



REVISTA DE INGENIERIA DE LA FACULTAD DE INGENIERIA, UNIVERSIDAD NACIONAL DE COLOMBIA - SEDE MEDALLA

DYNA

ISSN: 0012-7353

ISSN: 2346-2183

Universidad Nacional de Colombia

Bustamante-Rúa, Oswaldo; Rivera-Madrid, Ismael Eduardo; Palacio, Juan Luis; Lopera, Alex Arbey; Rincón-Fulla, Marlon; Osorio-Correa, Adriana; Menéndez-Aguado, Juan María

Study on alumina ball size distribution in white cement grinding

DYNA, vol. 88, no. 218, 2021, July-September, pp. 19-23

Universidad Nacional de Colombia

DOI: <https://doi.org/10.7440/res64.2018.03>

Available in: <https://www.redalyc.org/articulo.oa?id=49671325002>

- How to cite
- Complete issue
- More information about this article
- Journal's webpage in [redalyc.org](https://www.redalyc.org)

UNEN [redalyc.org](https://www.redalyc.org)

Scientific Information System Redalyc

Network of Scientific Journals from Latin America and the Caribbean, Spain and Portugal

Project academic non-profit, developed under the open access initiative

# Study on alumina ball size distribution in white cement grinding

Oswaldo Bustamante-Rúa<sup>a</sup>, Ismael Eduardo Rivera-Madrid<sup>b</sup>, Juan Luis Palacio<sup>b</sup>, Alex Arbey Lopera<sup>b</sup>, Marlon Rincón-Fulla<sup>b,c</sup>, Adriana Osorio-Correa<sup>d</sup>, & Juan Maria Menéndez-Aguado<sup>e</sup>

<sup>a</sup> Universidad Nacional de Colombia, sede Medellín, Facultad de Minas, CIMEX, Colombia. mobustam@unal.edu.co

<sup>b</sup> Institución Universitaria Pascual Bravo, Medellín, Colombia. ismael.rivera@pascualbravo.edu.co, jlpalaci@pascualbravo.edu.co, a.lopera1420@pascualbravo.edu.co, mrfulla@pascualbravo.edu.co

<sup>c</sup> Universidad Nacional de Colombia, sede Medellín, Escuela de Física, Colombia. mrinconf@unal.edu.co

<sup>d</sup> Grupo Procesos Físicoquímicos Aplicados, Facultad de Ingeniería, Universidad de Antioquia, Medellín, Colombia. adriana.osorio@udea.edu.co

<sup>e</sup> Escuela Politécnica de Mieres, Universidad de Oviedo, Oviedo, Spain. maguado@uniovi.es

Received: August 7<sup>th</sup>, 2020. Received in revised form: March 19<sup>th</sup>, 2021. Accepted: April 12<sup>th</sup>, 2021.

## Abstract

This paper presents the results of the characterization of alumina ball size distribution in the mill of a crushing and grinding plant. The mill was unloaded and the balls were sieved. Then, the results were adjusted to three kinds of models; two of them are the traditional models of Rosin-Rammler and Gates-Gaudin-Schumann. The third model applied was the Swebrec distribution function, which has been successfully used on fragmented rock size distribution. The best adjustment for the ball size distribution was obtained by the Swebrec function, with a correlation coefficient of 0.9984.

**Keywords:** ball size distribution; grinding; size distribution model.

# Estudio sobre la distribución de tamaño de bola de alúmina en molienda de cemento blanco

## Resumen

En este trabajo se presentan los resultados de la caracterización de la distribución del tamaño de bolas de alúmina en el molino de una planta de molienda y trituración. Se descargó el molino y se tamizaron las bolas. Luego, los resultados se ajustaron a tres tipos de modelos; dos de ellos son los modelos tradicionales de Rosin-Rammler y Gates-Gaudin-Schumann. El tercer modelo aplicado fue la función de distribución de Swebrec, que se ha utilizado con éxito en la distribución del tamaño de rocas fragmentadas. El mejor ajuste para la distribución del tamaño de bolas se obtuvo mediante la función de Swebrec, con un coeficiente de correlación de 0,9984.

**Palabras clave:** distribución de tamaño de bola; molienda; modelo de distribución de tamaño.

## 1. Introduction

In cement manufacturing, the grinding process is affected by many factors, one of which is ball size distribution (BDS). In fact, not getting a good ball size distribution could negatively affect the product's output as well as its quality [1]. Knowing the distribution of the ball size at the beginning of the process would help the system to reach a steady state in a shorter period of time, making the grinding more efficient and reducing the mills' energy consumption. Several studies show that ball size distribution affects the

mill's efficiency and productivity [2-11]. Also, in industrial mills when the dust reaches some degree of fineness, using big balls in the mill can result in low productivity, high energy consumption and a non-compliance product. Therefore, a reasonable distribution of the ball size is the key to reach the crushing and grinding balance.

Nowadays, there are several mathematical models for modeling particle size distribution. One of them is the Rosin-Rammler distribution function (also known as Weibull distribution), which is a two-parameter distribution widely used to describe or model both fine and coarse particles [12].

**How to cite:** Bustamante-Rúa, O., Rivera-Madrid, I.E., Palacio, J.L., Lopera, A.A., Rincón-Fulla, M., Osorio-Correa, A. and Menéndez-Aguado, J.M., Study on alumina ball size distribution in white cement grinding. DYNA, 88(218), pp. 19-23, July - September, 2021.

This distribution was originally used by Rosin and Rammler to describe the size distribution of particles generated by crushing and grinding [13]; García et al. [14] successfully applied the Rosin-Rammler [13] and the Gates-Gaudin-Schumman (GGS) distribution models [15,16] to agglomerated cork. There is another kind of distribution that was initially applied for modeling fragmented rock by blasting. This is known as the Swebrec function [17,18], which has three parameters and has been also used to model fines [19]. José A. Sanchidrián et al [20] used different data of fragmented rock of sizes from 0.002mm to 2000mm aiming to demonstrate which distribution function best describes fragmented rock, and found that, in terms of coefficient of determination R<sup>2</sup>, Rosin-Rammler is the best two-parameter function to describe rock fragments and, among the three-parameter distribution models, Swebrec is the best one, with an R<sup>2</sup> very closed to 1. Recently Shuhua Liu et al [21] showed that the size distribution of particles of copper tails dust fits the RR and Swebrec models. On the other hand, recent studies related to the ball wear kinetics with the aim of obtaining the ball size distribution, have been addressed in the references [22-24].

Due to the importance given to these distributions in recent as well as in old studies, a comparison of the three models – Rosin-Rammler, Swebrec and Gates-Gaudin-Schumman - was carried out in order to define which one best describes an alumina ball size distribution in an industrial mill. None of these models have been used to model a BSD because of the difficulty sieving the balls to gather experimental data. For the present research, the balls sieving was possible due to the fact that the mill was stopped for maintenance.

## 2. Mathematical models used to determine the ball size distribution

As already mentioned, we will apply three distribution models, of two and three parameters, used for both coarses and fines, but it was difficult to find literature for their application on BSD.

The three-parameter model that we will implement for this research is the Swebrec Distribution model described below.

Considering that cumulative passing function  $F(x)$  can be expressed as:

$$F(x) = \frac{1}{1 + f(x)} \quad (1)$$

and taking into consideration that  $f(x_{max}) = 0$  and  $f(x_{50}) = 1$ , the function is fixed at  $x_{max}$  and  $x_{50}$ .

Finally,  $f(x)$  is:

$$f(x) = \left[ \frac{\ln\left(\frac{x_{max}}{x}\right)}{\ln\left(\frac{x_{max}}{x_{50}}\right)} \right]^b \quad (2)$$

As a consequence, the Swebrec distribution function can be described as:

$$F(x) = \frac{1}{1 + \left[ \frac{\ln\left(\frac{x_{max}}{x}\right)}{\ln\left(\frac{x_{max}}{x_{50}}\right)} \right]^b} \quad (3)$$

where  $F(x)$  is cumulative passing at  $x$  size;  $x_{max}$  is maximum size;  $x_{50}$ , size passing 50% weight and  $b$  the curve waving parameter.

The Gades-Gaudin-Schumman is another model to be used for this research, since it is frequently applied in crushing and grinding, defined by:

$$F(x) = \left( \frac{x}{K} \right)^n \quad (4)$$

where  $F(x)$  is the sample fraction finer than size  $x$ ,  $x$  is the particle diameter,  $K$  is the maximum diameter of the particles (size modulus) expressed in mm, and  $n$  (distribution modulus) is an adjustable and dimensionless parameter.

Applying a logarithm on both sides of Eq. (4) we obtain a linear model whose equation corresponds to a straight line with slope  $n$ , as follows:

$$\ln[F(x)] = n \ln(x) - n \ln(K) \quad (5)$$

Finally, the third model to be applied is the Rosin-Rammler model, which is defined by:

$$G(x) = e^{-\left(\frac{x}{K}\right)^n} \quad (6)$$

where  $G(x)$  is the distribution function and  $x$  is the particle size expressed in mm,  $K$  is the size modulus (mm), and  $n$  is a measurement of the particle size variability,  $K$  and  $n$  being adjustable characteristic parameters of the distribution.

Eq. (6) can be re-written in the following way:

$$\ln\{-\ln[G(x)]\} = n \ln(x) - n \ln(K) \quad (7)$$

## 3. Experimental procedure and initial condition

During the mill maintenance stop of a crushing and grinding plant, the ball bed for one mill was unloaded with its 31424 tons of alumina balls.

The mill was initially uploaded with 31543 tons of balls before starting the grinding process. It was then reloaded during the process with balls of a single size (38.10 mm). The ground material used was silica sand. Figs. 1a and 1b shows an interior view of the mill employed in this study, before and after the maintenance stops, respectively.

The mill inner diameter  $D$  (usable diameter) is 242 cm and the mill inner length (usable length) is 650 cm. The balls of the mill were sieved through meshes of (38.10 mm), (31.75 mm), (25.40 mm), (19.05 mm), and (12.70 mm), and weighed before and after the maintenance stops, in order to determine the experimental ball size distribution (BSD).



Figure 1: (a) picture of the mill before being unloaded; (b) picture of the mill after being unloaded.  
Source: The Authors.

Table 1.  
Ball load and unload distribution.

Unload Distribution			Load Distribution		
Ball content inside the mill by diameters according to the ball unload (Kg)			Ball# content inside the mill by diameters according to the ball load (Kg)		
	Quantity (kg)	%		Quantity (kg)	%
Rubble	290	0.92%	Rubble	0	0.00%
Diameter 12.70 mm	1531	4.87%	Diameter 12.70 mm	878	2.78%
Diameter - 19.05 mm	3601	11.46%	Diameter - 19.05 mm	3667	11.63%
Diameter - 25.40 mm	9506	30.25%	Diameter - 25.40 mm	13866	43.96%
Diameter - 31.75 mm	12401	39.46%	Diameter - 31.75 mm	8150	25.84%
Diameter - 38.10 mm	4095	13.03%	Diameter - 38.10 mm	4981.5	15.79%
<b>Total ball</b>	<b>31424</b>	<b>100.00%</b>	<b>Total ball</b>	<b>31543</b>	<b>100 %</b>

Source: The Authors.

#### 4. Results and Discussion

Table 1 shows the initial percentage of the ball load before starting the grinding process, whose values are 15.79%, 25.84%, 43.96%, 11.63%, 2.78% for the ball sizes 38.10 mm, 31.75 mm, 25.40 mm, 19.05 mm and 12.70 mm, respectively. During the unloading process, it was determined that the percentage of the balls sieved by using the widest mesh was around 13.03%, while for the other mesh sizes -[31.75 mm, 38.10 mm], [25.40 mm, 31.75 mm], [19.05 mm, 25.40 mm], [12.70 mm, 19.05 mm] and [0.0 mm, 12.70 mm]- the percentages were 39.46%, 30.25%, 11.46%, 4.87% and 0.92%, respectively. From this standpoint, it can be seen that the 38.10-mm and 25.40-mm balls had a higher wearing. On the other hand, balls of 31.75 mm, 19.05 mm and 12.70 mm had a lower wearing during this process.

The adjustment equation for the Swebrec function is given by:

$$P(x) = 100 \times \left\{ \frac{1}{1 + \left[ \frac{\ln\left(\frac{38.10}{x}\right)}{0.2211} \right]^{3.40}} \right\} \quad (8)$$

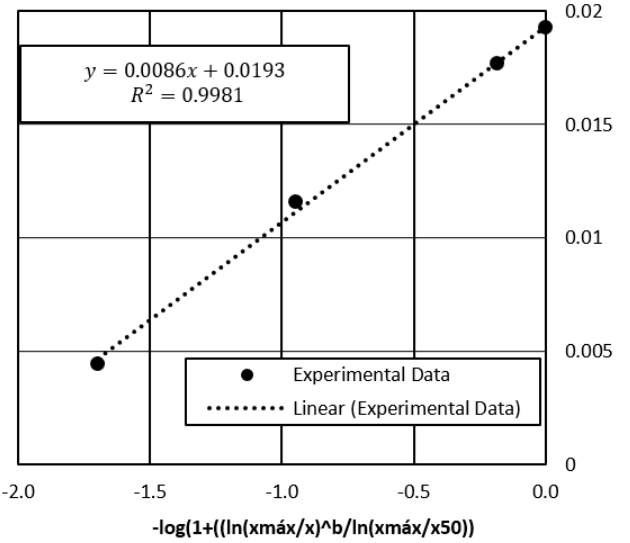


Figure 2: Linear adjustment for Swebrec distribution model.  
Source: The Authors.

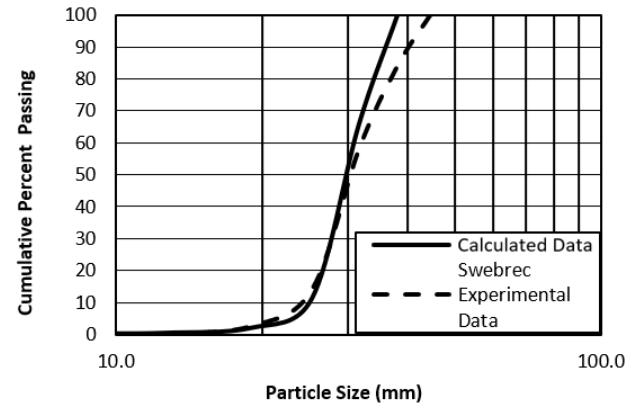


Figure 3: Experimental data adjustment to Swebrec distribution model.  
Source: The Authors.

The adjustment of the experimental data for the BSD of the Swebrec distribution is presented in Figs. 2 and 3.

From Figs. 2 and 3, it can be seen that the Swebrec model is in high agreement with the experimental data, since the correlation coefficient obtained in the adjustment scheme is 0.9981.

In Fig. 3, parameter  $x_{50}$  obtains a value of 30.54 and parameter  $b$  obtains a value of 3.40. Some studies report that when  $b \rightarrow 1$ , the curve inflexion point tends to  $x = x_{max}$ ; also, if parameter  $b$  decreases, the curve inflexion point tends to  $x = x_{50}$  [17].

From Figs. 4, 5, 6 and 7, it can be seen that the Gates-Gaudin-Schumann and Rosin-Rammler models are in high agreement with the experimental data, since the correlation coefficients obtained in the adjustment scheme are 0.9407 and 0.9914, respectively. If we analyze the correlation coefficient  $R^2$ , it can be seen that even though the linear adjustment of the GGS model is very good, the one of the Rosin-Rammler is much better, only exceeded by the Swebrec distribution model.

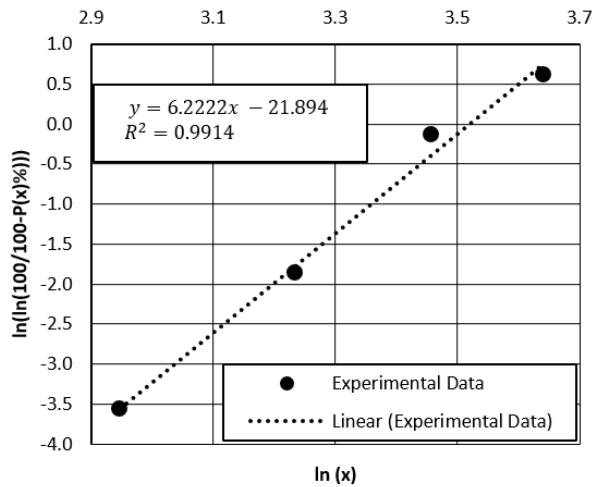


Figure 4: Linear adjustment for Rosin-Rammler distribution model.  
Source: The Authors.

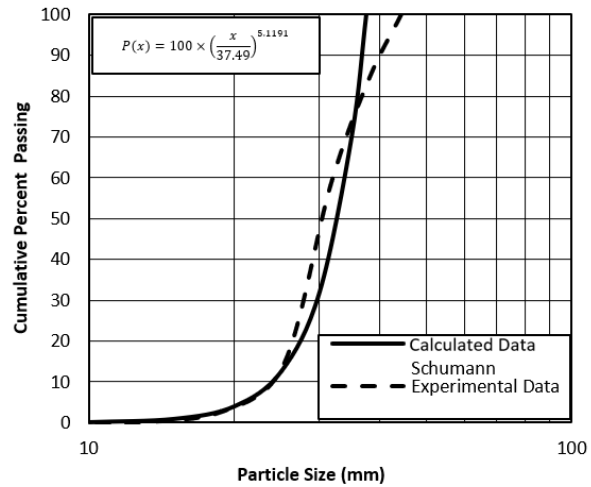


Figure 7: Experimental data adjustment to Swebrec distribution model.  
Source: The Authors.

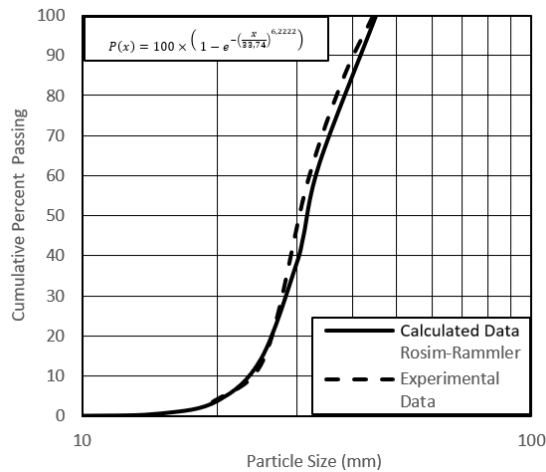


Figure 5: experimental data adjustment for Rosin-Rammler distribution model.  
Source: The Authors.

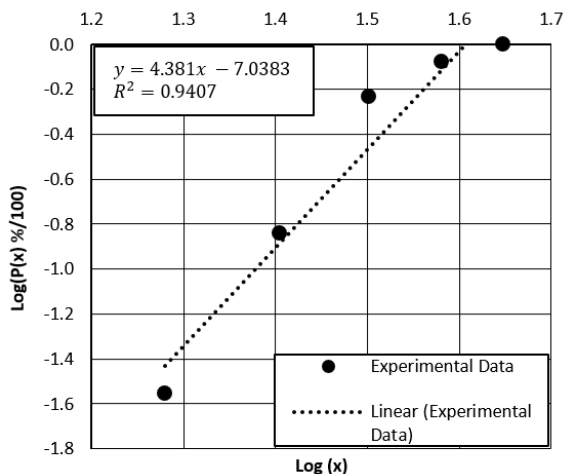


Figure 6: Linear adjustment for Gates-Gaudin-Schumman distribution model.  
Source: The Authors.

In this sense, it could be suggested that the Swebrec function it is a sounding alternative for modelling BSD functions in contrast to traditional functions that describe the ceramic ball wear process in white cement industrial mills.

## 5. Conclusions

The distribution functions that are used the most for characterizing materials are the Gates-Gaudin-Schumman and Rosin-Rammler functions, however, deciding which is the best for modeling the particle size distribution is non-trivial, since the choice of the best distribution function strongly depends on the material studied. That is the case of Gates-Gaudin-Schumman distribution function, which is mainly used in coal preparation studies for what it was originally developed. In this research, from the standpoint of the correlation coefficient in particular, we can conclude that the Rosin-Rammler model is better than the Gates-Gaudin-Schumman for modeling a BSD in the white cement industry, due to the fact that their correlation coefficients were of 0.9914 and 0.9407, respectively. Nevertheless, the proposal of implementing a more modern model was very successful, which was the case of the Swebrec distribution model. This model obtained a correlation coefficient of 0.9981, being better than the Rosin-Rammler and the Gates-Gaudin-Schumman models.

In general, we can conclude that during the grinding process for the production of white cement, the Swebrec function can be chosen as a model of ball size distribution in order to characterize the ball size and thus, to improve the mill performance.

## Acknowledgements

IER, JLP, AL and MRF thank to Institución Universitaria Pascual Bravo for the financial support within the research project entitled: “Estudio espectral y de la respuesta óptica de sistemas de pocas partículas confinadas en nanoestructuras semiconductoras cuasi bidimensionales” with grant number IN201907.

# References

- [1] Yunqing, Z., Effect of ball size distribution on grinding efficiency. *Grind. Technol.*, pp. 37-39, 2000.
- [2] Cook, T.M. and Courtney, T.H., The effects of ball size distribution on attritor efficiency. *Metallurgical and Materials Transactions A*, 26, pp. 2389-2397, 1995. DOI: 10.1007/BF02671252.
- [3] Katubilwa, F.M., Effect of ball size distribution on milling parameters, MSc. Thesis, Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, South Africa, 2008.
- [4] Shin, H., Lee, S., Jung, H.J. and Kim, J-B., Effect of ball size and powder loading on the milling efficiency of a laboratory-scale wet ball mill. *Ceramics International*, 39, pp. 8963-8968, 2013. DOI: 10.1016/j.ceramint.2013.04.093.
- [5] François, M., Katubilwa, M. and Moys, M.H., Effect of ball size distribution on milling rate. *Minerals Engineering*, 22, pp.1283-1288, 2009. DOI: 10.1016/j.mineng.2009.07.008.
- [6] Hlabangana, N., Danha, G. and Muzenda, E., Effect of ball and feed particle size distribution on the milling efficiency of a ball mill: an attainable region approach. *South African Journal of Chemical Engineering*, 25, pp. 79-84, 2018. DOI: 10.1016/j.sajce.2018.02.001.
- [7] Chimwani, N., Mulenga, F.K. and Hildebrandt, D., Ball size distribution for the maximum production of a narrowly-sized mill product. *Powder Technology*, 284, pp. 12-18, 2015. DOI: 10.1016/j.powtec.2015.06.037.
- [8] Bwalya, M.M., Moys, M.H., Finnie, G.J. and Mulenga, F.K., Exploring ball size distribution in coal grinding mills. *Powder Technology*, 257, pp. 68-73, 2014. DOI: 10.1016/j.powtec.2014.02.044.
- [9] Razavi-Tousi, S.S. and Szpunar, J.A., Effect of ball size on steady state of aluminum powder and efficiency of impacts during milling. *Powder Technology*, 284, pp. 149-158, 2015. DOI: 10.1016/j.powtec.2015.06.035.
- [10] Cho, H., Kwon, J., Kim, K. and Mun, M., Optimum choice of the make-up ball sizes for maximum throughput in tumbling ball mills. *Powder Technology*, 246, pp. 625-634, 2013. DOI: 10.1016/j.powtec.2013.06.026.
- [11] Erdem, A.S. and Ergun, S.L., The effect of ball size on breakage rate parameter in a pilot scale ball mill. *Minerals Engineering*, 22, pp. 660-664, 2009.
- [12] Djamrani, K.M. and Clark, I.M., Characterization of particle size based on fine and coarse fractions. *Powder Technol.*, 93, pp. 101-108, 1997. DOI: 10.1016/S0032-5910(97)03233-6.
- [13] Rosin P. and Rammler, E., The laws governing the fineness of powdered coal. *J. Inst. Fuel*, 7, pp. 29-36, 1933.
- [14] Macías-García, A., Cuerda-Correa, E.M. and Díaz-Díez, M.A., Application of the Rosin-Rammler and Gates-Gaudin-Schumann models to the particle size distribution analysis of agglomerated cork. *Mater. Charact.*, 52, pp. 159-164, 2004. DOI: 10.1016/j.matchar.2004.04.007.
- [15] Gates, A.O., Kick vs. Rittinger: an experimental investigation in rock crushing performed at Purdue University. *Trans AIME*, 52, pp. 875-909, 1915.
- [16] Schumann, J., Principles of comminution I: size distribution and surface calculations. *Trans. AIME*, pp. 1189, 1940.
- [17] Ouchterlony, F., The Swebrec function: linking fragmentation by blasting and crushing. *Min. Technol. Trans. Inst. Min. Metall. A*, 114, pp. 29-44, 2005. DOI: 10.1179/037178405X44539.
- [18] Ouchterlony, F., Sanchidrián, J.A. and Moser, P., Percentile Fragment size predictions for blasted rock and the fragmentation-energy fan. *Rock Mech Rock Eng* 50, pp. 751-779, 2017. DOI: 10.1007/s00603-016-1094-x.
- [19] Osorio, A.M., Menéndez-Aguado, J.M., Bustamante, O. and Restrepo, G.M., Fine grinding size distribution analysis using the Swebrec function. *Powder Technol*, 258, pp. 206-208, 2014. DOI: 10.1016/j.powtec.2014.03.036.
- [20] Sanchidrián, J.A., Ouchterlony, F., Segarra, P. and Moser, P., Size distribution functions for rock fragments. *International Journal of Rock Mechanics and Mining Sciences*, 71, pp. 381-394, 2014.
- [21] Liu, S., Li, Q. and Song, J., Study on the grinding kinetics of copper tailing powder. *Powder Technol*, 300, pp. 105-113, 2018. DOI: 10.1016/j.powtec.2018.02.025.
- [22] Kwon, J., Jeong, J. and Cho, H., Simulation and optimization of a two-stage ball mill grinding circuit of molybdenum ore. *Adv Powder Technol.*, 27, pp. 1073-1085, 2016. DOI: 10.1016/j.appt.2016.03.016.
- [23] Lee, H., Kim, K. and Lee, H., Analysis of grinding kinetics in a laboratory ball mill using population-balance-model and discrete-element-method. *Adv Powder Technol.*, 30, pp. 2517-2526, 2019. DOI: 10.1016/j.appt.2019.07.030.
- [24] Bürger, R., Bustamante, O., Fulla, M.R. and Rivera, I.E., A population balance model of ball wear in grinding mills: an experimental case study. *Minerals Eng.*, 128, pp. 288-293, 2018. DOI: 10.1016/j.mineng.2018.09.004.

**O. Bustamante-Rúa**, is titular profesor at Universidad Nacional de Colombia, Sede Medellín. PhD. in Engineering Sciences with a Mention Extractive Metallurgy - Mining from the Universidad de Concepción - Chile, in 2002. BSc. in Mining and Metallurgy Engineering. CIMEX. ORCID: 0000-0002 -1692 -991X

**I.E. Rivera-Madrid**, is full time professor at Institución Universitaria Pascual Bravo, Medellín, Colombia. PhD. in Engineering, mention in Materials Science and Technology, in 2014, from the Universidad Nacional de Colombia. BSc. in Mathematics. ORCID: 0000-0003-0184-4697

**J.L. Palacio**, is full time professor at Institución Universitaria Pascual Bravo, Medellín, Colombia. Currently a PhD. student in Sciences-Physics from the Universidad Nacional de Colombia. BSc. in Physics Engineering. ORCID: 0000-0003-0613-4301

**A.A. Lopera**, is full time professor at Institución Universitaria Pascual Bravo, Medellín, Colombia. PhD. in Engineering Science and Technology of Materials, in 2017 from the Universidad Nacional de Colombia. BSc. in Physics Engineering. ORCID: 0000-0001-5491-6274

**M. Rincón-Fulla**, is full time professor at Institución Universitaria Pascual Bravo, Medellín, Colombia. PhD. in Sciences-Physics, in 2017 from the Universidad Nacional de Colombia. BSc. in Physics Engineering. ORCID: 0000-0002-7972-3636

**A.M. Osorio-Correa**, is full time profesor at Universidad de Antioquia. PhD. in Engineering, in 2014 from the Universidad de Antioquia. BSc. Chemical Engineering. PFA. ORCID: 0000-0002-6413-9023

**J.M. Menéndez-Aguado**, is titular professor at Universidad de Oviedo, España. DSc. in Mining Engineering from the Universidad de Oviedo, España. ORCID: 0000-0002-6216-6984