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# Risk prioritization based on the combination of FMEA and dual hesitant fuzzy sets method

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## Abstract

**Paper aims:** This paper proposes the combination of the quality tool FMEA (Failure Modes Effects and Analysis) with the DHFS (Dual Hesitant Fuzzy sets) technique to process judgements with hesitation and hence conduct the prioritization of failure modes considering a group decision making problem.

**Originality:** There are no studies that combine the FMEA tool with the DHFS technique.

**Research method:** Firstly, this paper presents a review of the current FMEA literature. Then, the group decision model is presented combining the FMEA and the DHFS. Finally, an illustrative example in the context of supplier failure modes is brought to guide future applications of the proposal.

**Main findings:** The paper presents a model that combines the FMEA tool with the DHFS. It allows considering different risk factors weights in a group decision process with experts from several areas. The model is also able to deal with the different types of hesitations present in the judgements.

**Implications for theory and practice:** The traditional FMEA does not deal with individual judgments of different decision makers. The new proposal can be easily applied in different contexts of potential failure modes analysis considering different types of hesitation in group decision making, such as medical and humanitarian.

## Keywords

FMEA. Dual Hesitant Fuzzy Sets. Risk evaluation. Supplier failure modes. Group decision.

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## 1. Introduction

The quality management processes seek to cope with changes in an increasingly complex and dynamic environment while ensuring that organizations are able to deliver products and services that comply and satisfy their customers (Fonseca, 2015). The ISO 9000 series of international standards has become an important reference and a key management structure for all types of organizations around the world (Fonseca & Domingues, 2017). The update of the ISO 9001: 2015 standard presents a thinking based on risk management and process-based approaches, which highlights the growing emphasis on risk management within quality management programs (Sitnikov et al., 2017).

Failure Modes and Effects Analysis (FMEA) is one of the most well-known quality management tools used to deal with risk assessment in continuous improvement programs for products, processes, and services (Kumru & Kumru, 2013). The literature presents a large number of studies that apply the FMEA in several problems, such as: applications to reduce medical errors (Li et al., 2017; Bonfant et al., 2010; Chiozza & Ponzetti, 2009);



development of new products (Lorenzi & Ferreira, 2018; Lin et al., 2015; Chin et al., 2008); approaches for supplier selection problems (Arabsheybani et al., 2018; Foroozesh et al., 2018; Chen & Wu, 2013), among others.

The traditional FMEA risk prioritization number (RPN) is based on the decision maker's judgment about the risk factors severity, occurrence, and detection. Although the FMEA technique is widely diffused, many studies present proposals for the improvement of the technique by adding more risk factors and/or incorporating weights on them (Fattahi & Khalilzadeh, 2018; Chang, 2016; Liu et al., 2012). In addition, the evaluation of the risk factors is difficult to be precisely determined, since it involves imprecision that needs to be treated (Rahmatin et al., 2018; Liu et al., 2013; Liu et al., 2015a). In order to overcome this problem, some papers combine fuzzy techniques with the FMEA, such as: Fuzzy Cognitive Map (Rezaee et al., 2018), Fuzzy MOORA (Arabsheybani et al., 2018), Fuzzy Inference System (Geramian et al., 2017), Fuzzy VIKOR (Liu et al., 2015b), Technique for Order of Preference by Similarity to Ideal - Fuzzy TOPSIS (Song et al., 2013) and Analytic Hierarchy Process - Fuzzy AHP (Abdelgawad & Fayek, 2010).

The literature presents the FMEA as an efficient tool that must be conducted as a group decision making problem, which involves the interrelationship between the departments of an organization (Liu et al., 2015c; Chin et al., 2009). However, most techniques do not deal with the group decision making process and do not allow decision makers to hesitate in their judgments. Hence, techniques based on the hesitant fuzzy representations (Zolfaghari & Mousavi, 2018; Chang et al., 2018) and intuitionistic fuzzy (Tooranloo et al., 2018; Mirghafoori et al., 2018; Can, 2018) were proposed to consider group decision making in FMEA applications, however, each representation deals with hesitation in its own way (Calache, 2018). The Hesitant Fuzzy representations are used when there are doubts in the judgment, being able to activate one or more linguistic terms for the variable (Zhang et al., 2016). The Intuitionist Fuzzy representation is used to add a margin of hesitation in the definition of the fuzzy numbers that represent the linguistic terms (Nguyen, 2016). To explore the advantages of both representations Zhu et al. (2012) proposed the Dual Hesitant Fuzzy Sets (DHFS). The DHFS are more flexible than the existing fuzzy sets, which enables to deal with much more information in real world problems (Singh, 2017). Also, there are no studies that combine the FMEA tool with the DHFS technique.

Therefore, this paper presents a new model that combines the FMEA tool with the Dual Hesitant Fuzzy technique. The advantages of the proposed model over the FMEA proposals found in the literature are that it deals simultaneously with: different risk factor weights; individual judgments that represent the knowledge of experts from several different areas; it considers imprecision in the judgments through fuzzy set theory; it uses linguistic terms to represent the judgments of decision makers; it considers the hesitation from the linguistic terms parameterization and; it considers the hesitation in the judgments caused by the decision makers. The article is structured as follows: Section 2 presents the basic concepts of the traditional FMEA; Section 3 discusses the group decision making applications in FMEA. The Dual Hesitant Fuzzy Sets technique and the aggregation operator used are exhibited in Section 4. Section 5 presents the proposed model that integrates the FMEA application with the DHFS approach to deal with group decision making. An illustrative application is developed in Section 6 and, finally, Section 7 presents the conclusions and future works.

## 2. FMEA

FMEA is an analytical method of risk assessment that seeks to identify, prioritize, and determine the causes and effects associated with failure modes (Fattahi & Khalilzadeh, 2018). Potential failure modes are defined as the way in which a component, a system, or a subsystem can potentially fail to achieve or deliver the functionality described for the item (Xu et al., 2002). FMEA is not only limited to failures, but includes errors in general, which are the inability to function in a certain way or to operate in an undesired way regardless of the cause (Jiang et al., 2017). In this way, the FMEA is presented as a powerful tool that has been applied in several industrial sectors (Liu et al., 2013), such as the aerospace sector (Yazdi et al., 2017; Chaudhuri et al., 2013), product development (Zhu et al., 2018; Chang, 2016), and several applications in healthcare (Liu et al., 2017; Wang et al., 2016).

The most important indicator used by the traditional FMEA is the Risk Priority Number (RPN), calculated according to the following equation:  $RPN = O \times S \times D$ , where O, S and D respectively represent the risk factors of Occurrence, Severity, and Detection (Geramian et al., 2017). These factors are evaluated by experts that give scores from 1 to 10 for each factor related to each risk analyzed, according to the example presented in Table 1. The results obtained by multiplying the risk factors are used to prioritize the analyzed risks (Liu et al., 2012).

Some literature review studies were performed to analyze applications of the FMEA tool: Liu et al. (2013) presents the main limitations that are being addressed by the new proposals of combinations of techniques with the FMEA; Kabir & Papadopoulos (2018) present a review of the application of fuzzy representations with the

Table 1. Typical rates for failure mode index.

Rating	Occurrence	Severity	Detection
1	Almost never	No severity	Certainty of detection
2	Extremely Low	Extremely Low	Extremely high
3	Very low	Very low	Very High
4	Low	Low	High
5	Moderately Low	Moderately Low	Average
6	Average	Average	Moderately Low
7	Moderately High	Moderately High	Low
8	High	High	Very low
9	Very High	Very High	Extremely Low
10	Extremely High	Extremely High	Almost impossible

FMEA tool for safety and reliability engineering; Ng et al. (2017) reviews the integration of FMEA with other quality tools for problem solving; Sutrisno et al. (2013) carried out a survey of Improvement strategy in FMEA; Chrysostom & Dwivedi (2013) raised the methodologies used in the FMEA and pointed the intuitionistic fuzzy as being the representation used to deal with the problem of group decision making. Despite the literature review work presented, no study has mapped the techniques used to deal with the problem of group decision making, which will be addressed in the next section.

### 3. Group decision making in the FMEA

As pointed out in the literature, the procedures for risk assessment and prioritization of failure modes by the FMEA tool can usually be considered as a multi criteria group decision making problem with the imprecision of information (Wang et al., 2018). In this way, the Fuzzy-Set-Based representations have been incorporated in several multi criteria decision-making techniques in order to overcome the limitations pointed out in the traditional FMEA method to deal with imprecise data (Hu et al., 2019; Huang et al., 2018; Liu et al. 2013).

Table 2 presents the articles that address the group decision making problem in the FMEA tool. It shows the proposal of each article and the factors considered in the problem scope. Therefore, it is possible to identify the advantages of the proposed model when compared with the other studies. The proposed combination of FMEA with the DHFS representation can cope with all aspects analyzed in Table 2, such as: the definition of different weights for the risk factor; collection of individual judgments in order to consider a holistic problem view; fuzzy logic is used to deal with imprecisions; the decision makers use linguistic terms to represent their judgments; the hesitation in the definition of the linguistic terms parameterization and the hesitation in the judgments of the decision makers are addressed by the DHFS representation.

The studies in Table 2 present a variety of techniques, representations of information and application contexts. With the exception of TOPSIS, all techniques were applied only once. In addition, it is possible to perceive a predominance of techniques that aggregate different judgments from different decision makers and few consensus approaches that involve the minimization of divergence among decision makers. Concerning the aggregation operators, the most commonly applied are those based on weighted average, for instance, Huang et al. (2017) use the weighted averaging operator of linguistic distribution assessments; Chai et al. (2016) apply the linguistic weighted average operator; Wang et al. (2016) use the intuitionistic fuzzy ordered weighted average operator; Tooranloo (2016) employ the intuitionistic fuzzy weighted average operator; Chaudhuri et al. (2013) implement the ordered weighted averaging operator.

Regarding the information representations, a great predominance of the linguistic variables based on the fuzzy set theory was observed. However, it was not found studies that deals with Dual-Hesitant Fuzzy sets in failure modes analysis.

### 4. Dual hesitant fuzzy

The fuzzy set theory initially proposed by Zadeh (1965) obtained wide acceptance in several fields of study, as well as in the risk management through the application of fuzzy FMEA. Many generalized forms of fuzzy sets have been proposed to deal with the imprecision of these problems such as: intuitionistic fuzzy (Tooranloo et al., 2018; Zhu et al., 2018); hesitant fuzzy (Chang et al., 2018; Soyer et al., 2016); type-2 fuzzy (Akyuz & Celik, 2018; Bahrebar et al., 2018), among others.

Table 2. FMEA articles and group decision making.

Paper	Proposal	Pondering risk factors weights	Consider individual judgments	Deals with imprecision	Use linguistic terms for judgments	Hesitation in the linguistic terms parameterization	Hesitation in the judgment
Jenab & Dhillon (2005)	Group-based failure effects analysis.			✓			
Chin et al. (2009)	Failure mode and effects analysis using a group-based evidential reasoning approach.		✓				
Zhang & Chu (2011)	Fuzzy-RPNs-based method integrating weighted least square method.		✓	✓	✓		
Chaudhuri et al. (2013)	A group decision making approach using numeric and linguistic data.		✓	✓	✓		
Ko (2013)	2-tuple linguistic representational model for constructing HOQ-based failure modes and effects analysis.	✓		✓	✓		
Helvacioğlu & Ozen (2014)	Integrates and apply the Fuzzy TOPSIS technique in the FMEA tool.	✓	✓	✓	✓		
Liu et al. (2014a)	A new risk priority model is proposed for evaluating the risk of failure modes based on fuzzy set theory and MULTIMOORA method.	✓	✓	✓	✓		
Liu et al. (2014b)	Risk priority model using interval 2-tuple hybrid weighted distance (ITHWD) measure.	✓	✓	✓	✓		
Liu et al. (2014c)	Risk assessment methodology using intuitionistic fuzzy hybrid weighted Euclidean distance operator.	✓	✓	✓	✓	✓	
Lolli et al. (2015)	FlowSort-GDSS is proposed to sort the failure modes into priority classes by involving multiple decision-makers.	✓		✓			
Liu et al. (2015c)	FMEA approach combining interval 2-tuple linguistic variables with gray relational analysis.	✓	✓	✓	✓		
Chai et al. (2016)	Interval type-2 fuzzy sets applied with the FMEA tool.	✓		✓	✓		
Wang et al. (2016)	Integrates Complex PROportional ASessment (COPRAS) and analytic network process (ANP) method and interval-valued intuitionistic fuzzy context.	✓	✓	✓	✓	✓	
Hajiagha et al. (2016)	Propose an extension of VIKOR method under Fuzzy Belief Structure for FMEA.	✓	✓	✓	✓		

Table 2. Continued...

Paper	Proposal	Pondering risk factors weights	Consider individual judgments	Deals with imprecision	Use linguistic terms for judgments	Hesitation in the linguistic terms parameterization	Hesitation in the judgment
Tooranloo (2016)	Proposed a model for failure mode and effects analysis based on intuitionistic fuzzy approach.	✓	✓	✓	✓	✓	
Chang (2016)	Propose an integrated method, combining multiattribute failure mode analysis (MAFMA) and 2-tuple representation.	✓	✓	✓	✓		
Guo (2016)	Combines intuitionistic fuzzy sets (IFSs) with evidence theory to analyze the potential failure modes.	✓	✓	✓	✓	✓	
Huang et al. (2017)	Applies linguistic distribution assessments and employs an improved TODIM in FMEA.	✓	✓	✓	✓		
Yazdi et al. (2017)	Extend FMEA by considering a group decision-making under the fuzzy environment.		✓	✓	✓		
Liu et al. (2017)	Failure Mode and Effect Analysis Using Cloud Model Theory and PROMETHEE Method.		✓	✓	✓		
Foroozesh et al. (2018)	Utilizes IVFSs and mean-variance-skewness concepts by a group of supply chain-experts to solve a selection problem.			✓	✓		
Zhu et al. (2018)	Failure Mode and Effects Analysis Considering Consensus and Preferences Interdependence.	✓	✓	✓	✓		
Zhang et al. (2019)	Proposes a consensus based group decision making framework for FMEA.	✓	✓	✓	✓		✓
Wang et al. (2018)	A risk evaluation and prioritization method for FMEA with prospect theory and Choquet integral.		✓	✓	✓		
Chang et al. (2018)	Integrates the ordered weighted geometric (OWG) operator and hesitant fuzzy linguistic term sets.		✓	✓	✓		✓
Our proposal	Combining FMEA with DHFS.	✓	✓	✓	✓	✓	✓

Zhu et al. (2012) proposed a new generalization of the fuzzy sets, the dual hesitant fuzzy sets (DHFS). This representation seeks to combine the intuitionist and hesitant concepts, integrating the advantage of each one of them. The Hesitant Fuzzy Sets were proposed by Torra (2010) and allow the membership degree of an element in a set to be associated with several possible values, enabling the decision maker's hesitation in the definition of the variables that represent his judgment (Zhang et al., 2017). As in intuitionistic fuzzy sets, DHFS also has degrees of membership and non-membership functions; however, these two functions are expressed by several determined numbers rather than a single number, modeling the real-world problems more accurately

than other generalizations of fuzzy theory. Specifically, DHFS is very useful for group decision-making problems, when it is difficult to determine the membership and non-membership functions (Yu et al., 2016). Recently, this approach has been widely used for multicriteria decision-making problems, with the development of new models and theories (Zhang et al., 2017). Calache et al. (2021) present a literature review of dual hesitant fuzzy sets applications.

Zhu et al. (2012) define the concept of dual hesitant fuzzy as an extension of hesitant fuzzy sets. Given a fixed set  $U$ , a Dual Hesitant Fuzzy set  $D$  in  $X$  is represented as:  $\tilde{D} = \{x, \tilde{h}_{\tilde{D}}(x), \tilde{g}_{\tilde{D}}(x) | x \in U\}$ , in which  $\tilde{h}_{\tilde{D}}(x) \in \tilde{g}_{\tilde{D}}(x)$  are two sets of some value in the range  $[0, 1]$  denoting the membership and non-membership degrees of the element  $x \in U$  to the set  $D$ , respectively, with the conditions:  $0 \leq \gamma, \eta \leq 1$  e, for all  $x \in U$   $\gamma \in \tilde{h}_{\tilde{D}}(x), \eta \in \tilde{g}_{\tilde{D}}(x), \gamma^+ \in \tilde{h}_{\tilde{D}}^+(x) = \cup_{\gamma \in \tilde{h}_{\tilde{D}}(x)} \text{Max}\{\gamma\}, \eta^+ \in \tilde{g}_{\tilde{D}}^+(x) = \cup_{\eta \in \tilde{g}_{\tilde{D}}(x)} \text{Max}\{\eta\}$ .

The basic operations and properties of the DHFS sets were also presented by Zhu et al. (2012). Given three elements DHFS,  $d, d_1$  and  $d_2$ , and a non-negative integer  $n$ , then the basic operations can be presented as Yu et al. (2016):

Union- Sum:

$$d_1 \oplus d_2 = \cup_{\tilde{\gamma}_1 \in \tilde{h}_1, \tilde{\gamma}_2 \in \tilde{h}_2, \tilde{\eta}_1 \in \tilde{g}_1, \tilde{\eta}_2 \in \tilde{g}_2} \{\{\tilde{\gamma}_1 + \tilde{\gamma}_2 - \tilde{\gamma}_1 \tilde{\gamma}_2\}, \{\tilde{\eta}_1 \tilde{\eta}_2\}\} \quad (1)$$

Intersection-Multiplication:

$$d_1 \otimes d_2 = \cup_{\tilde{\gamma}_1 \in \tilde{h}_1, \tilde{\gamma}_2 \in \tilde{h}_2, \tilde{\eta}_1 \in \tilde{g}_1, \tilde{\eta}_2 \in \tilde{g}_2} \{\{\tilde{\gamma}_1 \tilde{\gamma}_2\}, \{\tilde{\eta}_1 + \tilde{\eta}_2 - \tilde{\eta}_1 \tilde{\eta}_2\}\} \quad (2)$$

Multiplication by  $n$ :

$$nd = \cup_{\tilde{\gamma} \in \tilde{h}, \tilde{\eta} \in \tilde{g}} \{\{1 - (1 - \tilde{\gamma})^n\}, \{\tilde{\eta}^n\}\} \quad (3)$$

Potentialiation by  $n$ :

$$d^n = \cup_{\tilde{\gamma} \in \tilde{h}, \tilde{\eta} \in \tilde{g}} \{\{\tilde{\gamma}^n\}, \{1 - (1 - \tilde{\eta})^n\}\} \quad (4)$$

Based on the concepts proposed by Zhu et al. (2012), Wang et al. (2014) has developed some aggregation operators, including the Dual Hesitant Fuzzy Weighted Average (DHFWA), which is used to calculate weighted averages of judgments based on DHFS linguistic terms (Zeng et al., 2018; Li, 2014).

Let  $\tilde{d}_j = (\tilde{h}_j, \tilde{g}_j) (j = 1, 2, \dots, n)$  be a DHFS set of elements and let  $\omega = (\omega_1, \omega_2, \dots, \omega_n)$  be the vector of weights related to  $\tilde{d}_j$  with  $\sum_{j=1}^n \omega_j = 1$ , then the DHFWA aggregation operator can be represented as follows (Wang et al., 2014):

$$DHFWA(\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_n) = \oplus_{j=1}^n (\omega_j \tilde{d}_j) = \cup_{\tilde{\gamma}_j \in \tilde{h}_j, \tilde{\eta}_j \in \tilde{g}_j} \left\{ \left\{ 1 - \prod_{j=1}^n (1 - \tilde{\gamma}_j)^{\omega_j} \right\}, \left\{ \prod_{j=1}^n (\tilde{\eta}_j)^{\omega_j} \right\} \right\} \quad (5)$$

Let  $\tilde{d}_j = (\tilde{h}_j, \tilde{g}_j) (j = 1, 2, \dots, n)$  be a set of DHFS elements, the score function  $s(d_j)$  is defined as follows:

$$S(d_j) = \frac{1}{\# \tilde{h}} \sum_{\tilde{\gamma}_j \in \tilde{h}_j} \tilde{\gamma}_j - \frac{1}{\# \tilde{g}} \sum_{\tilde{\eta}_j \in \tilde{g}_j} \tilde{\eta}_j \quad (6)$$

Where the  $\# \tilde{h}$  and  $\# \tilde{g}$  are respectively the number of membership ( $\tilde{\gamma}_j \in \tilde{h}_j$ ) and non-membership ( $\tilde{\eta}_j \in \tilde{g}_j$ ) elements considered in the calculus.

For example, if  $S(d_3) > S(d_1) > S(d_2)$ , then,  $\tilde{d}_3 \succ \tilde{d}_1 \succ \tilde{d}_2$ .

## 5. Proposed model

Figure 1 presents the proposed model for the application of Dual Hesitant Fuzzy Sets for group decision making in the FMEA tool. The proposed model has four main steps: characterization of failure modes, decision makers' judgments, aggregation of judgments and ranking of risks based on the RPN value. They are detailed as follows.

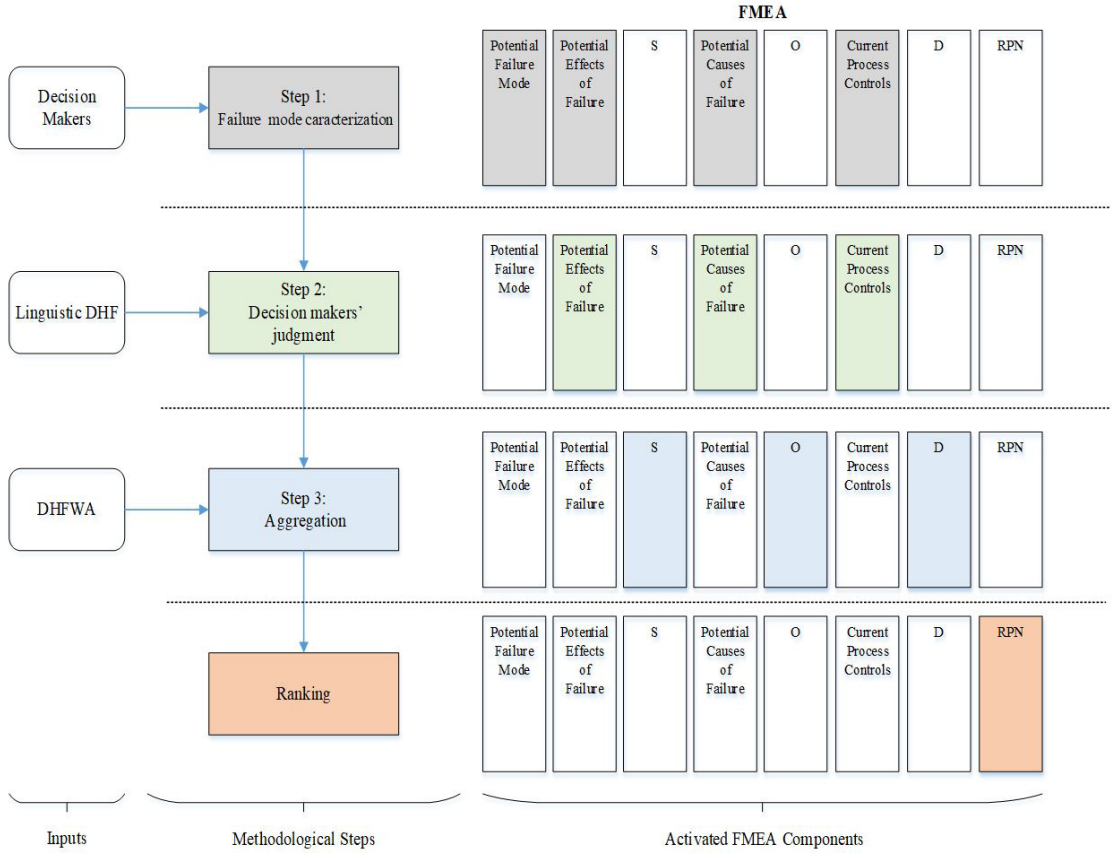


Figure 1. Proposed model for DHF application in FMEA.

Characterization of failure modes: this step consists in defining the risks in focus. To contribute for a better evaluation of decision makers in relation to severity, occurrence and detection, failure modes are detailed, respectively, in relation to the effects of the failure, the potential causes of failure and the current control processes. Qualitative quality management tools, such as the relationship diagram and the Ishikawa diagram are typically applied for this step (Tummala & Schoenherr, 2011; International Electrotechnical Commission, 2009). This step also defines the decision makers who will be responsible for the analysis and judgments of failure modes;

II) Decision makers' judgment: based on the characterization of failure modes, the decision makers should make individual judgments concerning the relative importance of the risk factors (Severity, Occurrence, and Detection) and their level of failure.

Let  $\tilde{w}_{kj} = (\tilde{h}_{kj}, \tilde{g}_{kj})$  be a group of DHFS elements, the individual assessment  $\tilde{w}_{kj}$  on the weight of the risk factors represented by  $j = 1(Severity), 2(Occurrence), 3(Detection)$  is given by  $n$  decision makers  $DM_k (k = 1, 2, \dots, n)$ . The linguistic terms and respective intuitionist numbers used by decision makers to assess the importance of risk factors are presented in Table 3 as described in Liu et al. (2015a). The judgments collected following the DHFS approach can be presented according to the matrix given by Equation 7.

Severity Occurrence Detection

$$\begin{matrix} DM_1 \\ DM_2 \\ \vdots \\ DM_n \end{matrix} \begin{pmatrix} \tilde{w}_{11} & \tilde{w}_{12} & \tilde{w}_{13} \\ \tilde{w}_{21} & \tilde{w}_{22} & \tilde{w}_{23} \\ \vdots & \vdots & \vdots \\ \tilde{w}_{n1} & \tilde{w}_{n2} & \tilde{w}_{n3} \end{pmatrix} \quad (7)$$



Table 3. Language variables and respective intuitionist numbers applied to assess the weight of risk factors.

Linguistic Terms	Membership degree	Non-membership degree
Very Low Importance (VL)	0.10	0.85
Low Importance (L)	0.30	0.65
Medium Importance (M)	0.50	0.50
High Importance (H)	0.75	0.20
Very High Importance (VH)	0.90	0.05

Table 4. Linguistic terms and intuitive fuzzy numbers for assessing the level of failure mode in relation to risk factors.

Linguistic Terms	Severity and Occurrence		Detection	
	Membership degree	Non-membership degree	Membership degree	Non-membership degree
(EL) Extremely low	0.00	0.90	1.00	0.00
(VL) Very Low	0.10	0.75	0.90	0.10
(L) Low	0.25	0.60	0.75	0.10
(ML) Medium Low	0.40	0.50	0.60	0.25
(M) Medium	0.50	0.45	0.50	0.45
(MH) Medium High	0.60	0.25	0.40	0.50
(H) High	0.75	0.10	0.25	0.60
(VH) Very High	0.90	0.10	0.10	0.75
(EH) Extremely High	1.00	0.00	0.00	0.90

For each decision maker  $k$ , an individual matrix  $\tilde{A}_k$  ( $k = 1, 2, \dots, n$ ) should be used to collect the judgments  $\tilde{a}_{ij}$  on the level of failure modes  $FM_i$  ( $i = 1, 2, \dots, m$ ) in relation to the risk factors  $j = 1, 2, 3$ . Let  $a_{ij} = (h_{kij}, g_{kij})$  be a group of DHFS elements, the matrix given by Equation 8 can be used to collect the DHFS judgments using the linguistic terms and respective fuzzy numbers presented in Table 4 (Liu et al., 2015a).

$$\tilde{A}_k = \begin{matrix} & \begin{matrix} \text{Severity} & \text{Occurrence} & \text{Detection} \end{matrix} \\ \begin{matrix} FM_1 \\ FM_2 \\ \vdots \\ FM_m \end{matrix} & \begin{pmatrix} \tilde{a}_{11}^k & \tilde{a}_{12}^k & \tilde{a}_{13}^k \\ \tilde{a}_{21}^k & \tilde{a}_{22}^k & \tilde{a}_{23}^k \\ \vdots & \vdots & \vdots \\ \tilde{a}_{m1}^k & \tilde{a}_{m2}^k & \tilde{a}_{m3}^k \end{pmatrix} \end{matrix} \quad (8)$$

- III) Aggregation of the judgments: the judgments collected in the previous phase will be aggregated through the application of the dual hesitant fuzzy aggregation operator (DHFWA), and the scores are calculated according to Equations 5 and 6, related to  $k = 1, 2, \dots, n$  decision makers. As a result of this aggregation, a vector of weights is obtained,  $W_j$  ( $j = 1, 2, 3$ ) that represent the importance of risk factors, and an aggregated matrix of judgments  $A_{ij}$  ( $i = 1, 2, \dots, m; j = 1, 2, 3$ ) of the failure modes, according to the matrix given by Equation 9.

$$\begin{matrix} & \begin{matrix} \text{Severity} & \text{Occurrence} & \text{Detection} \end{matrix} \\ \begin{matrix} FM_1 \\ FM_2 \\ \vdots \\ FM_m \end{matrix} & \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} & A_{m3} \end{pmatrix} \end{matrix} \quad (9)$$

The results obtained by the aggregation are contained in the range between  $[-1, 1]$ . To enable RPN calculation, the aggregated results need to be converted to scales compatible with traditional FMEA. For the translation of values  $W_j$  which represent the weights of the risk factors, Equation 10 must be used and then the values must be weighted, resulting in the weight vector  $\omega_j$  ( $j = 1, 2, 3$ )  $\in [0, 1]$ . Then this weight vector should be normalized such that  $\sum_{j=1}^3 \omega_j = 1$ . For the translation of values  $A_{ij}$ , which represent the aggregate evaluations for each failure mode  $i$  in relation to each risk factor  $j$ , Equation 11 must be applied, resulting in the value  $R_{ij}$  ( $i = 1, 2, \dots, m; j = 1, 2, 3$ )  $\in [1, 10]$ .

$$\omega_j = 0.5 w_j + 0.5 \quad \forall j = 1, 2, 3. \quad (10)$$

$$R_{ij} = 4.5 A_{ij} + 5.5 \quad \forall i = 1, 2, \dots, m; j = 1, 2, 3. \quad (11)$$

IV) Ranking of the failure modes: the value of the  $RPN_i$  ( $i = 1, 2, \dots, m$ ) for each failure mode is calculated by means of the weighted multiplication according to Equation 12. The failure modes with higher values of  $RPN_i$  should be prioritized for mitigation.

$$RPN_i = [(R_{i1})^{S_1} \cdot (R_{i2})^{S_2} \cdot (R_{i3})^{S_3}] \quad \forall i = 1, 2, \dots, m \quad (12)$$

## 6. Illustrative application

The analysis of potential failure modes of a supplier is an important activity of supply chain management, due to the impact that potential failure modes have on the performance of a supply chain (He & Yang, 2018; Wu et al., 2006). For the evaluation of supplier potential failure modes, the FMEA tool is widely applied, for example, Valinejad & Rahmani (2018) evaluate the failure modes in the internet service providers; Foroozesh et al. (2018) assess sustainable-suppliers for manufacturing services; Ghadge et al. (2017) present an application in a printed circuit board supply chain; among others. In this way, an illustrative application related to supplier potential failure modes was developed following the steps proposed by the model described in Section 5.

Characterization of failure modes: assume a group of decision makers, composed of a production manager (DM1), a purchasing manager (DM2) and a quality manager (DM3) with respective weights [0.4; 0.3; 0.3], which evaluated 6 potential failure modes, presented in Table 5, in relation to the traditional FMEA risk factors (Severity, Occurrence and Detection). DMs used quality tools such as relationship diagram and Ishikawa diagram to analyze failure modes. These tools complement the FMEA in managing information and improving the failure modes understanding;

II) Decision-makers' judgment: Table 6 presents the judgments of each decision maker on the risk factors importance using the linguistic terms in Table 3.

Each of the decision makers made judgments on the levels of failure modes according to risk factors using the linguistic terms in Table 4. The results of these assessments are presented in Tables 7 to 9 with the abbreviations of the linguistic terms provided by the decision makers.

In this case, each of the decision makers has the autonomy to judge the failure modes according to their respective points of view. In this way, differences between judgments can happen. The procedure of judgment aggregation, presented in the next step, seeks to consider these different points of view from different areas of the organization to obtain a more systemic view of the impacts of failure modes. In addition, the process of collecting judgments and presenting information in tables can contribute to the presentation of these different points of view that can be used in a consensus search for the development of improvement action plans.

III) Aggregation of the judgments: In order to elucidate the application of the presented equations, the severity weight calculation is detailed. Equation 5 is applied to obtain the aggregated values for the weight of the risk factors as shown in Equation 13. Then, Equation 6 is used to find the dual hesitant fuzzy score, as presented by Equation 14. The translated score is obtained through the use of Equation 10, as demonstrated in Equation 15. The normalized value is obtained by the sum as presented in Equation 16. The same procedure is carried out for the calculation of the other risk factors weights, and the results are exhibit in Table 10.

$$DHFWA(\tilde{w}_{11}, \tilde{w}_{21}, \tilde{w}_{31}) = \left\{ \left\{ 1 - (0.9^{0.4} * 0.75^{0.3} * 0.9^{0.3}) \right\}, \left\{ 0.05^{0.4} * 0.20^{0.3} * 0.05^{0.3} \right\} \right\} = \{0.868\}, \{0.076\} \quad (13)$$

$$S(W_1) = 0.868 - 0.076 = 0.792 \quad (14)$$

$$\omega_1 = 0.5 * 0.792 + 0.5 = 0.896 \quad (15)$$

$$S_I = \frac{0.896}{0.896 + 0.532 + 0.489} = 0.372 \quad (16)$$

Table 5: Supply failure modes used in FMEA evaluation.

	Potential Failure Effect	Potential Cause of Failure	Current process of control
<b><math>FM_1</math></b> - Supplier's inability to meet quality requirements	-Production of non-conforming products; -Commitment of the image with customers;	-Lack of process control; -Material used inappropriate;	- Sampling inspection.
<b><math>FM_2</math></b> - Can not provide competitive pricing	- Loss of product quality. - Higher production costs; - Decrease in profit margin.	-Lack of training - High transportation cost	- Benchmarking
<b><math>FM_3</math></b> - Low supplier financial health	- Increase in the cost of production; - Late delivery; - Fines for delay; - Do not deliver products;	- Inefficient management; - Low sales volume.	- Audit programs.
<b><math>FM_4</math></b> - Inability to cope with changes in volume demand	-Commitment of the image with the client. - Do not deliver products; - Late delivery; - Fines for delay;	-Ineffective communication; -Lack of capacity.	- Communication by e-mail and telephone;
<b><math>FM_5</math></b> - Lack of integration between customer and supplier	- Unbalance of production; - Late delivery; - Fines for delay; -Commitment of the image with the client;	-Ineffective communication;	- No control process.
<b><math>FM_6</math></b> - Failure to meet delivery requirements	- Increased cost of raw material. - Delays in production; - Late delivery; - Fines for delay; -Commitment of the image with the client.	-Cargo theft.	- Monitoring of arrival times in the logistics sector.

Table 6. Judgments of the importance of risk factors according to decision makers.

	Severity	Occurrence	Detection
<b><math>DM_1</math></b>	VH	H, VH	H
<b><math>DM_2</math></b>	H	M	H
<b><math>DM_3</math></b>	VH	H	M, H

Table 7. Judgments of the  **$DM_1$**  on the levels of failure modes according to each risk factor.

<b><math>DM_1</math></b>	Severity	Occurrence	Detection
<b><math>FM_1</math></b>	H, VH	L, ML	M
<b><math>FM_2</math></b>	VH, EH	M, MH	MH, H
<b><math>FM_3</math></b>	H, VH	M	H
<b><math>FM_4</math></b>	H	H, VH	MH
<b><math>FM_5</math></b>	M, MH	H	L, ML
<b><math>FM_6</math></b>	H, VH	L, ML	ML, M

Table 8. Judgments of the  $DM_2$  on the levels of failure modes according to each risk factor.

$DM_2$	Severity	Occurrence	Detection
$FM_1$	H, VH	ML, M	M, MH
$FM_2$	H	M	H, VH
$FM_3$	M, MH	ML, M	VH
$FM_4$	M	H	M, MH
$FM_5$	L, ML	VH	M
$FM_6$	H, VH	VL	ML, M, MH

Table 9. Judgments of the  $DM_3$  on the levels of failure modes according to each risk factor.

$DM_3$	Severity	Occurrence	Detection
$FM_1$	VH, EH	VL, L	L, ML, M
$FM_2$	L, ML, M	ML, M	MH, H
$FM_3$	L, ML	L, ML	MH, H
$FM_4$	MH, H	H, VH	H, VH, EH
$FM_5$	MH, H	H, VH	M; MH
$FM_6$	H	VL, L	M, MH

Table 10. Aggregated weights of risk factors.

	Severity	Occurrence	Detection
Aggregated score	0.792	0.532	0.489
Translated and weighted score	0.372	0.318	0.309

In the same way, the calculation for the aggregated scores for the  $FM_i$  in the risk factor severity is detailed as follows. The DHFWA aggregated value is obtained by Equation 5, as show in Equation 17. Then, the DHF score calculation presented in Equation 18 is obtained using Equation 6. To get the translated score for the failure modes levels, Equation 11 is used, as presented by Equation 19. The aggregated scores are presented in Table 11.

$$\begin{aligned}
 DHFWA(\tilde{a}_{11}^1, \tilde{a}_{11}^2, \tilde{a}_{11}^3) &= \left\{ \left\{ 1 - (0.75^{0.4} * 0.75^{0.3} * 0.90^{0.3}) \right\}, \left\{ 0.10^{0.4} * 0.10^{0.3} * 0.10^{0.3} \right\} \right\}; \\
 &\left\{ \left\{ 1 - (0.90^{0.4} * 0.75^{0.3} * 0.90^{0.3}) \right\}, \left\{ 0.10^{0.4} * 0.10^{0.3} * 0.10^{0.3} \right\} \right\}; \\
 &\left\{ \left\{ 1 - (0.75^{0.4} * 0.90^{0.3} * 0.9^{0.3}) \right\}, \left\{ 0.10^{0.4} * 0.10^{0.3} * 0.10^{0.3} \right\} \right\}; \\
 &\left\{ \left\{ 1 - (0.75^{0.4} * 0.75^{0.3} * 0.9^{0.3}) \right\}, \left\{ 0.10^{0.4} * 0.10^{0.3} * 0.0^{0.3} \right\} \right\}; \\
 &\left\{ \left\{ 1 - (0.75^{0.4} * 0.75^{0.3} * 1.0^{0.3}) \right\}, \left\{ 0.10^{0.4} * 0.10^{0.3} * 0.0^{0.3} \right\} \right\}; \\
 &\left\{ \left\{ 1 - (0.90^{0.4} * 0.75^{0.3} * 1.0^{0.3}) \right\}, \left\{ 0.10^{0.4} * 0.10^{0.3} * 0.0^{0.3} \right\} \right\} \\
 &= \{0.810, 0.868, 0.856, 1.0, 1.0, 1.0\} \{0.1, 0.1, 0.1, 0.0, 0.0, 0.0\}
 \end{aligned} \tag{17}$$

Table 11. Aggregated scores for failure mode levels in each risk factor.

	Aggregated score			Translated score		
	Severity	Occurrence	Detection	Severity	Occurrence	Detection
$FM_1$	0.872	-0.230	0.152	9.424	4.464	6.184
$FM_2$	0.799	0.099	-0.297	9.096	5.944	4.162
$FM_3$	0.420	-0.047	-0.430	7.391	5.289	3.564
$FM_4$	0.487	0.714	-0.209	7.692	8.712	4.559
$FM_5$	0.228	0.733	0.264	6.525	8.798	6.689
$FM_6$	0.714	-0.420	0.096	8.712	3.610	5.931

$$S(A_{ij}) = \frac{1}{6}(0.810 + 0.868 + 0.856 + 1.0 + 1.0 + 1.0) - \frac{1}{6}(0.10 + 0.10 + 0.10 + 0.0 + 0.0 + 0.0) = 0.872 \quad (18)$$

$$R_{ij} = 4.5 * 0.872 + 5.5 = 9.425 \quad (19)$$

IV) Ranking of failure modes: the value of the  $RPN_i$  ( $i = 1, 2, \dots, m$ ) for each failure mode computed by Equation 12 and the respective priority order of mitigation is given in Table 12 below. For this illustrative application failure mode of Lack of integration between customer and supplier ( $FM_5$ ) is presented with higher mitigation priority.

It should be noted that the proposed model has a compensatory nature in which it considers the aggregated judgments of the decision makers and the risk factors weights. Thus, all the risk factors are used for the priority definition. For example,  $FM_1$  has a higher score in the severity and detection factors than  $FM_4$ , but the score of occurrence in  $FM_4$  is much bigger than in  $FM_1$ . Although the value of the RPN of the two failure modes are very close, the  $FM_4$  RPN is higher due to the compensation between the evaluated criteria.

Explanation of the problem using FMEA enables team discussions in the search for consensus for continuous improvement actions. For example, the most critical failure mode was  $FM_5$  with the highest RPN, presented in Table 12. The results in Table 11 show that the most critical risk factors are related to potential cause and control. Thus, based on the information in Table 5, DMs should discuss these results and action plans aimed at 1) improving communication with suppliers to address the potential cause of the failure and 2) creating a control process for this failure mode.

Table 12. RPN result and failure modes Ranking.

Failure Modes	RPN	Ranking
$FM_1$	6.509	3
$FM_2$	6.226	4
$FM_3$	5.294	6
$FM_4$	6.794	2
$FM_5$	7.217	1
$FM_6$	5.833	5

## 7. Conclusion

Current quality management is highly concerned with managing the potential failures of a process, system, or product. This concern was reaffirmed with the update of ISO 9001: 2015. To deal with the evaluation and

prioritization of potential failures, the FMEA tool is widely used in the literature. The techniques based on hesitant fuzzy and intuitionistic fuzzy representations have been applied to deal with the group decision making problem, however, each one deals with a type of hesitation. Therefore, the Dual Hesitant Fuzzy technique was proposed to combine the advantages of Hesitant Fuzzy and Intuitionistic Fuzzy, providing a greater ability to deal with hesitations.

Thus, this article presented a new proposal that integrates the FMEA tool with the Dual Hesitant Fuzzy technique for group decision making to deal with the hesitation in the evaluation and prioritization of failure modes. It provides a more appropriate treatment of expert hesitation than other FMEA approaches found in the literature, based on Hesitant Fuzzy or Intuitionistic Fuzzy. Models based on Hesitant Fuzzy deal only with hesitation related to the activation of one or more linguistic terms for a variable, while those based on Intuitionistic Fuzzy deal only with the hesitation regarding the definition of the fuzzy number representing the linguistic term. This proposal based on the Dual Hesitant Fuzzy addresses the hesitation by means of these two combined ways. In addition, an application model was proposed following four steps: characterization of failure modes, judgments of decision makers, aggregation of judgments and ranking of risks based on the RPN weighted calculation. Finally, an illustrative application for failure modes in the context of supply risk management was developed to elucidate the steps of the proposed model.

Through the illustrative application, it is possible to verify that the proposed model combining FMEA and DHFS can easily be replicated in several real problems. The main practical implications of this combination are:

- ability to deal with group decision making considering individual judgments;
- the treatment of the subjectivity generated by hesitation in the evaluation of risk factors;
- the use of linguistic terms to represent the judgments of decision makers;
- it deals with decision makers with different importance and;
- it considers risk factors with varying weights according to the addressed context.

As a limitation of the study, the proposed model does not deal with the interrelation between failure modes and does not verify the interrelationship between the risk factors. In this way, the proposed model can be improved in future studies to overcome these limitations. Finally, additional studies can be conducted in different application contexts in order to further explore the applicability of the proposed model.

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