Bolema: Boletim de Educação Matemática
ISSN: 0103-636X
ISSN: 1980-4415
UNESP - Universidade Estadual Paulista, Pró-Reitoria de Pesquisa; Programa de Pós-Graduação em Educação Matemática

Prastiti, Tri Dyah

Problem-Based Learning on the Learning Perseverance of Indonesian Senior High School Students in Solving Mathematical Problems

Bolema: Boletim de Educação Matemática, vol. 34, no. 68, 2020, pp. 1206-1220
UNESP - Universidade Estadual Paulista, Pró-Reitoria de Pesquisa; Programa de Pós-Graduação em Educação Matemática

DOI: 10.1590/1980-4415v34n68a17

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

Problem-Based Learning on the Learning Perseverance of Indonesian Senior High School Students in Solving Mathematical Problems

Aprendizagem Baseada em Problemas na Perseverança da Aprendizagem de Estudantes Indonésios do Ensino Médio na Resolução de Problemas Matemáticos

Tri Dyah Prastiti*

ORCID iD 0000-0002-1452-2489

Abstract

Students should be encouraged to achieve high order thinking skills, in which one of the ways to build the thinking skills is through solving mathematical problems. This study aimed to observe the effectiveness of a problem-based learning method and its effect on students' perseverance in solving mathematical problems. The subjects of this research were science students at a Senior High School (SMAN) 2 Surabaya, Indonesia, selected with clustered random sampling. The research was conducted with an experimental design, and the problem-based learning method was applied in a Science 1 Class (experiment class), while a Science 4 Class was taught by following the conventional learning method designed by the teacher (control class), where the learning process was centered on the teacher and focused on the discipline. The instruments used in this research were the learning set for both learning methods, initial and final problem sets. The research results showed that the application of problem-based learning methods resulted in better learning outcomes. Moreover, the analysis of covariance showed that the application of a problem-based learning method is effective to develop students' perseverance in solving mathematical problems.

Keywords: High order thinking skill. Problem solving. Problem-based learning. Perseverance. High school.

Resumo

Os alunos devem ser incentivados a alcançar habilidades de pensamento de alta ordem, e uma das maneiras de desenvolver habilidades de raciocínio é resolvendo problemas matemáticos. Este estudo teve como objetivo observar a eficácia do método de aprendizagem baseado em problemas e seu efeito na perseverança dos alunos na resolução de problemas matemáticos. Os sujeitos desta pesquisa foram estudantes de ciências da Escola Secundária Senior (SMAN) 2 Surabaya, Indonésia, selecionados com amostragem aleatória agrupada. O método de aprendizado baseado em problemas foi aplicado na aula de ciências 1 (aula experimental), enquanto a aula de ciências 4 foi ministrada seguindo o método de aprendizado convencional, projetado pelo professor (aula de controle). Os instrumentos utilizados nesta pesquisa envolveram as ferramentas de aprendizagem vinculadas aos dois métodos de aprendizado, o baseado em problemas e o convencional, valendo-se de conjuntos de problemas, iniciais e finais. A pesquisa permitiu concluir que a aplicação do método baseado em problemas apresentou melhores resultados de aprendizado. Além disso, a análise de covariância mostrou que a aplicação do método de aprendizado baseado em problemas é eficaz no desenvolvimento da perseverança dos alunos, no que concerne à

* PhD in Mathematics Education from Universitas Negeri Malang (UM). Associate Professor of Mathematics Education Department, Faculty of Education and Teacher Training, Universitas Terbuka (UT), Surabaya, East Java, Indonesia. Address for correspondence: Kampus C Universitas Airlangga, Jl. Mulyorejo, Kecamatan Mulyorejo, 60115, Surabaya, East Java, Indonesia. E-mail: tridyahprastiti@ecampus.ut.ac.id.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

solução de problemas matemáticos.

Palavras-chave: Habilidades de pensamento de alta ordem. Solução de problemas. Aprendizagem baseada em problemas. Perseverança. Ensino Médio.

1 Introduction

Learning mathematics is essential as the subject does not only teach students about numbers and equations but also conducts their ability to think or know as a thinking skill. When learning mathematical problems, aside of learning to find a solution to the problems, the learning methods should also be able to encourage students to achieve high order thinking skills. Research has showed that students with high order thinking skills is likely to be more successful, whether in academic or in other work fields (RESNICK, 1987; SCHOENFELD, 1999; ZOHAR; DORI, 2003). However, the learning process in solving mathematical problems is affected by the learning methods applied by the teacher. Research by Prastiti et al. (2017), showed that learning methods with various problem types approach would improve students' ability in problem-solving. Moreover, teachers who support and facilitate their students in finding and presenting the solution and/or the alternative solutions to the mathematical problem would improve their students' thinking skills (HO; HEDBERG, 2005; PIMTA; TAYRUAKHAM; NUANGCHALERM, 2009).

The internal factors, such as learning motivation, would also affect the students' ability in solving mathematical problems (PIMTA; TAYRUAKHAM; NUANGCHALERM, 2009), as motivation is correlated with the students' positive attitudes toward the learning process. Research by Lerch (2004) showed that the students' positive attitude in the form of perseverance would improve their problem-solving ability. Perseverance has been mentioned in one of the U.S. Standard of Mathematical Practices (SMPs), which the first practice is stated as: "Make sense of the problems and persevere in solving them" (USA, 2010). The students' ability to persevere in solving mathematical problems was also increasingly expected, aside from constructing the mathematical justification and making sense of the problem (USA, 2010), in which a shift in the mathematics curriculum would be important (DAVIS et al., 2013). Furthermore, perseverance is an important attitude in solving mathematical problems, as the process requires a series of processes. Polya (1981) described that solving mathematical problems consisted of understanding the problem, devising a plan, carrying out the plan, and is followed by looking back at the result.

The research on high order thinking skills for high school students in Indonesia showed

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

an unsatisfying result, especially on the subject of mathematics (PRASTITI; TRESNANINGSIH; MAIRING, 2018; MAHENDRA, 2015; KURNIAWATI; DAFIK; FATAHILLAH, 2016; MAIRING, 2017). The condition thus demanded a suitable learning method to increase the students' problem-solving ability, from which would arise higher order thinking skills for the student. The problem-based learning is one of the promising methods which has showed the capability to improve the students' ability to solve mathematical problems (HO; HEDBERG, 2005; KING; GOODSON; ROHANI, 2016). In problem-based learning, the students' learning process is oriented on the mathematical problems, discussing the solution in group, presenting the obtained solution, and finally drawing up the conclusions (ARENDS, 2012). In this study, we observe the application of problem-based learning methods in high school students in Surabaya, Indonesia. We observed the method's effectiveness regarding the learning outcomes and learning perseverance compared to the conventional learning method designed by the teacher, so we could find the feasibility of the method to be applied in the curriculum.

2 Methods

This research was done in 8 learning activities for each learning methods, from September to December 2018. A more detailed research conduction is as described in the following subsections.

2.1 Research Design

We used a quantitative approach with experimental research and covariance analysis (RUTHERFORD, 2001) in this research. The independent variable was the learning method (X_i), consisted of problem-based learning with regards in students' perseverance (X_1) and the practiced teacher-centered learning (conventional learning method) (X_2). The dependent variable was the students' mathematical problem-solving abilities, and the control variable (covariate) is the students' initial problem-solving abilities (A_{ij}). The experimental research with covariance analysis were aimed to elucidate the students' problem-solving towards different learning methods in this research. The covariance analysis model is as follows:

$$y_{ij} = \mu + X_i + \beta A_{ij} + \epsilon_{ij}$$

with $i = 1,2$ and $j = 1, 2, 3, \dots,$ total number of students in each class

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

2.2 Instruments

The instruments in this research were the learning sets in two learning methods (problem-based learning and conventional learning method), initial mathematical problem set, and final mathematical problem set.

The initial mathematical problem set is as follows:

In a geometric sequence, with the infinite geometric series of 1, the subtraction of the second term with the first term is -4/9, and the common ratio is positive, please define the formula for n-term in the sequence!

The final mathematical problem set is a trigonometry problem, and the questions are as follow:

1. Prove that $-\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}=-2\cos 2A$
2. a. On the ABC triangle it is known that $\cos A.\cos B=\sin A.\sin B$ and $\cos A.\cos B=\sin A.\sin B$ Define the shape of the ABC triangle and the degree of each angle.

   b. Is there any other way to solve problem 2 (a)? If yes, please describe.
3. Andi and his friend went for a vacation in a village south of Malang city by car. On their way to the village, Andi realized that his car was nearly out of gas, then he decided to stop and looked at the map, searching for the closest gas station. The map showed that the closest gas station was located 10 km away at 30º (figure below).

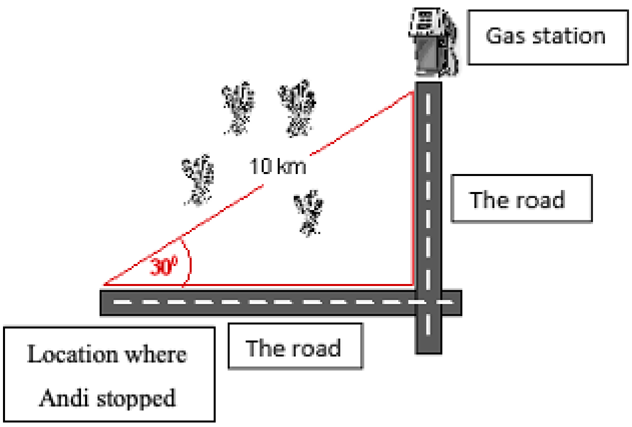

Figure 1 – Research question
Source: Prepared by the author

Andi brought his cell phone with the GPS feature which enabled him to go through the road or even through the meadow without worrying about getting lost. However, if they decided to go across the meadow and got too far away from the road, the GPS signal

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

could be lost and they might end up getting lost anyway. Andi estimated that the remaining gas was only enough to take them as far as 12 km. In the end, the options they could take were only going through the meadow; only following the road; or driving following the road halfway and driving through the meadow for the rest of the remaining way. From the story above:

a. Which route should Andi take so that they do not get lost and arrive in the gas station? Please elaborate your answer.
b. If you were Andi's friend, please give an alternative route that Andi could take to arrive in the gas station. Please describe your answer.

2.3 Research Participants

The subjects of this research were students of a Science Class in SMAN 2 Surabaya Senior High School, Indonesia. Out of eight Science Classes in the school, two of the classes were the participants selected for the research through clustered random sampling. The result of clustered random sampling selected Science Class 4 for experiment class (X_1), which applied the problem-based learning method; and Science Class 1 for the control class (X_2), which applied the conventional learning method designed by the teacher (teacher-centered and discipline-focused learning approach). The total number of students in Science Class 1 and 4 were 39 and 40, respectively.

The problem-based learning in the experiment class was conducted in five steps, which were: (1) The teacher explains the aim of the study and motivate the students on the importance of the learning outcomes; (2) The teacher accompanies students to define and organize the given mathematical problems; (3) The teacher encourages students to gather information related to the mathematical problems and experiment on available problem-solving approaches; (4) The teacher form learning groups between the students to conduct their own mathematical problems and accompany them to solve the problem through discussion between students; and (5) The teachers help students reflect on and evaluate the conducted mathematical problems and its solution. In the control class, the conventional learning was conducted in the form of teacher-centered learning, where the teacher gives the lecture and the students exclusively listen and are expected to learn independently.

The problem-based learning method with regards to the students' perseverance is considered effective if there is significantly more students who showed mathematical problem-solving ability in the experiment class compared to the control class.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

2.4 Research Procedures

The research was conducted in 9 steps, which were deciding the research topic, conducting literature review to define the research questions, developing research hypothesis, determining research participants, developing research instruments, collecting the data, analyzing the data, and finally drawing up research conclusions (LODICO et al., 2006).

2.5 Data Collection

There were two types of data in this research, which were the students' initial ability and final ability in solving mathematical problems. The first data was collected by giving out the initial mathematical problem set to the research participants before applying the learning methods. The second data was obtained by giving out the final mathematical problem set after applying the learning methods. The solutions found by the students were scored with a holistic rubric, in which the maximum score on each mathematical problem was 4.

Table 1 – Rubric holistic in solving mathematical problems

Score	Description
0	a. Student could not find any solution. b. Student writes everything which had been known and asked, but does not show any understanding to the problem.
1	a. Student correctly writes everything which had been known and asked, writes a solution to the problem, but the given solution is wrong. b. Student shows an effort to achieve the sub-objectives, but does not succeed. c. Student answers correctly, but could not show the method.
2	a. Student uses a wrong method and gets a wrong answer, but shows an understanding to the problem. b. Student writes a correct answer, but the method is not understandable or wrong.
3	a. Student has applied the correct method, but understands some parts of the problem wrongly, or ignore some conditions of the problem. b. Student has applied the correct method, but: (i) gives a wrong solution without any explanation, or (ii) does not write the answer. c. Student writes the correct answer and correct method, but the application is not fully correct.
4	a. Student has applied the correct method and writes the correct answer. b. Student applied the correct method and writes the correct answer, but there are few mistakes in the calculation.

Source: Charles et al. (1987)

2.6 Data Analysis

The analysis of covariance was used to analyze the data. The analysis assumed that the covariate was independent from other variables, non-normal distribution errors, and equal variances (RUTHERFORD, 2001). Two hypothesis groups were then drawn in this research,

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

which were:

First hypothesis group:

$H_0: \beta = 0$; There is no linear correlation between covariate to the problem-solving ability.$H_1: \beta \neq 0$; There is a linear correlation between covariate to the problem-solving ability.

Second hypothesis group:

$H_0: \mu_{X_1} = \mu_{X_2}$; The average score between control and experimental class is equal.

$H_1: \mu_{X_1} \neq \mu_{X_2}$; The average score between control and experimental class is different.

3 Results

In the problem-based learning, students were encouraged to conduct discussion and find the solution through teamwork consisting of 4-5 students. The teachers acted as motivators and provided scaffolding for each group. The result of discussion was then presented in front of the class regarding the different solutions or method to solve the given problems. In both classes, they were asked to do the same initial and final mathematical problem sets, and the given solution by the students were scored by using holistic rubric with the score ranging from 0 to 4. The result showed that the average total score for the students' ability in solving the final mathematical problems were 8.20 for the treatment class and 7.75 for the control class. In problems 2b and 3b of the final problem set, the obtained average score was below the initial problem set, as students found difficulties in determining a different method to solve the problem as required (Table 2).

Table 2 – The average score of students' ability in mathematical problem-solving

Class	**Initial Problem Set**	**Final Problem Set**				
		1	**2a**	**2b**	**3a**	**3b**
Experiment	1.87	2.40	2.50	0.40	2.50	0.40
Control	2.15	2.43	2.45	0.20	2.43	0.25

Source: Prepared by the author

We also conducted the assumption test before testing the hypothesis. The initial students' ability to solve mathematical problems (covariate) was obtained by giving out the same initial mathematical problem set in both experiment and control class, so that the covariate could be independent from the treatment. The assumption test showed that the first assumption was fulfilled.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

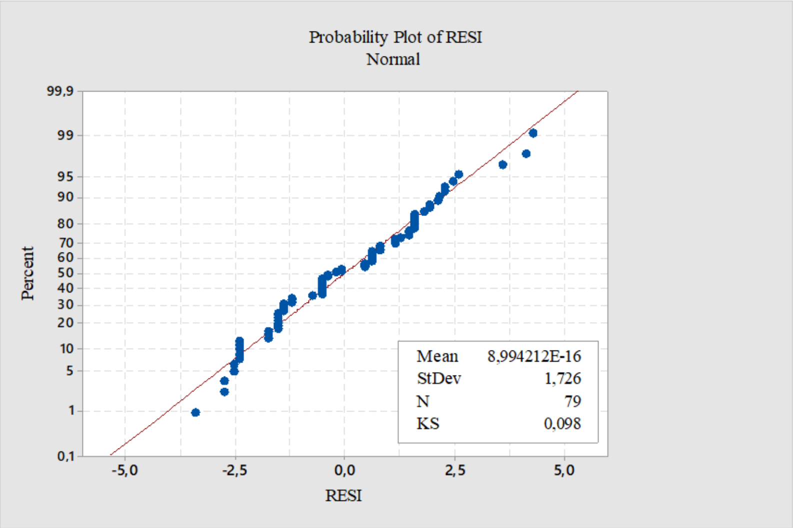

Figure 2 – The normality test
Source: Prepared by the author

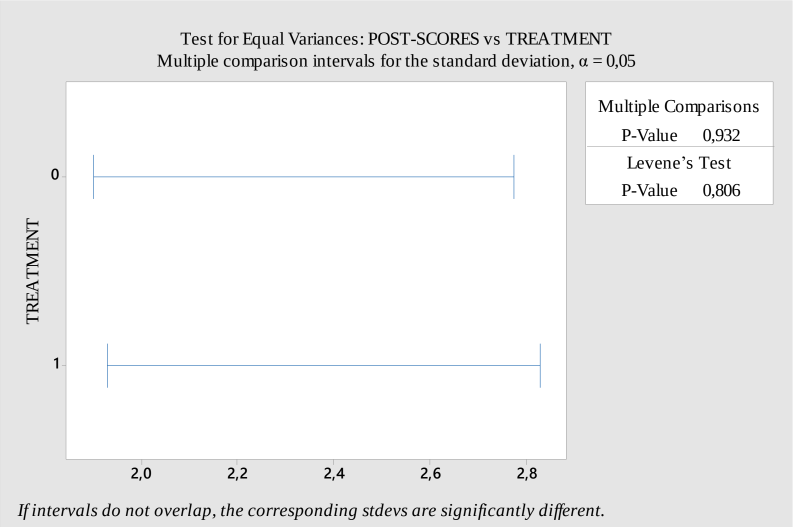

Figure 3 – The equal variances test
Source: Prepared by the author

The Kolmogorov-Smirnov test for error normality resulted in $p - value = 0.06 \geq 0.05 = \alpha$, which showed that the error normality assumption was fulfilled (Figure 2). Furthermore, the Levene-test resulted in $p - value = 0.806 \geq 0.05 = \alpha$, which showed that the equal variances assumption was fulfilled (Figure 3). The result of the tested hypothesis is as follows:

General Linear Model: post vs pre-treatment

Table 3 – Analysis of variance

Source	df	Adj SS	Adj MS	F-Value	P-Value
Pre-treatment	1	160.00	159.997	52.32	0.000
Post-treatment	1	23.93	23.934	7.83	0.007
Error	76	232.43	232.43	3.058	
Lack of fit	4	33.23	8.307	3.00	0.024
Pure Error	72	199.20	199.20	2.767	
Total	78	396.99	396.99		

Source: Prepared by the author

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

Table 4 – Model summary

S	R-Sq	R-sq (adj)	R-sq (pred)
1.74878	41.45%	39.31%	36.24%

Source: Prepared by the author

Table 5 – Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	3.304	0.677	4.88	0.000	
Pre-treatment	2.330	0.322	7.23	0.000	1.05
Post-treatment	-0.565	0.202	-2.80	0.007	1.05

Source: Prepared by the author

The test on the first hypothesis to β (initial problem-solving ability coefficient) resulted in $p - value = 0 < 0.05 = \alpha$. The result concluded that the H_0 was rejected, which means that there was a linear correlation between covariate with the problem-solving ability at a 95% degree of confidence. The test on the second hypothesis to the treatment (learning methods) resulted in $p - value = 0.007 < 0.05 = \alpha$, which showed that the H_0 was rejected. The test thus concluded that there was a significant difference to the problem-solving ability of both methods at a 95% degree of confidence. The overall research result thus showed that students who learn with problem-based learning method (experiment class) had better problem-solving abilities when compared to those who learned with the conventional method (control class).

4 Discussion

The research supported problem-based curriculum materials for mathematics learning (LI; LAPPAN, 2014), while Edson et al. (2019) added that the problem-based-curricula will provide an avenue for promoting deep mathematical understanding and reasoning. In this research, we observed the application of problem-based learning in high school students, and the result showed that the problem-based learning method is effective in improving the students' mathematical problem-solving ability. Furthermore, Stein; Remillard; Smith (2007) added that the method would improve the students' mathematical modelling, reasoning, and conceptual understanding aside from problem solving. The result is also in accordance to Sahrudin (2014) and Sari (2014), which showed that problem-based learning would improve the students' problem-solving ability.

In this research, the conventional learning method practiced by the teacher was through teacher-centered and discipline-focused learning. Herbel-Eisenmann; Steele; Cirillo (2013) described that in the teacher-centered learning, instruction by the teacher continues to dominate mathematic classrooms, while Cazden (2001) added that students would experience few

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

opportunities to share their ideas or ask questions in the teacher-centered learning. Although teachers may begin the class with open-ended questions, conventional learning discourse often shifts to a more traditional initiate-response-evaluate pattern, thus revealing the challenges the teacher experience in moving to open ended participation (CADY; MEIER; LUBINSKI, 2006; LARSSON; RYVE, 2012; LEIKIN; ROSA, 2006; TRUXAW; DEFRANCO, 2008). Other prior researches have shown that the approach to tackle problems faced by conventional teacher-centered learning is through supportive student inquiry teaching by the teachers (e.g., HUFFERD-ACKLES; FUSON; SHERIN*,* 2004; LEATHAM et al., 2015; SMITH; STEIN, 2011; STAPLES, 2007). These studies showed that with proper support, teachers can establish a classroom that fosters inquiry-based exploration of rich tasks (SULLIVAN et al., 2013). The problem-based learning then provides a solution to tackle the problems found in the conventional learning by providing a learning approach that focus on students inquiry towards mathematical problem-solving.

One of the early researches on problem-based learning was done by Polya (1973), which showed that the students' ability to solve mathematical problems could be taught by imitating problems, which occurred in daily activities. In this research, mathematical problems inspired from daily activities were used to build the problem sets. However, building the problem set for problem-based learning in mathematics has its own challenge as well. The challenge includes making sense of understandings from a problem-based perspective on the teaching and learning of mathematics (ARBAUGH et al., 2006; COLLOPY, 2003; LESTER; CAI, 2016). The complexity of building mathematical problem sets in the problem-based learning method is allegedly caused by the unsatisfying result of the students' ability to find an alternative solution to the problem. Choppin (2011), Lappan (1997), and Lubienski (1999) stated that teaching with problem-based curriculum materials places greater demands on the role of teaching mathematics. The result thus suggested that there should be preliminary learning activities which introduce students and teachers to problem-based learning. Research by Choppin (2011) has suggested that teachers also required support in their understanding "that not only included an articulation of the designers' intent, the mathematics, and the task features, but also of how the task features afforded opportunities for students to engage with mathematical concepts. Research on designing a learning method from show-and-practice to problem-based learning has been done by Lappan et al. (1985).

In this research, we can see that students still found difficulties in finding alternative solutions to the given mathematical problems. This showed that the students did not have the high order thinking skill yet. However, the unsatisfying result is not surprising, as the high order

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

thinking skill, which would let students grasp the subtlety of ideas, required time and persevering efforts. Lappan and Phillips (2009) mentioned that the time required to fully develop a particular mathematical idea, the extent to which students grasp the mathematical subtlety of ideas, and the degree to which students reach useful closure of the idea require careful consideration to the mathematical challenge and the positioning of the problem within a carefully sequenced set of problems. However, the condition might improve if the problem-based learning method is applied continuously, as the method can develop the students' ability to think creatively (BAHAR; MAKER, 2015).

In the problem-based learning methods, teachers were suggested to act as motivators for the students, while also providing scaffolding if required. In this research, students became more motivated to find the solution for the mathematical problems, which resulted in better learning perseverance and outcomes. The research by Lerch (2004) and Pimta; Tayruakham; Nuangchalerm (2009) also showed that the students' motivation would affect their learning outcomes, and that motivation from teachers would positively affect the students' learning process. Moreover, Edson et al. (2019), added that teachers should support the students' communication of mathematical ideas over problems, units, and grades. Support from teachers would motivate student and at the same time help teachers assess their students' learning progress, since the problem-based learning with group model discussion would require more complex formative assessments as students interact and influence each other's thinking (WIRKALA; KUHN, 2011).

5 Conclusion

The application of problem-based learning methods was effective in improving the students' ability to solve mathematical problems, while also improving their learning perseverance as well. The research showed that even though the high school students' ability to solve mathematical problems were still unsatisfying, the application of a problem-based learning method gave them significantly better learning outcomes in comparison to the conventional method. The application to integrate problem-based learning method to the mathematics curriculum for high school student is then suggested.

Acknowledgments

The author would like to thank Universitas Terbuka Indonesia for supporting the

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

research. There are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

References

ARBAUGH, F.; LANNIN, J.; JONES, D. L.; PARK-ROGERS, M. Examining instructional practices in Core-Plus lessons: Implications for professional development. **Journal of Mathematics Teacher Education**, Heidelberg, v. 9, n. 6, p. 517-550, 2006.

ARENDS, R. I. **Learning to teach** 9. ed. New York: The McGraw-Hill Companies, 2012.

BAHAR, A.; MAKER, C. J. Cognitive backgrounds of Problem Solving: A comparison of open ended vs. closed mathematics problems. **Eurasia Journal of Mathematics, Science and Technology Education**, Derbyshire, v. 11, n. 6, p. 1531-1546, 2015.

CADY, J.; MEIER, S. L.; LUBINSKI, C. A. The mathematical tale of two teachers: A longitudinal study relating mathematics instructional practices to level of intelectual development. **Mathematics Education Research Journal**, Heidelberg, v. 18, n. 1, p. 3–26, 2006.

CAZDEN, C. **Classroom discourse**: The language of teaching and learning 2. ed. Portsmouth: Heinemann, 2001.

CHARLES, R.; LESTER, F.; O'DAFFER, P. **How to evaluate progress in problem solving**. Reston: The National Council of Teachers of Mathematics, 1987.

CHOPPIN, J. Learned adaptations: Teachers' understanding and use of curricular resources. **Journal of Mathematics Teacher Education**, Heidelberg, v. 14, n. 5, p. 331–353, 2011.

COLLOPY, R. Curriculum materials as a professional development tool: How a mathematics textbook affected two teachers' learning. **The Elementary School Journal**, Chicago, v. 103, n. 3, p. 227–311, 2003.

DAVIS, J.; CHOPPIN, J.; ROTH-MCDUFFIE, A.; DRAKE, C. **Common core state standards for mathematics**: Middle school mathematics teachers' perceptions. 2013. Available at: http://www.warner.rochester.edu/files/warnercenter/docs/commoncoremathreport.pdf. Access in: 17 feb. 2019.

EDSON, A. J.; PHILLIPS, E.; SLANGER-GRANT, Y.; STEWART, J. The arc of learning framework: An ergonomic resourc for design and enactment of problem-based curriculum. **International Journal of Educational Research**, Amsterdam, v. 92, p. 118-135, 2019.

FRANKE, M. L.; WEBB, N. M.; CHAN, A. G.; ING, M.; FREUND, D.; BATTEY, D. Teacher questioning to elicit students' mathematics thinking in elementar classrooms. **Journal of Teacher Education**, Thousand Oaks, v. 60, n. 4, p. 380–392, 2009.

HERBEL-EISENMANN, B. A.; STEELE, M. D.; CIRILLO, M. (Developing) teacher discourse moves: A framework for professional development. **Mathematics Teacher Educator**, Reston, v. 1, n. 2, p. 181–196, 2013.
HO, K. F.; HEDBERG, J. G. Teachers' pedagogies and their impact on students' mathematical problem solving. **Journal of Mathematical Behaviour**, Amsterdam, v. 24, p. 238–252, 2005.

HUFFERD-ACKLES, K.; FUSON, K.C.; SHERIN, M.G. Describing levels and componentes of a math-talk learning community. **Journal for Research in Mathematics Education**, Reston, v. 35, n. 2,

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

p. 81–116, 2004.

KING, F. J.; GOODSON, L.; ROHANI, F. **Higher order thinking skills**. Centre for Advancement of Learning and Assessment. 2016. Available at: http://www.cala.fsu.edu/files/higher_order_thinking_skills.pdf. Access in: 17 feb. 2019.

KURNIAWATI, E. Y.; DAFIK, D.; FATAHILLAH, A. Analisis Pola Berpikir Kreatif Siswa Kelas X IPA 2 SMAN 2 Jember dalam memecahkan masalah open-ended bangun datar dan bangun ruang. **Edukasi**, Jember, v. 3, n. 1, p. 18-23, 2016.

LAPPAN, G. The challenges of implementation: Supporting teachers in reforming the third R: Changing the school mathematics curriculum. **American Journal of Education**, Chicago, v. 106, n. 1, p. 207–239, 1997.

LAPPAN, G.; FITZGERALD, W. M., PHILLIPS, E. D.; WINTER, M. J. **Middle grades mathematics project**. Menlo Park: Addison-Wesley, 1985.

LAPPAN, G.; PHILLIPS, E. D. Challenges in US mathematics education through a curriculum development lens. **Journal of the International Society for Design and Development in Education**, Nottingham, v. 1, n. 3, p. 1–19, 2009.

LARSSON, M; RYVE, A. Balancing on the edge of competency-oriented versus procedural-oriented practices: Orchestrating whole-class discussions of complex mathematical problems. **Mathematics Education Research Journal**, Heidelberg, v. 24, n. 4, p. 447–465, 2012.

LEATHAM, K. R.; PETERSON, B. E.; STOCKERO, S. L.; VAN-ZOEST, L.R. Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. **Journal for Research in Mathematics Education**, Reston, v. 46, n. 1, p. 88–124, 2015.

LEIKIN, R.; ROSA, S. Learning through teaching: A case study on the development of a mathematics teacher's proficiency in managing an inquiry-based classroom. **Mathematics Education Research Journal**, Heidelberg, v. 18, n. 3, p.44–68, 2006.

LERCH, C. M. Control decisions and personal beliefs: Their effect on solving mathematics problem. **Journal of Mathematical Behaviour**, Amsterdam, v. 23, n. 1, p. 21-36, 2004.

LESTER, F. K.; CAI, J. Can mathematical problem solving be taught? Preliminary answers from 30 years of research. *In*: FELMER P.; PEHKONEN E.; KILPATRICK, J. (ed.). **Posing and solving mathematical problems, research in mathematics education**. Cham: Springer International Publishing, 2016. p. 117–135.

LI, Y.; LAPPAN, G. **Mathematics curriculum in school education**. Dordrecht: Springer, 2014.

LODICO, M. G.; SPAULDING, D. T.; VOEGTLE, K. H. **Methods in Educational Research**: From Theory to Practice. San Fransisco: John Wiley & Sons, 2006.

LUBIENSKI, S. T. Problem-centered teaching. **Mathematics Teaching in the Middle School**, Reston, v. 5, n. 4, p. 250–255, 1999.

MAHENDRA, Y. B. Proses berpikir kreatif siswa kelas XA SMA Negeri 1 gedangan dalam memecahkan masalah matematika pada materi peluang dengan soal higher order thinking ditinjau dari tingkat berpikir kreatif. **MATHEdunesa**, Surabaya, v. 1, n. 4, p. 131-140, 2015.

MAIRING, J. P. Kemampuan siswa SMA dalam menyelesaikan masalah sistem persamaan linear tiga variabel. **Aksioma**, Metro, p. 15-26, 2017.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

PIMTA, S. L; TAYRUAKHAM, S.; NUANGCHALERM, P. Factors influencing mathematics problem solving ability of sixth grade students. **Journal of Social Sciences**, New York, v. 5, n. 4, p. 381–385, 2009.

POLYA, G. **How to solve it**. 2. ed. Princeton: Princeton University Press, 1973.

POLYA, G. **Mathematical discovery**: On understanding, learning, and teaching problem solving. New York: John Wiley & Sons, 1981.

PRASTITI, T. D.; YUMIATI; MAIRING J. P.; HANDAYANI, E. P. Pengembangan Lembar Kerja Mahasiswa Berbasis REACT pada Tutorial Statistika Pendidikan di Universitas Terbuka. **Jurnal AdMathEdu**, Yogyakarta, v. 7, n. 1, p. 61-72, 2017.

PRASTITI, T. D.; TRESNANINGSIH, S.; MAIRING, J. P. Tingkat Kemampuan Berpikir Kreatif Matematis Siswa SMAN di Suabaya. **Jurnal AdMathEdu**, Yogyakarta, v. 8, n. 1, p. 83-94, 2018.

RESNICK, L. **Education and learning to think**. Washington: National Academy Press, 1987.

RUTHERFORD, A. **Introducing anova and ancova**: A GLM approach. London: Sage Publications, 2001.

SAHRUDIN, A. Implementasi strategi pembelajaran discovery untuk meningkatkan kemampuan pemecahan masalah matematis dan motivasi belajar siswa SMA. **Jurnal Pendidikan Unsika**, Karawang, v. 2, n. 1, p. 1–12, 2014.

SARI, N. Peningkatan kemampuan pemecahan masalah matematis melalui pembelajaran berbasis masalah dan pembelajaran konvensional pada mahasiswa STMIK di kota Medan. **Jurnal Saintech**, Jakarta, v. 6, n. 4, p. 106–111, 2014.

SCHOENFELD, A. Looking toward the 21st century: Challenges of educational theory and practice. **Educational Researcher**, Washington, v. 28, n. 7, p. 4-14, 1999.

SMITH, M.; STEIN, M. K. **Five practices for orchestrating productive mathematics discussions**. Reston: NCTM, 2011.

STAPLES, M. Supporting whole-class collaborative inquiry in a secondary mathematics classroom. **Cognition and Instruction**, Germantown, v. 25, n. 2, p. 161–217, 2006.

STEIN, K. M; REMILLARD, J.; SMITH, M. How curriculum influences student learning. *In*: FRANK, K.; LESTER, J. (ed.). **Second handbook of Research on Mathematics Teaching and Learning**. Charlotte: Information Age Publishing, 2007. p. 319–369.

SULLIVAN, P.; AULERT, A.; LEHMANN, A.; HISLOP, B.; SHEPHERD, O.; STUBBS, A. Classroom culture, challenging mathematical tasks and student persistence. *In*: STEINLE, V.; BALL, L.; BARDINI, C. (ed.). **Proceedings of the 36th Teacher moves for supporting student reasoning anual conference of the Mathematics Education Research Group of Australasia**. Melbourne: insert publisher, 2013. p. 618–625.

TRUXAW, M. P.; DEFRANCO, T. Mapping mathematics classroom discourse and its implications for models of teaching. **Journal for Research in Mathematics Education**, Reston, v. 39, n. 5, p. 489–525, 2008.

USA. **Common core state standards for mathematics**. 2010. Available at: http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf. Access in 17 feb. 2019.

ISSN 1980-4415
DOI: http://dx.doi.org/10.1590/1980-4415v34n68a17

WIRKALA, C.; KUHN, D. Problem-based learning in K-12 education: Is it effective and how does it achieve its effects? **American Educational Research Journal**, Thousand Oaks, v. 48, n. 5, p. 1157–1186, 2011.

ZOHAR, A.; DORI, Y.A. Higher-order thinking skills and low achieving students: Are they mutually exclusive? **Journal of the Learning Sciences**, Germantown, v. 12, n. 2, p. 145-181, 2003.

Submetido em 23 de Março de 2019.
Aprovado em 12 de Julho de 2020.