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# Exact minimization of the energy losses and the CO<sub>2</sub> emissions in isolated DC distribution networks using PV sources

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## Abstract

This paper addresses the optimal location and sizing of photovoltaic (PV) sources in isolated direct current (DC) electrical networks, considering time-varying load and renewable generation curves. The mathematical formulation of this problem corresponds to mixed-integer nonlinear programming (MINLP), which is reformulated via mixed-integer convex optimization: This ensures the global optimum solving the resulting optimization model via branch & bound and interior-point methods. The main idea of including PV sources in the DC grid is to minimize the daily energy losses and greenhouse emissions produced by diesel generators in isolated areas. The GAMS package is employed to solve the MINLP model, using mixed and integer variables; also, the CVX and MOSEK solvers are used to obtain solutions from the proposed mixed-integer convex model in the MATLAB. Numerical results demonstrate important reductions in the daily energy losses and the harmful gas emissions when PV sources are optimally integrated into DC grid.

**Keywords:** minimization of greenhouse gas emissions; renewable energy resources; daily demand curves; convex optimization; diesel generators.

# Minimización exacta de las pérdidas de energía y las emisiones de CO<sub>2</sub> en redes de distribución DC aisladas empleando fuentes fotovoltaicas

## Resumen

Este paper aborda la ubicación y el tamaño óptimos de las fuentes fotovoltaicas (PV) en redes eléctricas aisladas de corriente continua (CC), considerando la carga variable en el tiempo y las curvas de generación renovable. La formulación matemática de este problema corresponde a la programación no lineal de enteros mixtos (MINLP), que es reformulada mediante optimización convexa de enteros mixtos. Esto asegura el óptimo global resolviendo el modelo de optimización resultante a través de métodos de punto interior y ramificación. La idea principal de incluir fuentes fotovoltaicas en la red de CC es minimizar las pérdidas diarias de energía y las emisiones de efecto invernadero producidas por los generadores diésel en áreas aisladas. El paquete GAMS se emplea para resolver el modelo MINLP, utilizando variables mixtas y enteras. Además, los solucionadores CVX y MOSEK se utilizan para obtener soluciones del modelo convexo de enteros mixtos propuesto en MATLAB. Los resultados numéricos demuestran importantes reducciones en las pérdidas diarias de energía y las emisiones de gases nocivos cuando las fuentes fotovoltaicas se integran de manera óptima en la red de CC.


**Palabras clave:** minimización de gases de efecto invernadero; fuentes de generación renovable; curvas de demanda diaria; optimización convexa, generadores diésel.

## 1. Introduction

Electrical distribution networks are the power system component responsible for providing electrical service to

end-users in medium- and low-voltage levels in urban and rural areas [1,2]. These grids are typically constructed with a radial structure using AC technologies to reduce investment costs and simplify the coordination of the protective devices

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[3]. However, due to the recent advances in power electronic converters, renewable energy resources, and energy storage devices, is possible to operate electrical distribution networks using DC technologies [4-6], with the following advantages:

- Low power losses in comparison with AC distribution networks [7].
- Easy static and dynamic analysis since the reactive power and frequency concepts are non-existent in DC distribution networks [8].
- The possibility of obtaining convex power flow models via conic constraints with the possibility of ensuring zero duality gap when compared with exact nonlinear non-convex models [9].

In the specialized literature, the analyses of DC networks are made in the context of providing electric distribution applications in urban and rural areas [1,10]. The main approaches correspond to: a) optimal location and sizing of distributed generators in DC grids [11], b) optimal location and operation of battery energy storage systems [12-14], c) optimal reconfiguration of DC feeders [15], d) and planning of DC networks [1], among others. Based on the importance of these studies, we focus on the problem of the optimal location and sizing of distributed generators in DC networks considering daily load and PV curves to minimize the number of daily energy losses and the harmful greenhouse gas emissions to the atmosphere. Next, some recent works regarding DC distribution networks are presented below.

The authors in [16] have presented optimization models to locate and size distributed generators in DC grids using mixed-integer nonlinear programming model, which is solved with optimization tools available in the general algebraic modeling system, i.e., GAMS, and heuristic optimization methods such as black hole optimizer, and population-based incremental learning. References [17] and [18] have proposed convex optimization models to place and size distributed generators in DC grids considering conic and semidefinite approximations considering the peak load conditions. For their part, authors in [12] and [19] have presented MINLP models to operate battery energy storage systems in DC grids, and their solutions are reached with heuristic algorithms and convex reformulations. As another contribution, the authors of [9] have proposed a conic reformulation of the power flow problem for DC grids considering conic constraints by transforming the hyperbolic relation between voltages and currents in the power balance equations into conic constraints [20]. Additional studies regarding DC distribution networks are listed in Table 1.

Considering the previous state-of-the-art, there is a gap in which this research contributes through a novel mixed-integer convex programming model to minimize greenhouse gas emissions in rural distribution networks with the optimal installation and sizing of PV sources. The main advantage of

the proposed reformulation from the exact MINLP model is to reach global optimum, which can ensure in the mixed-integer convex model. This model uses a combination of the Branch & Bound and the interior point methods with zero duality gap compared to the exact formulation [34]. The proposed convex model's effectiveness and robustness are tested in two radial DC distribution tests, which are composed of 33 and 69 node test feeders. Besides, this paper makes comparisons with MINLP solvers available in the GAMS optimization tool.

The remainder of this research is organized as follows: Section 2 presents the exact MINLP formulation for the optimal placement and sizing of PV sources in DC distribution grids for rural application to minimize the amount of greenhouse gas emissions the atmosphere produced by diesel generators. Section 3 presents the conic transformation of the power balance equations using the product's hyperbolic equivalent between continuous variables, becoming the exact MINLP model into a mixed-integer convex one. Section 4 describes the main aspects of the mixed-integer convex programming using the Branch & Bound method. Section 5 describes the main characteristics of the IEEE 33- and IEEE 69-node test feeders as well as the daily load and PV generation curves, respectively. Section 6 presents the numerical results in both test feeders using the CVX tool and the MOSEK solver in the MATLAB programming environment and their comparisons with the GAMS MINLP solvers [11]. Section 7 shows the main conclusions derived from this study and further works.

## 2. Exact MINLP formulation

A mixed-integer nonlinear programming model allows describing the problem of the optimal location and sizing of PV sources in DC distribution networks to minimize the daily energy losses considering the calculation of the amount of greenhouse gas emissions to the atmosphere by diesel generators. In such a model the continuous part is related to the power flow variables, i.e., currents, voltages, and powers; and the integer part is related with the possibility of locating a PV source or no in a particular node of the grid [35].

The objective functions for the MINLP model to locate and size PV sources in DC grids can be the represented as follows.

$$\min E_{losses} = \sum_{ij \in E} \sum_{t \in T} R_{ij} i_{ij,t}^2 \Delta t \quad (1)$$

$$\min G_{emissions} = \sum_{t \in T} \sum_{k \in N} CGG_{emissions}^0 p_{0k,t} \Delta t \quad (2)$$

where  $E_{losses}$  represents the objective function value associated with the daily energy losses,  $R_{jk}$  is the resistive parameter of the line between nodes  $j$  and  $k$ ;  $i_{jk,t}$  is the current that flows in the branch that connects nodes  $j$  and  $k$  at time  $t$ .  $G_{emissions}$  represents the objective function value regarding the amount of greenhouse gas emissions to the atmosphere produced by diesel generators during a typical day of operation;  $CGG_{emissions}^k$  is the coefficient of greenhouse gas emissions in the diesel generator connected at node 0.  $p_{0k,t}$  is

Table 1.  
Main topics under development in DC networks.

Topic	Reference
Convergence analysis of power flow methods for DC networks.	[7,8,21-24]
Heuristic optimal power flow methods for DC networks	[25-28]
Control of power electronic converters in DC microgrids	[29-33]

Source: Authors

the amount of power generation sent from the slack node for the first line, i.e., the line that connects nodes 0 to  $k$ .  $\Delta t$  is the length of the fraction of time where the power generation is assumed as a constant. This period is typically 30 or 60 minutes. Note that  $T$  and  $E$  are the sets that contains all the periods in which is divided the typical operative day and the set that contains all the branches of the network, respectively. The total nodes of the network are denoted by  $N$ .

**Remark 1:** In this research we select as objective function the minimization of the energy losses in all the branches of the network, i.e., Equation (1) for a typical day of operation, and the amount of greenhouse emissions calculated as a function of the amount of power injected by the slack source, i.e., applying Equation (2).

The problem of the optimal location and sizing of PV sources in distribution grids for rural areas must accomplish the power balance equations and the capacities of the devices, among other constraints. The set of constraints is presented below.

$$p_{jk,t} - R_{jk} i_{jk,t}^2 - \sum_{m:(k,m)} p_{km,t} = p_{k,t} - y_k^{pv} p_{k,t}^{pv,nom}, \quad \{\forall (j,k) \in E, \forall t \in T\} \quad (3)$$

$$v_{k,t}^2 = v_{j,t}^2 - 2R_{jk} p_{jk,t} + R_{jk}^2 i_{jk,t}^2, \quad \{\forall (j,k) \in E, \forall t \in T\} \quad (4)$$

$$p_{jk,t} = v_{j,t} i_{jk,t}, \quad \{\forall (j,k) \in E, \forall t \in T\} \quad (5)$$

$$-i_{jk}^{max} \leq i_{jk,t} \leq i_{jk}^{max}, \quad \{\forall (j,k) \in E, \forall t \in T\} \quad (6)$$

$$v_{min} \leq v_{k,t} \leq v_{max}, \quad \{\forall k \in N, \forall t \in T\} \quad (7)$$

$$0 \leq y_k^{pv} \leq x_k^{pv} p_k^{pv,max}, \quad \{\forall k \in N\} \quad (8)$$

$$\sum_{j \in N} x_k^{pv} \leq N_{max}^{pv}, \quad (9)$$

where  $p_{jk,t}$  ( $p_{km,t}$ ) is the power flow in the branch that connects nodes  $j(k)$  and  $k(m)$  in the period  $t$ ;  $p_{k,t}$  is the power demand at node  $k$  in the period  $t$ ;  $y_k^{pv}$  is the size of the PV source connected at node  $k$ ;  $p_{k,t}^{pv,nom}$  represents the generation value of the PV source at node  $k$  in the period  $t$ , note that this curve is normalized in percentage;  $v_{j,t}$  ( $v_{k,t}$ ) is the voltage value at node  $j(k)$  at time  $t$ ;  $i_{jk}^{max}$  is the thermal bound associated with the caliber of the conductor in the line that connects nodes  $j$  and  $k$ ;  $v_{min}$  and  $v_{max}$  are the minimum and maximum voltage limit for all the nodes of the grid at any period;  $p_k^{pv,max}$  is the maximum size allowed for a PV source connected a node  $k$ ;  $x_k^{pv}$  is the binary variable associated with the location ( $x_k^{pv} = 1$ ) or not ( $x_k^{pv} = 0$ ) of a PV source at node  $k$ ; and  $N_{max}^{pv}$  is the number of PV sources available for installation in the DC network.

**Remark 2:** The MINLP model defined from (1) to (9) is a mixed-integer non-convex optimization model due to the square variables present in Equations (2) and (3) as well as the product of these in (4). However, this

model can be transformed into a mixed-integer convex one using the conic representation of the power balance equations presented in [36].

The optimization model (1)-(9) can be interpreted as follows: Equation (1) formulates the objective function of the optimization problem which is related with the minimization of the amount of the daily energy losses; Equation (2) determines the quantity of greenhouse gas emissions emitted by diesel generators that feeds rural distribution grids using DC technologies; Equation (3) is known in the specialized literature as the branch power flow constraint that guarantees the power balance at each node of the grid at each period of time; Equation (4) defines the voltage drop at each branch of the network as a function of the power and current flows at each period of time; Equation (5) shows the definition of the power in an electrical element as function of its voltage and current (i.e., Tellegen's theorem); Inequality constraints (6) and (7) determine the thermal limits associated with the calibers of the conductors in all the branches and the voltage regulation bounds in the nodes of the grid, respectively. Inequality constraint (8) determines if a PV source is installed or no at node  $k$ , while inequality constraint (9) limits the number of PV sources that can be connected to the DC grid.

**Remark 3:** The optimization model presented from (1) to (9) is only applicable to radial DC distribution networks since it was developed based on the concept of branch power flow proposed in [37] where only exists the possibility of having one path between each node and the slack source.

The transformation of the exact MINLP model (1)-(9) into a mixed-integer convex model using conic constraints will be presented in the next section.

### 3. Mixed-integer convex reformulation

The transformation of the MINLP model (1)-(9) into a convex one with binary and continuous variables is made through the usage of auxiliary variables that allow changing voltages and currents to rewrite Equations (3) to (4) as affine expressions. Let us define  $u_{k,t} = v_{k,t}^2$  and  $l_{jk,t} = i_{jk,t}^2$ ; using them in Expressions (3) and (4) it is possible to build affine planes; however, the main complication of the model is the power definition in (5). To transform this equation into a convex one, let us use the hyperbolic equivalent of the product between two variables as follows (note that sets notation was eliminated for easy comprehension of the mathematical procedure):

$$\begin{aligned} p_{jk,t}^2 &= v_{j,t}^2 i_{jk,t}^2, \\ p_{jk,t}^2 &= u_{j,t} l_{jk,t} = \frac{1}{4} (u_{j,t} + l_{jk,t})^2 - \frac{1}{4} (u_{j,t} - l_{jk,t})^2, \\ (2p_{jk,t})^2 + (u_{j,t} - l_{jk,t})^2 &= (u_{j,t} + l_{jk,t})^2, \\ \left\| \begin{matrix} 2p_{jk,t} \\ u_{j,t} - l_{jk,t} \end{matrix} \right\| &= u_{j,t} + l_{jk,t}. \end{aligned} \quad (10)$$

Equation (10) is a conic equality constraint that is still non-convex due to the presence of the equality symbol [17]. However, as described in [38], this symbol can be replaced by a low equal symbol, which allows transforming it into a convex constraint, as represented below.

$$\|2p_{jk,t}\| \leq u_{j,t} + l_{j,k,t}, \quad (11)$$

Now, with expression (11) and the auxiliary variables previously defined, the optimization model (1)-(9) is transformed from a MINLP structure to a mixed-integer convex one as presented below:

**Objective function:**

$$\min G_{emissions} = \sum_{t \in T} \sum_{k \in N} CGG_{emissions}^0 p_{0k,t} \Delta t \quad (12)$$

**Set of constraints:**

$$p_{jk,t} - R_{jk} l_{jk,t} - \sum_{m:(k,m)} p_{km,t} = p_{k,t} - y_k^{pv} p_{k,t}^{pv,nom}, \quad \{\forall(j,k) \in E, \forall t \in T\} \quad (13)$$

$$u_{k,t} = u_{j,t} - 2R_{jk} p_{jk,t} + R_{jk}^2 l_{jk,t}, \quad \{\forall(j,k) \in E, \forall t \in T\} \quad (14)$$

$$\|2p_{jk,t}\| \leq u_{j,t} + l_{j,k,t}, \quad \{\forall(j,k) \in E, \forall t \in T\} \quad (15)$$

$$-i_{jk}^{max} \leq i_{jk,t} \leq i_{jk}^{max}, \quad \{\forall(j,k) \in E, \forall t \in T\} \quad (16)$$

$$v_{min} \leq v_{k,t} \leq v_{max}, \quad \{\forall k \in N, \forall t \in T\} \quad (17)$$

$$0 \leq y_k^{pv} \leq x_k^{pv} p_k^{pv,max}, \quad \{\forall k \in N\} \quad (18)$$

$$\sum_{j \in N} x_k^{pv} \leq N_{max}^{pv}, \quad (19)$$

**Remark 3:** The mixed-integer convex model can be solved using a hybrid optimization algorithm based on the Branch & Bound method combined with a modification of the interior point method for convex models with the main advantage that the global optimum finding is guarantee [34].

The main aspects of the solution methodology will be presented in the next section.

#### 4. Solution methodology

The mixed-integer convex reformulation proposed in this research is a convex reformulation problem with integrality constraints on some variables [17]. Therefore, we take advantage of the fact that the mixed-integer convex reformulation proposed is a convex problem, which can be solved efficiently with some integer programming solvers such as the Branch & Bound (B&B) algorithm [34].

The B&B algorithm in each bifurcation (i.e.,  $x_k^{pv}$  takes the value of “0” or “1”) generates a convex problem, which is solved with an interior-point method. In a child bifurcation (B1, B2, ..., BN) conforming to a B&B tree (which is a convex problem), the problem must be solved from its main fork (B0). This produces a series of secondary branch problems, which are solved for entire partitions where their

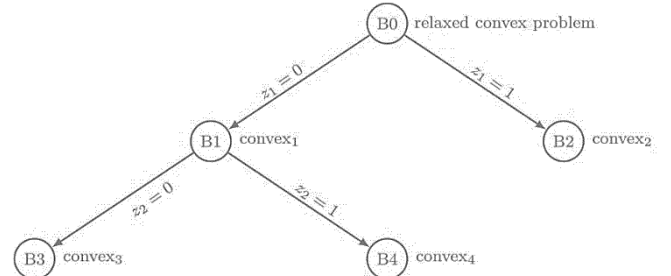


Figure 1. Schematic representation of the B&B algorithm for the convex problem. Source: Authors

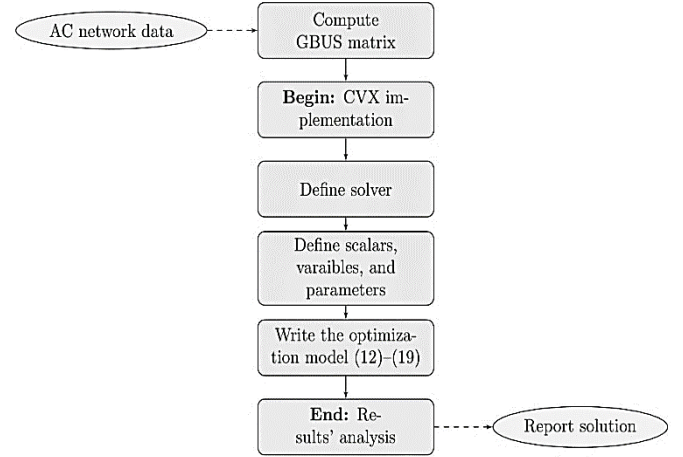


Figure 2. Flow chart of the proposed optimization approach. Source: Authors

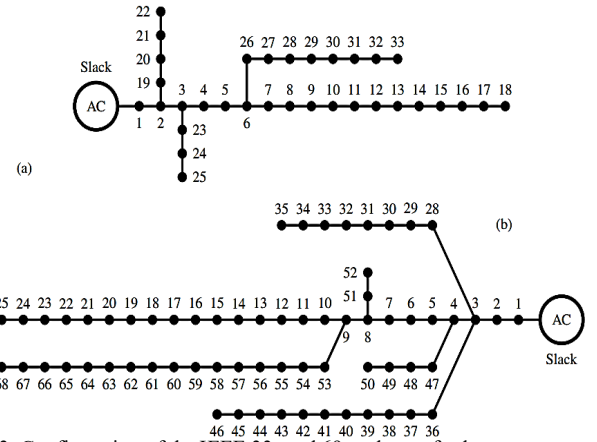


Figure 3. Configuration of the IEEE 33- and 69-node test feeders. Source: Authors

primary branch is a lower bound for the convex problem. This methodology is efficient despite having many variables and continues until reaching the best binary solution, which is the global optimum of the problem. This methodology is illustrated in Fig. 1.

Finally, Fig. 2 shows the flow chart of the proposed optimization approach for exact minimization of the energy losses and the CO2 emissions in isolated DC distribution networks using PV sources.



Table 2.  
Data for building load and PV curves

Time (h)	Demand (p.u)	Solar (p.u)	Time (h)	Demand (p.u)	Solar (p.u)
1	0.4240	0	13	0.8013	1.000
2	0.4108	0	14	0.7899	0.951
3	0.3999	0	15	0.7774	0.521
4	0.4083	0	16	0.7704	0.255
5	0.4744	0	17	0.8022	0.035
6	0.5301	0	18	0.8926	0
7	0.5669	0	19	10.000	0
8	0.6326	0.049	20	0.9682	0
9	0.7202	0.249	21	0.8890	0
10	0.7805	0.300	22	0.7832	0
11	0.8268	0.683	23	0.6175	0
12	0.8369	0.835	24	0.5212	0

Source: Authors

## 5. DC test feeders

To validate the proposed mixed-integer convex optimization model's effectiveness and robustness for locating and sizing PV sources in DC grids, two radial test feeders composed of IEEE 33- and 69-node are considered. The electrical configuration between nodes in both test feeders is depicted in Fig. 3.

The parametric information of these DC distribution grids can be consulted in [22]. In addition, the daily load and the normalized PV curves are listed in Table 2.

Regarding the greenhouse gas emissions, the information about diesel generators presented in [39]. Here, the most relevant gas emissions for medium size diesel generators (those with capabilities lower than 10 MW) is the carbon dioxide, i.e., CO<sub>2</sub>, with an average emissions rate of 612.35 kg/MWh.

## 6. Computational validation

The evaluation of the exact MINLP model and the proposed mixed-integer convex model are made in the GAMS optimization software and the CVX with the MOSEK solver in MATLAB [17], respectively. We implement both optimization models in a personal computer AMD Ryzen 7 3700U, 2.3 GHz, 16 GB RAM with 64-bits Windows 10 Home Single Language. All algorithms developed in this paper are available at File Exchange.

### 6.1 IEEE 33-node test feeder

Here we validate the effectiveness and robustness of the proposed optimization approach to optimal place and size PV sources in DC distribution grids. For that purpose, it is considered the possibility of installing three PV sources into the grid; also, two GAMS solvers named BONMIN and COUENNE were used to compare the results of the MIC approach. Table 3 presents the comparisons among different methods.

Numerical results in Table 3 allows noting that: (i) the proposed mixed-integer convex model reaches the minimum value regarding the minimization of the daily energy losses when distributed generators are in nodes 13, 24, and 30 with nominal generation rates of 994.9 kW, 1040.8 kW, and 1115.2 kW, respectively. Those values correspond to a

reduction of 32.31 %. (ii) the GAMS solvers COUENNE and BONMIN reach local optimal solutions that allow reducing the daily energy losses about 32 % and 32.14 %; however, their differences with the global optimal solution are very small, since these (i.e., GAMS solvers) identifies in their solutions nodes in the neighborhood of the global optimal solution reached by the proposed MIC approach; (iii) regarding the minimization of the amount of the CO<sub>2</sub> emissions, the reductions reached by the COUENNE, BONMIN, and MIC approaches are 23.83 %, 23.85 %, 23.84 %, respectively.

The results mentioned above imply that the GAMS solvers and the MIC approaches are comparable regarding minimizing the daily energy losses and the amount of CO<sub>2</sub> emissions. However, the main advantage of the proposed MIC model is the global optimum solution, while the GAMS solvers do not ensure the global optimum finding due to the non-convexities in the exact MINLP model.

### 6.2 IEEE 69-node test feeder

To determine the efficiency of the mixed-integer convex (MIC) model to locate and size PV nodes in the IEEE 69-node test feeder, we evaluate the possibility of installing three distributed generators along the DC test feeder. Table 4 reports the optimal location reached by the proposed MIC method for several PV sources installed along the grid. It is important to note that no comparisons are made because the GAMS solvers BONMIN and COUENNE do not converge for this test feeder. This is because the GAMS tries to recover the solution reached by their relaxed model; however, it is imprecise and does not meet the minimum gap. Hence the solution does not converge.

Table 3.  
Numerical results in the IEEE 69-node test feeder.

Approach	Location-Size node (MW)	Losses (kWh/Day)	Emissions (tons/Da)
0	---	1629.7604	38.8163
COUENNE	11 (0.9782)	1108.2271	29.5648
	24 (0.9693)		
	30 (1.0428)		
BONMIN	13 (0.8989)	1105.9545	29.5590
	24 (1.0980)		
	14 (0.8347)		
MIC	13 (0.9949)	1103.2300	29.5607
	24 (1.0408)		
	30 (1.1152)		

Source: Authors

Table 4.  
Numerical results with the proposed MIC approach in the IEEE 69-node test feeder.

Number of PVs	Location-size node (MW)	Losses (kWh/Day)	Emissions (tons/Day)
0	---	1838.5134	40.7180
1	61	1255.0720	34.0297
2	17 (0.5359)	1215.2164	32.6849
	61 (2.0258)		
3	18 (0.5353)	1208.7995	30.9829
	49 (0.8643)		
	61 (2.0254)		

Source: Authors

Numerical results in Table 4 shows the following: (i) node 61 is the most sensitive node to locate PV sources with nominal rates higher than 2 MW since this allows a higher reduction in the number of daily energy losses; (ii) the maximum reduction of energy losses and CO<sub>2</sub> emissions is reached with three PV sources located at nodes 18, 49, and 61, with reductions of about 34.25 % and 23.91 %, respectively; (iii) the results demonstrate that the reduction of the number in the daily energy losses is directly connected with the amount of CO<sub>2</sub> emissions since the optimal location of PV sources makes possible a better distribution of the power injections in the sources of the DC grid. This result implies that power injections in the diesel source are reduced, and as can be seen in eq. (2); hence, the amount of CO<sub>2</sub> is also reduced due to its linear relation.

**Remark 4:** *The reduction in the daily energy losses presents a saturation while the number of PV increases due to their effect is only restricted to the periods between 7 and 18 (see Table 2), where the PV source can generate power. Note that the difference between one and two PV sources is only 39.8556 kWh/Day, which is also lower than the solution reached by the two and three generators, i.e., 6.4168 kWh/Day.*

## 7. Conclusions and future works

The optimal location and sizing PV sources in electric distribution networks operated using DC technologies have been addressed in this research from exact mathematical optimization. The exact MINLP model was transformed into a mixed-integer convex model with the main advantage that the global optimum can be ensured by applying the Branch & Bound method combined with an interior point approach for conic programming.

Numerical results in the IEEE 33-node test feeder show that the proposed MIC approach found a better reduction of daily energy losses than BONMIN and COUENNE solvers in the GAMS optimization package that were stuck in local optimal solutions.

For the IEEE 69-node test feeder, it was observed that depending on the number of the PV sources the amount of CO<sub>2</sub> emissions and the daily energy losses presents a saturation regarding their possible reductions. Besides, it was noted that for two PV sources, the reduction of daily energy losses was about 33.90 %; and for three PV sources, the total reduction was about 34.25 %, i.e., the additional gain when an additional PV source installed was only 0.35 %.

As future works, the following research can be conducted: (i) to extend the proposed MIC approach to AC to the location of renewable energy resources considering the presence of battery energy storage systems align the AC grid; and (ii) the application of the proposed MIC to the problem of the dynamic reactive power compensation in AC grids considering FACTS.

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