tag{1}$$

We construct purely empirical copulas by computing the relative frequency of households located in different quantiles of the joint distribution of income and wealth. More formally, the empirical copula can be described as:

$$C\left\{\widehat{F_1}(x_1),\widehat{F_2}(x_2)\right\}=\frac{1}{N+1}\sum\nolimits_{i=1}^{N}1\left(X_{1,i}\le x_1,X_{2,i}\le x_2\right) \tag{2}$$

In addition to the previous estimations, we also provide smoothed kernel estimators for the empirical copula density. The kernel estimator is more efficient than the purely empirical approach and provides a clearer depiction of the graph, which makes the copula more comprehensible. The estimates are based on the following formula:

$$\hat{c}(u,v)=\frac{1}{Nh^2}\sum\nolimits_{i=1}^{N}K\left(\frac{u-\widehat{F_{x1}}(X_{1i})}{h},\frac{v-\widehat{F_{x2}}(X_{2i})}{h}\right) \tag{3}$$

Where $\hat{c}$ is the estimated copula density on u and v (pseudo-observations from the uniform marginal distributions), K is a primitive for K: R $\rightarrow$ R; $\int K-1$ and h is the kernel bandwidth. We take Gaussian functions for K for simplicity, although other functions can also be used to estimate the copula (e.g. Charpentier et al., 2007). We use a bandwidth of 0.045.[11]

Addressing the “boundary bias” of the kernel estimator for copulas is important for income and wealth, given that the dependence between them is larger at the top and bottom of the joint distribution, i.e. close to the boundaries. To deal with this bias, we use the “Mirror Image” technique (Deheuvels & Hominal, 1979; Schuster, 1985), consisting of adding observations using the “reflection” principle (see Appendix).

TESTING EQUALITY BETWEEN COPULAS

To deeply analyse the dependence structure between income and wealth, we use the test by Rémillard and Scaillet (2009) test equality between copulas. We perform the test with two goals. The first is to compare the dependence structure between income and wealth across the four considered countries. The second is to analyse which household characteristics influence the observed dependence pattern in Uruguay.

The test statistic relies on estimating rankings of individuals in each marginal distribution. Intuitively speaking, the statistic compares the pattern of concordances among individual rankings within the marginal distributions of two or more random variables between two populations.

The rank of each individual in the marginal distribution of each l random variable in a m population can be defined as,

$$Um_{il}=\frac{N_m}{1+N_m}F_{l,m}(X_{il}) \qquad \begin{array}{l} l=1,..K;K\ge 2 \\ m=1,2 \end{array} \tag{4}$$

[11]There is still a debate in optimal bandwidth regarding kernel estimation of copulas.

$i = 1,..Nm$

where Nm is the size of population *m*. $F_{l,m}$ (X_{il}) denotes the cdf of the random variable *l* in population *m* evaluated at X_{il}. To obtain the statistic we first compute the sample analogous of Um_{il}, which is defined as,

$$\hat{U}m_{il} = \frac{rank(X_{il})}{1+N_m} \text{ with } rank(X_{il}) = \sum\nolimits_{j=1}^{N} 1(X_{il} \geq X_{jl}) \tag{5}$$

The null hypothesis is that two copulas (m = 1,2) are equal, and the test statistic proposed by Rémillard and Scaillet (2009) is based on the Cramér-von Mises principle and given by:

$$S = \left(\frac{1}{N_1} + \frac{1}{N_2}\right)^{-1} x \left[\begin{array}{l} \frac{1}{N_1^2}\sum\nolimits_{i=1}^{N_1}\sum\nolimits_{j=1}^{N_1}\prod\nolimits_{l=1}^{K}\left(1-\widehat{U}1_{il} \vee \widehat{U}1_{jl}\right) \\ +\frac{1}{N_2^2}\sum\nolimits_{i=1}^{N_2}\sum\nolimits_{j=1}^{N_2}\prod\nolimits_{l=1}^{K}\left(1-\widehat{U}2_{il} \vee \widehat{U}2_{jl}\right) \\ -\frac{2}{N_1 N_2}\sum\nolimits_{i=1}^{N_1}\sum\nolimits_{j=1}^{N_2}\prod\nolimits_{l=1}^{K}\left(1-\widehat{U}1_{il} \vee \widehat{U}2_{jl}\right) \end{array}\right] \tag{6}$$

where a v b stands for *max* (a, b). The distribution of the test statistic is obtained via simulation. S_j is defined as the value of the test statistic in the *j-th* replication, and the p-value of the test is obtained as,

$$\frac{1}{J}\sum_{j=1}^{J} 1\left(\widetilde{S_j} > S\right). \tag{7}$$

In this paper, we apply this test to a bivariate case (K = 2), where income and wealth are the two variables of interest. The application of the test in this case is not free of additional obstacles. First, the sample size of the Monte Carlo simulations provided by Rémillard and Scaillet (2009) are considerably smaller than those of the surveys we are analysing. Second, we need to account for the stratified design of the surveys under analysis by using sample weights in the test formula. Finally, to simulate the distribution for the test statistic, we follow these authors and draw univariate and independent standard normal for each margin.

RESULTS

In this section, we present the dependence structure between income and wealth by comparing Uruguay with the rest of the countries analysed. We first compare the synthetic indexes and the results of the non-parametric estimation of copulas together with the Rémillard and Scaillet (2009) test to examine the joint distribution of income and wealth. Then, we explore the determinants of income and wealth in Uruguay and give some insights about the influence of those factors over the observed pattern of dependence between these variables.

Table 8.
Income and Wealth Correlation

	Pearson	Spearman	QI_1 & QW_1	QI_5 & QW_5
Uruguay	0.25	0.37	0.069	0.096
Uruguay (no bus)	0.29	0.36	0.069	0.096
Chile	0.37	0.28	0.053	0.094
Spain	0.51	0.40	0.060	0.089
U.S.	0.54	0.60	0.089	0.121

Notes: After tax income is considered in all cases except for Spain. All the imputation sets for each survey as well as sample weights were used.

We start by analysing the correlation index shown in Table 8. The Pearson index is 0.25 in Uruguay, the lowest value among the countries under analysis. The figure for Uruguay is close to the one estimated for Chile. Table 8 also shows the Spearman index for Uruguay at 0.37. The value is higher than Chile and, instead, similar to that of Spain.[12] Pearson indices assumes a linear correlation between variables; hence, a low value could be interpreted as a poor linear fit. In other words, the relationship between the variables is far from constant along the whole distribution. When looking at Pearson, Uruguay ranks at the bottom of the table. Instead, when looking at Spearman, Chile takes this place. This change in the ranking could be explained by the fact that Pearson assumes a linear correlation, which may not be suitable for Uruguayan data. While wealth distribution is similar in Chile and Uruguay, income is more equally distributed in the latter, hence a linear approximation will show a poor goodness of fit.

Table 8 also shows an indicator assessing the dependence of income and wealth at the bottom and at the top of the joint distribution. The QI1 & QW1 statistic is defined as the proportion of households belonging to the lowest 20% of income and wealth simultaneously. The QI5 & QW5 is analogous except for the top 20%. If the relationship between both variables is the same along the whole joint distribution, then the proportion of households in the QI1 & QW1 and QI5 & QW5 would have been around 0.04 (considering the variables in the $[0,1]^2$ domain). These statistics clearly

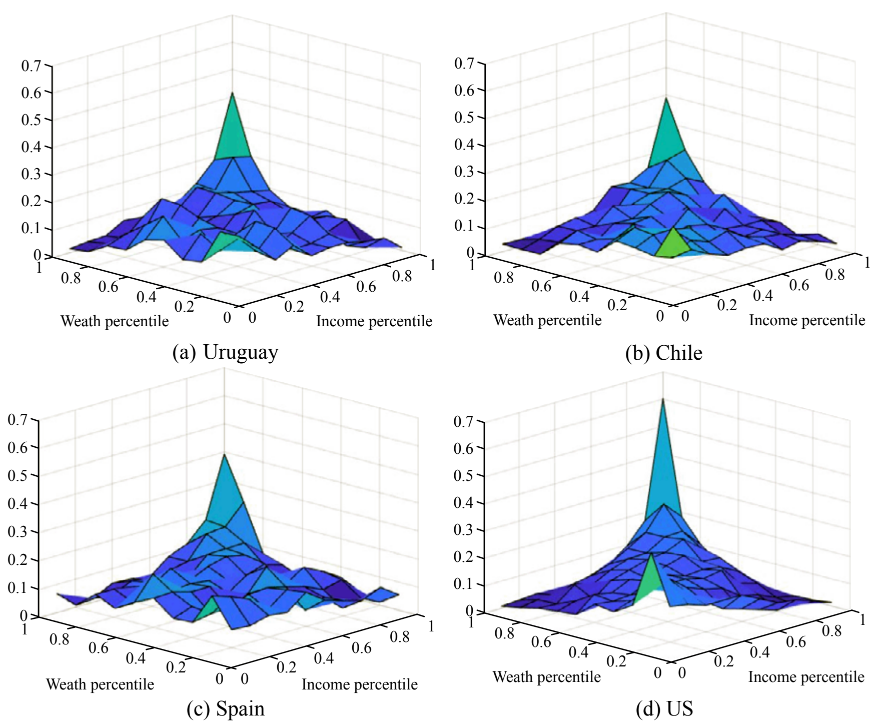
[12]The estimates for Spain (using the 2011 wave of the EFF) are well above those of various previous studies (Arrondel et al., 2014; Jäntti et al., 2015) that use the 2008 wave. The difference could be rationalized by the impact of the Great Recession on income and wealth distribution.

capture a dependence between income and wealth and unveil that the dependence is stronger at the top than at the bottom of the distribution.

INCOME AND WEALTH COPULAS

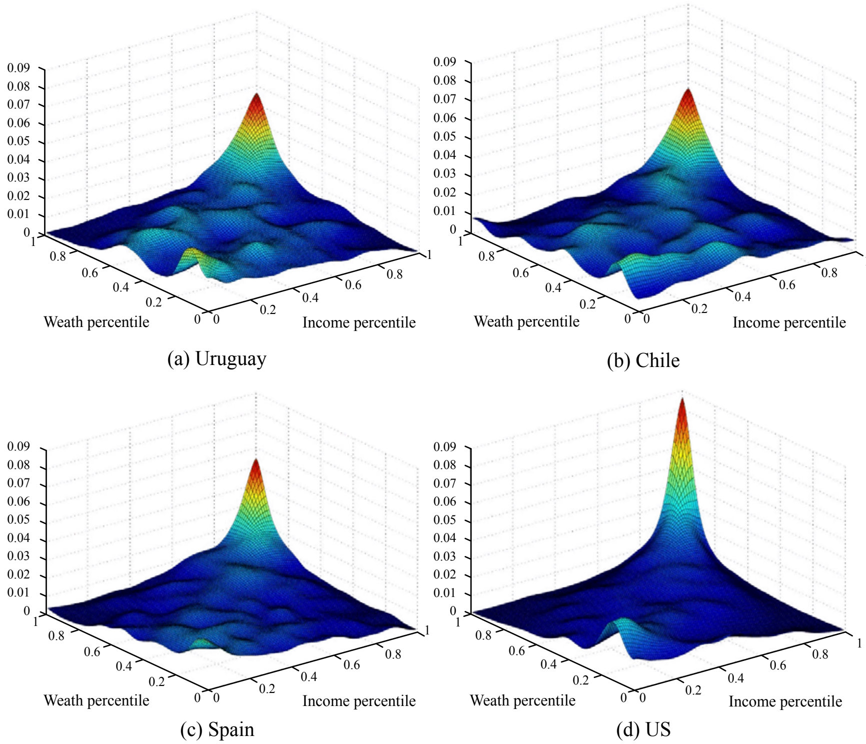
Figure 3 depicts densities of the empirical copulas in which we divide income and wealth distribution into 10 percentiles. The colour scale indicates the magnitude of the joint density. As with previous findings, the estimated copula for Uruguay has a sharp peak located at the top 10% of the joint distribution and a smaller peak at the opposite pole. A flatter density exists in the remaining areas of the distribution of income and wealth. This copula is visually similar to the ones estimated for Chile and Spain but different to the U.S., mostly due to the latter exhibiting a large peak at the top deciles of the joint distribution.

Figure 3.
Empirical Copulas For Income and Wealth



Note: Each sample is divided in 10 percentiles of income and wealth. All the imputation sets as well as sample weights for each survey were used.

Figure 4.
Kernel Smoothed Copulas For Income and Wealth

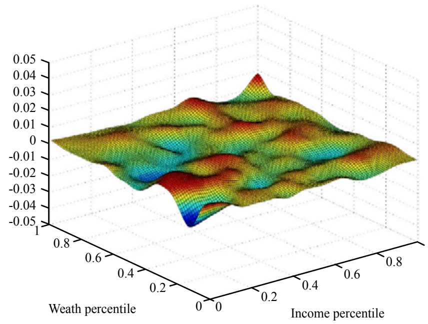


Note: Gaussian kernel copulas were built considering a bandwidth of 0.045. All the imputation sets as well as sample weights for each survey were used.

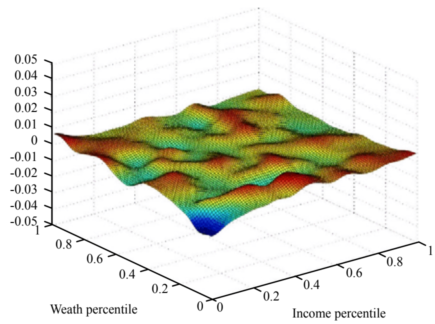
Smoothed kernel copulas are shown in Figure 4. It is still possible to notice the highest peak at the top of the joint distribution and the "smaller" one close to [0,0]. The peak at the top of the joint distribution is remarkably higher and sharper in the U.S. than in the other countries. To improve the visualization of the differences between the copulas, in Figure 5 we plot the simple difference between the estimation of the density copula for the "other" country and for Uruguay at each point. Those graphs also suggest that the pattern of dependence between income and wealth is similar in Chile, Spain, and Uruguay, but very different in the U.S.

Previous results provide only visual comparisons. To formally assess whether the copulas are statistically different, we use a non-parametric test of equality between copulas. Table 9 includes the p-values from the Rémillard and Scaillet (2009) test using 1,000 replications for each pair of countries. The hypothesis of equality

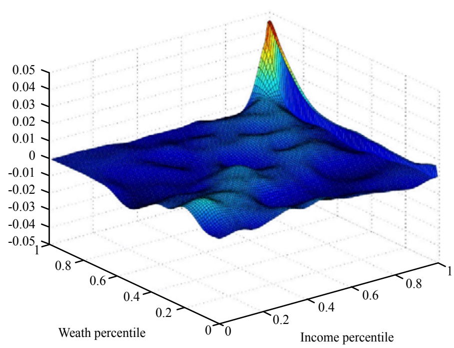
Figure 5.
Differences Between Kernel Estimated Copulas: Chile, Spain. and the U.S. vs. Uruguay



(a) Chile minus Uruguay



(b) Spain minus Uruguay



(c) US minus Uruguay

Note: Kernel copulas were built considering a bandwidth of 0.045. All the imputation sets as well as sample weights for each survey were used.

between copulas is rejected in all cases, save for the Spain-Uruguay pair.[13] We can link this result to the Spearman correlation shown in Table 8, where Uruguay and Spain are the closest to each other with estimated values of 0.37 and 0.4, respectively. This result could also be linked to the analysis of the marginal distribution of income, which shows a close relationship between Uruguay and Spain. A considerable limitation for analysing the previous result is that income collected in Spain is before taxes while in Uruguay it is after taxes. Accounting for taxes will certainly affect the copula, but it will most likely not change the shape: dependence may still be stronger at the top and bottom of the distribution. If one accepts that the Spanish tax system is not highly progressive and not that different from

[13]Despite focusing on Uruguay, we provide the p-value estimations from testing the equality between copulas for the rest of copula pair (Chile-Spain, Chile-U.S., Spain-U.S.).

Table 9.
Rémillard and Scaillet (2009) Test for Equality Between Each Pair of Copulas (p-values)

	Uruguay	**Chile**	**Spain**
Chile	0.001		
Spain	0.195	0.000	
U.S.	0.000	0.000	0.000

Notes: The null establishes that both copulas are equal. p-values are computed via simulation using 1,000 replications. Sample weights were used in all cases.

other EU countries, as suggested by the OECD report (see OECD, 2012), the pattern for copulas observed in countries such as Germany or Italy (analysed by Jäntti et al., 2015) should also be observed in Spain. Nevertheless, as taxes have an equalizer effect on income distribution, dependence at the top of the joint distribution will probably become less relevant. The latter could affect the result derived from the copula test.

SOURCES OF HETEROGENEITY AND THEIR INFLUENCE ON THE DEPENDENCE BETWEEN INCOME AND WEALTH IN URUGUAY

This section aims to analyse the main household determinants for income, wealth, and its joint distribution. We perform the analysis in three steps. First, we estimate separate mean regressions for income and wealth using household characteristics as covariates. Second, we build smoothed kernel copulas using the residuals obtained from the previous regressions. Third, we test the equality between copulas for income and wealth estimated in the section ‘Income and wealth copulas’ and the ones built using the residuals from the regressions. The two last steps allow us to examine the dependence between income and wealth once the effect of the covariates has been considered. We do not follow an identification strategy to estimate causal effects. Hence, our results are exploratory in nature as is most of the recent research on the topic.

In the first step, we estimate a set of mean regressions that considers each potential source of heterogeneity separately for each variable. We include the average age of household members aged 18 or older and its square as covariates to capture age dependent effects. Family composition effects are included through the number of household members; a dummy for the presence of children under 16; and family structure distinguishing between couple, single male, single female without children, and single female with children. The effect of education is measured

Table 10.
Mean Regressions. Dependent Variable: Net Wealth (In Logs)

Age	0.00817***					0.00673***	0.00598***	0.00384***	0.00380***
	[0.00151]					[0.00146]	[0.00142]	[0.00135]	[0.00136]
Age squared	-0.00599***					-0.00473***	-0.00273*	-0.00106	-0.00101
	[0.00145]					[0.00145]	[0.00143]	[0.00135]	[0.00136]
Number of hh's members		-0.00726***				-0.00142	0.00772***	0.00853***	0.00848***
		[0.00277]				[0.00282]	[0.00286]	[0.00277]	[0.00277]
Male		-0.0583***				-0.0529***	-0.0238	-0.0214	-0.0218
		[0.0153]				[0.0151]	[0.0148]	[0.0142]	[0.0143]
Female without children		-0.0617***				-0.0640***	-0.0575***	-0.0460***	-0.0461***
		[0.0127]				[0.0127]	[0.0123]	[0.0115]	[0.0115]
Female without children		-0.0878***				-0.0787***	-0.0536***	-0.0473***	-0.0480***
		[0.00850]				[0.00825]	[0.00866]	[0.00827]	[0.00825]
Children under 16 at home		-0.0461***				-0.0391***	-0.0294***	-0.0255***	-0.0241***
		[0.00894]				[0.00894]	[0.00866]	[0.00811]	[0.00822]
Years of schooling			0.0159***				0.0175***	0.0146***	0.0142***
			[0.00117]				[0.00130]	[0.00111]	[0.00108]

(Continúa)

Table 10.
Mean Regressions. Dependent Variable: Net Wealth (In Logs)

Main residence inherited				0.0232**				0.0278***	0.0285***
				[0.0111]				[0.0107]	[0.0107]
Other real estates inherited				0.280***				0.232***	0.232***
				[0.0308]				[0.0290]	[0.0290]
Business inherited				0.348***				0.320***	0.322***
				[0.0673]				[0.0654]	[0.0654]
Montevideo					0.0405***				0.0177**
					[0.00897]				[0.00802]
Constant	13.26***	13.57***	13.35***	13.48***	13.49***	13.34***	13.11***	13.17***	13.17***
	[0.0350]	[0.0149]	[0.0110]	[0.00469]	[0.00576]	[0.0339]	[0.0370]	[0.0339]	[0.0340]
R-squared	0.017	0.025	0.078	0.146	0.007	0.037	0.123	0.230	0.231
Observations	3,471	3,471	3,471	3,471	3,471	3,471	3,471	3,471	3,471

Note: Robust standard errors in brackets*** p<0.01, ** p<0.05, * p<0.1. Omitted category for family structure is couple. *, **, *** denotes significance at 10%, 5%, and 1% level respectively. We use the 10 imputation sets provided in EFHU database and compute statistics following Rubin's (1987) rules.

Table 11.
Mean Regressions. Dependent Variable: Income (In Logs)

Age	0.0308***					0.0211***	0.0165***	0.0157***	0.0153***
	[0.00582]					[0.00569]	[0.00479]	[0.00481]	[0.00477]
Age squared	-0.0332***					-0.0235***	-0.0116**	-0.0109**	-0.0105**
	[0.00549]					[0.00544]	[0.00457]	[0.00457]	[0.00454]
Number of hh's members		0.0273***				0.0112	0.0659***	0.0672***	0.0665***
		[0.0102]				[0.0105]	[0.00916]	[0.00917]	[0.00918]
Male		-0.396***				-0.398***	-0.223***	-0.215***	-0.220***
		[0.0465]				[0.0465]	[0.0397]	[0.0395]	[0.0393]
Female without children		-0.461***				-0.421***	-0.382***	-0.369***	-0.369***
		[0.0391]				[0.0399]	[0.0335]	[0.0333]	[0.0330]
Female without children		-0.610***				-0.621***	-0.471***	-0.466***	-0.473***
		[0.0481]				[0.0480]	[0.0409]	[0.0412]	[0.0409]
Children under 16 at home		-0.194***				-0.208***	-0.150***	-0.147***	-0.131***
		[0.0305]				[0.0304]	[0.0254]	[0.0253]	[0.0252]
Years of schooling			0.105***				0.105***	0.102***	0.0970***
			[0.00265]				[0.00267]	[0.00270]	[0.00273]

(Continúa)

Table 11.
Mean Regressions. Dependent Variable: Income (In Logs)

Main residence inherited				-0.146***				-0.0932***	-0.0858***
				[0.0386]				[0.0321]	[0.0316]
Other real estates inherited				0.535***				0.228***	0.235***
				[0.0562]				[0.0467]	[0.0462]
Business inherited				0.376***				0.189**	0.211***
				[0.0982]				[0.0825]	[0.0814]
Montevideo					0.392***				0.203***
					[0.0277]				[0.0228]
Constant	11.98***	12.77***	11.54***	12.58***	12.45***	12.40***	11.02***	11.06***	11.03***
	[0.144]	[0.0372]	[0.0295]	[0.0152]	[0.0167]	[0.146]	[0.129]	[0.130]	[0.129]
R-squared	0.020	0.097	0.307	0.036	0.057	0.107	0.388	0.395	0.409
Observations	3,485	3,485	3,483	3,485	3,485	3,485	3,483	3,483	3,483

Note: Robust standard errors in brackets, *** p<0.01, ** p<0.05, * p<0.1 Omitted category for family structure is couple. *, **, *** denotes significance at 10%, 5% and 1% level respectively. We use the 10 imputation sets provided in EFHU database and compute statistics following Rubin's (1987) rules.

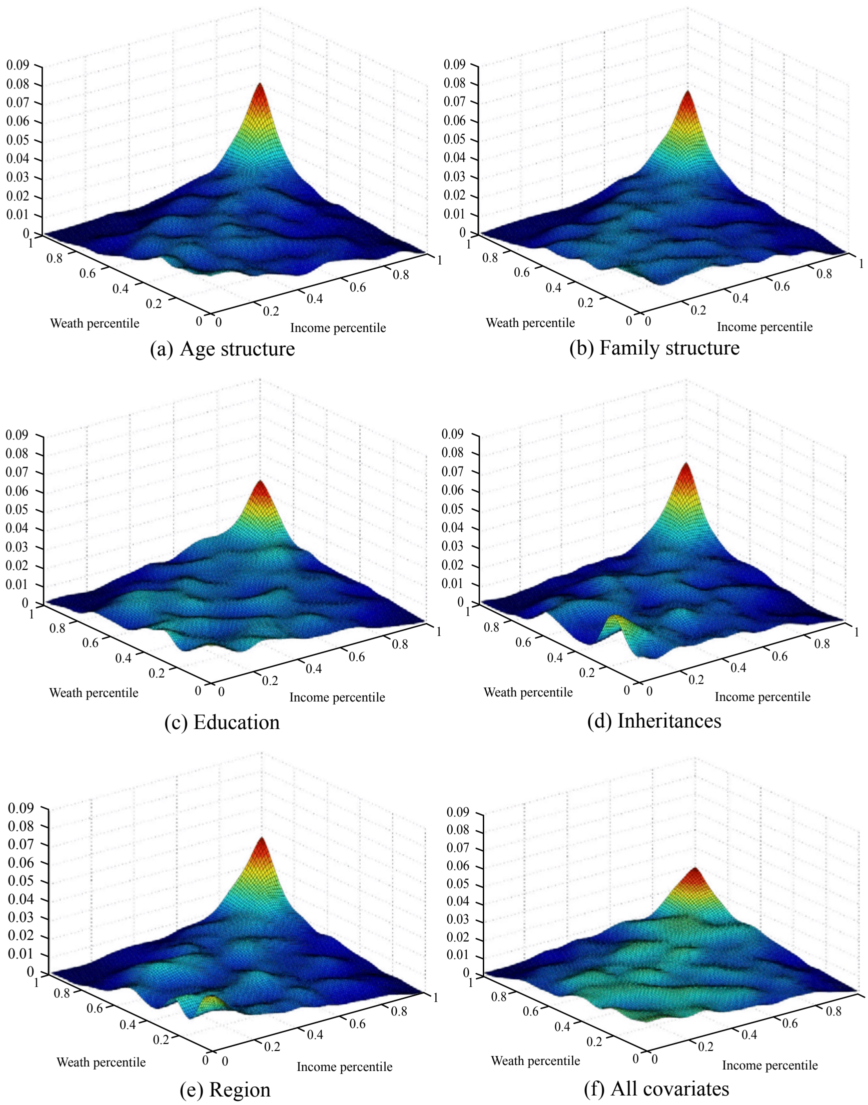
by the years of schooling of the reference person. The effect of inheritances is captured through a set of dummy variables that consider whether the household inherited the main residence, other real estate and/or business equity (the survey does not collect data on the value of the inherited asset). We also include a geographical reference as a source of heterogeneity. Arrondel et al. (2014) perform a similar analysis for EU countries through the estimation of generalized ordered probit models, linking the household position in the wealth distribution to that in the income distribution.

Table 10 and 11 depict the estimates of the mean regression for wealth and income, respectively. The first columns of each table show the estimates for each one of the group of covariates separately and the last column shows the estimated result when all the groups of covariates are included. When looking at the last column of each table, it can be seen that all groups of covariates are statistically significant for both variables. The effect of age can be seen in income. However, for wealth, age is significant at the 10^{th} and the square of age is non-significant. Education is a major determinant for both variables, though the partial effect is larger for income than for wealth. Inheritances are the main determinant for wealth within the considered sources of heterogeneity. They explain 15% of wealth variance when considering this set of dummies as the only covariates. Inheritances also influence income, but their explanatory power is considerably lower at 3.6 %. While the inheritance of other real estate and business positively affects average income, the mean income of those who had inherited their main residence is lower than those who had not. Family structure also play a significant role. Both, average wealth and income increases with the number of household members, although the effect on income is sharper. Mean income is higher for couples than for singles. The average wealth of couples and single male households are greater than for single females. Average income and net wealth are lower when children under 16 years old are living in the house. Households in Montevideo (the capital and main city) retrieve, on average, a higher income and have a higher level of wealth.

Once the mean regressions for income and wealth are estimated, we build copulas using each regressions' residuals. The copulas are depicted in Figure 6. Panels *a* to *e* show the copulas using the residuals for each group of covariates separately. Panel *f* shows the copula when all covariates are included simultaneously in the regression. Visually, they retain the main characteristics observed in the section 'Income and wealth copulas'. There is a main peak at the top of the joint distribution and a "flat" relationship elsewhere, perhaps the small peak at the [0; 0] of the joint distribution is absent. This can be seen in Figure 7, which depicts the difference between each of the copulas shown in Figure 6 and the copula using the observed income and wealth estimated in the previously-mentioned section.

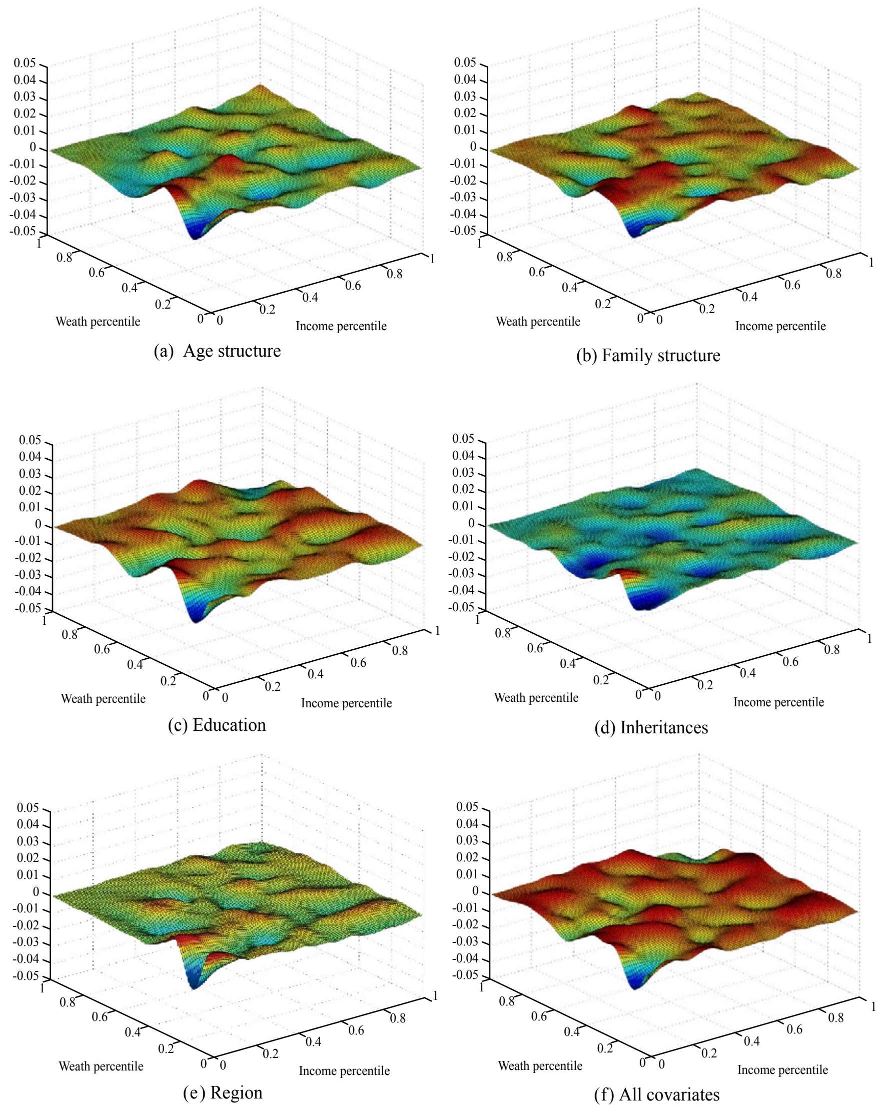
Finally, we perform the Rémillard and Scaillet (2009) test comparing the copulas estimated using the residuals of the regressions with the copula built with observed income and wealth. The first column in Table 12 shows the p-value from testing the equality between the copula built with the regressions considering each

Figure 6.
Kernel Estimated Copulas For Residuals From Income and Wealth Mean Regressions



Note: Kernel copulas were built considering a bandwidth of 0.045. All the imputation sets as well as sample weights for each survey were used.

Figure 7.
Observed Versus Residuals: Differences Between Kernel Estimated Copulas



Note: Kernel copulas were built considering a bandwidth of 0.045. All the imputation sets as well as sample weights for each survey were used.

group of covariates separately (age, family structure, education, inheritances, and region) and the one built with the observed income and wealth. The second column shows the p-value from testing the equality between the copula estimated with the observed income and wealth against the copulas built with the residuals from regressions that sequentially add each factor (group of covariates). The last column tests the equality between copulas of residuals at each sequential step vs the previous one.

Table 12.
Rémillard and Scaillet (2009) Test for Equality Between Two Copulas (p-values)

Factor	Each factor	Sequential	Each added factor
Age	0.197		
Family structure	0.332	0.214	0.960
Education	0.000	0.000	0.000
Inheritances	0.868	0.000	1.000
Region	0.340	0.000	1.000

Notes: The null establishes that both copulas are equal; p-values are computed via simulation using 1,000 replications. 1 Test of equality between copulas of observed income and wealth versus residuals from a regression that includes each factor. 2 Test of equality between copulas of observed income and wealth versus residuals from regressions that sequentially add each factor. 3 Test of equality between copulas of residuals at each sequential step versus residuals of the previous one.

The data shows that education is the covariate with the highest influence on the dependence between income and wealth. Furthermore, the evidence from the tests reveals that, among all the covariates considered, education is the only one which significantly influences the shape of the copula. When removing the effect of education, Figure 6 shows that the dependence between income and wealth at the [0,0] corner vanishes, while the strong dependence observed at the [1,1] corner is reduced.

Despite the fact the test suggests that other factors are not statistically significant to explain the shape of the copula, we can analyse some differences between the copulas that consider the influence of those variables by looking at figures 6 and 7. By removing the influence of age structure, the peak at the bottom is smoothed but the peak at the top becomes slightly sharper. By addressing the differences between regions, we can conclude that households at the [0,0] and [1,1] corners of the distribution are more frequent in Montevideo than in the rest of the country. When considering the residuals of regressions that include all covariates, the corresponding copula is statistically different and flatter than that of the observed income and wealth. The Rémillard and Scaillet (2009) test indicates that education drives this result. Visually, the copula of the residuals does not show the peak at the bottom of the joint distribution while the peak at the top is also reduced. Nevertheless, the peak at the [1,1] corner is still relevant in magnitude and it reveals evidence that the strong correspondence between high wealth and top income households can only be partially explained by the household characteristics considered.

The literature has analysed the role of education as one of the main determinants for wealth and income distribution. For instance, more educated households show higher levels of financial literacy, which allows them to benefit from financial market participation. However, financial knowledge and education are unevenly distributed across households, thus impacting on wealth and income distribution (Lusardi et al., 2017). In a study using U.S. data, Cagetti (2003) found that less educated consumers are more impatient and, thus, accumulate less precautionary savings. This creates a skewed wealth distribution with a right tail.

CONCLUDING REMARKS

In this paper, we analyse income and wealth distribution in Uruguay using data from EFHU, a household financial survey. We can summarize our results with three main findings. First, based on the marginal distribution for income and wealth, we conclude that wealth is more concentrated than income. Second, the analysis of copulas for income and wealth indicates that dependence between them is not constant along the whole joint distribution and, instead, is more relevant at the bottom and the top. When comparing the Uruguayan copulas to those from Chile, Spain, and the U.S., countries that have conducted financial surveys which served as basis for designing EFHU, we found similar results. Third, for Uruguay, we estimate mean regressions for income and wealth to assess the main sources of wealth and income heterogeneity. Despite household composition and bequests being among the main sources of heterogeneity for income and wealth, education is the most relevant explanatory variable for both the marginal distribution and the dependence structure of income and wealth.

The analysis could be extended at least in two important directions. First, to deeply analyse the role of education as the main variable shaping the distribution. This accounts for understanding not only the role of education, but also the channels through which consumption/savings behaviour and earning dynamics are affected. From a public policy view, if one aims to reduce income and wealth inequality, enhancing access to education could be a promising path. Second, the analysis provides some insights on wealth and income taxation. The more positive the dependence between both variables, the easier for a tax scheme targeting income to also tax wealth (Jäntti et al (2015)). In Uruguay, there is a high dependence at the top and at the bottom of the distribution, which could make progressive income taxes even more effective to reduce inequality.

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APPENDIX

Following Charpentier et al. (2007), the "mirror image" technique consists of adding observations to reflect each point with respect to the edges and corners of the unit square. In the bivariate case, a copula is the joint CDF of $F(x_1; x_2) = C(Fx_1(X_1); F_{x2}(X_2))$. When estimating the copula, the original dataset was converted to $(U^i;V^i) = C(\hat{F}_{x1}(X_{1i});\hat{F}_{x1}(X_{2i}))$ for i = 1; 2….N, and empirical CDF are used to estimate the marginal distributions:

$$\widehat{F_{x1}}(X_{1i}) = \frac{1}{N+1}\sum_{i=1}^{N} 1(X_{1,i} \leq x_1)$$

$$\widehat{F_{x2}}(X_{2i}) = \frac{1}{N+1}\sum_{i=1}^{N} 1(X_{2,i} \leq x_2)$$

More formally, the technique involved adding: $\left(\pm\widehat{U}_i, \pm\widehat{V}_i\right)$, $\left(\pm\widehat{U}_i, 2-\widehat{V}_i\right)$, $\left(2-\widehat{U}_i, \pm\widehat{V}_i\right)$, $\left(2-\widehat{U}_i, 2-\widehat{V}_i\right)$ such that the kernel smoothed version for the copula density is:

$$\begin{aligned}
c(u,v) = \frac{1}{Nh^2}\sum_{l=1}^{N} & K\left(\frac{u-\widehat{U}_l}{h}\right)K\left(\frac{v-\widehat{V}_l}{h}\right) + K\left(\frac{u+\widehat{U}_l}{h}\right)k\left(\frac{v-\widehat{V}_l}{h}\right) \\
& + K\left(\frac{u-\widehat{U}_l}{h}\right)K\left(\frac{v+\widehat{V}_l}{h}\right) + K\left(\frac{u+\widehat{U}_l}{h}\right)K\left(\frac{v+\widehat{V}_l}{h}\right) \\
& + K\left(\frac{u-\widehat{U}_l}{h}\right)K\left(\frac{v-2+\widehat{V}_l}{h}\right) + K\left(\frac{u+\widehat{U}_l}{h}\right)K\left(\frac{v-2+\widehat{V}_l}{h}\right) \\
& + K\left(\frac{u-2+\widehat{U}_l}{h}\right)K\left(\frac{v-\widehat{V}_l}{h}\right) + K\left(\frac{u-2+\widehat{U}_l}{h}\right)K\left(\frac{v+\widehat{V}_l}{h}\right) \\
& + K\left(\frac{u-2+\widehat{U}_l}{h}\right)K\left(\frac{v-2+\widehat{V}_l}{h}\right)
\end{aligned}$$

where K is a primitive for $K: R \to R, \int K = 1$ and h is a bandwidth sequence such that

$h_N \to 0$ when $N \to \infty$.