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Indispensability, causation and explanation\*

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# Indispensability, causation and explanation\*

Sorin BANGU

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**ABSTRACT:** When considering mathematical realism, some scientific realists reject it, and express sympathy for the opposite view, mathematical nominalism; moreover, many justify this option by invoking the causal inertness of mathematical objects. The main aim of this note is to show that the scientific realists' endorsement of this causal mathematical nominalism is in tension with another position some (many?) of them also accept, the doctrine of methodological naturalism. By highlighting this conflict, I intend to tip the balance in favor of a rival of mathematical nominalism, the mathematical realist position supported by the 'Indispensability Argument' —but I do this indirectly, by showing that the road toward it is not blocked by considerations from causation.

**Keywords:** Indispensability, Causation, Explanation, Nominalism, Realism, Causal Eliminativism.

**RESUMEN:** Al considerar el realismo matemático, algunos realistas científicos lo rechazan y expresan su simpatía por la aproximación opuesta, el nominalismo matemático; es más, muchos justifican esta opción invocando la inercia causal de los objetos matemáticos. El propósito principal de esta nota es mostrar que la adhesión de los realistas científicos a este nominalismo matemático causal entra en tensión con otra posición que algunos (¿muchos?) de ellos también aceptan, la doctrina del naturalismo metodológico. Al subrayar este conflicto, pretendo desequilibrar la balanza a favor de un rival del nominalismo matemático, la posición realista matemática que apoya el 'Argumento de la Indispensabilidad' —pero lo hago indirectamente, mostrando que el camino hacia esta posición no queda bloqueado por consideraciones sobre la causación—.

**Palabras clave:** Indispensabilidad, Causación, Explicación, Nominalismo, Realismo, Eliminativismo Causal.

It is silly to agree that a reason for believing that  $p$  warrants accepting  $p$  in all scientific circumstances, and then to add 'but even so it is not *good enough*'. Such a judgment could only be made if one accepted a trans-scientific method as superior to the scientific method.

H. Putnam (1971, 356)

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\* Some of the ideas in this paper constitute developments and refinements of suggestions I made in my 2012 book. I presented a version of the paper at the Annual Philosophy of Science Conference in Dubrovnik in 2013, and I thank that audience (especially Mark Colyvan and Nic Fillion) for constructive criticism. I'm grateful for comments on the most recent version to my audience at the workshop in Santiago de Compostela in Dec. 2016, in particular to Matteo Plebani, Xavier Donato, Otávio Bueno, Concha Martínez-Vidal, José Sagüillo, Susan Vineberg and José Luis Falguera.



## 1. Introduction

Without assuming any particular theory of causation, let us call *causal mathematical nominalism* (CMN) the kind of mathematical nominalism that denies ontological status to mathematical objects on the ground of their causal inertness.<sup>1</sup> Some scientific realists believe that their realism cohabitates well with mathematical nominalism —as Colyvan put it recently, this “marriage” is “a popular option amongst philosophers of science” (2006, 225)— and in many cases this nominalism is motivated by causal considerations. In this paper I claim that a scientific realist’s sympathy for CMN is in tension with another position often endorsed by scientific realists —*methodological naturalism*, where the version of the doctrine relevant here is traditionally associated with Quine. By highlighting this conflict, my intention is to mount an *indirect* defense of what is perceived as the most serious opponent of nominalism, the mathematical realist doctrine supported by the so-called ‘Quine (-Putnam) Indispensability Argument’.<sup>2</sup> To insist, the line of thinking I develop below is not meant to argue directly for indispensabilist mathematical realism; I only hope to show that it remains a viable option for a scientific realist.

In essence, I try to show that the conjunction of the following three positions is inconsistent (more precisely, that 1. and 3. taken together are in tension with 2.):

1. *Scientific* realism.
2. Causal *mathematical* nominalism (i.e., a form of mathematical antirealism).
3. Methodological naturalism.

Furthermore, I also gesture toward the idea that, given this tension, a reasonable option for the scientific realist is to drop 2., the nominalist component. This does open the door largely to becoming a mathematical realist indeed —although, as I said, my goal here is more modest than to (re)establish the Indispensability Argument (in any of its forms).<sup>3</sup> However, this seems to me a goal worth reaching, since indispensabilist mathematical realists keep hearing the refrain ‘no causal efficacy, no ontological rights’; thus, it is important to demonstrate that they have no reasons to worry about CMN.

The harmony of the marital relation between scientific realism and mathematical nominalism has preoccupied others before, and the papers by Saatsi (2007), Busch (2011) and Busch and Morrison (2015) also deserve special mention. Yet these works don’t tackle the issue in the way I intent to do it below. First, they don’t ask about the very motivation for adopting CMN, and in particular don’t discuss the ‘eliminativist’ idea I take up in section IV.2. Second, the subsequent discussion examines the *indispensabilist mathematical realist* options of a *scientific realist* who is, specifically, a *methodological naturalist*. I’m going to be very careful about the specifics of these positions (and the labels) here,

<sup>1</sup> I regard CMN as a metaphysical (ontological) position, and I emphasize this since this causal inertness has of course well-known epistemological implications; they are of secondary relevance here. See (Benacerraf 1973).

<sup>2</sup> H. Field, a prominent mathematical nominalist, counts the Indispensability Argument as “the most compelling argument that has been offered against the nominalist position”, and also as “the only non-question-begging argument” (1980, 4) for mathematical realism.

<sup>3</sup> That would be to *show* that 1 and 3, together with some other assumptions (see below), entail (a certain version of) the negation of 2.

since I grant that Busch and Morrison may be right to answer ‘no’ to the question they ask in their title ‘Should a Scientific Realist be a Platonist?’ —but note that I grant this under the assumption that ‘Platonist’ refers to *traditional* mathematical realism. Here, however, I will *not* discuss Platonism at all. In fact, I’ll consistently avoid using the label ‘Platonist’ altogether, because, unlike the authors above (and many others in fact), I take it that there are *significant* differences between (traditional) mathematical Platonism and (the contemporary) indispensabilist-naturalist mathematical realism. Although both realisms agree that mathematical entities are ‘abstract’, i.e., causally inert (‘intangible objects’, as Quine once called them), they disagree about a lot else. One major difference is that for an indispensabilist realist “mathematical objects exist contingently” (Colyvan 2001, 4; Bangu 2012, 59), while this is of course not the case for a Platonist. The rejection of the aprioricity of mathematical truth is another, as is the acceptance of the blurring of the analytic-synthetic distinction.<sup>4</sup>

To summarize: while I too suspect that a *typical* scientific realist may be able to dodge mathematical Platonism, the kind of scientific realist I consider here is *not* the generic type discussed in the above-mentioned papers, but one who is a methodological naturalist, and thus contemplates ‘marriage’ to an indispensabilist-naturalist mathematical realist (again, as different from a Platonic relation).

The structure of this paper is as follows. In section II, I will sketch the Indispensability Argument, emphasizing the role of Quinean methodological naturalism; then, in section III, I introduce CMN. Section IV examines a metaphysical issue usually left untouched in the literature —one’s fundamental justification for adopting CMN— and finds that this ultimate ground traces back to (simplistic) views about scientific explanation, at odds with scientific practice. I conclude in section V.

## 2. Indispensability and naturalism

Originating in Quine’s writings, and occasionally endorsed by Putnam,<sup>5</sup> the Indispensability Argument has been discussed extensively in the recent literature; thus, there is no need for a very detailed exposition of it. In a simplified form, it runs as follows:<sup>6</sup> mathematical entities are indispensable to (the formulation of) our best scientific theories, and we ought to have ontological commitment to all (and only) the entities that are so indispensable. The gist of the argument is clear, but for my purposes here it needs some unpacking. The

<sup>4</sup> This contemporary (post-)Quinean indispensabilist form of mathematical realism is sometimes called *scientific platonism*; see Paseau (2007). There are other differences, discussed in Bangu (2012), where I also remarked that not just any form of scientific realism fits indispensabilist mathematical realism, but only a *certain* version of the realist doctrine, which I called ‘posit realism’. This is a subtlety I don’t discuss here. See Psillos (1999), and Chakravartty (2013), for how the scientific realist doctrine is currently understood.)

<sup>5</sup> Putnam (1971, 347) is the clearest endorsement. But see Bueno’s reading of this passage (in his (2018)), along the lines of Putnam (2012).

<sup>6</sup> See Colyvan (2001). Baker (2009) calls the version of the argument in which mathematical entities are deemed indispensable to explanations, the ‘Enhanced’ Indispensability Argument. Leng (2010), Pincock (2012) and Bangu (2012) contain recent substantial discussions of both versions.



following is a version that renders more visible several features not immediately discernible in the above abridged variant:

Quinean naturalism advises that what we ought to include in our ontology can only be established by applying the criterion of ontological commitment to the regimented versions of our best (true) scientific theories. If propositions featuring existential quantification over mathematical entities are indispensable to the formulation of these theories, then we ought to have ontological commitment to these entities. This conclusion follows under the additional assumption that confirmational holism is a viable view of scientific confirmation.<sup>7</sup>

A now conspicuous aspect of the Indispensability Argument is its intended audience: the scientific realists, i.e., those philosophers who hold (a) that our best, well-confirmed scientific theories are to be taken literally, (b) that they are true, and (c) that their central terms refer.<sup>8</sup> Such philosophers believe, for instance, that the unobservable entities posited by our best scientific theories (e.g., the electron field, the space time-curvature, and genes) should be granted full ontological rights. Yet, even more specifically, I aim at those scientific realists who *also* believe that mathematical objects should be denied such rights (and this mainly for CMN-related concerns.)

In addition to (i) scientific realism, (ii) indispensability and (iii) holism, another key-component of the Indispensability Argument as I understand it here is (iv) the criterion of ontological commitment. As is well known, this is a two-step recipe for detecting what a theory (often implicitly) says there is: it urges that we are committed to those objects taken to be values of the variables that a true theory quantifies existentially over —that is, the commitment is to the truth-makers of these theoretical statements.<sup>9</sup> (This is the second step; the first consists in rewriting the theory in first order logic, and all this under the additional assumptions that (v) this ‘regimentation’ is possible, and (vi) the framework allowing it, first-order logic, is the best one suited for such a task.)

I have quickly parsed these six assumptions here in order to bring more sharply in view a seventh one, which plays the central role in what follows: Quine’s doctrine of methodological naturalism, the very foundation on which the Indispensability Argument is built. Although there has been a great deal of discussion about what this doctrine amounts to,<sup>10</sup> here I take it to consist in a couple of well-known general-methodological theses: that philosophy is not prior, but continuous, to science, which means that natural science, and not a supra-scientific ‘first philosophical’ tribunal, is the arena where philosophical questions have to be asked and answered; in short, “the recognition that it is within science itself, and not in some prior philosophy, that reality is to be identified and described.” (Quine 1981, 21). Most relevant in this context is the thesis that philosophical methodology should be

<sup>7</sup> From Bangu (2012, 147).

<sup>8</sup> To stress, one who is *not* a scientific realist (i.e., one who is agnostic, for instance, about electrons and other unobservables) will dismiss mathematical objects (hence the Indispensability Argument) as a matter of course. Like other indispensabilists, I admit that what follows has no force against such a philosopher.

<sup>9</sup> This way of presenting the argument has very recently come under scrutiny; Azzouni (2010, 25) discusses whether the idea of invoking ‘truth-makers’ is viable. Although I think it is, this would require a separate argument, so I’ll ignore this complication here; see Beebe and Dodd 2005 for discussion.

<sup>10</sup> See Burgess and Rosen (1997), Maddy (1997, 2005, 2007), Paseau (2005, 2013). For detailed exegesis of Quine’s own views, see Hookway (1988) and Hylton (2007).

informed by, and try to emulate, scientific methodology. This leads to the idea that scientific practice takes precedence over philosophical critique —or, as Maddy put it, in case of conflict “it is the philosophy that must give” (1997, 161). The brand of naturalism relevant here has been described by Burgess and Rosen (1997, 65) as follows:

The naturalists’ commitment is ... to the comparatively modest proposition that when science speaks with a firm and unified voice, the philosopher is either obliged to accept its conclusions or to offer what are recognizably scientific reasons for resisting them.

A naturalist philosopher of this (‘modest’) methodological kind determines her ontological commitments not by following any of the traditional metaphysical doctrines (such as ‘materialism’, or ‘physicalism’), but by seeking to find what is assumed to exist by the best scientific theories. These are the theories judged *within the scientific practice* as being, on balance, the best (the simplest, of widest scope, most predictive and explanatory, etc.) Moreover, as the proponents of this form of naturalism constantly stress, the doctrine is not blindly subservient to science. On the contrary, it leaves open (even encourages) philosophical interference within scientific matters. However, not just any interference is allowed, but only that which results in a betterment of science. What *is* opposed is the interference motivated by the constraints dictated by adherence to a particular philosophical doctrine (and CMN is a case in point here; this remark will become relevant later on.)

### 3. *Causal mathematical nominalism and the ‘Eleatic Principle’*

With this sketch of the Indispensability Argument in place, we can now move on to describe CMN. This view makes *causal efficacy* the decisive condition of ontological commitment. Thus, an entity’s ability to *participate* in causal interactions is postulated to be the necessary and sufficient condition for its existence. From now on, and following the literature (Oddie 1982; Colyvan 2001, 2005; Azzouni 2004, 2010), I will refer to this criterion as the ‘Eleatic Principle’.<sup>11</sup> Following other mathematical realists, and for the sake of the argument here, I’ll bracket out the sufficient condition as unproblematic, and I shall focus only on the necessary condition.<sup>12</sup>

Let me add two caveats before I begin to discuss the problems announced at the outset. First, this formulation of CMN leaves out one complication: the fact that the view is often associated with an additional requirement, spatio-temporal localization.<sup>13</sup> I mention

<sup>11</sup> Oddie (1982) coined the phrase ‘the Eleatic Principle’. Advocates of it include, among many others, Armstrong (1978, 5), Ellis (1990, 22), Field (1989, 68) and, more recently Azzouni (2004, 10): “I ... argue for a version of the Eleatic Principle, the claim that, in some sense, everything there is has causal powers.”

<sup>12</sup> Colyvan (2001, ch. 3) makes an argument against the necessity part of the Eleatic Principle based on inductive considerations. Since I agree with Azzouni’s (2004, 155) and Marcus (2015) criticisms, I won’t discuss it here.

<sup>13</sup> Both Rosen (2012) and Rodriguez-Pereyra (2011) discuss these characterizations together; less emphasis on them is laid by Chihara’s entry on ‘Nominalism’ in Shapiro (2005). Armstrong (1980, 149) also endorses “the doctrine that reality consists of nothing but a single all-embracing spatio-temporal

it here, although what I have to say below will not be affected by taking it into account; I believe that concentrating only on causal efficacy captures the spirit of this nominalist position more precisely. As the terms ‘causation’ and ‘spatio-temporal’ are typically understood, they are virtually extensionally equivalent: anything that is located in space-time has causal powers, and vice-versa. But then, one might ask, what about *space-time itself*? Or, even more specifically, what about the *individual points* of the space-time manifold: are *they* located in space-time as well? Do *they* interact causally with other objects, or with other such points? These are difficult and pertinent questions,<sup>14</sup> yet here I’ll put them aside by formulating CMN in a slightly restricted way.

The second remark: as is well-known, one wishing to resist ontological commitment to mathematical abstracta has several kinds of nominalism to choose from.<sup>15</sup> *Causal* nominalism, however, is *prima facie* more attractive than other nominalisms, because it is perhaps the best articulated version of this doctrine. One can appreciate this aspect of CMN if one reflects on what grounds the early nominalism of Quine and Goodman (1947). What they posit as the foundation for their “refus[al] to admit the abstract objects that mathematics needs” (1947, 105) is “a philosophical *intuition* that cannot be justified by appeal to anything more ultimate.” (1947, 105; my emphasis). Yet to invoke such intuition amounts to little more than just foot stomping —and, at least by comparison, CMN does better. Not only does it offer a rationale for refusal, but the one it proposes is very appealing, and turned out to be decisively convincing for many scientific realists.

#### 4. *Why believe the Eleatic Principle?*

Commendable as it may be as a philosophical strategy, the endeavor to offer a justification for adopting a fundamental metaphysical position —here, CMN— is also risky, as it opens up the possibility of further questioning. In this section I’ll do just that, and examine some possible justifications of the Eleatic Principle. Since the principle determines what ontological stance one must embrace, a supporter of a rival stance is entitled to examine it, just as the nominalists have scrutinized all the seven premises of the Indispensability Argument. So, let us ask, why should one take causal efficacy as a *necessary* condition for existence? The nominalist is not out of answers here; yet, I argue, advancing them leads to serious tensions with naturalism.

##### 4.1. CAUSATION AND EXPLANATION

When confronted with the key-question ‘what is special about causal efficacy?’, one typically brings in the issue of scientific explanation. So, the argument goes, if one is interested

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system”. And, according to Colyvan (2001, 22), “This conception of naturalism has an important consequence (at least according to Armstrong): We should believe in only causally active entities (or perhaps, more generously, *potentially* causally active entities).”

<sup>14</sup> They have actually been asked, by various people critical of Field (1980). See Rosen (2012) for discussions of them, as well as for the existence of some possible exceptions to the identification of causal efficacy and spatio-temporal localization.

<sup>15</sup> Burgess and Rosen (1997) contains a book-length presentation of the nominalist options.

in *explaining* natural phenomena, and one believes that the phenomena to explain are *only* the result of causal interactions, then one maintains that

(C) Only causal relations and entities have explanatory power.<sup>16</sup>

Therefore, according to claim (C), we need not grant ontological status to causally inert entities and structures (mathematical ones included), since to do so is to take onboard more than is necessary to formulate the best scientific explanations. This is an important point, since now it is time to recall the constraints of methodological naturalism.

I will argue that the causal mathematical nominalist's acceptance of claim (C) is subject to a couple of difficulties. And, given its role in supporting the Eleatic Principle (hence CMN), one should conclude that these positions themselves are significantly weakened. Moreover, both these difficulties will be traced back to a conflict with scientific practice, hence to a conflict with methodological naturalism.

The first problem is that many have argued that claim (C) is *false*, by invoking counterexamples from science. A second difficulty, deeper and yet less discussed, can be presented. It does not claim that (C) is wrong; instead, it objects that it may not even be *meaningful*—by assessing it from the standpoint of scientific practice (more precisely, fundamental physics). I'll take the two difficulties in turn.

So, the first challenge to (C) comes from the multitude of examples in the literature in which what does the explanatory work involves, at least *prima facie*, *more* than causal interactions—that is, the explanandum doesn't have an *exclusively* causal nature. Here perhaps the most famous case of an explanation which does not (seem to) appeal to causal interactions is Newton's explanation of uniform motion—it happens when *no* forces act on a body. Even a minimal acquaintance with this literature reveals that many authors have taken, upon the examination of such examples, a definite stance against (C). To choose one name from a rather long list, Psillos says that “Causal inertia does not imply explanatory inertia” (2010, 951). The philosophical commentary on scientific explanation is rife with such pronouncements and examples abound. Since it is virtually impossible to discuss even one in the space of this paper, the best I can do is list several of them; see also Mancosu (2008). So, as examples, one can mention:

— *topological* explanations: Putnam's (1975) famous square-peg-not-fitting-into-a-round hole, Kitcher's (1989) knot, Colyvan and Lyon's (2008) honeycomb, Pincock's (2012) bridges.

<sup>16</sup> This is the conclusion of what Brown (2012, 11-12) calls (without endorsing) the “standard argument”: “Numbers and other mathematical entities are outside of space and time, so they can't causally interact with us. Unlike gravity, which makes things fall, and germs, which make us sick, numbers cannot make things happen. Since they cannot make things happen, they cannot explain them.” Note that the claim “[mathematicalia] cannot make things happen” may not be as obvious as it seems. Earlier work by Steiner (1975) points out the possibility that, for instance, a (relation proved by a) mathematical theorem (understood as an abstract entity) might have some sort of causal effect. Rosen (2012) explains: “Suppose John is thinking about the Pythagorean Theorem and you ask him to say what's on his mind. His response is an event—the utterance of a sentence; and one of its causes is the event of John's thinking about the theorem. Does the Pythagorean Theorem ‘participate’ in this event? There is surely *some* sense in which it does.”

- *equilibrium* explanations: Garfinkel's (1981) predator-prey, Sober's (1983) sex-ratio.
- *number theoretic* explanations, drawing on the concept of (co-)primeness (such as Baker's cicada (2005), or variations on the theme of the tile puzzle —why can't one cover a rectangular bathroom floor with, say, 23 square tiles of equal area?— see Shapiro (2000, 217).
- *optimization* explanations: Bangu's (2012, 2013) maximum-volume box and the 'banana game', Baron's (2013) Levy walk, as well as a host of other
- *miscellaneous examples*, harder to classify, in:
  - Steiner (1978a, b)
  - Lipton (1991)
  - Colyvan (2001), especially his discussion of the Lorentz contraction in Special Relativity
  - Batterman (2002), on renormalization group explanation of universality and the dimensional explanations
  - Colyvan and Lyon (2008), on phase-spaces
  - Brown (2012), on spin
  - Lange (2013), on Mom's strawberries and the double pendulum.

Although the list is of course not exhaustive, it hopefully gives enough reasons to doubt (C).

Before I turn to the second difficulty, two observations. First, these examples have of course been challenged —together with the very idea that something else than causal relations has (or *can* have) explanatory power;<sup>17</sup> for instance, Lewis [1986] is a prominent exclusivist. It is however fair to say that this is a minority view, and that most philosophers today accept that something of mathematical nature, in addition to causal relations, can be explanatory in science. The other observation is that in order to reject (C), I only need to make the case for the claim that the explanatory power in those examples is ensured by some sort of mixture, of elements of both causal *and* non-causal/mathematical nature. If this is achieved, there is no need to argue for the further, stronger claim that it is mathematics *alone* that provides this power (some of the examples in the list above, e.g., Lange's, do attempt to show this).

So, if the argument from counterexample above holds, it is simply not the case that *only* causal relations (and entities) have explanatory power. Note that this *non-exclusivism* about explanation accepts the viability of causal explanation; the second difficulty arises once one *denies* this. This difficulty is more radical since it challenges the very idea of causal explanation in general —by doubting the meaningfulness of the key notion occurring in (C), that of 'causal relation'. What I'll do now is assess, on behalf of the naturalist, what this phrase means (if anything) in scientific language. (Note that in doing this I just proceed according to the Quinean naturalist maxim cited above, which urges that we should assess philosophical claims "within science itself".)

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<sup>17</sup> Recently, Lange (2013) has claimed that some of the examples above are in fact (exclusively) causal, while some of the ones he discusses are (what he dubs) 'distinctively mathematical' explanations (in which causes are 'cited' but not 'exploited'). Although I have worries about this notion (and his examples), I won't pursue them here.

To be sure, the causal nominalist may simply reject the very legitimacy of such an assessment. She may point out that the notion has an intuitive meaning in ordinary conversation—and this is enough. In this case, a conflict with naturalism ensues right away. But it is also possible that she may agree to such an assessment and, since this is the most charitable attitude to have in dealing with the nominalist, I'll assume that she does. Consequently, I will proceed to such an examination below. Its end result, however, will reveal that the nominalist finds herself, again, at odds with naturalism.

What leads to trouble is the simple remark that 'cause' is a term that does not seem to appear, or does virtually no work, in the canonic formulations of the fundamental scientific theories.<sup>18</sup> One then suspects that the reason for this situation is actually deeper. Two contemporary metaphysicians, Carroll and Markosian, present the issue neatly, as follows:

Some worry that the absence of the word 'causes' from *the formulation of fundamental theories of physics* is an indication that causation is merely a folk concept, maybe like the concept of a witch, that may get lots of use in ordinary conversation, but which has no application to the world since there are no witches. Our best physical theories include fundamental laws that are *equations* relating various properties to other properties but without explicitly stating that there are any causal connections or even that there would be certain causal connections if certain conditions were to come to pass. (2010, 43; my emphases)

Unsurprisingly, some methodological naturalists are sympathetic to considerations like these. They are thus favorable to *eliminativism* about causation, a view holding "that causation sentences don't succeed in describing the world, and so also hold that, strictly speaking, nothing causes anything else." (Carroll and Markosian 2010, 43). J. Shaffer summarizes the view in terms congruent with methodological naturalism:

The main argument for eliminativism is that *science* has no need of causation. The notion of causation is seen as *a scientifically retrograde relic* of Stone Age metaphysics. (Shaffer 2016; my emphases)

Note, moreover, that much like non-exclusivism about explanation, eliminativism is by no means an obscure line of thought. One can provide a long list of illustrious supporters of it, which include Ernst Mach and Bertrand Russell, whose famous description of causation—as "a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm" (1912, 1)—is alluded to above. Significantly, Quine himself sounds like an eliminativist: "... the notion of cause itself has no firm place in science. The disappearance of causal terminology from the jargon of one branch science and another has seemed to mark the progress in the understanding of the branches concerned." (1976, 242) Additionally, he also argues that "...a notion of cause is out of place in modern physics ... Clearly the term plays no role at the austere levels of the subject." (1974, 6) "Science at its most austere bypasses the notion [of cause]..." (1992, 76). Moreover, there is a resurgence

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<sup>18</sup> The well-known remark is Bertrand Russell's: "All philosophers, of every school, imagine that causation is one of the fundamental axioms or postulates of science, yet, oddly enough, in advanced sciences such as gravitational astronomy, the word 'cause' never occurs" (1912, 1).

of this form of skepticism about causation in the recent years. For A. Ahmed, for instance, “Causation is a pointless superstition” (2014, vii).<sup>19</sup>

Thus, since causation sentences can’t even describe the world, it is preposterous to require that they be used in *explaining* it —while this is exactly what (C) does. The charge against CMN is then that as long as the phrase ‘causal relation’ in (C) is empty, the claim itself is hollow. Once the core ideas of causation, and causal interaction, are removed from the (scientific) picture, what is left, as we saw, is *mathematical* “equations relating various properties to other properties.” This is bad news for the causal mathematical nominalist. Not only is the very notion of a causal relation (and causal efficacy) obscure, even meaningless, but once one tries to make sense of the causal discourse, by (charitably) examining what such a philosopher *may* be talking about when using the folk vocabulary of causation, one discovers that it is actually *mathematical equations*! As is clear, from here the road is wide open to argue for mathematical realism via an Indispensability Argument.<sup>20</sup>

#### 4.2. RELUCTANT REALISM

Even if these difficulties for claim (C) (and thus for CMN) are recognized, the nominalists may retreat by asking: is the alternative package (1. scientific realism + indispensabilist mathematical realism + 3. methodological naturalism) more stable? They point out that this may not be the case. After all, the scientist’s (and the naturalist scientific realist’s) commitment to mathematical objects is a mere byproduct; in essence, it occurs only because of the (perhaps only possible) way we do successful science. The suggestion is, ultimately, that science is somewhat forced, perhaps even ‘fooled’, into this ‘marriage’; and, as the nominalists see things, it is their duty to signal, and remedy, this ‘abuse’.

This strand of argument doesn’t come as a surprise to the indispensabilist realist; one may recall that this kind of mathematical realism is typically described as ‘reluctant’ realism. This, however, is no reason to worry, since the indispensabilist realist can address the charges issued above. First, given Quine’s earlier endorsement of nominalism, his change of mind is in fact a sign of intellectual honesty. Second, the conclusion of the Indispensability Argument is, strictly speaking, that science is *committed* to the existence of mathematical objects, not that it *intends* to prove their existence. Sure enough, both the scientific and the indispensabilist realist fully realize that physics, and science more generally, are *prima facie about* electrons and genes, etc., not numbers, sets and functions.<sup>21</sup> And yet, what does it mean to say that science is about these things? Physics, for instance, is about the electrons in the sense that it is an investigation that aims to discover *true statements* about these entities. (More precisely, quantum field theory is about the electron in so far as it asserts the true claim that ‘there are quantum fields, they have minimum energy excited states, and the electron is such a stable state, the negatively charged quantum of the electron field’.) Thus, given this aim, it turns out that physics’ best (holistically speaking) way to attain it is to use mathematics. The nominalist complaint —that science is somehow forced to marry mathematical abstraction, i.e., to make room for causally inert mathematical objects in its ontology of

<sup>19</sup> See also some of the essays in Price and Corry (2007). Norton’s paper proposed the ‘folk concept’ idea.

<sup>20</sup> Recall that a host of several other assumptions need to be defended for this argument to hold.

<sup>21</sup> The philosophers holding this view talk about the “nominalistic content” (Balaguer 1998, 141) of science. See Psillos (2010) for doubts about the meaningfulness of this notion.

causally efficacious electrons, fields and genes —is unfair. In fact, science gets a very good deal at the end of the day: at the cost of commitment to some mathematical abstraction, it gains, among other things, tremendous descriptive-representational power: how else to represent a quantum field other than as a mathematical object?<sup>22</sup>

At this point we can see what should be worrisome for the scientific realist. Her causal mathematical nominalism persuades her to adopt some illusory higher (first-philosophical, supra-scientific) perspective (expressed in claims like (C)) from which to oversee, and judge, scientific practice —or, more precisely, that part of the scientific practice having to do with the way theories and explanations are formulated. And this amounts to instituting an all-powerful, supra-scientific tribunal, in stark contrast with the gist of naturalism.

It is now hopefully clear for a *naturalist* scientific realist that mathematical realism offers a better option for her than causal nominalism; while the latter just doesn't square with her naturalism, the former integrates into the whole picture quite well. So, by now, both the advantages of mathematical realism, as well as CMN's tensioned relation with an important aspect of scientific practice (what scientists take to be explanatory, what vocabulary they use) should be obvious. Yet, one can still ask, do the nominalists then propose a revision of this practice? That is, can we categorize them as 'revolutionary' nominalists? (as Burgess and Rosen (1997) and Chihara (2005) call them). Even if this is so, I have already noted that naturalism is constitutively anti-dogmatic, not fearing 'revolutions', conflicts, or other calls for reform. When it comes to such disagreements, the key-question in settling them is *how* the revision of the scientific practice is justified. And, as is quite evident in this case, this revision is not proposed out of concern for the betterment of science —on the contrary, the scientific realist points out— but is grounded in extra/supra-scientific (philosophical-nominalist) concerns instead. Therefore, if one remains within the confines of naturalism, one must conclude that such concerns are just immaterial.

## 5. Conclusion

The argument here amounts to the claim that the package (1. *scientific* realism + 2. causal *mathematical* nominalism + 3. methodological naturalism) is inherently unstable: a naturalist scientific realist's mathematical nominalism, when motivated by causation-related worries, is in conflict with her methodological naturalism. As is immediate, this is not an argument against nominalism per se; it only shows that the scientific realist has a choice to make, between her sympathy for this causal form of nominalism, on one hand, and her methodological naturalism, on the other. It is of course up to the scientific realist to decide which option is the most commendable here; but to my mind, it seems that accepting the second alternative is a better way out —i.e., that dropping CMN (rather than methodological naturalism) is a more sensible choice, especially since, as I briefly argued in section IV.2, the alternative package (1. + 3. + indispensabilist realism) does *not* face similar cohabita-

<sup>22</sup> The reference to (relativistic) *quantum* fields is not accidental; unlike the classical fields, such fields assign to space-time locations *mathematical* objects (i.e., operators, whose eigenvalues can be *other mathematical objects*, e.g., scalars, vectors, spinors or tensors). Thus one strongly suspects that quantum field theories are *not amenable in principle* to the same nominalization strategies employed by Field 1980. (See also Malament 1982.)



tion problems.<sup>23</sup> In fact, taking the first option, i.e., to abandon methodological naturalism instead, seems to me a decision with many more worrisome consequences (e.g., the return of traditional metaphysics), whose analysis would deserve a separate paper.

To close. The familiar refrain ‘no causal efficacy, no ontological rights’ has inhibited many naturalist scientific realists’ tendency to become mathematical realists. This paper purports to show that although tempting, this causal type of mathematical nominalism turns out to be rather problematic. In a nutshell, I have offered reasons to the effect that a scientific realist attracted by the specific form of the (Quine-Putnam) indispensabilist mathematical realism discussed here (recall, as different from the traditional Platonism) has nothing to worry when it comes to the challenge from CMN. Thus, I suggest, the scientific realist initially impressed by the Eleatic Principle should now feel encouraged to seek ‘divorce’ from the causal mathematical nominalist, and to consider ‘marrying’ a more stable ‘partner’, the indispensabilist mathematical realist. Finally, recall that the arguments presented here have proceeded in a charitable fashion, by accepting the sufficiency condition of the Eleatic Principle;<sup>24</sup> thus, only the necessary condition has been scrutinized, i.e., the idea that the scientific realists’ ontology contains *only* causally efficacious entities. In the end then, the extension of this ontology such that it includes causally inert mathematical objects is entirely compatible with scientific realism construed within the bounds of methodological naturalism.

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<sup>23</sup> To repeat, this does not amount to showing that the Indispensability argument holds; (re)establishing this (i.e., that 1. + 3. + some other assumptions *entail* mathematical realism) was *not* my goal here.

<sup>24</sup> In light of eliminativism, the Principle is a non-starter, so this may sound as conceding too much. However, note that no argument has been made to the effect that a naturalist *must* be an eliminativist. While this is an alternative indeed, being a non-exclusivist is enough for the point of this paper to hold. In fact, a mixt position may be the best option here: non-exclusivism when dealing with explananda in the domain of familiar objects, and eliminativism when it comes to fundamental physics.

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