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A sinusoidal equation as an alternative to classical growth functions to describe growth profiles in turkeys

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ABSTRACT. Because of the relatively long growing cycle and the high cost of research into turkey production and nutrition, the potential benefits from modelling growth in this avian species are considerable. Though there are many studies aimed at evaluating animal growth models, the number of studies targeting growth models in turkeys is quite limited. In this paper we present a sinusoidal function to describe the evolution of growth in turkeys as a function of time based on data published by Aviagen. The new function was evaluated with regard to its ability to describe the relationship between body weight and age in turkeys and was compared to four standard growth functions: the Gompertz, logistic, Lopez, and Richards. The results of this study show that the new sinusoidal function precisely describes the growth dynamics of turkeys. Fitting the functions to different data profiles nearly always led to the same or less maximized log-likelihood values for the sinusoidal equation, indicating its suitability in describing growth data from turkeys.

Key words: growth functions; sinusoidal equation; turkeys.

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Introduction

Turkey meat is an excellent protein source and has a good price-quality ratio (Roberson et al., 2003). Therefore, it is important to know the factors influencing the productive performance of this species, the yield and quality of the carcass (Nestor, Anderson, Hartzler & Velleman, 2005). Representation of biological concepts through the simulation of growth dynamics enables us to better adapt management and nutrition to the requirements of the animals, while taking into account the interaction between genotype, nutrition and environmental conditions (Thornley & France, 2007). Growth is a fundamental property of biological systems and can be defined as an increase in body size per time unit. Understanding the economic importance of various traits such as live weight, weight gain, rate of maturity, and age and live weight at which maximal growth occurs has led researchers to carry out detailed studies targeting the weight-age relationship (Ersoy, Mendes & Aktan, 2006). For this reason, different mathematical growth models have been applied and developed (Gompertz, 1925; Von Bertalanffy, 1957; Richards, 1959; López, France, Dhanoa, Mould & Dijkstra, 2000; France, Dijkstra & Dhanoa, 1996). Research on the characteristics of livestock growth also provides useful and practical information for breeding purposes (Maruyama, Potts, Bacon & Nestor, 1998; Aggrey, 2004). Two important traits are the genetic potential for growth and the time to reach maturity. Successful determination of various growth parameters is important when selecting animals at early phases of their growth by using parameter predictions. Certain authors have reported that growth curve parameters can be used as direct breeding criteria in improving some of the associated traits in addition to describing growth in animals (Akbaş, 1996; Lawrence & Fowler, 2002; Landgraft et al., 2002). The growth curve for describing live weight is usually of sigmoidal shape, with small but increasing gains at the beginning, acceleration up to a certain age (inflection point), followed by decreasing gains as weight reaches its maximum. Modelling animal growth has been a topic of noticeable interest over the past fifty years. Traditionally, mathematical equations, usually referred to as growth functions, have been used to relate body weight (BW) to age

or cumulative feed intake (Fitzhugh, 1976; Darmani Kuhi, Kebreab, López, & France, 2002; Darmani Kuhi, Kebreab, López, & France, 2003a,b; Darmani Kuhi, Kebreab, López, & France, 2004; Porter et al., 2010).

Because of the relatively long growing cycle and the high cost of research on production and nutrition, the potential benefits from modelling growth in this avian species are noteworthy (Firman, 1994). Though there are many studies aimed to evaluate growth models in animals, the number of studies targeting growth models in turkeys is quite limited compared to other poultry species (Ersoy, Mendeş, Geflügelk & Keskin, 2007). The objective of the present study is to introduce a new sinusoidal function into poultry science by applying it to temporal growth data from turkeys, and comparing its fitting performance with that of four standard growth functions, viz. the Gompertz, logistic, Lopez and Richards.

Material and methods

Growth functions

The functions used to describe the growth curves of turkeys are presented in Table 1. The Gompertz, logistic, Lopez, Richards and sinusoidal equations were fitted to the data to model the relationship between body weight and age.

Data source and statistical analysis

Five time course profiles (Table 2) from the Management Handbook of Aviagen (2013) were used in this study to investigate the relationship between BW and age in different male strains of turkeys.

Statistical analyses were performed using the non-linear procedure of MATLAB 7.13.0 and the Gauss–Newton algorithm. Comparison of models was carried out by analyzing model behaviour when fitting the curves using nonlinear regression and assessing statistical performance. The maximized log-likelihood (MLE), estimated error variance (MSE), Akaike information criterion (AIC) and Bayesian information criterion (BIC) were used to evaluate the general goodness-of-fit of each model to the different data profiles.

Table 1. Properties of the growth functions considered.

Growth function	Functional form*	Time at inflexion point (t^*)	Weight at inflexion point (W^*)
Gompertz	$W = W_0 \exp \left[\left(\ln \frac{W_f}{W_0} \right) (1 - e^{-bt}) \right]$	$\frac{1}{b} \ln \left(\ln \frac{W_f}{W_0} \right)$	$0.368 W_f$
Logistic	$W = \frac{W_0 W_f}{W_0 + (W_f - W_0) e^{-bt}}$	$\frac{1}{b} \ln \left(\frac{W_f - W_0}{W_0} \right)$	$0.5 W_f$
Lopez	$W = \frac{(W_0 K^n + W_f t^n)}{(K^n + t^n)}$	$K \left[\frac{n-1}{n+1} \right]^{1/n}$	$\left[\left(1 + \frac{1}{n} \right) W_0 + \left(1 - \frac{1}{n} \right) W_f \right]$
Richards	$W = \frac{W_0 W_f}{[W_0^n + (W_f^n - W_0^n) \exp(-bt)]^{1/n}}$	$\frac{1}{b} \ln \left(\frac{W_f^n - W_0^n}{n W_0^n} \right)$	$\frac{2 W_f}{(n+1)^{1/n}}$
Sinusoidal	$W = a_1 + a_2 [\sin(ct - \theta)]^n$	$\left \frac{\theta}{c} \right $	a_1

* W = body weight, W_0 = initial weight, W_f = asymptotic weight; t = months of age, b , K , and n are parameters that define the position, scale and shape of the growth curve. For the sinusoidal, a_1 is the vertical offset (height of the baseline or weight at inflexion), a_2 is the amplitude (the height of each peak above the baseline), θ/c is the phase shift or time at inflexion and n is a parameter [for more details see Figure 1 and Darmani Kuhi et al. (2018)].

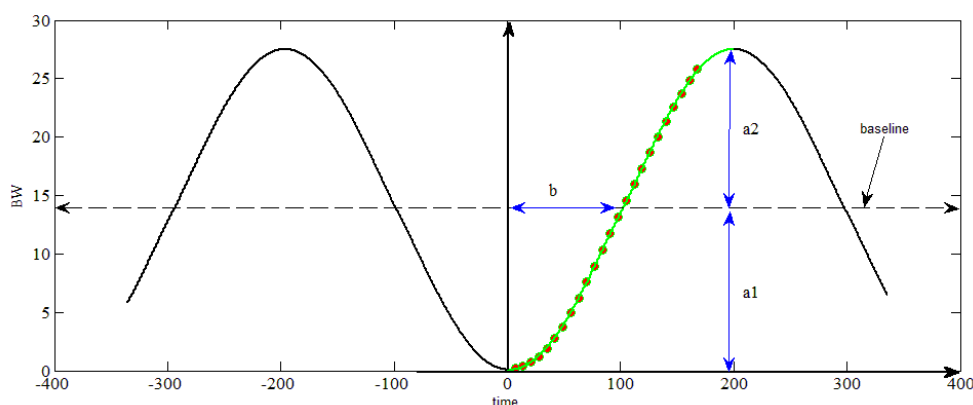


Figure 1. Graph of the sinusoidal function showing its fit to the data of B.U.T.6 (Male). a_1 is the height of the baseline; a_2 is the height of each peak above the baseline; θ/c (denoted as b) is the horizontal offset of the base point where the curve crosses the baseline as it ascends from initial weight.

Table 2. Time course profiles for male turkeys (Management Handbook of Aviagen, 2013) used in this study.

Strain	B.U.T 6	B.U.T Premium	B.U.T 10	Nicholas 300	Bronze
Age (d)	Live weight (kg)				
7	0.18	0.17	0.17	0.17	0.17
14	0.39	0.38	0.36	0.37	0.36
21	0.73	0.70	0.65	0.67	0.66
28	1.22	1.17	1.07	1.11	1.11
35	1.90	1.80	1.64	1.69	1.72
42	2.75	2.61	2.35	2.42	2.48
49	3.77	3.57	3.20	3.29	3.40
56	4.94	4.67	4.16	4.28	4.44
63	6.22	5.86	5.21	5.36	5.57
70	7.57	7.11	6.31	6.50	6.76
77	8.96	8.39	7.45	7.67	7.98
84	10.36	9.68	8.59	8.85	9.20
91	11.76	10.96	9.74	10.03	10.42
98	13.16	12.23	10.88	11.21	11.63
105	14.55	13.51	12.03	12.39	12.84
112	15.95	14.80	13.19	13.58	14.07
119	17.33	16.09	14.35	14.78	15.29
126	18.70	17.38	15.51	15.98	16.52
133	20.04	18.64	16.65	17.15	17.72
140	21.33	19.87	17.76	18.29	18.88
147	22.56	21.04	18.83	19.39	20.00
154	23.72	22.16	19.85	20.44	21.06
161	24.81	-	-	-	-
168	25.82	-	-	-	-

Results and discussion

The estimated parameters for the five equations are given in Table 3 and equation behaviour is illustrated in Figure 2.

Table 3. Estimated parameters for the given data profiles obtained using the different growth functions.

Function*	W_0	W_f	b	K	n	a_1	a_2	c	θ
B.U.T 6									
Gompertz	0.215	34.26	0.017	-	-	-	-	-	-
Logistic	0.814	27.60	0.034	-	-	-	-	-	-
Richards	0.015	41.05	0.011	-	-0.314	-	-	-	-
Lopez	0.061	46.22	-	150.7	2.153	-	-	-	-
Sinusoidal	0.077	27.52	-	-	1.058	13.681	13.839	-0.016	1.539
B.U.T premium									
Gompertz	0.200	31.29	0.017	-	-	-	-	-	-
Logistic	0.693	24.24	0.036	-	-	-	-	-	-
Richards	0.005	41.36	0.010	-	-0.364	-	-	-	-
Lopez	0.032	45.59	-	158.7	2.087	-	-	-	-
Sinusoidal	0.149	25.02	-	-	1.072	12.418	12.605	-0.016	1.518
B.U.T 10									
Gompertz	0.190	28.51	0.017	-	-	-	-	-	-
Logistic	0.630	21.86	0.036	-	-	-	-	-	-
Richards	0.010	37.91	0.010	-	0.356	-	-	-	-
Lopez	0.052	42.22	-	163.7	2.071	-	-	-	-
Sinusoidal	0.146	22.84	-	-	1.067	11.362	11.480	-0.016	1.527
Nicholas 300									
Gompertz	0.195	29.35	0.017	-	-	-	-	-	-
Logistic	0.648	22.51	0.036	-	-	-	-	-	-
Richards	0.011	38.88	0.010	-	-0.353	-	-	-	-
Lopez	0.055	43.39	-	163.4	2.073	-	-	-	-
Sinusoidal	0.149	23.51	-	-	1.066	11.699	11.817	-0.016	1.527
Bronze									
Gompertz	0.190	29.73	0.017	0.000	-	-	-	-	-
Logistic	0.660	23.04	0.036	0.000	-	-	-	-	-
Richards	0.005	39.30	0.010	-	-0.364	-	-	-	-
Lopez	0.032	43.31	-	158.6	2.087	-	-	-	-
Sinusoidal	0.144	23.77	-	-	1.072	11.798	11.976	-0.016	1.518

* W_0 is initial weight, W_f is final weight, and b, K, n, a_1, a_2, c and θ are parameters.

The predicted values for initial weight and the behaviour of the model in fitting the data (Table 3 and Figure 2) indicated that the logistic equation was inadequate. The logistic showed a trend to overestimate initial weights for all data sources. The trend for the Richards was underestimation of initial weights. The W_0 values for the Lopez were close to the expected initial average BW. For final (asymptotic) BW (W_f), there were magnitude differences between the different functions. Estimates of final body weight with the Lopez and Richards were higher than with the Gompertz, logistic and sinusoidal equations and appeared to be overestimates. The differences between functions with respect to growth rate, maturation rate and relative growth rate reflect existing differences among the functions with respect to their abilities to fit the data.

In general, a comparison between models based on the calculated statistical criteria (Tables 4 and 5) indicated some relevant differences between functions. The logistic equation gave higher values of these statistics than the other growth functions. These statistical criteria clearly demonstrate the suitability and superiority of the sinusoidal, Lopez and Richards equations over the others.

Growth curves are critical for the understanding and formulation of breeding programs because they shift in response to selection. Nonlinear functions have been used extensively to represent changes in size with age, so that the genetic potential of animals for growth can be evaluated (Ozoje, Peters, Caires & Kizilkaya, 2015).

Early estimation of weight at maturity and growth rate relative to body size can be of importance for selection purposes, given their association with other traits and the economy of production (Butts, Backus, Lidvall, Corrick & Montgomery, 1980; Butts, Lidvall, Backus & Corrick, 1980; Tawah and Franke, 1985). Rate of maturing, rate of gain and mature size are directly related to the economics of production and as such are important traits which have attracted the attention of breeders and livestock scientists. Exploitation of these parameters in growth models through curve fitting using live-weight-age data could improve economic returns positively (Salako, 2014).

Comparison of the growth functions based on their behaviour (Figure 2) showed that, with exception of the logistic, the other functions gave a suitable fit to the data profiles. Here, the interesting choice lies between the sinusoidal and Richards equations. Based on maximized log-likelihood, MSE, AIC and BIC criteria and depending on the strain, the sinusoidal equation showed superiority over the other growth functions (Tables 3 and 4).

Table 4. Goodness-of-fit of the growth functions to the turkey data profiles.

Strain	Statistical criterion [*]	Gompertz	Logistic	Lopez	Richards	Sinusoidal
B.U.T 6	MLE	16.60	-39.25	73.10	78.34	54.40
	MSE	0.0293	0.3004	0.0028	0.0022	0.0061
	AIC	-19.20	92.50	-128.2	-138.7	-86.81
	BIC	-28.35	83.35	-140.0	-150.4	-101.2
B.U.T premium	MLE	15.81	-29.64	57.50	63.33	70.23
	MSE	0.0285	0.2253	0.0043	0.0033	0.0024
	AIC	-17.61	73.28	-96.99	-108.7	-118.5
	BIC	-26.76	64.14	-108.8	-120.4	-132.8
B.U.T 10	MLE	22.65	-23.68	67.82	72.39	82.36
	MSE	0.0209	0.1718	0.0027	0.0022	0.0014
	AIC	-31.29	61.36	-117.6	-126.8	-142.7
	BIC	-40.44	52.21	-129.4	-138.5	-157.1
Nicholas	MLE	21.72	-24.87	67.08	71.50	82.42
	MSE	0.0218	0.1813	0.0028	0.0022	0.0014
	AIC	-29.43	63.74	-116.2	-125.0	-142.8
	BIC	-38.58	54.59	-127.9	-136.8	-157.2
Bronze	MLE	18.03	-27.40	59.70	65.53	72.80
	MSE	0.0258	0.2035	0.0039	0.0030	0.0021
	AIC	-22.06	68.81	-101.4	-113.1	-123.6
	BIC	-133.5	-10.93	-228.5	-221.0	-159.8

*MLE = the maximized log-likelihood estimate, MSE = mean squared error, AIC = Akaike information criterion and BIC = Bayesian information criterion.

Table 5. Comparison between the general goodness-of-fit of the models to the turkeys data profiles based on various statistical criteria^{*}.

Compared models	Statistical criterion [†]			
	MLE	MSE	AIC	BIC
Sinusoidal vs. Gompertz	100	100	100	100
Sinusoidal vs. logistic	100	100	100	100
Sinusoidal vs. Lopez	80	80	80	60
Sinusoidal vs. Richards	80	80	80	60

^{*}Numbers in the table are the percentage of cases in which fits of the sinusoidal equation to data were superior to the equation according to the criteria specified in the columns. [†]MLE = the maximized log-likelihood estimate, MSE = mean squared error, AIC = Akaike information criterion and BIC = Bayesian information criterion.

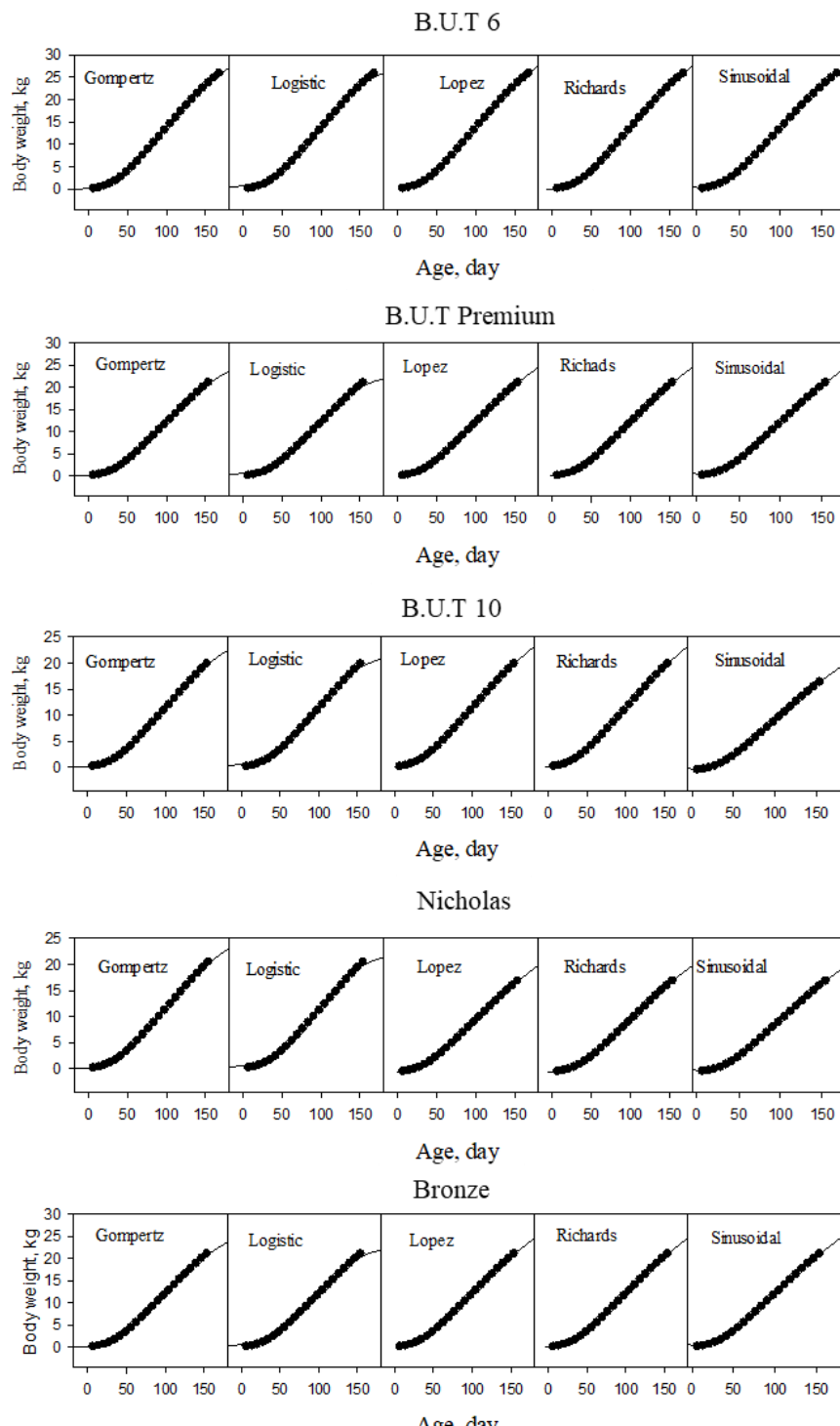


Figure 2. Plots of live weight (g) against age (d) showing the fit of the different growth functions to the turkey data profiles.

Conclusion

In conclusion, comparison of the growth functions in terms of goodness of fit criteria revealed that flexible growth functions (e.g. the sinusoidal equation) were the most appropriate functions to describe the age-related changes in body weight in turkeys. This result is especially important when the behaviour of a particular data set is not defined previously (Darmani-Kuhi et al., 2003; Beiki, Pakdel, Moradi-shahrabak & Mehrban, 2013). Nevertheless, selection of the best function requires special attention to characterize the growth patterns of animals raised under different environmental conditions (Narinc, Emre, Mehmet & Tulin, 2010). Therefore, it seems timely to compare the fit of different functions before selecting the one which performs most accurately.

References

- Aggrey, S. E. (2004). Modelling the effect of nutritional status on pre-asymptotic and relative growth rates in a random-bred chicken population. *Journal of Animal Breeding and Genetics*, 121(4), 260–268. doi: 10.1111/j.1439-0388.2004.00462.x
- Antony, N. B., Emmerson, D. A., Nestor, K. E., & Bacon, W. L. (1991). Comparison of growth curves of weight selected populations of turkeys, quail and chickens. *Poultry Science*, 70, 13–19. doi: 10.3382/ps.0700013
- Aviagen. (2013). *Ross PS Management Handbook*. Recovered from: <http://en.aviagen.com/assets/Uploads/RossPSHandbook2013i.pdf>.
- Beiki, H., Pakdel, A., Moradi-Shahrbabak, M., & Mehrban, H. (2013). Evaluation of growth functions on Japanese quail lines. *Journal of Poultry Science*, 50, 20–27. doi: 10.2141/jpsa.0110142
- Butts, W. T. Jr., Backus, W. R., Lidvall, E. R., Corrick, J. A., & Montgomery, R. F. (1980a). Relationships among definable characteristics of feeder calves, subsequent performance and carcass traits. I. Objective measurements. *Journal of Animal Science*, 51, 1297–1305. doi: 10.2527/jas1981.5161297x
- Butts, W. T. Jr., Lidvall, E. R., Backus, W. R., & Corrick, J. A. (1980b). Relationships among definable characteristics of feeder calves, subsequent performance and carcass traits. II. Subjective scores. *Journal of Animal Science*, 51, 1306–1313. doi: 10.2527/jas1981.5161297x
- Darmani Kuhi, H., Kebreab, E., López, S., & France, J. (2002). A derivation and evaluation of the von Bertalanffy equation for describing growth in broilers over time. *Journal of Animal and Feed Sciences*, 11(1), 109–125. doi: 10.22358/jafs/67795/2002
- Darmani Kuhi, H., Kebreab, E., López, S., & France, J. (2003a). A comparative evaluation of functions for the analysis of growth in male broilers. *Journal of Agricultural Science*, 140, 451–459. doi: 10.1017/S0021859603003149
- Darmani Kuhi, H., Kebreab, E., López, S., & France, J. (2003b). An evaluation of different growth functions for describing the profile of live weight with time (age) in meat and egg strains of chicken. *Poultry Science*, 82, 1536–1543. doi: 10.1093/ps/82.10.1536
- Darmani Kuhi, H., Kebreab, E., López, S., & France, J. (2004). A comparative evaluation of functions for describing the relationship between live-weight gain and metabolizable energy intake in turkeys. *Journal of Agricultural Science*, 142, 691–695. doi: 10.1017/S0021859605004880
- Darmani Kuhi, H., Shabanpour, A., Mohit, A., Falahi, S., & France, J. (2018). A sinusoidal function and the Nelder-Mead simplex algorithm applied to growth data from broiler chickens. *Poultry Science*, 97(1), 227–235. doi: 10.3382/ps/pex299
- Ersoy, İ. E., Mendeş, M., & Aktan, S. (2006). Growth curve establishment for American Bronze turkeys. *Archives Animal Breeding*, 49(3), 293–299. doi: 10.5194/aab-49-293-2006
- Ersoy, İ. E., Mendeş, M., Geflügelk, A., & Keskin, S. (2007). Estimation of parameters of linear and nonlinear growth curve models at early growth stage in California Turkeys. *Archiv für Geflügelkunde*, 71(4), S. 175–180.
- Firman, J. D. (1994). Turkey growth modelling: metabolic approach. *Journal of Applied Poultry Research*, 3, 373–378. doi: 10.1093/japr/3.4.373
- Fitzhugh, H. A. (1976). Analysis of growth curves and strategies for altering their shape. *Journal of Animal Science*, 42, 1036–1051. doi: 10.2527/jas1976.4241036x
- France, J., Dijkstra, J., & Dhanoa, M. S. (1996). Growth functions and their application in animal science. *Annales de Zootechnie*, 45: 165–174. doi: 10.1051/animres:19960637
- Gompertz, B. (1825). On the nature of the function expressive of the law of human mortality, and on a new method of determining the value of life contingencies. *Philosophical Transactions of the Royal Society*, 36, 513–585. doi: 10.1098/rstl.1825.0026
- Hurwitz, S., Talpaz, H., Bartov, I., & Plavnik, I. (1991). Characterization of growth and development of male British United turkeys. *Poultry Science*, 70, 2419–2424. doi: 10.3382/ps.0702419
- Landgraft, S., Roehe, R., Susenbeth, A., Baulain, U., Knaph, P. W., Plastov, G. S., & Kalm E. (2002). Biological growth model as a new selection strategy for improvement of feed efficiency in swine. *Veterinarija ir Zootechnika*, 18(40), 1392–2130.

- Laudadio, V., Tufarelli, V., Dario, M., D'Emilio, F. P., & Vicenti, A. (2009). Growth performance and carcass characteristics of female turkeys as affected by feeding programs. *Poultry Science*, 88, 805–810. doi: 10.3382/ps.2008-00082
- Lawrence, T. L. J., & Fowler, V. R. (2002). *Growth of Farm Animals* (2nd ed.). Wallingford, UK: CAB International.
- López, S., France, J., Dhanoa, M. S., Mould, F., & Dijkstra, J. (2000). A generalized Michaelis-Menten equation for the analysis of growth. *Journal of Animal Science*, 78, 1816–1828. doi: 10.2527/2000.7871816x
- Maruyama, K., Potts, W. J. E., Bacon, W. L., & Nestor, K. E. (1998). Modeling turkey growth with the relative growth rate. *Growth, Development, and Aging*, 62(4), 123–139.
- Narinc, D., Emre, K., Mehmet, Z. F., & Tulun, A. (2010). Comparison of non-linear growth functions to describe the growth in Japanese quail. *Journal of Animal and Veterinary Advances*, 9, 1961–1966. doi: 10.3923/javaa.2010.1961.1966
- Nestor, K. E., Anderson, J. W., Hartzler, D., & Velleman, S. G. (2005). Genetic variation in pure lines and crosses of large-bodied turkeys. 4. Body shape and carcass traits. *Poultry Science*, 84, 1825–1834. doi: 10.1093/ps/84.12.1825
- Ozoje, M. O., Peters, S. O., Caires, K. C., & Kizilkaya, K. (2015). *Growth curve analyses of three turkey genotypes in the hot humid tropics using a Bayesian mixed model approach*. Orlando, FL: ADAS-ASAD Joint Annual meeting.
- Porter, T., Kebreab, E., Darmani Kuhi, H., López, S., Strathe, A. B., & France, J. (2010). Flexible alternatives to the Gompertz equation for describing growth with age in turkey hens. *Poultry Science*, 89, 371–378. doi: 10.3382/ps.2009-00141
- Richards, F. J. (1959). A flexible growth function for empirical use. *Journal of Experimental Botany*, 10, 290–300. doi: 10.1093/jxb/10.2.290
- Roberson, K. D., Rahn, A. P., Balander, R. J., Orth, M. W., Smith, D. M. V., Booren, B. L., Booren, A. M., Osburn, W. N., & Fulton, R. M. (2003). Evaluation of the growth potential, carcass components and meat quality characteristics of three commercial strains of tom turkeys. *Journal Applied Poultry Research*, 12, 229–236. doi: 10.1093/japr/12.2.229
- Salako, A. E. (2014). Asymptotic nonlinear regression models for the growth of White Fulani and N'dama cattle in Nigeria. *Livestock Research for Rural Development*, 26(5), Article #91. <http://www.lrrd.org/lrrd26/5/sala26091.html>
- Tawah, L. C., & Franke, D. E. (1985). Growth parameters and reproduction in purebred and crossbred beef cattle. *Journal of Animal Science*, 61(1), 8.
- Thornley, J. H. M., & France, J. (2007). *Mathematical models in agriculture: Quantitative methods for the plant, animal and ecological sciences* (2nd ed.). Wallingford, UK: CAB International.
- Von Bertalanffy, L. (1957). Quantitative laws for metabolism and growth. *Quarterly Review of Biology*, 32, 217–231. doi: 10.1086/401873