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Matemática

A new version of energy for involute of slant helix with bending energy in the Lie groups

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Abstract:

In this paper, we study energy of involute curves for slant helix in the Lie group. With this new representation, we illistrate some figures of energy by elastica. Finally, we have an original and satisfactorily connection between energy of the curve on Lie groups.
Keywords: energy, lie group, involute curves.

Introduction

The innovation that Lie group brings to mathematics is that it has three different structures of mathematical form that enable us set a connection between these different forms. Primarily, it has structure of group. Further, the elements belonging to this group form a topological space such that it can be defined as being a certain case of a topological group. Lastly, the elements also form an analytic manifold (Catmull & Clark, 1978; Crouch & Silva, 1995; Capovilla, Chryssomalakos, & Guven, 2002;).

Lie groups play a key role not only in physical systems also in mathematical studies such as loop groups, gauge groups, and Fourier integral's groups operators that occur as symmetry groups and phase spaces (Esprito, Fornari, Frensel, & Ripoll, 2003). Lie groups are also useful in mechanics (Milnor, 1976;). Since incomprehensible inviscid fluid motion and rigid body motion correspond to geodesic flow of left (or right) invariant metric defined on a Lie group (Arnold, 1966; Kolev, 2004).

The study of computing an energy of given vector field depending on the structure of the geometrical spaces has earned such attention in the last couple years. It has been shown that these types of computations have numerous applications in various fields and thus multidisciplinary subjects have been evolved. For instance, Wood (Wood, 1997) studied on the unit vector field's energy firstly. Gil-Medrano (Gil, 2001) worked on relation between energy and volume of vector fields. (Chacon, Naveira, & Weston, 2001; Chacon & Naveira, 2004) investigated on the energy of distrubutions and corrected energy of distrubutions on Riemannian manifolds. Altin computed energy of a Frenet vector fields for a given nonlightlike curves (Altin, 2011). Körpınar discussed timelike biharmonic particle's energy in Heisenberg spacetime (Körpınar, 2014). Also, curves and its flows are researched simply by various experts (Asil, 2007; Turhan & Altay, 2014; Turhan & Altay, 2015; Yeneroğlu, 2016).

The corresponding theory for the energy of curvature-based energy is considered to be at its early stages of evolution. Some of the prolific fields and pioneering studies for this theory can be found in mathematical physics, membrane chemistry, computer aided geometric design and geometric modeling, shell engineering, biology and thin plate (Kirchhoff, 1850; Roberts, Schleif, & Dlugosz, 2007; Weber, 1961). One of the well-

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known functional and related work is 'bending energy functional', which appeared firstly Bernoulli-Euler elastica formulation for energy (Euler, 1744; Einstein, 1920; Guven, Valencia, & Vazquez, 2014).

In this study we compute the energy of the particle lying on the 3-dimensional Lie group to investigate its connection between mass-energy and motion-energy concept which is a topic of special relativity (Einstein, 1905a; 1905b). Furthermore we gain a different perspective by calculating curve's energy on Lie groups to see the relation between energy of particle and curvature-based bending energy functional. The method we use for computing the energy of vector fields is that considering a vector field as a map from manifold M to the Riemannian manifold (TM, p_s) where TM is tangent bundle of a Riemannian manifold and p_s is a Sasaki metric induced from TM naturally.

We organize the manuscript by starting to state fundamental definitions and proposition for a Lie group. Then we recall interpretation of geometrical meaning of the energy for unit vector fields. Based on these relations we compute the energy of curves defined on the Lie group. Finally, we give examples about particle's energy for different cases by computing their value and drawing thier graph in Figure 1-6.

Material and methods

For a Lie group R with a bi-invariant metric $\langle\rangle$ and Levi-Civita connection ∇ of Lie group R if h shows the Lie algebra of R then we get isomorphism between h and TjR where j is natural element of R. Let $\langle\rangle$ be a bi-invariant metric on R then it is obtained, Equation 1:

$$\langle \mathbf{K}, [\mathbf{L}, \mathbf{Y}] \rangle = \langle [\mathbf{K}, \mathbf{L}], \mathbf{Y} \rangle \qquad (1)$$

(1)

and Equation 2

$$D_{\mathbf{K}} \mathbf{L} = \frac{1}{2} [\mathbf{K}, \mathbf{L}] \qquad (2)$$

(2)

for all $\mathbf{K}, \mathbf{L}, \mathbf{Y} \in h.$

For an arc-lenghted curve $\gamma : I \subset \mathrm{R} \to \mathbf{R}$ and orthonormal basis [K_1, K_2, ... K_n] of h it is written that $\mathbf{P} = \sum_{a=1}^{n} p_a \mathbf{K}_a$ and $\mathbf{Q} = \sum_{a=1}^{n} q_a \mathbf{K}_a$ for any two vector fields P and Q along the curve γ, where $p_a : I \to \mathrm{R}$ and $q_a = I \to \mathrm{R}$ are smooth fucntions. Let P and Q be any two vector fields then Lie bracket is written in the following form Equation 3:

$$[\mathbf{P}, \mathbf{Q}] = \sum_{a=1}^{n} p_a q_a [\mathbf{K}_a, \mathbf{K}_b] \qquad (3)$$

(3)

and for a vector field P the covariant derivative along the curve γ is stated by Equation 4:

$$D_{\gamma'} \mathbf{P} = \mathbf{P}' + \frac{1}{2} [\mathbf{T}, \mathbf{P}], \qquad (4)$$

(4)

where:

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$\mathbf{P}' = \sum_{a=1}^{n} \frac{d\mathbf{p}}{dt}\mathbf{K}_a$ and $\mathbf{T} = \gamma'$ and (Ciftci, 2009). Frenet apparatus of the curve γ can be represented by elemets $(\mathbf{T},\mathbf{N},\mathbf{B},\kappa,\tau)$ for a 3-dimensional Lie group R.

Definition 2.1: For a parametrized curve $\gamma : I \subset \mathbf{R} \to \mathbf{R}$ and Frenet apparatus $(\mathbf{T},\mathbf{N},\mathbf{B},\kappa,\tau)$ we have Equation 5:

$$\boldsymbol{\tau}_{\mathbf{R}} = \frac{1}{2}\langle [\mathbf{T},\mathbf{N}], \mathbf{B} \rangle \quad (5)$$

(5)

or equivalently Equation 6:

$$\boldsymbol{\tau}_{\mathbf{R}} = \frac{1}{2\kappa^2\tau}\left\langle \mathbf{T}'', [\mathbf{T},\mathbf{T}'] \right\rangle + \frac{1}{4\kappa^2\tau}\left\| [\mathbf{T},\mathbf{T}'] \right\|^2. \quad (6)$$

(6)

For a parametrized arc-lenghted curve $\gamma : I \subset \mathbf{R} \to \mathbf{R}$ in 3-dimensional Lie group. If $\gamma'(s)$, $\gamma''(s)$, $\gamma'''(s)$, $\gamma''''(s)$ are linearly dependent for all $s \in I$ then it is said that γ is Frenet curve of osculating order three. It can be constructed to an orthonormal Frenet frame for each Frenet curve of order three in the following way Equation 7:

$$\begin{aligned} \nabla_{\mathbf{T}}\mathbf{T} &= \kappa\mathbf{N} \\ \nabla_{\mathbf{T}}\mathbf{N} &= (\tau - \tau_{\mathbf{R}})\mathbf{B} - \kappa\mathbf{T} \quad (7) \\ \nabla_{\mathbf{T}}\mathbf{B} &= (\tau_{\mathbf{R}} - \tau)\mathbf{N}, \end{aligned}$$

(7)

where:

Lie group R has the Levi-Civita connection ∇.

Proposition 2.2: For a 3-dimensional Lie group Rinduced with a bi-invariant metric we have following statements that can be obtained for different Lie groups (Ciftci, 2009), Equation 8:

$$\begin{aligned} &(\mathbf{i})\ \tau_{\mathbf{R}} = 0 \ \text{ if R is Abelian group.} \\ &(\mathbf{ii})\ \tau_{\mathbf{R}} = 1 \ \text{ if R is } SU^2. \quad (8) \\ &(\mathbf{iii})\ \tau_{\mathbf{R}} = \frac{1}{2} \ \text{ if R is } SO^3. \end{aligned}$$

(8)

Results and discussion

Energy on the frenet vector field

Definition 3.1.: Let $\alpha : I \subset R \to \mathbf{R}$ be an arc length parametrized curve. Then α is called a slant helix if its principal normal vector makes a constant angle with a left-invariant vector field X which is unit length (Okuyucu, Gök, Yaylı, & Ekmekçi, 2013).

Definition 3.2.: Let $\alpha : I \subset R \to \mathbf{R}$ be an arc length parametrized curve with the Frenet apparatus $(\mathbf{T},\mathbf{N},\mathbf{B},\kappa,\tau)$. Then the harmonic curvature function of the curve α is defined by Equation 9:

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$$H = \frac{\tau - \tau_{G}}{\kappa}, \qquad (9)$$

(9)

where:

$\tau_{G} = \frac{1}{2} g([\mathbf{T}, \mathbf{N}], \mathbf{B}).$

Proposition 3.3.: If the curve a is a slant helix in R, then the axis of α is Equation 10:

$$\mathbf{X} = [\frac{H\kappa(1+H^2)}{H'}\mathbf{T} + \mathbf{N} + \frac{\kappa(1+H^2)}{H'}\mathbf{B}]\cos\varpi, \qquad (10)$$

(10)

where:

$\varpi \neq \frac{\pi}{2}$ is a constant angle.

Definition 3.4.: For two Riemannian manifolds (M, ρ) and $(N, \tilde{h})$ the energy of a differentiable map $f:(M,\rho) \to (N, \tilde{h})$ can be defined as Equation 11:

$$\varepsilon nergy(f) = \frac{1}{2}\int_M \sum_{a=1}^{n} \tilde{h}(df(e_a), df(e_a))v, \qquad (11)$$

(11)

where:

$\{e_a\}$ is a local basis of the tangent space and v is the canonical volume form in M (Altin, 2011).

Let T^1M be the unit tangent bundle endowed with the restriction of the Sasaki metric on TM. Then the energy of a unit vector field X is defined to be the section's energy of $X: M \to T^1M$. For the bundle projection $\omega: T^1M \to M$, vertical/horizontal splitting induced by the Levi-Civita connection can be stated as $T(T^1M) = V \oplus H$. Further, we write $TM = F \oplus G$ where F shows the line bundle generated by X and G is the orthogonal complement.

Proposition 3.5.: Let $Q: T(T^1M) \to T^1M$ be the connection map. Then following two conditions hold:

i) $\omega \circ Q = \omega \circ d\omega$ and $\omega \circ Q = \omega \circ \tilde{\omega}$, where $\tilde{\omega}: T(T^1M) \to T^1M$ is the tangent bundle projection;

ii)for $\rho \in T_xM$ and a section $\xi: M \to T^1M$; we have Equation 12:

$$Q(d\xi(\rho)) = \nabla_{\rho}\xi, \qquad (12)$$

(12)

where:

$\nabla$is the Levi-Civita covariant derivative (Chacon & Naveira, 2004).

Definition 3.6.: For $\varsigma_1, \varsigma_2 \in T_{\xi}(T^1M)$, we define Equation 13:

$$\rho_S(\varsigma_1, \varsigma_2) = \rho(d\omega(\varsigma_1), d\omega(\varsigma_2)) + \rho(Q(\varsigma_1), Q(\varsigma_2)). \qquad (13)$$

(13)

This yields a Riemannian metric on TM: As we know p_s is called the Sasaki metric that also makes the projection $\omega: T^1M \to M$ a Riemannian submersion.

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Definition 3.7.: Angle is known as the angle between arbitrary Frenet vectors given any curve K. For an initial point the angle between Frenet vectors can be stated with the help of the curvature function of the curve K as the following Equation 14:

$$A_i = \int_{\vartheta}^{s} \left\| \nabla_{\mathbf{V}_1} \mathbf{V}_i \right\| du, \qquad (14)$$

(14)

where:

V_i represents Frenet vector.

Energy of spherical images in lie groups and bending energy functional

In the theory of relativity, all the energy moving through an object contributes to the body's total mass that measures how much it can resist to acceleration. Each kinetic and potential energy makes a highly proportional contribution to the mass (Carmelli, 1965). In this study not only we compute the energy of surface curves but we also investigate its close correlation with bending energy of elastica which is a variational problem proposed firstly by Daniel Bernoulli to Leonard Euler in 1744. Euler elastica bending energy formula for a space curve in the 3-dimensional Frenet curvature along the curve is known as Equation 15:

$$H_B = \frac{1}{2}\int \kappa^2 ds. \qquad (15)$$

(15)

Definition 4.1.: Let α be an arc-lengthed regular curve in R. Then the curve μ is called the involute of the curve α if the tangent vector field of the curve α is perpendicular to the tangent vector field of the curve μ. That is $<\mathbf{T},\mathbf{T}_\mu>=0$ where T and T_u are the tangent vector fields of the curves α and μ, respectively.

Theorem 4.2.: Let α be an arc-lengthed regular curve in R and μ be an involute of α. Then α is a slant helix in a three dimensional Lie group if and only if μ is a general helix.

Theorem 4.3.: Energy of Frenet vectors by using Sasaki metric is stated by Equation 16 a t 20 .

$$\varepsilon nergy(\mathbf{T}_\mu) = \frac{1}{2}\int_0^{s^*} (1 + (\tau - \tau_{\mathbf{R}})^2 + \kappa^2) ds, \qquad (16)$$

(16)

$$\varepsilon nergy(\mathbf{N}_\mu) = \frac{1}{2}\int_0^{s^*} (1 + ((\frac{1}{\sqrt{1+H^2}})')^2 + (\frac{(\tau_{\mathbf{R}} - \tau)H}{\sqrt{1+H^2}} \qquad (17)$$

(17)

$$-\frac{\kappa}{\sqrt{1+H^2}})^2 + ((\frac{H}{\sqrt{1+H^2}})')^2) ds, \qquad (18)$$

(18)

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$$\varepsilon nergy(\mathbf{B}_\mu) = \frac{1}{2}\int_0^{s^*} (1 + ((\frac{H}{\sqrt{1+H^2}})')^2 + ((\frac{H\kappa}{\sqrt{1+H^2}}) \quad (19)$$

(19)

$$+ (\frac{1}{\sqrt{1+H^2}})(\tau_{\mathbf{R}} - \tau))^2 + ((\frac{1}{\sqrt{1+H^2}})')^2)ds. \quad (20)$$

(20)

Proof.: From definition tangent indicatrix, we have Equation 21 at 23:

$$\mathbf{T}_\mu(s^*) = \mathbf{N}(s), \quad (21)$$

(21)

$$\mathbf{N}_\mu(s^*) = -\frac{1}{\sqrt{1+H^2}}\mathbf{T} + \frac{H}{\sqrt{1+H^2}}\mathbf{B}, \quad (22)$$

(22)

$$\mathbf{B}_\mu(s^*) = \frac{H}{\sqrt{1+H^2}}\mathbf{T} + \frac{1}{\sqrt{1+H^2}}\mathbf{B}. \quad (23)$$

(23)

Lemma 4.4.: $\varepsilon nergy(\mathbf{T}_\mu) - H_B = \frac{1}{2}(s^* + \int_0^{s^*}(\tau - \tau_{\mathbf{R}})^2 ds).$

Final considerations

Now, we consider following results for involute of slant helix in Lie group.

i) Let R be an Abelian group. Thus we have $\tau_{\mathbf{R}} = 0$ and we get following graph respectively for the energy of Frenet fields.

ii) Let R be SU^2 . Thus we have $\tau_{\mathbf{R}} = 1$ and we get following graph respectively for the energy of Frenet fields.

iii) Let R be SO^3 . Thus we have $\tau_{\mathbf{R}} = \frac{1}{2}$ and we get following graph respectively for the energy of Frenet fields.

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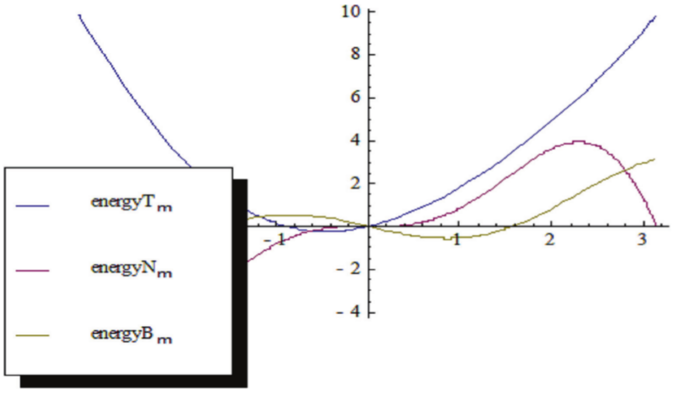

FIGURE 1.
Energy in Abelian group.

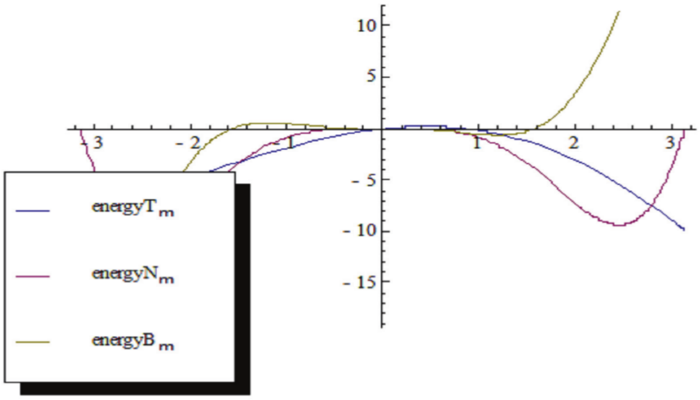

FIGURE 2.
Energy in SU^2.

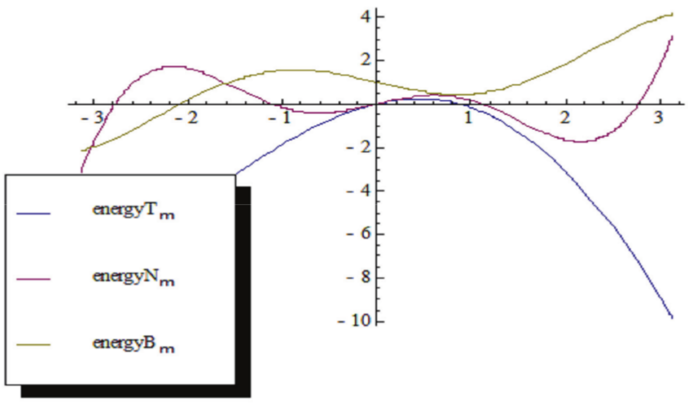

FIGURE 3.
Energy in SO3.

Corollary 4.5.: T_u, N_u, B_u have not constant energy in the Lie group R with a bi-invariant metric.

Theorem 4.6.: Angle of Frenet vectors can be respectively given by using Def. 3.4 as Equation 24 a t 28 :

$$A_1 = \int_0^{s^*} ((\tau - \tau_{\mathbf{R}})^2 + \kappa^2)^{\frac{1}{2}} ds, \quad (24)$$

(24)

$$A_2 = \int_0^{s^*} (((\frac{1}{\sqrt{1+H^2}})')^2 + (\frac{(\tau_{\mathbf{R}} - \tau)H}{\sqrt{1+H^2}} \quad (25)$$

(25)

$$- \frac{\kappa}{\sqrt{1+H^2}})^2 + ((\frac{H}{\sqrt{1+H^2}})')^2)^{\frac{1}{2}} ds. \quad (26)$$

(26)

$$A_3 = \int_0^{s^*} (((\frac{H}{\sqrt{1+H^2}})')^2 + ((\frac{H\kappa}{\sqrt{1+H^2}}) \quad (27)$$

(27)

$$+ (\frac{1}{\sqrt{1+H^2}})(\tau_{\mathbf{R}} - \tau))^2 + ((\frac{1}{\sqrt{1+H^2}})')^2)^{\frac{1}{2}} ds. \quad (28)$$

(28)

Now, we consider following results for angle in Lie group.

i) Let R be an Abelian group. Thus we have $\tau_{\mathbf{R}} = 0$ and we get following graph respectively for the angle of Frenet fields.

ii) Let R be SU2. Thus we have $\tau_{\mathbf{R}} = 1$ and we get following graph respectively for theangle of Frenet fields.

iii) Let R be SO3. Thus we have $\tau_{\mathbf{R}} = \frac{1}{2}$ and we get following graph respectively for the angle of Frenet fields.

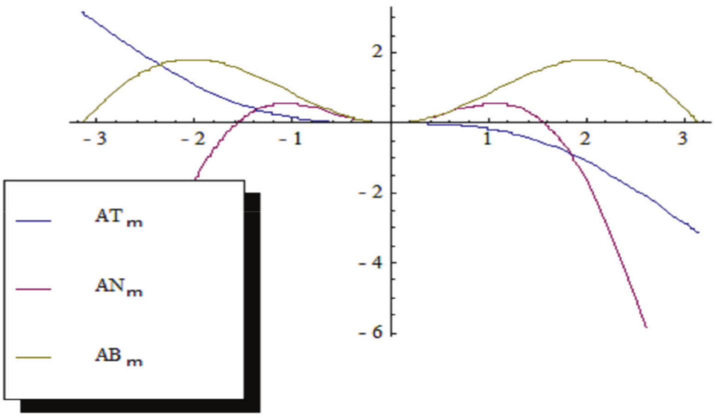

FIGURE 4.
Angle in Abelian group.

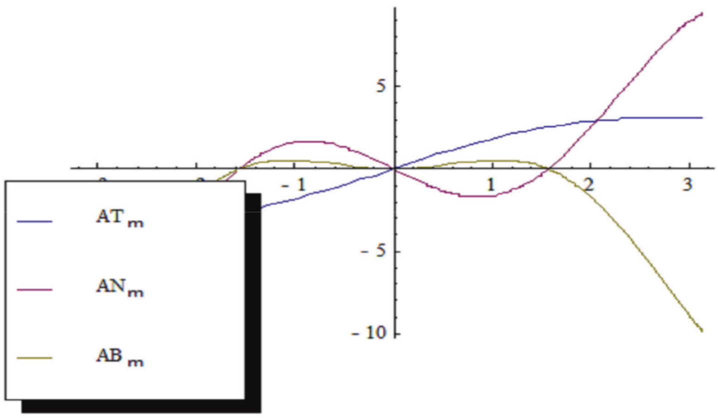

FIGURE 5.
Angle in SU^2 .

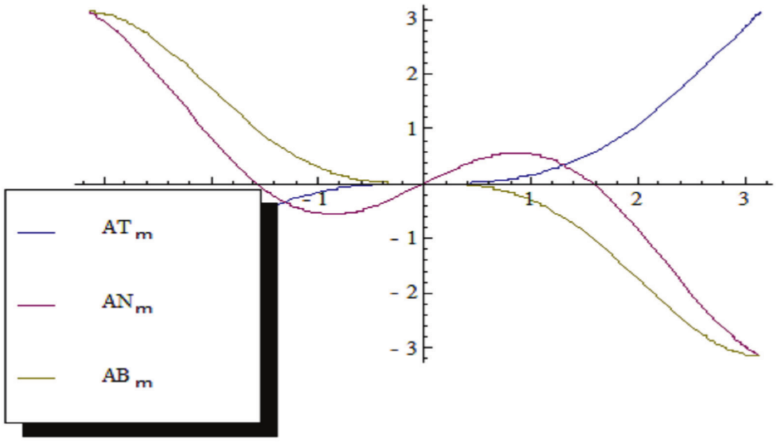

FIGURE 6.
Angle in SO^3 .

Conclusion

Lie groups and energy play an significant role in geometric design and theorical physics.

In this work, we study energy of involute curves for slant helix in the Lie group. With this new representation, we illistrate some figures of energy by elastica. Also, we have an original and satisfactorily connection between energy of the curve on Lie groups.

In the light of these results, we will study energy of magnetic curves in the Lie groups. We also aimed to obtain other useful and original results about the relationship between the spherical indicatrices of magnetic curves and its energy.

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