ADJUSTMENT OF A REGIONAL ALTIMETRIC NETWORK, IN BRAZIL, TO ESTIMATE NORMAL HEIGHTS AND GEOPOTENTIAL NUMBERS

Giuliano Sant'Anna Marotta[1] - ORCID: 0000-0003-1073-0683

[1] Universidade de Brasília, Instituto de Geociências, Observatório Sismológico, Brasília – DF, Brasil.
E-mail: marota@unb.br

Abstract:

In 2018, the Brazilian Institute of Geography and Statistics (IBGE) defined the normal height values and geopotential numbers for the High Precision Altimetric Network (HPAN), creating the need for developing procedures able to adapt works correlated with the Brazilian Geodetic System (BGS). In that context, and considering the state and municipal altimetric networks, it is necessary to estimate normal heights. To this end, this paper proposes to estimate normal height values and geopotential numbers for the Altimetric Network of the Federal District (AN-DF). Computational procedures involving the Least Squares Method were established and applied in a geometric levelling survey, which includes 200 stations distributed throughout the study area. The obtained results allowed estimating the normal heights of the network stations with accuracy up to 0.032m. However, because significant differences were found in the adjusted and known values of some used HPAN stations, we recommend a rigorous analysis of these stations before using them. In any case considering that the geometric levelling and the applied procedures were carried out correctly, it is suggested that the values estimated for all stations of the AN-DF in this work may be used.

Keywords: Geopotential numbers; Normal heights; Regional altimetric network.

1. Introduction

The High Precision Altimetric Network (HPAN) belongs to the Brazilian Geodetic System (BGS) and is maintained by the Brazilian Institute of Geography and Statistics (IBGE). This network covers the entire Brazilian territory, having nowadays two vertical datums, Imbituba and Santana, and continuous data collection via geometric leveling technique, where new stations are implanted. Therefore, to incorporate new stations into the HPAN, all stations in the network adjustment are used to estimate and/or update their heights using the Least Squares Method (LSM).

Like the HPAN, a Gravimetric Network (GN), which also belongs to the BGS, has been continually updated, with the acquisition of new gravimetric data. At first, the GN was developed to enable and improve the computation of geoid heights. However, to give a physical meaning to the heights, new gravimetric data are being acquired, mainly in the HPAN stations. As a result, the IBGE published new heights, now called normal heights, for the HPAN stations in 2018 (IBGE 2018), where gravimetric observations were included in the functional model for the network adjustment, following the recommendations of the International Association of Geodesy (IAG).

The HPAN update is in agreement with the works performed in many parts of the world. In this context, Freitas and Blitzkow (1999) promoted a reflection on the physical meaning of height, the problems and the practical solutions that can be implemented to define a vertical datum for the Geocentric Reference System for the Americas (SIRGAS). Drewes et al. (2002) showed that height type, reference surface, realization and maintenance of the reference system are presented as main topics for defining the vertical reference system for SIRGAS, and recommended introducing two height types, the geometric (ellipsoidal) and normal (physical). Luz (2004) investigated the possibility of using gravimetric data to define the physical heights in Brazil and to connect them with the South American Height Systems associated with SIRGAS. To contribute with the integration of the Brazilian vertical network into the global height system, Palmeiro and Freitas (2010) developed a methodology to measure/ determine the physical heights by computing the geopotential numbers using the Geographical Information System to integrate the BGS information. Severo et al. (2013) determined and evaluated the corrections of different physical heights (dynamics, normal and orthometric) in a leveling belonging to HPAN, located in the northeastern Rio Grande do Sul. Also, Luz (2016) reviewed the main concepts related to vertical geodetic reference systems and physical heights, to prepare the first HPAN adjustment considering the geopotential differences.

Considering the exposition and also that states and cities in Brazil have their geodetic networks referred to the BGS, they need to be updated for each new adjustment and/or change in the national geodetic network. Therefore, it is necessary to establish procedures for estimating normal heights for these networks. Consequently, this research aims at developing a procedure to adjust the Altimetric Network of the Federal District (AN-DF), Brazil, and to estimate the normal heights and the geopotential numbers to be associated with the BGS.

2. Normal Height and Geopotential Number

Normal heights (H_N) can be characterized as a type of physical height (H_F) since they are associated with the terrestrial gravity field (g). Therefore, this height can be expressed by the geopotential number (C_P), defined as the difference between the gravity potential on geoid level (W_0) and the gravity potential at the point (W_P) considered. This relationship can be expressed as (Torge 1991; Freitas and Blitzkow 1999; Gemael 1999; Luz 2016; IBGE 2018):

$$C_P = W_0 - W_P = -\int_0^P dW = \int_0^P g.dH \qquad (1)$$

In practice, height is determined by surveys involving geometric leveling technique and, therefore, Equation 1 must be expressed by a discrete model. Then, considering a finite sum of sections of height differences (ΔH), with

the average gravity values ($\overline{g}$) observed for each section, we have (Gemael 1999):

$$C_P = W_0 - W_P \approx \sum_{i=1}^{n} \overline{g}_i . \Delta H_i \qquad (2)$$

Due to propagation errors on the geometric leveling technique to compute heights, Luz (2016) and IBGE (2018) suggested that is preferable to compute geopotential differences (ΔC_{AB}), for a small distance between points (A and B). It allows geopotential numbers (C) to be computed adjusting the observations of these differences (ΔC), because:

$$\Delta C_{AB} = C_B - C_A = -(W_B - W_A) \approx \overline{g}_{AB} . \Delta H_{AB} \qquad (3)$$

According to Torge (1991), C describes better the behavior of masses in the gravity field and could be used in several applications, such as hydraulic engineering and oceanography. However, C contradicts the demand for a height system that works on the metric unit. In this case, the physical height (H_F) can be expressed as (Freitas and Blitzkow 1999; Torge 1991; Gemael 1999; Severo et al. 2013; Luz 2016; IBGE 2018):

$$H_F = \frac{C}{G} \qquad (4)$$

Considering a height using the geoid as a reference, we have an orthometric height (H) as definition while G (Equation 4) must represent the average gravity measured along the plumb line (Figure 1). Thus, becoming necessary to know the gravity values inside the Earth. However, H can not be precisely estimated as the density distribution in the Earth interior is not known.

As an alternative to solve the problem for determining H , IBGE (2018) in consonance with Drewes et al. (2002), defined the use of normal heights (H_N), according to the definitions established by SIRGAS. In this case, Equation 4 can be expressed as:

$$H_N = \frac{C}{\gamma_v} \qquad (5)$$

γ_v represents the normal gravity average value, along the vertical (Figure 1), and can be expressed as (Heiskanen and Moritz 1967; Luz 2016; IBGE 2018):

$$\gamma_v = \gamma_0 . \left[1 - \frac{H_N}{a} . \left(1 + f + m - 2.f.sin^2 \varphi \right) + \left(\frac{H_N}{a} \right)^2 \right] \qquad (6)$$

$$\gamma_0 = \gamma_a . \frac{1 + k.sin^2 \varphi}{\sqrt{1 - e^2.sin^2 \varphi}} \qquad (7)$$

$$k = \frac{b.\gamma_b - a.\gamma_a}{a.\gamma_a} \qquad (8)$$

$$m = \frac{\omega^2.a^2.b}{GM} \qquad (9)$$

φ and H_N are the geodetic latitude and the normal height of a point of interest; a , b , e and f are the major and minor axes, the first eccentricity and flattening, respectively; γ_a , γ_b and γ_0 are the normal gravity on the equator, pole and the point considered, respectively; and ω and GM are the angular velocity and the geocentric gravitational constant. All parameters are associated with the adopted reference ellipsoid.

Given the above, according to Figure 1 and IBGE (2018), γ_v is the normal gravity average value along the vertical, H_N is measured along the so-called normal vertical direction and, because it does not consider the real Earth, the height does not refer to the geoid, but to the quasi-geoid.
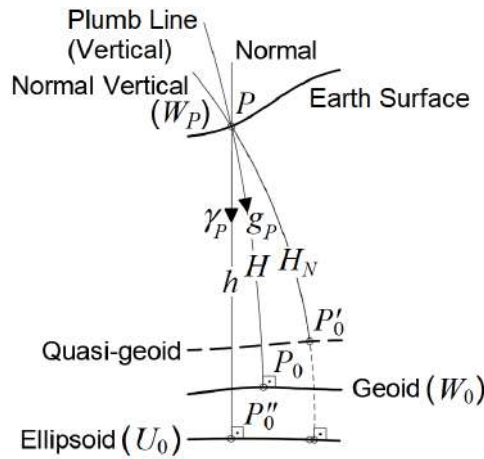
Figure 1: Geometric (h), orthometric (H) and normal (H_N) heights at point P. Adapted from Torge (1991), Freitas and Blitzkow (1999) and IBGE (2018).

3. Altimetric Network Adjustment and Evaluation

According to Equations 2 and 3, C as a parameter to be estimated can be used to compute the physical height, since ΔH and $\overline{g}$ are considered observed values. Subsequently, H_N can be estimated by Equation 5.

In this research H_N values were estimated by a functional model defined by developing the Equations 3 and 5 and then C values were computed by Equation 3. Therefore, the functional model considering geometric leveling and gravimetric observations for distinct points (A and B), can be expressed as:

$$F(X) = \Delta H_{AB} = \frac{H_{N_B} \cdot \gamma_{\nu_B} - H_{N_A} \cdot \gamma_{\nu_A}}{\overline{g}_{AB}} \tag{10}$$

The estimation of the parameters in Equation 10, using the Constrained LSM (CLSM), considers the following:

$$\Delta H_{AB} = L_b + V = F(X_0 + X) = F(X_a) \mid \sum_{i=1}^{n} V_i^2 \to 0 \tag{11}$$

where, L_b is the vector of observed values (ΔH_{AB}); V, the vector of residual values; X_0, the vector of initial parameters; X, vector of corrections; and X_a, vector of adjusted parameters.

For the CLSM, the condition equations refer to the normal heights of stations used as reference ($H_{N_{REF}}$). Therefore, X can be expressed as (Gemael, 1999):

$$X = -\left(A^T P A + C_C^T P_C C_C\right)^{-1}\left(A^T P L + C_C^T P_C \varepsilon_C\right) \tag{12}$$

where, A and C_C are the matrices of partial differential equations, which consider the functional model and the condition equations (functional model of injunction) related to the parameters, respectively; L, vector of the differences between the computed (L_0) and observed (L_b) values; P and P_C are the weight matrices of the observed and reference (injunction) values, respectively; and the ε_C, the closure error vector of the model.

As demonstrated in Equations 5, 6 and 10, H_N and γ_ν are interdependent. Therefore, it is necessary to develop the adjustment by LSM involving an interactive process, until the *a priori* and *a posteriori* H_N values are not significantly different. The heights (H) used *a priori*, both for defining the initial parameter vectors, and for determining γ_ν, H_N and Σ_{H_N} (uncertainties), are derived from the direct computation of the leveling circuits (Equation 13), involving H_N values of the reference stations.

$$H = H_{N_{REF}} + \sum_{i=1}^{n} \Delta H_i \qquad (13)$$

The weights of the geometric leveling observations are defined, considering the nominal accuracy (σ) of the equipment used and the distance (D) between each observation section of height difference, to compute the variance of the observed values ($\Sigma_{\Delta H}$). It can be expressed as:

$$\Sigma_{\Delta H} = \left(\sigma . \sqrt{D_{km}} \right)^2 \qquad (14)$$

Considering a regional network, the variance ($\Sigma_{H_{N_{REF}}}$) close to zero was defined for the stations used as a reference. In this case, these stations were considered local vertical datums defined in the study area.

To evaluate the results of this research, we analyzed: the Σ_{H_N}, estimated after applying the *a posteriori* variance factor in the weight matrix; the differences between the estimated and known H_N values in stations adopted as verification; the *a posteriori* variance factor ($\hat{\sigma}_0^2$) using the Chi-square (χ^2) test, and the *a posteriori* standardized residual (V_i / σ_{V_i}) using the Tau (τ) test, both according to Cross (1983).

4. Computing the Normal Heights and the Geopotential Numbers of the Altimetric Network in the Federal District, Brazil

The Federal District region, in Brazil, between 48.25°W and 47.33°W and 15.45°S and 16.06°S (Figure 2), has a slightly wavy relief, ranging from 600 to 1340 meters above sea level. Several works on territorial planning have been carried out over time due to the continuous development of the region, among them, the implementation of a precision altimetric network to elaborate and update the cartographic and cadastral basis, and the execution of infrastructure projects as well. Despite having 136 stations distributed throughout the studied region, this network, known as RN-DF (Figure 2), is concentrated in urban and urban expansion areas. In addition to the mentioned stations, the RN-DF incorporates another 30 existing HPAN / IBGE stations, here called RN-IBGE, and 34 stations of the Planimetric Network (SAT-IBGE) (Figure 2), both members of the BGS. The main objective of using the BGS stations was to improve the spatial distribution of altimetric data in the region.

In 2016, the Federal District Development Company (TERRACAP) used the geometric leveling technique and conducted an altimetric survey of the 200 mentioned stations, from RN-DF, RN-IBGE, and SAT-IBGE. This survey required relative datum defined in specific RN-IBGE stations; implementation of altimetric safety points (RNPI-DF) with distance up to 2km (Figure 2) within each leveling circuits; measurements of height differences at horizontal distances less than 100m±20m; use of ground support for positioning the leveling rod; measurements on the leveling rod 50cm above the ground; double sections of leveling rod measurements, with direct and inverted position, changing the equipment position for each section; survey in closed circuits; and closure error smaller than 6mm. km$^{0.5}$. The altimetric survey used calibrated and certified Leica DNA3 digital levels and was conducted in 2016.

Figure 2 shows the data cited above that was used for estimating normal heights and geopotential numbers. The 728 RNPI-DF points (Figure 2) show the paths covered, comprising 1393 km of geometric leveling performed in 58 circuits for the 200 points considered in this study.

In addition to altimetric data, the region (Figure 3) has 1377 Ground Gravity Stations (GGS) provided/managed by the BGS/IBGE, the National Petroleum Agency (ANP) and the University of Brasilia (UnB). The GGS provided by the IBGE and ANP were acquired using the LaCoste & Romberg and Scintrex Autograv CG-5 gravity meter between 1990 and 2014. However, the GGS provided by the UnB were acquired using only the Scintrex Autograv CG-5 gravity meter between 2013 and 2017.

The GGS data were used to generate a map of Bouguer anomalies (A_{Bg}), shown in Figure 3. The anomalies,

which represent the varying physical properties of the rocks in the region, vary from -103,290 mGal to -130,702 mGal, and were used to estimate gravity values (g) for stations or points that did not have that information.
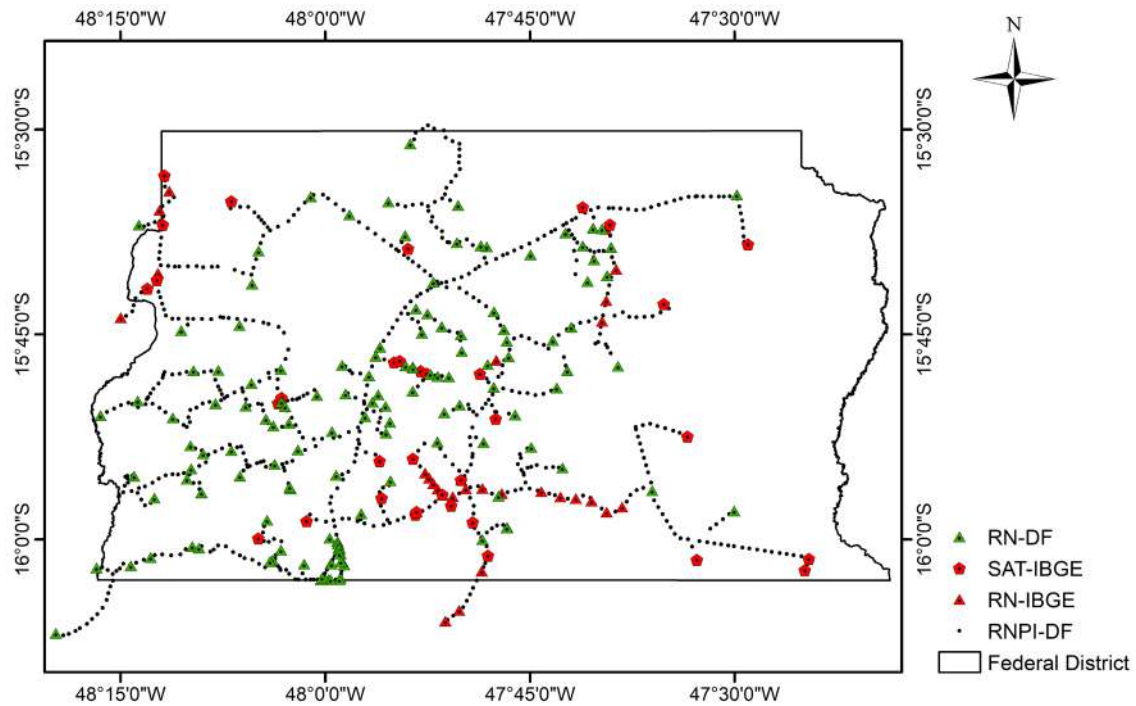

Figure 2: Distribution of the altimetric (RN-DF, RN-IBGE, SAT-IBGE, and RNPI-DF) stations in the study area.
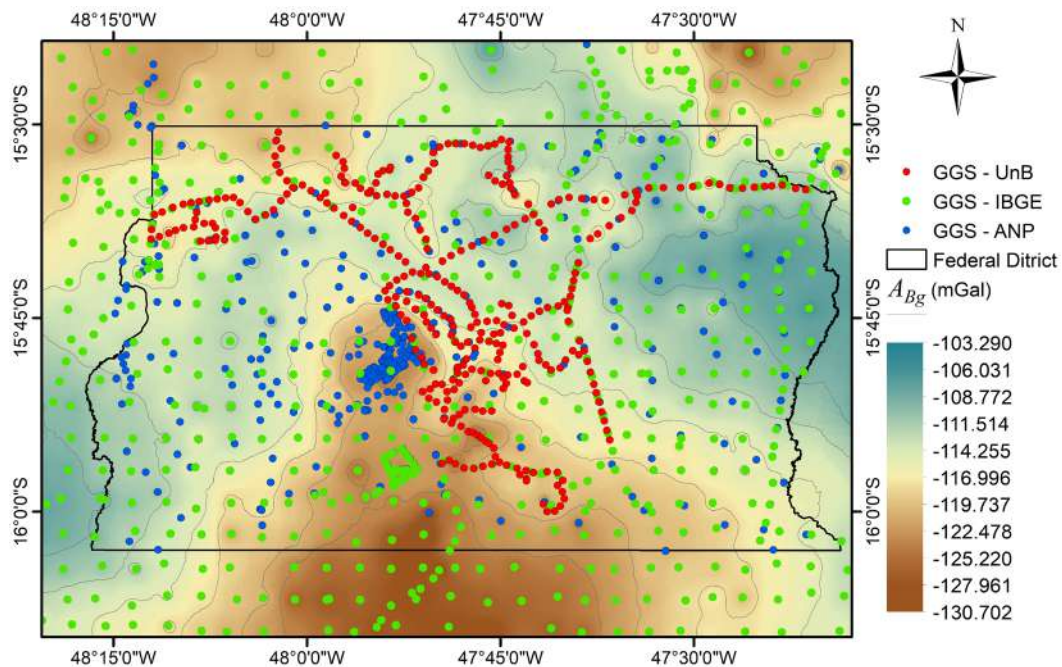

Figure 3: Bouguer anomaly and distribution of gravimetric (GGS) stations provided by IBGE, ANP, and UnB, in the study area.

4.1 Computational procedure

All routines used in this research were implemented based on the MATLAB® software. The flowchart detailing the computation sequence and generation of results is shown in Figure 4. ID (Figure 4) is the identifier of the Backsight and Foresight points (BS and FS), which represent the first and last readings, respectively, for each leveling section.

The flowchart in Figure 4 indicates that the information used in this research was separated into reference data, geometric leveling data, and gravity and position data.

The reference data comprises information from stations used as relative datum for computing the heights of the stations of interest. This information includes the identifier, or the name of the station, the geodetic coordinates, the normal heights, the geopotential numbers, and the gravity values. In the first moment, a single reference station (relative datum) was defined, both for computing *a priori* heights and for the final adjustment of the normal heights and geopotential numbers of the stations of interest. Subsequently, after analyzing the uncertainties of the final results, other reference stations were defined to improve accuracy. The uncertainties used for each reference station were extracted from the HPAN reports.

The geometric leveling data consists of information that allows computing the heights for the points of interest, starting from the reference stations. This information includes the identifier of the backsight (BS) and foresight (FS) points, the horizontal distance (D) between BS and FS, and height difference (ΔH). First, the heights and closure errors were computed. The errors were compared with the established tolerance value (*6mm. $km^{0.5}$*). After determining that the closure errors were within the tolerance interval, the normal heights, with their uncertainties, and geopotential numbers were adjusted.

The gravity and position data include information about all points used (RN-DF, RN-IBGE, SAT-IBGE, and RNPI-DF), including their respective identifier, geodetic coordinates, and gravity values. In the stations or points lacking gravity values (g), they were computed by interpolating the computed A_{Bg} values, using the values of ground gravity stations (Figure 3), adopting the following relationship:

$$A_{Bg} = g + C_{Fa} + C_{Bg} - \gamma \qquad (15)$$

$$C_{Fa} = 0.3086H \qquad (16)$$

$$C_{Bg} = -2\pi\rho GH \qquad (17)$$

where ρ is the average density of the Earth crust (2.67g.cm⁻³); G, the gravitational constant of Newton (6.67428e-11 m³Kg⁻¹S⁻²); C_{Fa}, the free-air correction; C_{Bg}, the Bouguer correction; and γ, the normal gravity value.

Interpolation was performed using the Inverse distance weighting (IDW) method applied to the computed A_{Bg} values. This data interpolation procedure is commonly adopted to compute geoid models and was used by Marotta and Vidotti (2017).

The height data computed using the reference stations, geometric leveling data and g computed, was also used *a priori* to compute γ_v, according to the flowchart in Figure 4.

With the requested data (Figures 2 and 3), as described in the previous sections, it was possible to estimate the values of H_N, Σ_{H_N} and C for stations and points of interest (Figure 4).

In this research, the GRS80 ellipsoid was used as a reference, and all the constants used, inherent to the presented equations, were given by Moritz (1984) and Petit and Luzum (2010).
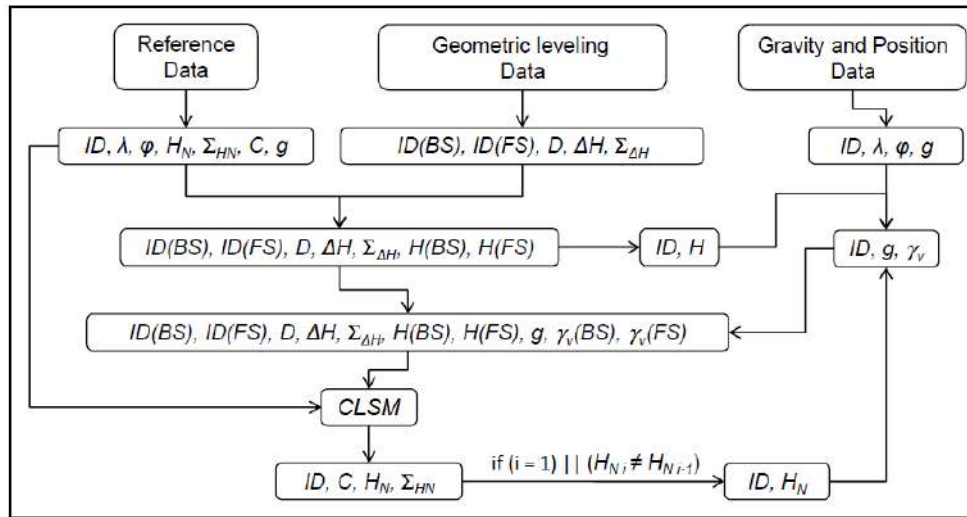
Figure 4: Flowchart of the sequence to estimate H_N, Σ_{H_N} and C.

4.2 Results

Figure 5 shows the adjustment results of the altimetric network of the Federal District using the uncertainties of H_N values estimated for all used stations and points (RN-DF, RN-IBGE, SAT-IBGE, and RNPI-DF), and taking the RN-IBGE station, 2223S, as reference. This station was chosen as a reference because it was the origin of the observations in the geometric leveling survey.

Considering the uncertainties of the H_N values estimated by the CLSM, which vary from 0.000 m to 0.037m (Figure 5), it can be seen that the stations and points farthest from the reference station (RN-IBGE 2223S) have less accurate results. For this reason, other reference stations were defined to improve the accuracy of the results for the whole study area. This choice considered this station as the origin of the observations, in the geometric leveling
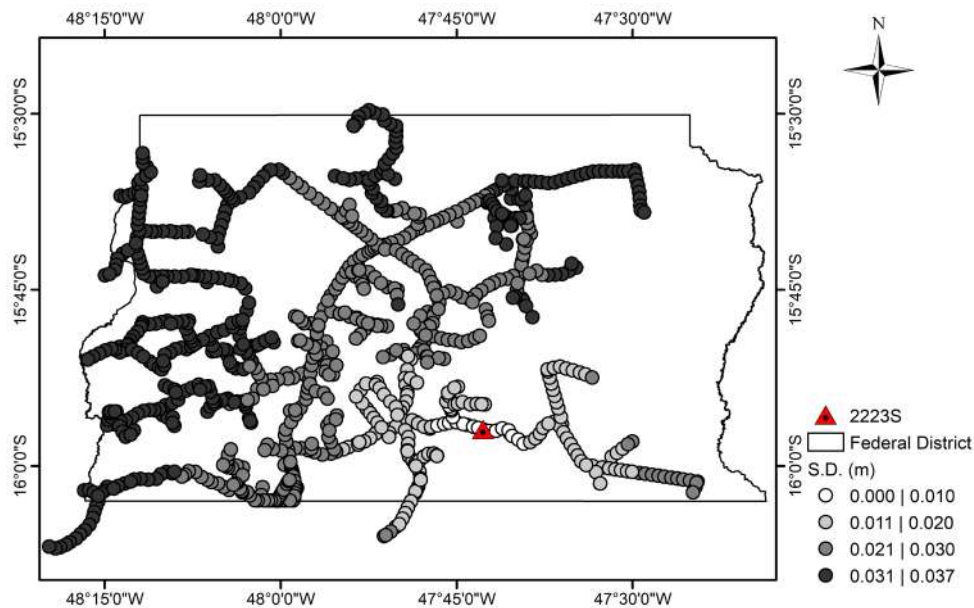

Figure 5: Standard deviations (S.D.) of the H_N values estimated for the stations and points used (RN-DF, RN-IBGE, SAT-IBGE, and RNPI-DF), using the 2223S RN-IBGE station as reference. The black polygon corresponds to the geographic boundary of the Federal District.

survey (RN-IBGE 2223S), and stations located in the central region of the study area (RN-IBGE 9305U) and/or that were used in more extensive and distant circuits (RN-IBGE 2263Z). Additionally, the differences between computed and known H_N were smaller than the uncertainties estimated for all selected reference stations.

On the applied CLSM performance, the $\hat{\sigma}_0^2$ estimated (0.158) was out of the χ^2 / gl range (0.888 to 1.119) defined for 578 degrees of freedom (gl) at 95% confidence level. As the $\hat{\sigma}_0^2$ was less than the χ^2 / gl lower limit and considering the absence of errors in the adopted procedures, it was suggested that the weight of the observations was underestimated. Therefore, the $\hat{\sigma}_0^2$ was used to compute the standard deviation of the estimated parameters.

After applying the CLSM, the maximum residual value was 0.011 m. Also, the maximum *a posteriori* standardized residual value was 3.502, smaller than the critical τ value (4.118) defined for 578 gl at 95%. Therefore, it was concluded that the observations used in this work did not present substantial errors.

Figure 6 shows the H_N estimated for the altimetric network, considering the stations 2223S, 9305U and 2263Z of the RN-IBGE as reference. Figure 7 shows the uncertainties of the H_N values ranging from 0.000 m to 0.032 m whereas Figure 8 shows the C computed directly, according to Equation 5, using computed γ_v (Equation 6) and estimated H_N, shown in Figure 6.

The results in Figures (5) and (7) show that the position and spatial distribution of the reference stations affected significantly the estimation of the uncertainties of the estimated H_N values.

The H_N values estimated using the three reference stations (2223S, 9305U and 2263Z) were compared with known values (Tables 1, 2 and Figure 9), according to IBGE (2018). The results for the 27 stations analyzed show that the differences (Table 1) range from 0.119m to -0.307m, with -0.008m average and 0.102m root-mean-square deviation (Table 2). Also, the largest differences between -0.124m (2226R) and -0.308m (1361T) are concentrated in the northeastern region of the studied area (Figure 9), close to the 9305U station (used as reference), in approximately 15km stretch of a geometric leveling circuit. In addition, the differences varied from 0.003m to 0.118m (Table 1) in stations (Figure 9) located near to 2223S (used as a reference).

Despite the large differences in the H_N values (Tables 1, 2 and Figure 9), the geometric leveling performed and used in this research was consistent. This result is firstly explained by the fact that all 58 circuits adjusted together, contemplating 30 RN-IBGE stations, 136 RN-DF stations, 34 SAT-IBGE stations, and 1393km total length, had differences smaller than 6mm. km$^{0.5}$ for each circuit and, secondly, the residual values and the statistical test applied showed no problems in the observations.
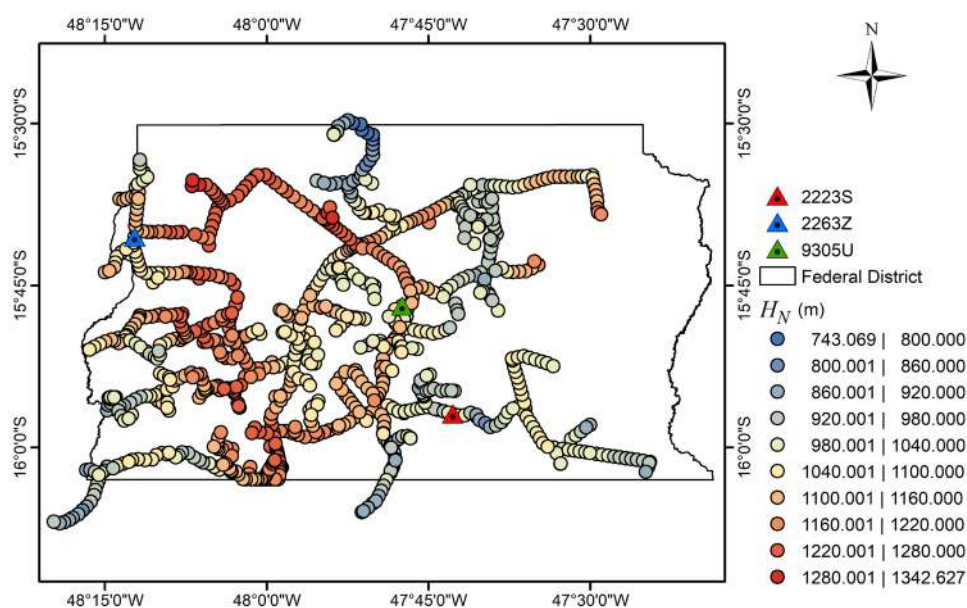

Figure 6: H_N values estimated for the stations and points (RN-DF, RN-IBGE, SAT-IBGE, and RNPI-DF), using stations 2223S, 2263Z and 9305U of the RN-IBGE as reference.

Furthermore, it is noteworthy that between 1953 and 2005, 256 RN-IBGE stations were implanted within the Federal District limits, close to roads that have been maintained or expanded along the years. And, according to IBGE information, only 70 out of 256 stations were considered in good conditions between 1995 and 2011 while the others were either destroyed or not found. In 2016, at the time of this research, only thirty out of 70 stations were found and used because they had reliable planimetric coordinate values estimated by GNSS positioning. Also, as described, it is possible that some of the RN-IBGE stations had their original positions changed, being, therefore, advisable to conduct a rigorous analysis of these stations by investigating the original leveling data measured by IBGE or by repeating some leveling circuits, including the stations with large differences between H_N as determined in this work.
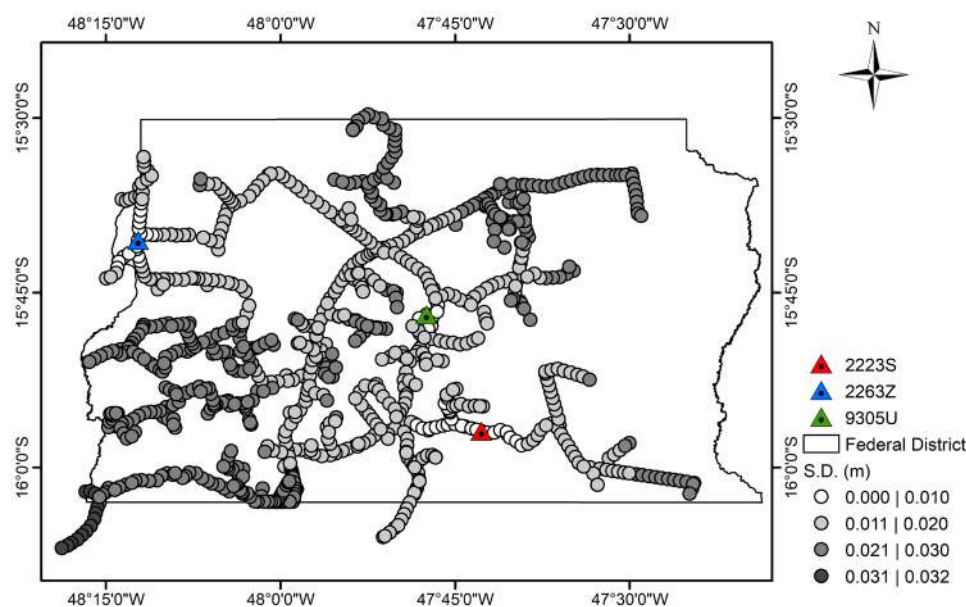

Figure 7: Standard deviations (S.D.) of the H_N values estimated for the stations and points (RN-DF, RN-IBGE, SAT-IBGE, and RNPI-DF), taking the stations 2223S, 2263Z and 9305U of the RN-IBGE as reference.
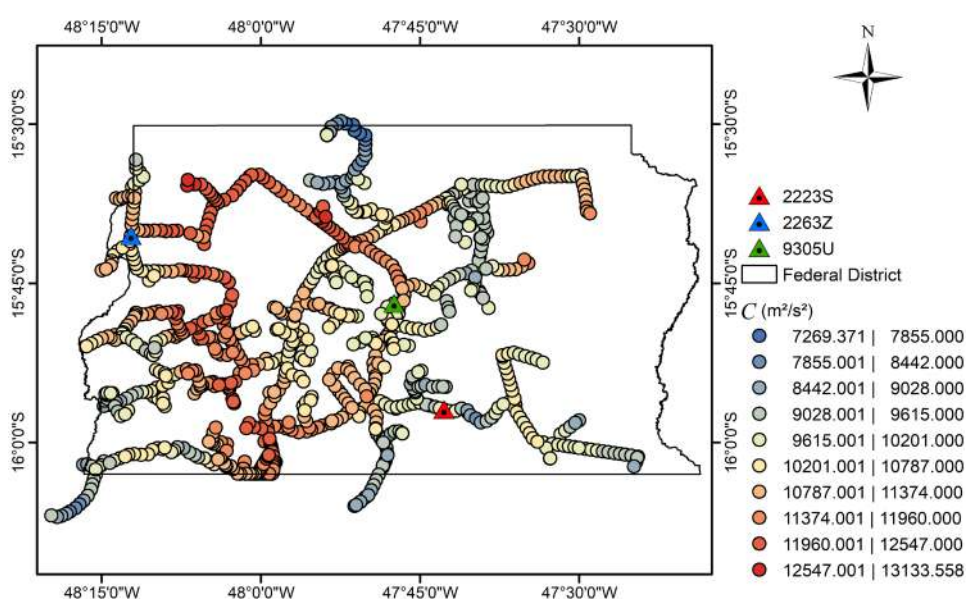

Figure 8: computed using H_N values estimated for the stations and points (RN-DF, RN-IBGE, SAT-IBGE, and RNPI-DF).

Table 1: Differences between the H_N estimated values, according to the adjustment considering the reference stations 2223S, 2263Z and 9305U, and known values ($H_{N\,IBGE}$) for the RN-IBGE stations used.

Station	Longitude (°)	Latitude (°)	$H_{N\,IBGE}$ (m)		H_N (m)		$H_N - H_{N\,IBGE}$ (m)
1190D	-47.808611	-16.038611	935.906	±0.071	935.933	±0.016	0.027
1190G	-47.836111	-16.086389	886.558	±0.071	886.536	±0.019	-0.023
1190H	-47.852778	-16.099444	924.429	±0.071	924.471	±0.019	0.042
1361T	-47.672385	-15.620219	961.315	±0.070	961.008	±0.022	-0.307
2223M	-47.807511	-15.937747	1041.224	±0.071	1041.251	±0.010	0.027
2223N	-47.784167	-15.943889	1010.868	±0.071	1010.867	±0.009	-0.001
2223R	-47.735833	-15.941667	971.820	±0.071	971.823	±0.006	0.003
2223T	-47.693611	-15.950000	990.627	±0.071	990.615	±0.005	-0.012
2223U	-47.674444	-15.953056	867.690	±0.071	867.626	±0.008	-0.064
2223V	-47.655833	-15.966667	897.657	±0.071	897.621	±0.010	-0.036
2223X	-47.636952	-15.960361	964.086	±0.071	964.053	±0.011	-0.033
2226N	-47.661611	-15.733908	923.904	±0.071	923.756	±0.019	-0.148
2226R	-47.656389	-15.708611	971.788	±0.071	971.664	±0.019	-0.124
223Q	-47.656667	-15.621389	950.450	±0.070	950.305	±0.022	-0.144
2263N	-48.249444	-15.729444	1158.403	±0.074	1158.382	±0.011	-0.022
2264D	-48.201944	-15.598333	1062.287	±0.075	1062.338	±0.012	0.051
2264F	-48.190000	-15.575000	1013.240	±0.075	1013.286	±0.014	0.046
2377H	-47.856667	-15.945278	1141.577	±0.071	1141.681	±0.013	0.104
2377P	-47.888056	-15.966389	1137.062	±0.071	1137.179	±0.015	0.116
2378A	-47.877500	-15.918889	1146.738	±0.071	1146.824	±0.015	0.086
2378B	-47.872500	-15.925000	1148.820	±0.071	1148.855	±0.015	0.036
2378C	-47.867500	-15.931667	1152.090	±0.071	1152.191	±0.014	0.101
2378D	-47.862222	-15.938333	1151.760	±0.071	1151.854	±0.014	0.094
4023A	-47.843889	-15.947778	1124.923	±0.071	1125.001	±0.012	0.078
4023B	-47.827839	-15.938263	1148.282	±0.071	1148.400	±0.011	0.119
4032J	-47.833611	-15.927500	1167.752	*	1167.707	±0.011	-0.045
9025U	-47.644444	-15.670278	925.112	±0.071	924.914	±0.020	-0.198

*represents unreported value.

Table 2: Descriptive statistics of the differences between the H_N estimated values, according to the adjustment considering the reference stations 2223S, 2263Z and 9305U, and known values ($H_{N\,IBGE}$) for the RN-IBGE stations used.

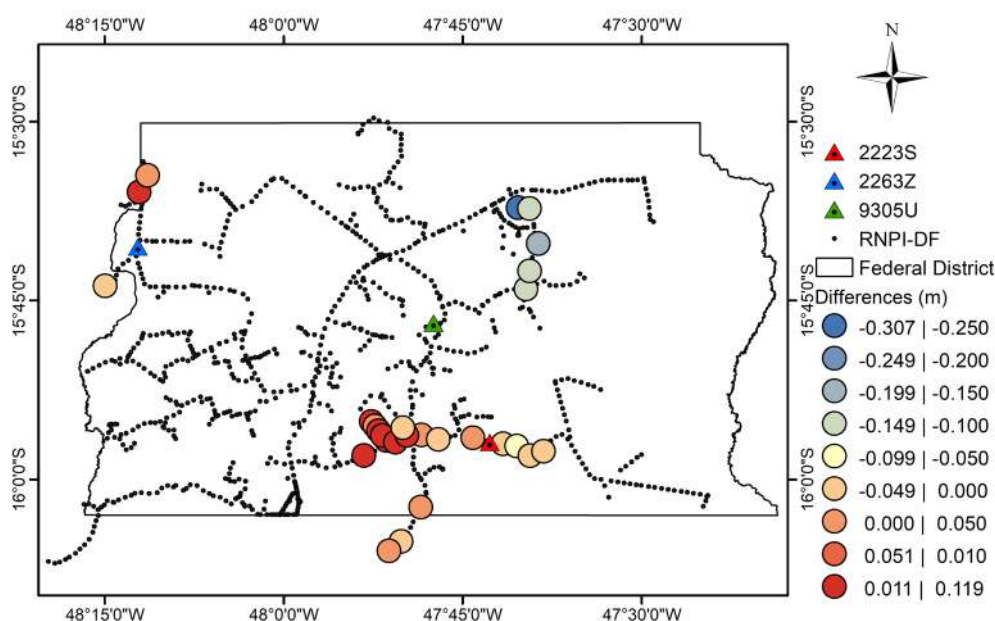| Statistics	$H_N - H_{N\,IBGE}$
Maximum Value (m)	0.119
Minimum Value (m)	-0.307
Average Value (m)	-0.008
Minimum Absolute Value (m)	0.001
Maximum Absolute Value (m)	0.307
Root-mean-square deviation Value (m)	0.102

Figure 9: Spatial distribution of the differences between the H_N estimated values, according to the adjustment considering the reference stations 2223S, 2263Z and 9305U, and known values for the RN-IBGE stations used.

5. Conclusion

This work presents procedures adopted for adjusting, using the geometric leveling, and estimating the normal heights and geopotential numbers for stations and points that are part of the Altimetric Network of the Federal District, Brazil.

The results show good accuracy of the estimated normal height values. Spatially, the accuracy increased (ranging from 0.000 m to 0.032 m) as the number of stations used as reference to compute the normal heights also increased.

Large differences were observed in the results when comparing the normal height values estimated in this research with the known values established by IBGE (2018). It is also observed that the greatest differences were concentrated in specific regions of the study site, in approximately 15km stretch of a geometric leveling circuit located near to one station used as a reference.

Despite the large differences in the normal height values, the geometric leveling performed and used in this research was consistent. This result is firstly explained by the fact that all 58 circuits adjusted together, contemplating 30 RN-IBGE stations, 136 RN-DF stations, 34 SAT-IBGE stations, and 1393km total length, with differences smaller than 6mm.km$^{0.5}$ for each circuit and, secondly, the residual values and the applied statistical test indicated no problems in the observations. Therefore, in the Federal District, it is suggested to use the normal heights and geopotential numbers computed in this research for the RN-DF stations, to meet internal demands. However, due to the large differences detected in the RN-IBGE stations in this study, it is also recommended to conduct a rigorous analysis of the original leveling data measured by IBGE or to repeat some leveling circuits, including those stations with detected large differences between normal heights.

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AUTHOR´S CONTRIBUTION

The author has developed all the work.

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