



Boletim de Ciências Geodésicas

ISSN: 1413-4853

ISSN: 1982-2170

Universidade Federal do Paraná

Freitas, Eliana Vieira de; Araki, Hideo  
SIMULATION OF URBAN GROWTH: A CASE STUDY FOR CURITIBA CITY, BRAZIL  
Boletim de Ciências Geodésicas, vol. 27, Esp., e2021019, 2021  
Universidade Federal do Paraná

DOI: <https://doi.org/10.1590/s1982-21702021000s00019>

Available in: <https://www.redalyc.org/articulo.oa?id=393968851009>

- How to cite
- Complete issue
- More information about this article
- Journal's webpage in redalyc.org

UFRJ  
redalyc.org

Scientific Information System Redalyc  
Network of Scientific Journals from Latin America and the Caribbean, Spain and  
Portugal

Project academic non-profit, developed under the open access initiative

## SIMULATION OF URBAN GROWTH: A CASE STUDY FOR CURITIBA CITY, BRAZIL

### *Simulação do crescimento urbano: estudo de caso para a cidade de Curitiba, Brasil*

Eliana Vieira de Freitas<sup>1</sup> - ORCID: 0000-0002-3394-1347

Hideo Araki<sup>1</sup> - ORCID: 0000-0001-7116-6102

<sup>1</sup>Universidade Federal do Paraná, Departamento de Geomática, Curitiba - Paraná, Brasil.

E-mail: elianavf98@gmail.com; haraki@ufpr.br

Received in 24<sup>th</sup> August 2020.

Accepted in 1<sup>st</sup> April 2021.

#### **Abstract:**

The present work seeks to model the simulation of the landscape of the city of Curitiba, through the use of cellular automata (CA) algorithms, together with the Geographic Information Systems (GIS) and Remote Sensing tools. Four models were elaborated, having as input data classified images from the years 2006, 2009, 2011 and 2014; and different time intervals between the initial and final landscape of the models. Geographic data of the city were also used, as well as the current legislation of the municipality, and such data contributed to the robustness of the modelling. Two validation tests were applied to verify the adequacy of the simulated models concerning the observed reality. The validation performed by the Multiple Resolutions Adjustment method indicated that the model elaborated with data from 2009 and 2011 reached the highest similarity index, being equal to 0.88. Thus, it was possible to carry out geosimulations and these indicated that, as long as the trends observed in the past are maintained, the urban expansion of the municipality will occur at the expense of vegetation.

**Keywords:** Landscape; Modelling; Cellular Automata; Geosimulations.

---

**How to cite this article:** FREITAS, E.V.; ARAKI, H. Simulation of urban growth: a case study for Curitiba city, Brazil. *Bulletin of Geodetic Sciences*. 27(spe): e2021019, 2021.



This content is licensed under a Creative Commons Attribution 4.0 International License.

## 1. Introduction

The accelerated urbanization process seen in recent decades has resulted in the concentration of 54% of the world population in urban areas, in contrast to only 33% in 1950; and it is estimated that the proportion of the urban population will reach 66% in 2050 (United Nations 2019). In Brazil, the urbanization process became more evident from the 1970s due to the rural exodus (Alves, Souza and Marra, 2011) and, currently, 84.72% of the population lives in urban areas (IBGE 2015). Curitiba is the eighth most populous city in the country and, according to IBGE 2019 data, the population increase was 0.83%, higher than the national average, which was equals to 0.79% (Agência IBGE 2019).

In the last 50 years, several computational models have been developed focusing at spatial-temporal phenomena, from fire propagation to vehicle and pedestrian flows, including the modelling urban modification and growth (Marceau and Benenson, 2011). There is a variety of models developed to represent the dynamic nature of urban growth considering cells, agents, neural networks, fractal geometry, among others (Bhatta, 2010). Among the models developed to represent the dynamic nature of urban growth, we highlight those that use cellular automata (CA) which form two-dimensional cell lattices.

Besides, CA models have become popular primarily because they are treatable, have incredible operational simplicity, generate dynamics that can reproduce traditional processes of diffusion changes and contain sufficient complexity to simulate unexpected, and surprising changes such as those observed in the urban landscape (Almeida et al. 2003). Olmedo et al. (2018) present a comparative description of models such as CA\_Markov, Dinamica EGO, Land Change Modeler, LucSim, Metronamica and SLEUTH.

Dinamica EGO contains a series of algorithms called operadores or “functors” that includes spatial algorithms available in GIS and other algorithms designed for spatial simulations, including cellular automata transition functions, and calibration and validation methods (Rodrigues and Soares-Filho 2018). Khatibiti et al (2018) report that the urban land use/land cover change simulation model with DinamicaEGO for the city of Karaj, Iran, is compatible with patches with areas larger than 25 hectares.

The first step in building a model of landscape dynamics is to map changing patterns of land use and land cover in the study region. In this sense, remote sensing techniques can provide reliable and spatially consistent information on large areas, as well as historical records of land use and coverage (Soares-Filho 1998). Knowing that the landscape is dynamic, the present study seeks to combine the tools of Geographic Information Systems and Remote Sensing with landscape simulation modelling processes, which use cellular automata, to perform a simulation of the future growth of the city of Curitiba. It is important to highlight that the simulation allows the construction of future scenarios, allowing planners to anticipate possible transformations in cities.

### 1.1 Objectives

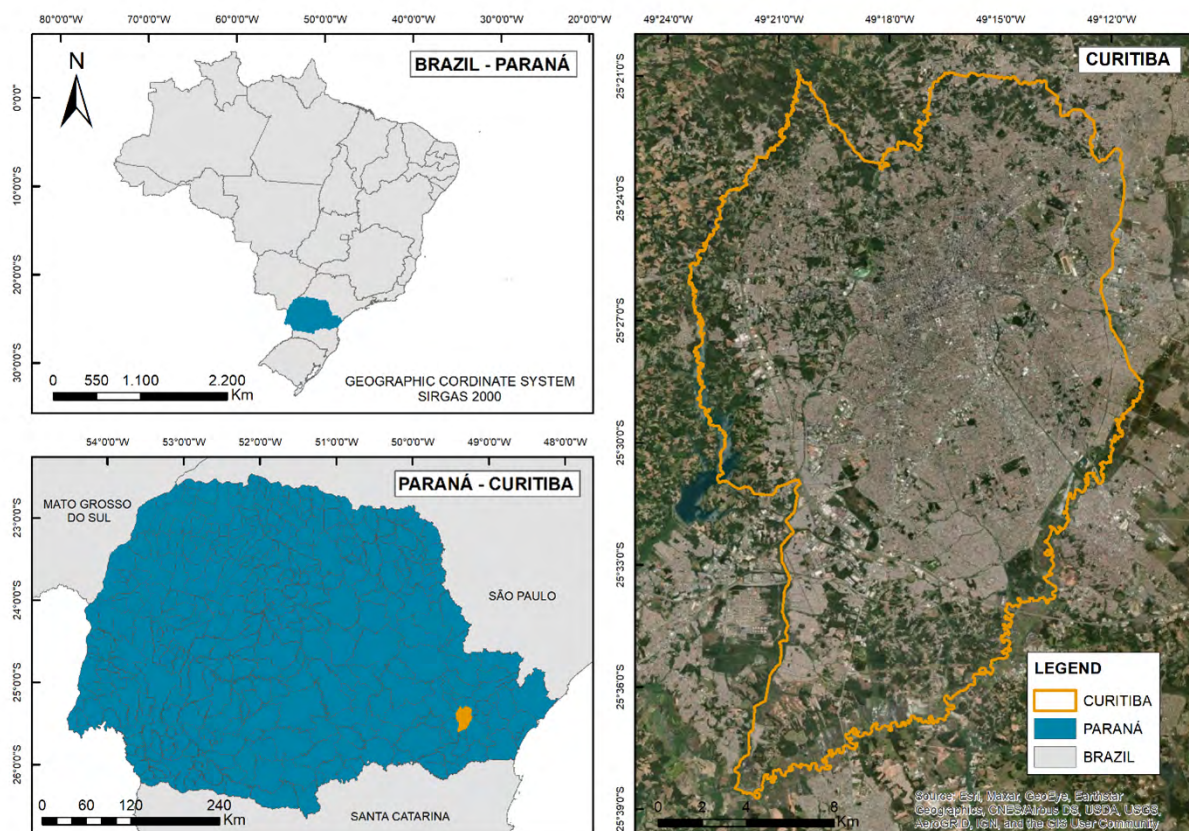
The study had as objectives: i) define static variables that model changes in land use and land cover; ii) analyze whether the static variables are dependent or independent; iii) perform tests with different input data, in order to arrive at the best landscape simulation model; iv) perform model validation using two different methods, fuzzy similarity and adjustment by multiple resolutions; e v) perform geosimulations in the city of Curitiba.

## 2. Study Area

The following criteria were used to choose the study area: consolidated legislation regarding land use and occupation and a complete and updated geographic database of the entire municipality.

The municipality of Curitiba (Figure 1) is located in the South of Brazil, is the capital of the State of Paraná. It has an area of 434.892 square kilometers and an average altitude of 935 meters in relation to sea level (Prefeitura de Curitiba 2021).

Curitiba is the eighth most populous city in the country with 1,933,105 inhabitants and it is estimated that in 2040 the population of Curitiba will consist of 2,004,739 inhabitants (IPARDES 2018).



Source: The authors.

**Figure 1:** Curitiba location map (study area).

## 3. Materials and Methods

### 3.1 Datasets

The materials used in this research were as follows:

- Landsat 5 and Landsat 8 images from the city of Curitiba for the years 2006, 2009, 2011 and 2014. The image bands corresponding to the regions of blue, green, red, near infrared and mid-infrared of the

electromagnetic spectrum were used, all with spatial resolution of 30 meters, free of cloud coverage and matching the same time of the year (October and November);

- Vector files of the city of Curitiba, such as hydrography, street axes, contour lines, and others useful spatial data; and
- Municipal legislation of Curitiba (Law 9800/2000 dealing with Soil Zoning, Use and Occupation; Law 13027/2000 dealing with the delimitation of the perimeter of the Environmental Protection Area - APA do Passaúna; Decree 250/2004 that partially changes the zoning of the Environmental Protection Area - APA do Iguaçu; Decree 250/2004 that amends and updates the economic ecological zoning of the Environmental Protection Area of Passaúna, among others).
- The modelling was performed using the software Dinamica EGO developed and made available by the Remote Sensing Center of the Federal University of Minas Gerais. The modelling environment (Dinamica EGO) has operators called functors. Each functor is responsible for performing a specific task, including operators of map algebra, spatial analysis and operators designed for spatial simulation (Rodrigues, Soares-Filho and Costa 2007).

## 3.2 Methods

### 3.2.1 Image acquisition and classification

All images were obtained for free through the United States Geological Survey (USGS) website, already converted to surface reflectance.

The attributes used in the classifications were three spectral indexes: Normalized Difference Vegetation Index (NDVI), Normalized Difference Built-Up Index (NDBI) and Modification of Normalised Difference Water Index (MNDWI). The expressions used to calculate these indices are presented below:

$$NDVI = \frac{NIR - Red}{NIR + Red} \quad (1)$$

$$NDBI = \frac{MIR - NIR}{MIR + NIR} \quad (2)$$

$$MNDWI = \frac{Green - MIR}{Green + MIR} \quad (3)$$

Where:

*Green* is the green band;

*Red* is the red band;

*NIR* is the near infrared band; and

*MIR* is the short-wavelength mid-infrared band.

Image classification was performed using a decision tree. The classes of interest were: urban, vegetation and water. The intervals used for class separation, in all images, were determined experimentally, through tests until all classes of interest were correctly classified by the decision tree. The classification was validated visually, by comparing the resulting image with the original image.

### 3.2.2 Definition of static variables (evidences)

One of the steps involved in the modelling process is to define static variables, that means, elements of the landscape that are not subject to change. The evidences were defined based on the municipal legislation in force in the municipality of Curitiba and on the cartographic basis through the aid of map algebra. Table 1 shows what these variables are and the criteria used to define them.

**Table 1:** Variables and criteria.

Static variables	Criteria
Areas where urban expansion is unlikely to reach	Water bodies, environmental protection areas, parks and forests. It also includes a military area.
Distance to main streets	Urban growth is expected to occur close to the main streets.
Distance to the “urban” class of the initial landscape	It is expected that the advance of the urban area will take place close to the already urbanized regions.
Distance to the “water” class of the initial landscape	This variable was used to model changes involving the “water” class.
Digital Terrain Model (DTM)	Used to analyze the relationship between altitude and changes involving vegetation.
Slope	The regions with the highest slope are expected to have less urban occupation.
Zones of Curitiba	Zoning establishes spatial planning, which is closely linked to the behavior of changes that occur in the urban landscape.
Zones of the Environmental Protection Area (APA) of Iguaçu and APA of Passaúna	These two variables were considered as complementary to the Zones of Curitiba. In the zoning of Curitiba, only the limits of these two conservation areas were considered. However, besides being areas of environmental preservation, there are areas of urban occupation together, which are guided by municipal legislation.

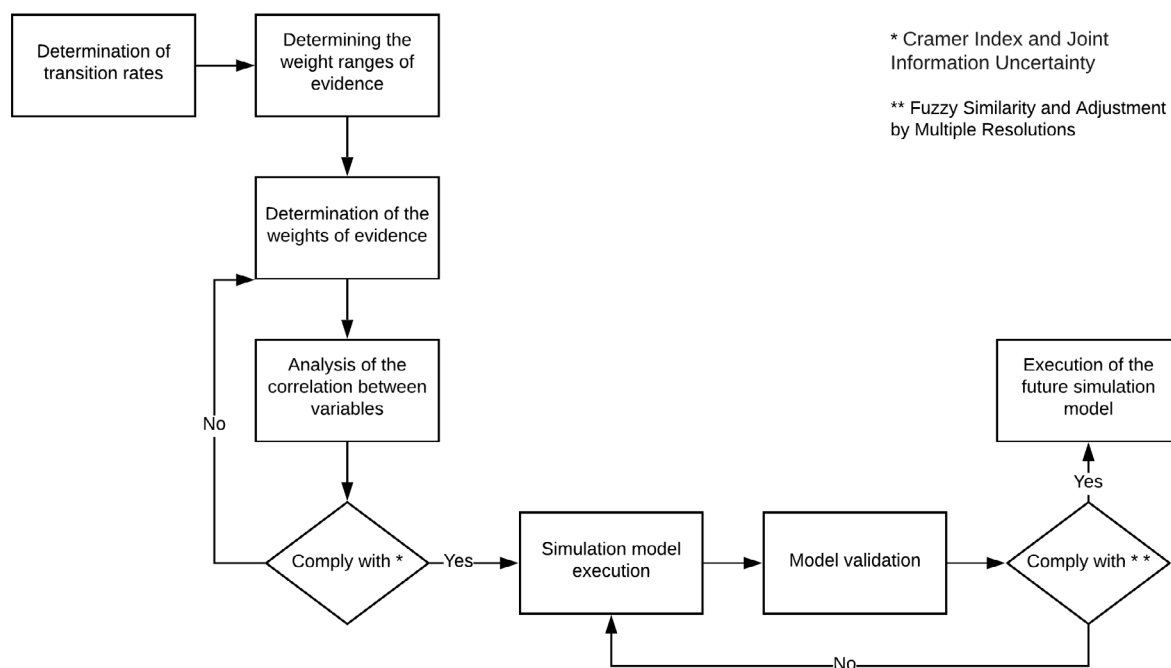
### 3.2.3 Landscape simulation modelling

With the classified images and static variables, it was possible to start the process of modelling the landscape simulation. Four models were elaborated with data from different periods, in order to arrive at the best set of input data for the simulation of the future landscape (Table 2).

**Table 2:** Elaborate models

Model	Initial landscape	Final landscape
1	2006	2009
2	2009	2011
3	2009	2014
4	2011	2014

Figure 2 presents a simplified scheme of the modelling process. The explanation of each step is described right after the image.



Source: The authors.

**Figure 2:** Modelling process.

The determination of the transition rates consists of calculating the transition matrix. Two transition matrices are determined. The first determines the transition between the final and initial landscape, called a single-step matrix. The second matrix is the multiple step matrix and it corresponds to a time step unit (year, month, etc.).

In the second stage, the intervals for the weights of evidence are determined, since the method of evidence weights is applied only to data categorized as, for example, zoning classes. For this reason, continuous data, such as distance to main streets, need to be categorized using distance ranges.

Then, the values of evidence weights for all static variables are calculated. After determining the weights of evidence, it is necessary to analyze the correlation between the static variables used in the modelling. In Dinamica EGO, this analysis can be performed using two indicators, the Cramer Index (V) and the Joint Information Uncertainty (U), which operate, respectively, with real values and percentages of overlapping areas between two maps of variables. This aims to assess the existence of dependence or spatial association between them (Bonham-Carter 1994). According to Bonham-Carter (1994) values less than 0.5, both for the Cramer index and for the Uncertainty of Joint Information, indicates that the variables do not have dependence; therefore, they can be used in modelling. If the values of V and U are higher than 0.5, one of the dependent variables must be eliminated.

The next step is execute the landscape simulation model, where the parameters of the Expander and Patcher functors were defined. It is important to know taht Expander is responsible for expanding existing stains, while Patcher is responsible for creating new stains (Rodrigues, Soares-Filho and Costa 2007). The following input data were used for the modelling process: initial landscape; static variables; weights of evidence and the multiple step matrix.

After the execution of the model, the model is validated by the Multiple Resolution Adjustment similarity method (F), introduced by Costanza (1989), which can be applied to a variety of spatial resolutions by changing the size of a sampling window. This sampling window scans the images and the average adjustment between the two scenes (real and simulated) for a particular window size. It is calculated by the following expression (Almeida 2004):

$$F_w = \frac{\sum_{s=1}^{tw} \left[ 1 - \sum_{i=1}^p \frac{|a_{i1} - a_{i2}|}{2w^2} \right]_s}{tw} \quad (4)$$

Where:

$F_w$  is the fit for the size window  $w \times w$ ;

$a_{i1}$  is the number of cells belonging to class in the simulated image within the sampling window;

$a_{i2}$  is the number of cells belonging to class in the real image within the sampling window;

$p$  refers to the number of different classes found in the sampling window;

$tw$  is the total number of windows sampled in a scene for a window of size  $w \times w$ .

For two identical scenes, a graph relating  $F_w$  and  $w$  will provide a straight line. However, if the scenes show the same proportion of land use classes with different spatial patterns, this line will gradually increase until  $F_w$  reaches a value of 1. If a reasonable adjustment of spatial patterns exists, this curve will fast grow asymptotically (Almeida 2004).

The second method used to evaluate the model's performance was based on the concept of fuzziness of location, where the representation of a cell is influenced by itself, and, to a lesser extent, by the cells in its vicinity (Hagen 2003).

The fuzzy similarity index used in this study was created by the Remote Sensing Center of the Federal University of Minas Gerais and represents an adaptation of the fuzzy similarity index created by Hagen (2003). It is a fuzzy similarity comparison test between the simulated map and the real map (Macedo et al. 2013).

In this study, the comparison was made by relating a difference map between the initial and final landscapes with a difference map between the final and the simulated landscape. A constant decay function was used to this purpose, calculated with the following window sizes: 1x1, 3x3, 5x5, 7x7, 9x9 and 11x11 (Macedo et al. 2013).

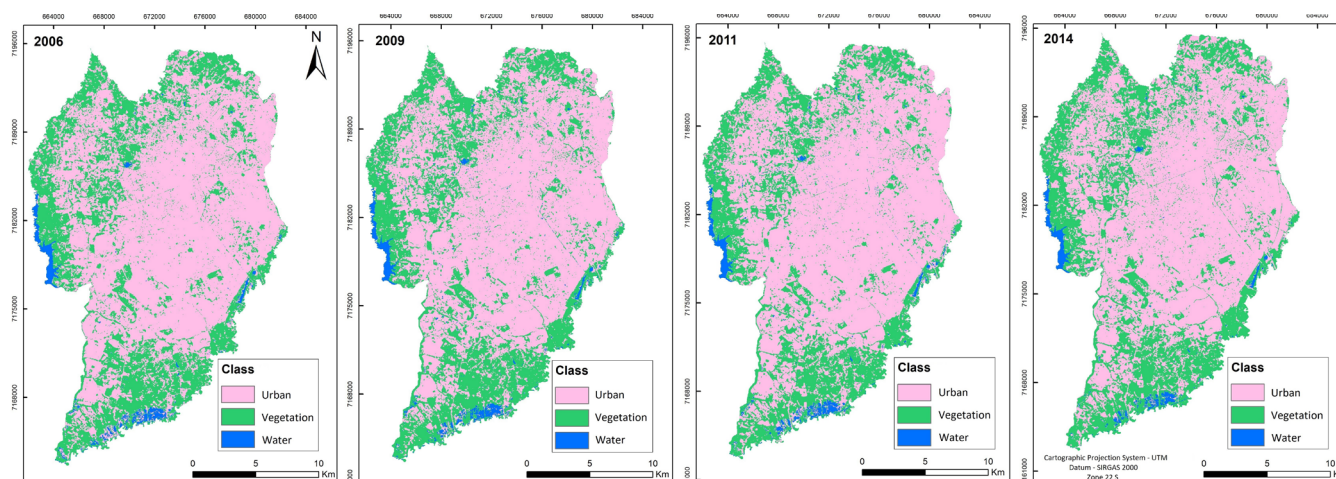
Finally, if the model does not meet the validation criteria it must be adjusted, otherwise it is possible to perform future simulations.

## 4. Results

### 4.1 Land cover maps

The land cover maps of the city of Curitiba, the result of the classification of Landsat images, and which were used in the landscape simulation models are shown in Figure 3.





Source: The authors.

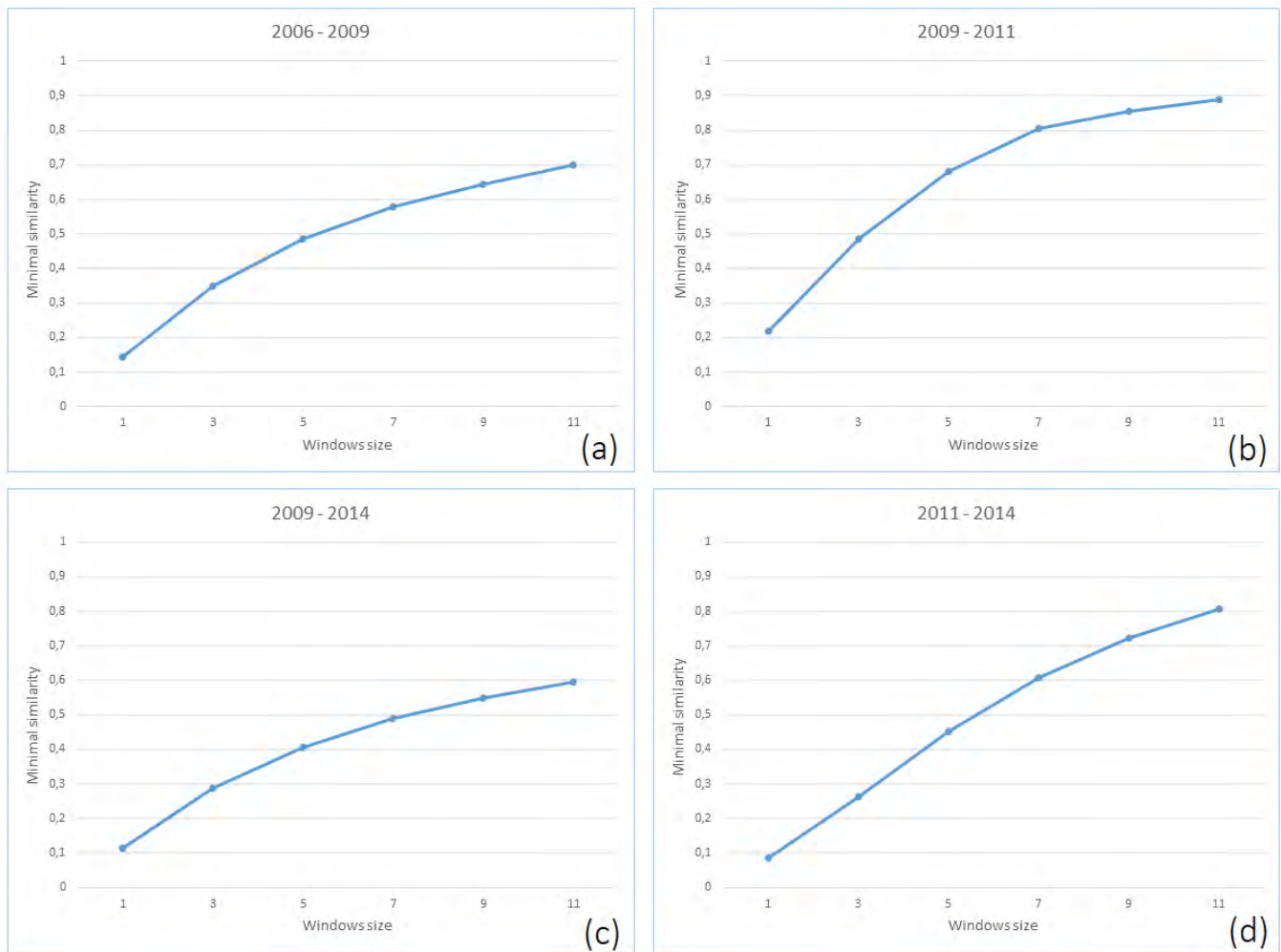
**Figure 3:** Land cover maps of Curitiba.

The class called “urban” reach all types of urban construction, asphalt and exposed soil. The “vegetation” class has all the vegetation coverings in the city, being forest masses or grass. The “water” class represents all types of water bodies in the municipality.

There were some confusions during the classification by decision tree, mainly between the “water” and “urban” classes. The NDBI index (used in the classification of urban areas) has values similar to the MNDWI index (used to classify water bodies) in some specific regions, such as shallow water surfaces and densely urbanized regions.

## 4.2 Modelling performed

The four models performed were elaborated in order to reach the data set that produced the best landscape simulation modelling. The choice of the best model was based on the analysis of similarity calculated using the Multiple Resolution Adjustment method (F) as shown in Figure 4. In the figure, the horizontal axis represents the window size ( $w$ ) and the vertical axis represents the adjustment for the window size  $w \times w$  ( $Fw$ ).



Source: The authors.

**Figure 4:** Similarity of the elaborated models. (a) similarity of the model elaborated with the images of 2006 and 2009, (b) similarity of the model elaborated with the images of 2009 and 2011, (c) similarity of the model elaborated with the images of 2009 and 2014 e (d) similarity of the model elaborated with the images of 2011 and 2014.

Oliveira (2015) says that minimum F values above 0.8 are considered acceptable. Looking at Figure 4 it is possible to notice that there are different degrees of similarities between the real landscape and the simulated landscape. Among the different models elaborated, two met the criterion of F above 0.8, being the model elaborated with data from 2009 and 2011, and the model elaborated with data from 2011 and 2014. However, the model that used the images from 2009 and 2011 had a higher F value, being equal to 0.88, so it was used in the next simulation process.

### 4.3 Modelling elaborated with data from 2009 and 2011

The application of the validation test showed that the data from 2009 and 2011 resulted in the best modelling, so these data were used in the elaboration of the simulation. Table 3 shows the area values of the landscape classes for 2009 and 2011, as well as the area of expansion/reduction of the classes and relative changes.

**Table 3:** Initial and final landscape area.

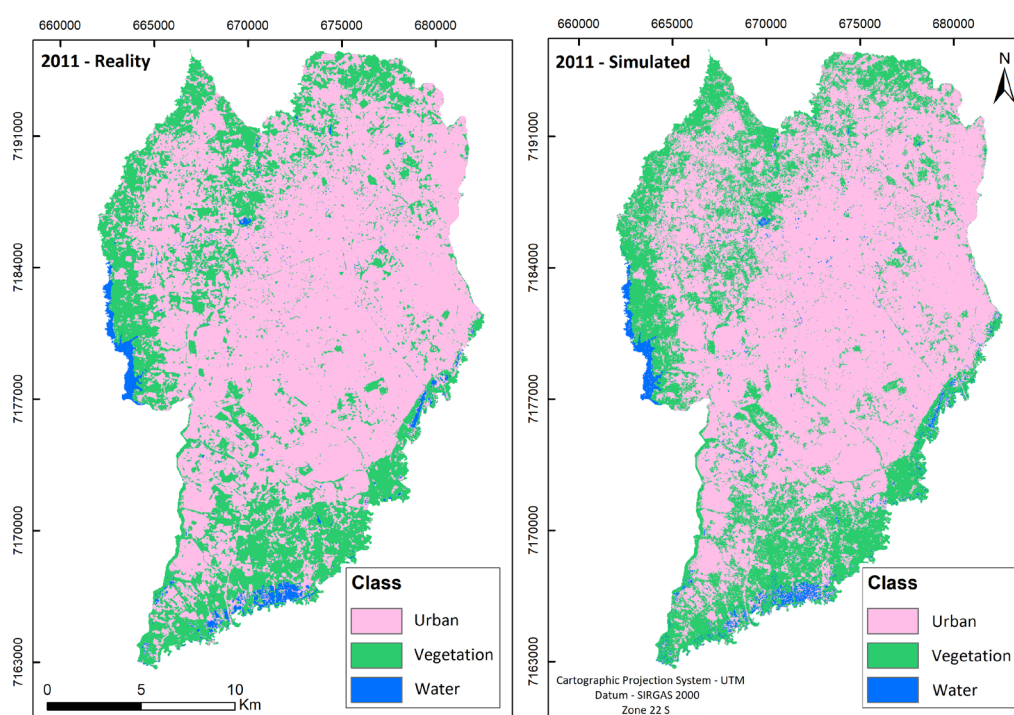
Class	Area (km <sup>2</sup> )		Expansion/Reduction (km <sup>2</sup> )	Relative change
	2009	2011		
Urban	282.93	299.59	16.66	5.89%
Vegetation	143.40	127.12	-16.28	-11.35%
Water	8.47	8.09	-0.38	-4.49%

Table 3 shows that in the period from 2009 to 2011 there was an increase in the urban area and a reduction in vegetation. It is interesting to note that the value of the urban area that increased was similar to the value of the green area that decreased, indicating that the urban expansion took place at the expense of vegetation.

The difference between the values of area of the “water” class in the two periods analyzed is due to recurrent drought and flood processes in some portions of the territory, notably in the southeastern portion, close to the Guaçu River where small temporary lakes are formed.

#### 4.3.1 Landscape Simulation Modelling

Figure 5 shows the result of the landscape simulation modelling for the year 2011 compared to the real landscape of the same year. Such modelling was performed in the environment of the software Dinamica EGO and had as input data, the initial landscape (land cover map of the year 2009), the static variables, the weights of evidence and the multiple stage matrix.



Source: The authors.

**Figure 5:** Comparison between the real landscape (left) and the simulated landscape (right).

Looking at Figure 5, we can see the similarity between the simulated landscape and the real landscape, however there were some confusions involving the “water” class and the “urban” class, the simulation map shows

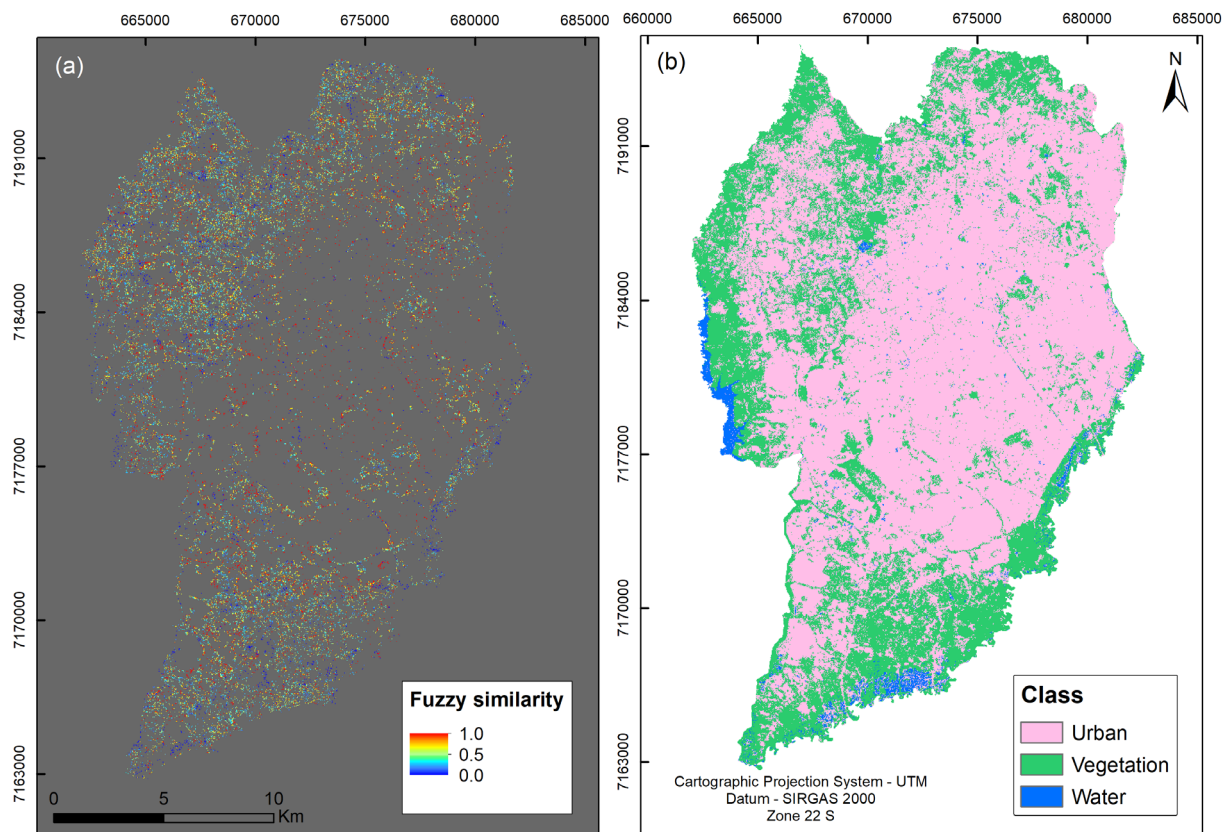
that changes were generated from “urban” to “water” erroneously. Since these errors were occasional, they have not harmed the final result.

Table 4 shows a comparison between the class area values for the 2011 simulation image and the 2011 classified image. It is possible to notice that the values are similar, what indicates the adequacy of the model.

**Table 4:** Real and simulated landscapes area.

Class	Area (km <sup>2</sup> )	
	2011 - Reality	2011 - Simulated
Urban	299.59	299.60
Vegetation	127.12	127.13
Water	8.09	8.07

Validation was performed using the Fuzzy similarity method (S) and the Multiple Resolution Adjustment method (F). The application of Fuzzy similarity results an image with values that vary between 0 and 1, where values above 0.5 indicate a good fit of the model (Oliveira 2015). Figure 6 shows a comparison between the result of applying the Fuzzy similarity and the simulated map of 2011.



Source: The authors.

**Figure 6:** Validation by fuzzy similarity. (a) Fuzzy similarity, where values above 0.5 indicate a better fit of the model and (b) Simulated landscape for the year 2011.

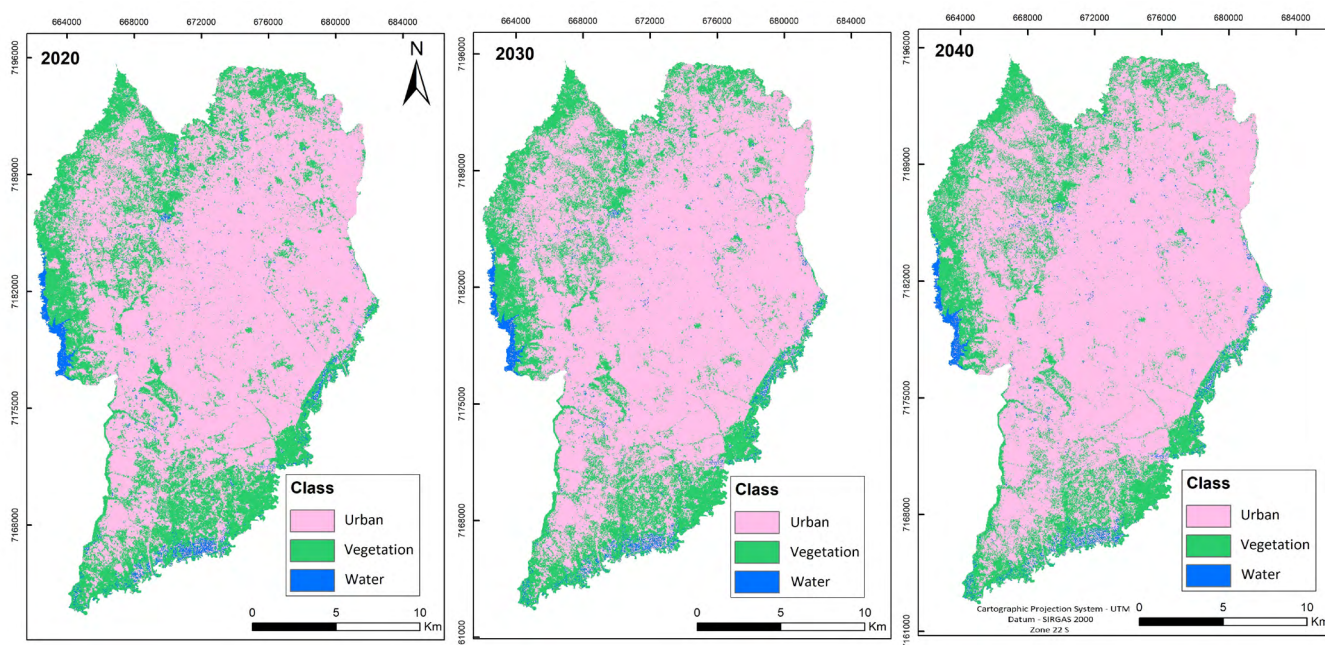


Figure 6 shows that the lowest rates of adequacy of the simulated landscape occurred mainly on the edges of the city of Curitiba, close to the regions that have bodies of water, as mentioned in item 4.3, causing confusion between “urban” and “water” during the classification stage. However, it was observed that the model had a good fit, with most of S values above 0.5.

Like validation performed by the Multiple Resolution Adjustment method, we can see in Figure 4 (b) that the similarity varies from 21% in a 1 by 1 window, to 88% in an 11 by 11 window. Considering that the cell size of Landsat images is 30 meters, we can say that the simulation reached an adequacy value above 50% in a spatial resolution of approximately 120 meters.

#### 4.4 Geosimulations

Still in the Dinamica EGO software environment and using as input data, the initial landscape (2009 land cover map), static variables, evidence weights and multiple transition rates, simulated landscapes (geosimulations) were generated for the years 2020, 2030 and 2040, as shown in Figure 7.



Source: The authors.

**Figure 7:** Geosimulations

Looking at Figure 7 it can be seen that the urban expansion did not reach the preservation areas, located in the northwest and southeast portion of the municipality. This is an indication that the static variables used are adequately modelling the urban expansion of the city.

The area of the geosimulations classes for the years 2020, 2030 and 2040 was also calculated, as shown in Table 5.

**Table 5:** Geosimulations class areas

Class	Area (Km <sup>2</sup> )		
	2020	2030	2040
Urban	306.66	318.70	328.39
Vegetation	120.22	108.48	99.05
Water	7.91	7.61	7.35

Looking at Table 5, we note that from 2020 to 2040 there is an increase in the area of the “urban” class of about 22 ; and in the same period there is a decrease in the area value of the “vegetation” class of approximately 21 . The same trend can be observed in the input data used in the modelling (real landscape of the years 2009 and 2011), Table 3 shows that the area value of the “urban” class was close to the value of “vegetation” that decreased.

These data may indicate that the expansion of the urban area occurs at the expense of vegetation, since the value of urban area that increased from 2020 to 2040 is close to the value of vegetation area that decreased in the same period.

For the “water” class there is also a decrease, from 2020 to 2040 the reduction was 0,55 . However, changes involving the water class were not considered in this work due to problems related to the NDBI and MNDWI indexes used in the classification of images and also due periods of drought and flood that occur in the city.

## 5. Discussion

The validation methods used here showed that the simulated landscape for the year 2011 reached an acceptable level of similarity, according to the criteria established by Oliveira (2015), where values of F above 0.8 and S above 0.5 are considered acceptable.

The static variables, which must be independent of each other for the application of the weight of evidence method, were analyzed and it was found that they met the correlation criteria established by Bonham-Carter (1994) who says that the correlation value both for the Cramer Index and for the Joint Information Uncertainty it should be less than 0.5.

Geosimulations showed that the static variables used in the modelling exercised their functions of associating or repelling changes. Looking at the future simulation maps, we can see that the expansion of the urban area did not reach the city’s permanent preservation areas, even in the simulated landscape of the year 2040. It is worth mentioning that these simulations represent reality in an approximate mode and are related to past trends. It means that if the trends of the past continue, the simulations will be more faithful to reality. Otherwise, the simulations will not be adequate.

Another critical point to be highlighted is the fact that the elaborated modelling applies only to the city of Curitiba since each city has its characteristics and the modelling of the spatial dynamics must take into account the attributes of each study site.

For future studies it would be pertinent: (a) consider other variables in addition to multitemporal images and the restrictions introduced by legislation regarding land use and occupation; (b) modelling for a grid with a resolution compatible with high spatial resolution orbital images; and (c) consider the Metropolitan Region of Curitiba (MRC) in modelling, as it is also constantly changing and many of the trends that occur in Curitiba also occur in the MRC. Also, these changes that occur in the MRC interferes in the landscape of Curitiba. In this case, it would be necessary to conduct a study for each city separately and have a geographic database of the entire MRC.

## 6. Conclusion

This article aimed to model future scenarios for the city of Curitiba. For this purpose, static variables were selected to model changes in the landscape over time, and in order for them to exercise their functions (repel or associate changes), the independence between them was verified through the use of two different indicators (Cramer Index and Uncertainty of Joint Information).

The experiments carried out were validated using the Multiple Resolution Adjustment method (Figure 4) which revealed that the experiment carried out with data from 2009 and 2011, resulted in the best modelling. Thus, a second validation was performed on this model, through the application of the Fuzzy similarity (Figure 6) such a test showed that this experiment modeled the changes in an acceptable way, allowing the execution of geosimulations.

Finally, geosimulations indicated that if the observed trend is maintained (Table 3), the urban area of Curitiba will continue to expand at the expense of vegetation.

## AUTHOR'S CONTRIBUTION

Eliana Vieira de Freitas wrote the manuscript and carried out the experiments. Hideo Araki contributed by discussing the theme and the results, and paper reviewing and refinement.

## REFERENCES

- Agência IBGE. 2019. IBGE divulga as estimativas da população dos municípios para 2019. [online] Available at: <<https://agenciadenoticias.ibge.gov.br/agencia-sala-de-imprensa/2013-agencia-de-noticias/releases/25278-ibge-divulga-as-estimativas-da-populacao-dos-municipios-para-2019>>. [Accessed 26 February 2020].
- Almeida, C. M. D. 2004. Modelagem da dinâmica espacial como uma ferramenta auxiliar ao planejamento: simulação de mudanças de uso da terra em áreas urbanas para cidades de Bauru e Piracicaba (SP), Brasil. (PhD Thesis) - Curso de Pós-graduação em Sensoriamento Remoto - Inpe - São José dos Campos, São Paulo. [online] Available at: <[http://www.dpi.inpe.br/gilberto/teses/tese\\_claudia.pdf](http://www.dpi.inpe.br/gilberto/teses/tese_claudia.pdf)>. [Accessed 23 November 2019].
- Almeida, C. M. et al. 2003. Stochastic cellular automata modelling of urban land use dynamics: empirical development and estimation. *Computers, Environment and Urban Systems*, Vol. 27, n. 5. doi: 10.1016/S0198-9715(02)00042-X.
- Alves, E., Souza, G. da S. and, Marra, R. 2011. Êxodo e sua contribuição à urbanização de 1950 a 2010. *Revista de Política Agrícola*, n. 2, pp.80-88.
- Bhatta, B. 2010. *Analysis of Urban Growth and Sprawl from Remote Sensing Data*. Springer-Verlag. doi: 10.1007/978-3-642-05299-6.
- Bohham-Carter, G. F. 1994. *Geographic Information System for Geoscientists*. Ottawa: Pergamon.
- Costanza, R. 1989. Model goodness of fit: A multiple resolution procedure. *Ecological Modelling*, pp.199-215. [online] Available at: <<https://www.pdx.edu/sites/www.pdx.edu/sustainability/files/Costanza%20EM%201989.pdf>>. [Accessed 23 November 2019]. n. 2, pp.80-88.
- Hagen, A. 2003. Fuzzy set approach to assessing similarity of categorical maps. *International Journal of Geographical Information Science*, April 2003. doi: 10.1080/13658810210157822.

- IBGE - Instituto Brasileiro de Geografia e Estatística. 2015. PNAD - Pesquisa Nacional por Amostra de Domicílios. [online] Available at: <<https://biblioteca.ibge.gov.br/visualizacao/livros/liv98887.pdf>> [Accessed 01 May 2020].
- IPARDES, 2018. Projeção da População dos Municípios do Paraná - 2018 - 2040. [online] IPARDES - Instituto Paranaense de Desenvolvimento Econômico e Social. Available at: <<http://www.ipardes.pr.gov.br/Pagina/Projecao-da-Populacao-dos-Municipios-do-Parana-2018-2040>> [Accessed 15 March 2021].
- Khatibiti, A. et al. 2018. Urban land use/cover changes toward sustainability. *Journal of Environmental Engineering and Landscape Management*, vol 26 Issue 1:1-7. ISSN 1648-6897.
- Macedo, R. D. C., et al. 2013. Spatial dynamic modelling of land cover and land use change associated with the sugarcane expansion. *Bulletin of Geodetic Sciences*, v. 19, n. 2. [online] Available at: <[https://www.scielo.br/scielo.php?script=sci\\_arttext&pid=S1982-21702013000200009](https://www.scielo.br/scielo.php?script=sci_arttext&pid=S1982-21702013000200009)>. [Accessed 23 November 2019]. doi: 10.1590/S1982-21702013000200009.
- Marceau, D.J.; Benenson, I. Ed. 2011. *Advanced Geosimulation Models*. Bentham Science Publishers. DOI: 10.2174/97816080522261110101.
- Oliveira, M. A. de. 2015. Modelagem Dinâmica Espacial das Classes de Uso e Cobertura da Terra de Venâncio Aires/RS. (Undergraduate thesis) - Curso de Engenharia Ambiental - Centro Universitário Univates - Lajeado - RS - Brazil. [online] Available at <<https://www.univates.br/bdu/bitstream/10737/983/1/2015MauroAndredeOliveira.pdf>>. [Accessed 26 February 2020].
- Olmedo, M. T. C. et al (Eds). 2018. *Geomatic Approaches for Modeling Land Change Scenarios*. ISBN 978-3-319-60801-3. Springer.
- Prefeitura de Curitiba, 2021. Perfil da Cidade de Curitiba. [online] Curitiba.pr.gov.br. Available at: <<https://www.curitiba.pr.gov.br/conteudo/perfil-da-cidade-de-curitiba/174>> [Accessed 15 March 2021].
- Rodrigues, H. O., Soares-Filho, B. S., Costa, W. L. de S. 2007. Dinamica EGO, uma plataforma para modelagem de sistemas ambientais. [online] Available at: <<http://marte.sid.inpe.br/attachment.cgi/dpi.inpe.br/sbsr@80/2006/11.06.17.59/doc/3089-3096.pdf>>. [Accessed 26 February 2020].
- Rodrigues, H.; Soares Filho, B. S. 2018. A short presentation of Dinamica EGO. In: Olmedo, M. T. C. et al (Eds). *Geomatic Approaches for Modeling Land Change Scenarios*. Springer. ISBN 978-3-319-60801-3.
- Soares-Filho, B. S. 1998. Modelagem da Dinâmica de Paisagem de uma Região de Fronteira de Colonização Amazônica. (PhD thesis) - Curso de Engenharia, Escola Politécnica da Universidade de São Paulo, São Paulo - SP, Brazil. [online] Available at: <[http://www.dpi.inpe.br/cursos/tutoriais/modelagem/referencias/tese\\_britaldo/tese.html](http://www.dpi.inpe.br/cursos/tutoriais/modelagem/referencias/tese_britaldo/tese.html)>. [Accessed 23 November 2019].
- United Nations. 2019. *Revision of World Population prospects*. Department of Economic and Social Affairs.
- USGS - United States Geological Survey. Landsat Surface Reflectance. n.d. [online] Available at: <[https://www.usgs.gov/land-resources/nli/landsat/landsat-surface-reflectance?qtscience\\_support\\_page\\_related\\_con=0#qt-science\\_support\\_page\\_related\\_con](https://www.usgs.gov/land-resources/nli/landsat/landsat-surface-reflectance?qtscience_support_page_related_con=0#qt-science_support_page_related_con)>. [Accessed 24 November 2019].