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Estimation of *R* for geometric distribution under lower record values

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Abstract: In this paper, the estimation of the stress-strength model $\mathrm{R} = \mathrm{P}(\mathrm{Y} < \mathrm{X})$, based on lower record values is derived when both X and Y are independent and identical random variables with geometric distribution. Estimating R with maximum likelihood estimator and Bayes estimator with non-informative prior information based on mean square errors and LINIX loss functions for geometric distribution are obtained. The confidence intervals of R are constructed by using exact, bootstrap and Bayesian methods. Finally, different methods have been used for illustrative purpose by using simulation. The main results are obtained and introduced through a set of tables and figures with discussions.

Keywords: Geometric distribution, stress-strength model, maximum likelihood estimator, Bayes estimator, means square errors loss functions, LINIX loss functions

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1. Introduction

The stress-strength model is one of the applied model in reliability, which has important practices in many fields especially in engineering applications. The statisticians deal with this model as X is strength and Y is a stress and the system working if, and only if, at any time the applied stress is lower than its strength. This model can be expressed as reliability function$\mathrm{R} = \mathrm{P}(\mathrm{Y} < \mathrm{X})$, where X and Y are independent and identical random variables. To estimate the reliability values in this model, we should first estimate the parameters of stress Y and strength X by using different methods of estimations, Hassan, Muhammed, and Saad (2015), Nkemnole and Samiyu (2017).

In geometric distribution, the probability of success is assumed to be the same for each trial. In such a sequence of trials, the geometric distribution is useful to model the number of failures before the first success. The distribution gives the probability that there are zero failures before the first success, one failure before the first success, two failures before the first success, and so on, Pitman (1993), Walck (2007).

The record values can be classified into lower and upper values, where the observer $\mathrm{X_i}$ is called lower value if it is smaller than all pervious subjects to experience. In other side, if $\mathrm{X_j}$ is exceed all the subjects to experience, it will call upper record.

Record values have been discussed in the statistical literature by many authors who explained how rerecord values are important in many fields. Chandler (1952) explained how the record values, record times and the inter record times based on the record values are obtained and used to form a model of extremes sequence of independent and identical distributed random variables. The record values and its kinds have introduced in Nagaraja (1988), Ahsanullah (1995, 2004).

Many authors have been focused on the stress-strength model R and they tried to apply it in different cases of studies. Birnbaum (1956), Kundu and Gupta (2005, 2006), Razaei, Tahmasbi, and Mahmoodi (2010), Hussian, (2013) studied the estimation of R for different distributions. The recorded values and more studies about R with different distributions and different methods of estimation found in Baklizi (2008, 2014), Essam (2012), Tarvirdizade, and Kazemzadeh Garehchobogh (2014).

This paper is organized as follows. In Section 2, maximum likelihood estimates and exact confidence interval of R are studied. Also, the asymptotic bootstrap confidence interval of R is established. In Section 3, the Bayes estimates of R against both squared error and LINEX loss functions are studied. Also, Bayes confidence interval is obtained. In Section 4, steps of simulation study are proposed. Results and discussion are shown in Section 5. Finally, conclusions appear in Section 6.

2. Likelihood inferences

In this section, maximum likelihood estimator (MLE) and confidence interval of R are derived. Also, the asymptotic bootstrap confidence interval of R is obtained.

2.1. Maximum likelihood estimator of R

According to Mohamed (2015), let Y be stress for the model of stress-strength is subjected to X as strength of the model. Assume $\mathrm{X} \sim \mathrm{P}(\mathrm{X}, \mathrm{P}_1)$ and $\mathrm{Y} \sim \mathrm{P}(\mathrm{Y}, \mathrm{P}_2)$ have geometric distribution with $x \in \{1,2,3, \ldots.\}$ and $0 < p_1 < 1.$

$$p(x) = (1 - p_1)^{x-1}\, p_1 \tag{1}$$

$$F(x) = 1 - (1 - p_1)^x \tag{2}$$

where $\mathrm{p}(.)$ And $\mathrm{F}(.)$ are the probability and cumulative density function.

Then the reliability function

$$R = \frac{p_2}{p_1 + p_2 - p_1\, p_2} \tag{3}$$

Let $\underline{r} = (r_0, \ldots \ldots, r_n)$ be the first independent set of lower record of data with size $(n+1)$from strength with geometric distribution with parameter p_1 and $\underline{s} = (s_0, \ldots \ldots, s_m)$ have the same features but with size $(m+1)$ from stress with geometric distribution with parameter p_2.

The likelihood function for both $\underline{\mathrm{r}}$ and $\underline{\mathrm{s}}$ are given by Arnold, Balakrishnan and Nagaraja (1998):

$$L_1(p_1|r\ ) = p(r_n) \prod_{i=0}^{n-1} \frac{p(r_i)}{F(r_i)}\,, \quad 0 < r_n < r_{n-1} < \cdots < r_0 < \infty \tag{4}$$

and

$$L_2(p_2|s\ ) = p(r_m) \prod_{j=0}^{m-1} \frac{p(s_j)}{F(s_j)}\,, \quad 0 < s_m < s_{m-1} < \cdots < s_0 < \infty. \tag{5}$$

The likelihood function of the observed record values $\underline{\mathrm{r}}$ and $\underline{\mathrm{s}}$ are:

$$L_1\big(p_1|\underline{r}\ \big) = (1 - p_1)^{r_n - 1} p_1^{n+1} \prod_{i=0}^{n-1} \frac{(1-p_1)^{r_i - 1}}{(1-(1-p_1))^{r_i}} \tag{6}$$

and

$$L_2(p_2|\underline{s}) = (1 - P_2)^{S_n - 1} P_2^{(m+1)} \prod_{j=0}^{m-1} \frac{(1-p_1)^{s_j - 1}}{(1-(1-p_1))^{s_j}} \tag{7}$$

Therefore, the joint log-likelihood function of $\underline{\mathrm{r}}$ and $\underline{\mathrm{s}}$ denoted by l is:

$$\ell = (r_n - 1)\, log\ (1-p_1) + (n+1)\, log\ p_1 + (s_m - 1) log\ (1-p_2) + (m+1) log\ p_2 + log\ \prod_{i=0}^{n-1}(1-p_1)^{r_i - 1} \prod_{j=0}^{m-1}(1-p_2)^{s_j - 1} \quad (8)$$

The maximum likelihood estimators of p_1 and p_2 are $\hat{p}_1$ and $\hat{p}_2$ according to the observed lower record values are obtaining by solving the equations as follow:

$$\frac{\partial \ell}{\partial p_1} = \frac{[-(r_n-1) - \sum_{i=0}^{n}(r_i - 1)]}{(1-\hat{\hat{p}}_1)} + \frac{(n+1)}{\hat{\hat{p}}_1} = 0 \quad (9)$$

and

$$\frac{\partial \ell}{\partial \mathrm{p}_2} = \frac{[-(\mathrm{s_m}-1) - \sum_{\mathrm{j}=0}^{\mathrm{m}}(\mathrm{s_j} - 1)]}{(1-\hat{\hat{\mathrm{p}}}_2)} + \frac{(\mathrm{m}+1)}{\hat{\hat{\mathrm{p}}}_2} = 0 \quad (10)$$

From (9) and (10), $\widehat{\mathrm{P}}_1$ and $\widehat{\mathrm{P}}_2$ are obtained as follow:

$$\hat{\hat{p}}_1 = \frac{(n+1)}{(r_n - 1) + (n+1) + \sum_{i=0}^{n}(r_i - 1)},$$

$$\hat{\hat{p}}_2 = \frac{(m+1)}{(s_m - 1) + (m+1) + \sum_{j=0}^{m}(s_j - 1)} \quad (11)$$

Hence, the maximum likelihood estimator of of R, denoted by $\widehat{\mathrm{R}}_{\mathrm{ML}}$, is given by substitution $\hat{p}_1$ and $\hat{p}_2$ in Eq. 3 as follows:

$$\hat{\mathrm{R}}_{ML} = \frac{\hat{\hat{p}}_2}{\hat{\hat{p}}_1 + \hat{\hat{p}}_2 - \hat{\hat{p}}_1 \hat{\hat{p}}_2} \quad (12)$$

2.2. Exact confidence interval of R

In this subsection, exact confidence interval of R based on the asymptotic properties and the general conditions of the MLE of $\hat{p}_1$ and $\hat{\mathrm{p}}_2$ is obtained (Lehmann, 1999). The asymptotic distribution of the MLE immediately comes from the Fisher information matrix of p_1 and p_2. That is,

As $n, m \to \infty$ *and* $\frac{n}{m} \to k$*, where* $0 < k < 1$*, then* $[\sqrt{n}(\hat{p}_1 - p_1), \sqrt{m}(\hat{p}_2 - p_2)] \xrightarrow{D} N_2(0, \delta(p))$,)

where

$$\delta(p) = I^{-1}(p) = \begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix}^{-1}$$

and the matrix$\mathrm{I}(P)$ is the Fisher information matrix of the parameter vector $P = (p_1, p)$.

The (ij)th element is defined as the second partial derivatives:

$$I_{ij} = \frac{\partial^2 l(P)}{\partial P_1 \partial P_2}, i, j = 1,2$$

From the asymptotic properties of the MLEs of $\mathrm{P}_1, \mathrm{P}_2$, one can easily get

$$\sqrt{n}(\hat{R} - R) = \sqrt{n}\,(\frac{\hat{\hat{p}}_2}{\hat{\hat{p}}_1 + \hat{\hat{p}}_2 - \hat{\hat{p}}_1 \hat{\hat{p}}_2} - \frac{P_2}{p_1 + p_2 - p_1 p_2}) \xrightarrow{D} N_2(0, \sigma^2) \quad (13)$$

where

$$\sigma^2 = E(\frac{\sqrt{n}\,\hat{\hat{p}}_2}{\hat{\hat{p}}_1 + \hat{\hat{p}}_2 - \hat{\hat{p}}_1 \hat{\hat{p}}_2} - \frac{\sqrt{n} P_2}{p_1 + p_2 - p_1 p_2})^2 \quad (14)$$

The maximum likelihood *(1 − ⊠) 100%* confidence interval of *R* is given by:

$$\hat{R} \pm Z_{1-\frac{\alpha}{2}} \hat{\sigma} \quad (15)$$

where $\widehat{\sigma}$ is the asymptotic standard deviation of R.

2.3. Asymptotic bootstrap confidence interval

In this subsection, the asymptotic bootstrap confidence interval of R is derived. Kotz and Pensky (2003) proposed the bootstrap method as an alternative way to construct a confidence interval.

The algorithm of the (1 − α) 100% confidence interval for R by using bootstrap method is illustrated below:

1- Use the estimators $\hat{p}_1$ and $\hat{p}_2$ in inverse of distribution of p_1 andp_2 to estimate the bootstrap sample $X_1^*, \dots\dots, X_n^*$ and $Y_1^*, \dots\dots, Y_m^*$, then compute the estimated value of R by MLE which is shown in Eq. 12.

2- Calculate the bootstrap MSE by:

$$\widehat{MSE}_B = \frac{1}{N}\sum_{k=1}^{N}(\bar{R}^j - \bar{R}) \quad (16)$$

3- The asymptotic (1 − α) 100% confidence interval is obtained by:

$$(\bar{R} - Z_{\frac{\alpha}{2}}\sqrt{\widehat{MSE}_B}, \bar{R} + Z_{\frac{\alpha}{2}}\sqrt{\widehat{MSE}_B}) \quad (17)$$

3. Bayesian inferences

In this section, the Bayes estimator of *R* is calculated by the mean squared error and LINEX loss functions. Also, Bayes confidence interval for *R* is obtained.

3.1. Bayes estimator of R based on mean square errors loss function

To get the Bayes estimator of $\hat{p}_1$ and $\hat{p}_2$ based on the mean square errors, we used the non-informative prior information for choosing the prior distributions. The steps to find the Bayesian estimator of $\hat{p}_1$ and $\hat{p}_2$, respectively, Mohamed (2015).

The non-informative priors of p_1 and p_2 by using Fisher information matrix for P_1 and P_2 are calculated as follows

$$I(P_1) = -E\left(\frac{\partial^2 l(P_1)}{\partial {P_1}^2}\right), I(P_2) = -E\left(\frac{\partial^2 l(P_2)}{\partial {P_2}^2}\right)$$

The prior distributions by non-informative of p_1 and p_2 are

$$\pi(p_1) \propto \frac{1}{p_1}, \pi(p_2) \propto \frac{1}{p_2} \tag{18}$$

The posterior distributions of p_1 and p_2, denoted by $\pi^*(p_1)$ and $\pi^*(p_2)$, are obtained by combining Eq. 6, Eq. 7 and Eq. 18 as follows

$$\pi^*(p_1) = \frac{(1-p_1)^{r_n-1} p_1^n \prod_{i=0}^{n-1} \frac{(1-p_1)^{r_i-1}}{(1-(1-p_1))^{r_i}}}{\int_0^1 (1-p_1)^{r_n-1} p_1^n \prod_{i=0}^{n-1} \frac{(1-p_1)^{r_i-1}}{(1-(1-p_1))^{r_i}} dp_1}, \tag{19}$$

$$\pi^*(p_2) = \frac{(1-P_2)^{S_n-1} P_2^m \prod_{j=0}^{m-1} \frac{(1-p_1)^{s_j-1}}{(1-(1-p_1))^{s_j}}}{\int_0^1 (1-P_2)^{S_n-1} P_2^m \prod_{j=0}^{m-1} \frac{(1-p_1)^{s_j-1}}{(1-(1-p_1))^{s_j}} dp_2}. \tag{20}$$

The Bayes estimates of p_1 and p_2, , under mean squared error loss function, denoted by $\hat{p}_{1MS}$ and $\hat{p}_{2MS}$, are calculated as follows

$$\hat{P}_{1MS} = \int_0^1 P_1 \pi^*(p_1) dp_1 = \int_0^1 \frac{(1-p_1)^{r_n-1} p_1^{n+1} \prod_{i=0}^{n-1} \frac{(1-p_1)^{r_i-1}}{(1-(1-p_1))^{r_i}}}{\int_0^1 (1-p_1)^{r_n-1} p_1^n \prod_{i=0}^{n-1} \frac{(1-p_1)^{r_i-1}}{(1-(1-p_1))^{r_i}} dp_1} d\,p_1, \tag{21}$$

and

$$\hat{P}_{2MS} = \int_0^1 p_2 \pi^*(p_2) d\,p_2 = \int_0^1 \frac{(1-P_2)^{S_n-1} P_2^{m+1} \prod_{j=0}^{m-1} \frac{(1-p_1)^{s_j-1}}{(1-(1-p_1))^{s_j}}}{\int_0^1 (1-P_2)^{S_n-1} P_2^m \prod_{j=0}^{m-1} \frac{(1-p_1)^{s_j-1}}{(1-(1-p_1))^{s_j}} dp_2} dp_2. \tag{22}$$

The mean square errors loss function of R based on the lower records data for X and Y denoted as $\hat{\mathrm{R}}_{\mathrm{MS}}$, can be obtained by substituting Eq. 21 and Eq. 22 in Eq. 3.

$$\hat{R}_{MS} = \frac{\hat{P}_{1MS}}{\hat{P}_{1MS} + \hat{P}_{2MS} - \hat{P}_{1MS}\hat{P}_{2MS}}$$

3.2. Bayes estimator of R based on LINEX loss function

In this subsection, the Bayes estimator of R under LINEX loss function, denoted by $\hat{R}_{LL}$ is obtained. $\hat{R}_{LL}$ is calculated as follows

$$\hat{R}_{LL} = -\frac{1}{a} lnE(e^{-aR}) = \frac{1}{n} ln \int_0^1 \int_0^1 e^{-aR} \pi^*(p_1)\pi^*(p_2) dp_1 dp_2$$

According to Lindley (1980) and after some calculations, the approximate Bayes estimator of R is

$$\hat{R}_{LL} = -\frac{1}{a} ln\, [e^{-a\hat{R}_{ML}} + \frac{1}{2}[R_{11}\sigma_{11} + R_{22}\sigma_{22} + L_{111}R_1{\sigma_{11}}^2 + L_{222}R_2{\sigma_{22}}^2]] \tag{23}$$

where

$a > 0$, $\sigma_{11} = \frac{P_1^2}{n}$, $\sigma_{22} = \frac{P_2^2}{m}$, $R_{11} = \frac{2(1-P_2)^2}{(P_1+P_2-P_1P_2)^2}$, $R_{22} = \frac{2P_1(1-P_1)}{(P_1+P_2-P_1P_2)^2}$, $L_{111} = \frac{\partial^3 \ell}{\partial P_1^3}$ *and* $L_{222} = \frac{\partial^3 \ell}{\partial P_2^3}$.

3.3. Bayes confidence interval of R

In this subsection, Bayes confidence interval for R is obtained. To derive the distribution of stress-strength R function based on Bayesian inferences, the posterior distributions of p_1 and p_2 must be found. the conjugate prior density functions of p_1 and p_2 is proportional with beta distribution as follows

$$\pi(p_1) \sim B(\alpha_1, \beta_1)\ , \pi(p_2) \sim B(\alpha_2, \beta_2) \tag{24}$$

After some calculations, the posterior distributions of p_1 is

$$\pi^*(p_1) = \frac{1}{B(a_1,b_1)}\ {p_1}^{a_1-1}\ (1-p_1)^{b_1-1} \tag{25}$$

where $a_1 = r_n + n + \alpha_1, b_1 = r_n + (k_1+1)\sum_{i=0}^{n-1} r_i + n - 1\ , k_1 = 0,1,2, \ldots$

And the posterior distribution for p_2 is

$$\pi^*(p_2) = \frac{1}{B(a_2,b_2)}\ {p_2}^{a_2-1}\ (1-p_2)^{b_2-1} \tag{26}$$

where $a_2 = s_m + m + \alpha_2, b_2 = s_m + (k_2+1)\sum_{j=0}^{m-1} s_j + m - 1\ , k_2 = 0,1,2, \ldots$

The Bayesian *(1 − α) 100%* confidence interval of R for p_1 and p_2, are

$$P(L_1 \le p_1 \le U_1|data) = 1-\alpha, (L_2 \le p_2 \le U_2|data) = 1-\alpha \quad (27)$$

Using Eq. 25, 26 and 27, the Bayesian confidence intervals (L_1, U_1) and (L_2, U_2) for p_1 and p_2 can be derived by solving the followings equations

$$\int_0^{L_1} \frac{1}{B(a_1,b_1)} \; {p_1}^{a_1-1} \; (1-p_1)^{b_1-1} d\, p_1 = \frac{\alpha}{2}, \quad (28)$$

$$\int_{U_1}^{0} \frac{1}{B(a_1,b_1)} \; {p_1}^{a_1-1} \; (1-p_1)^{b_1-1} d\, p_1 = \frac{\alpha}{2} \quad (29)$$

$$\int_0^{L_2} \frac{1}{B(a_2,b_2)} \; {p_2}^{a_2-1} \; (1-p_2)^{b_2-1} d\, p_2 = \frac{\alpha}{2}, \quad (30)$$

and

$$\int_{U_2}^{0} \frac{1}{B(a_2,b_2)} \; {p_2}^{a_2-1} \; (1-p_2)^{b_2-1} d\, p_2 = \frac{\alpha}{2} \quad (31)$$

Therefore, Bayes confidence interval for R is constructed by substitute Eq. 28, 29, 30 and 31 in Eq. 3.

4. Simulation study

In this section a simulation study is studied to compare the performance of MLE and Bayes estimates (under squared error and LINEX loss functions). The exact values of R are 0.714 and 0.95. The estimates of R through MLE and Bayes methods under lower record values are calculated for different sample sizes. Three different methods for confidence intervals are computed. The simulation study is performance according to the following steps:

1. Generate 10000 samples from**uniform(0,1)**, then find the 300 random sample according to geometric distribution through the transformation technique.
2. From each vector the first $(n+1)$ of lower record values $r_0, \ldots \ldots, r_n$ for the values of strength random variables **X** be selected,
3. Repeat the previous two steps to generate 5000 random samples of size 300 from geometric distribution and select from each vector the first $(m+1)$ of lower record values $s_0, \ldots \ldots, s_m$ for the values of stress random variable **Y**.
4. The MLE of p_1, p_2 are obtained from Eq. 11, then the MLE of R is obtained by substitute p_1 and p_2 in Eq. 12. The maximum likelihood confidence intervals of p_1 and p_2 are calculated with confidence level at α = 0.05 by using Eq. 15. The bootstrap confidence intervals are obtained from Eq. 17. Bayesian confidence intervals are obtained from Eq. 3, 28 and 29.
5. Compute Bayes estimator of R under mean squared error and LINEX loss functions.

5. Results and discussion

Simulation results are tabulated in Tables (1:6). We can observe the following results:

1. The coverage percentage of MLE is better than that of the Bayesian estimator at R =0.714 and 0.985 according to Tables (1,2).
2. The coverage percentage of Bayes LINEX loss function is better than that of Bayes under MSE at R =0.714 and 0.985 according to Tables (3,4).
3. The average length of the exact confidence intervals is shorter than the bootstrap and Bayes methods according to Tables (5,6).
4. When **n** and **m** increase, the coverage percentages decrease for different estimators at different values of p_1, p_2 according to Tables (1,6).

Table 1. Simulation results for maximum likelihood estimator and Bayes estimators under mean square errors at $p = 0.1, p_2 = 0.2$ and *R*=0.714.

n	m		MLE			BAYES	
		$\hat{R}_{ML}$	MSE	Converge	$\hat{R}_{MS}$	MSE	Converge
2	2	0.67	0.06	0.938	0.445	0.073	0.623
3	2	0.642	0.052	0.899	0.544	0.053	0.762
	3	0.658	0.025	0.922	0.599	0.101	0.839
4	2	0.636	0.0047	0.891	0.345	0.342	0.483
	3	0.602	0.013	0.843	0.268	0.435	0.375
5	3	0.637	0.0061	0.892	0.222	0.222	0.311
	4	0.6	0.03	0.84	0.677	0.814	0.948
6	4	0.607	0.012	0.85	0.558	0.353	0.782
	5	0.6674	0.02	0.935	0.454	0.111	0.636

Table 2. Simulation results for maximum likelihood estimator and Bayes estimators under mean square errors at $p_1 = 0.01, p_2 = 0.16$ and R=0.95.

n	m	MLE			BAYES		
		$\hat{R}_{ML}$	MSE	Converge	$\hat{R}_{MS}$	MSE	Converge
2	2	0.87	0.0145	0.916	0.179	0.818	0.188
3	2	0.829	0.0103	0.873	0.171	0.005	0.18
	3	0.867	0.0147	0.913	0.934	0.548	0.983
4	2	0.835	0.099	0.879	0.747	0.546	0.786
	3	0.894	0.0127	0.941	0.341	0.852	0.359
5	3	0.831	0.0102	0.875	0.946	0.681	0.996
	4	0.884	0.0134	0.931	0.074	0.004	0.078
6	4	0.901	0.0116	0.948	0.861	0.707	0.906
	5	0.871	0.0143	0.917	0.334	0.012	0.352

Table 3. Simulation results for Bayes estimators under mean square errors and LINEX loss function at $p_1 = 0.1, p_2 = 0.2$ and R=0.714.

n	m	BAYES with MSE			BAYES with LINEX loss function		
		$\hat{R}_{MS}$	MSE	Converge	$\hat{R}_{LL}$	MSE	Converge
2	2	0.445	0.073	0.623	0.697	0.569	0.976
3	2	0.544	0.053	0.762	0.566	0.288	0.793
	3	0.599	0.101	0.639	0.493	0.42	0.69
4	2	0.345	0.342	0.453	0.333	0.525	0.466
	3	0.268	0.435	0.375	0.465	0.453	0.651
5	3	0.677	0.814	0.648	0.54	0.791	0.756
	4	0.222	0.222	0.311	0.208	0.289	0.391
6	4	0.558	0.353	0.782	0.21	0.246	0.294
	5	0.454	0.111	0.636	0.697	0.249	0.976

Table 4. Simulation results for Bayes estimators under mean square errors and LINEX loss function at $p_1 = 0.01, p_2 = 0.16$ and R=0.95.

n	m	BAYES with MSE			BAYES with LINEX loss function		
		$\hat{R}_{MS}$	MSE	Converge	$\hat{R}_{LL}$	MSE	Converge
2	2	0.179	0.818	0.188	0.772	0.229	0.813
3	2	0.171	0.005	0.18	0.442	0.319	0.465
	3	0.934	0.548	0.563	0.54	0.044	0.568
4	2	0.747	0.546	0.786	0.811	0.256	0.854
	3	0.341	0.852	0.359	0.805	0.365	0.847
5	3	0.946	0.681	0.996	0.579	0.855	0.609
	4	0.074	0.004	0.078	0.27	0.068	0.284
6	4	0.861	0.707	0.606	0.727	0.423	0.765
	5	0.334	0.012	0.352	0.601	0.031	0.633

Table 5. Simulation results for exact, bootstrap and
Bayes confidence interval at $p_1 = 0.01$, $p_2 = 0.16$ and *R*=0.714.

n	m	95% CI of the length of Exact CI	95% CI of the length of Bootstrap CI	95% CI of the length of Bayes CI
2	2	1.765*10^-5	0.224	1.660*10^-4
3	2	2.823*10^-5	0.479	2.623*10^-4
	3	4.741*10^-5	0.201	4.657*10^-3
4	2	1.043*10^-3	0.431	1.232*10^-5
	3	4.899*10^-6	0.692	3.099*10^-5
5	3	2.839*10^-5	0.235	3.222*10^-4
	4	2.419*10^-6	0.459	2.543*10^-6
6	4	2.86*10^-7	1.397	2.543*10^-6
	5	7.696*10^-7	0.425	6.777*10^-7
	6	3.775*10^-7	0.302	4.723*10^-7

Table 6. Simulation results for exact, bootstrap and
Bayes confidence interval at
$p_1 = 0.01$, $p = 0.16$ and *R*=0.95.

n	m	95% CI of the length of Exact CI	95% CI of the length of Bootstrap CI	95% CI of the length of Bayes CI
2	2	3.025*10^-4	0.119	2.111*10^-3
3	2	6.603*10^-5	0.149	5.344*10^-4
	3	5.720*10^-7	0.137	7.502*10^-6
4	2	1.31*10^-6	0.147	2.222*10^-6
	3	5.637*10^-6	0.277	8.324*10^-6
5	3	4.127*10^-8	0.116	5.1322*10^-7
	4	1.291*10^-7	0.159	3.2657*10^-7
6	4	6.213*10^-8	0.186	5.121*10^-6
	5	1.917*10^-8	0.216	3.777*10^-8
	6	2.415*10^-8	0.115	1.400*10^-8

6. Conclusion

In this paper, the MLE and Bayesian estimators are derived for *R* when the stress and strength variables are independently geometric distributions based on lower record values. The exact, bootstrap and Bayesian confidence intervals are investigated.

Generally, the coverage percentage of MLE is better than coverage percentage of the Bayes estimator.

Regarding, the number of records n and m for stress-strength modelvaribles.it is observed that the coverage percentage is increase as n and m increase and vice versa.

The estimate values of MLE is better than coverage percentage of the Bayes estimator at *R* = 0.714 and 0.95.

The average confidence interval lengths of the exact method are shorter than the corresponding average confidence interval lengths of the bootstrap and Bayes method .

The MSEs of the Bayesian estimator under MSE loss function are less than the LINEX loss function.

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