Short note

May Rayleigh waves propagate with group- and phase-velocities of opposite sign in the valley of Mexico City?

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Resumen

Con base a la fórmula que determina la velocidad de grupo de las ondas sísmicas y utilizando resultados previamente publicados por los autores, se discute en forma teórica la ocurrencia de velocidades de grupo negativas de ondas de Rayleigh en modelos simplificados de la cuenca del Valle de México.

Palabras clave: Ondas de Rayleigh, velocidades del grupo negativas, Valle de México

Abstract

On the basis of the group-velocity formula the occurrence of negative group velocities of Rayleigh waves is theoretically discussed for simplified models of Mexico City's basin by using results of a preceding article of the authors.

Key words: Rayleigh waves, negative group velocities, Valley of Mexico City

Introduction

This short note is a complement of the article about unusual, equivocal Rayleigh dispersion curves by Malischewsky *et al.* (2017) in the Special Volume on the 30TH Anniversary of the 19 September 1985 Earthquake of *Geofisica Internacional*. It was not demonstrated, that the strange dispersion curves presented there lead to negative group velocities, but this is indeed the case as shown below. So this short comment in some way serves the popularization and understanding of negative group velocities, well-known in many fields of physics and techniques [see e. g. Nedopasov *et al.* (2017), Tamm *et al.*. (2017), Gérardin *et al.*. (2019)], in seismology as well [see Lysmer (1970), Forbriger (2017)] . Let us point out that in literature on normal mode theory solutions to the boundary value problem usually are discussed without an explicit source. The sign of propagation velocity then is not defined with respect to a source of wave energy. The

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term “negative group velocity” thus actually means that “group velocity has the opposite sign of phase velocity” and is often coupled with the conception of “backward waves” in literature. It is not the intention of this short comment to shed light onto this connection. We only note, that with Shevchenko (2007) there are three definitions for forward and backward waves, where the classical one is that a wave is considered a forward (backward) wave if the directions of its phase and group velocities are the same (opposite).

Group velocities

The formula for the group velocity of waves [see e. g. Malischewsky (2018).] is

$$U(F) = \frac{C^2}{C - F\,dC/dF} \tag{1}$$

with dimensionless phase velocity C and dimensionless frequency F, which are defined depending on the model under consideration. Already Lamb (1904) has pointed out that it is possible for the group velocity to be of opposite sign with respect to phase velocity. He discusses the solution of the wave equation in the presence of a source of energy and points out that wave groups always propagate away from the source, while the complete wave train is actually a superposition of harmonic waves with negative phase velocity. Having a look on formula (1) U(F) and C are of opposite sign only for anomalous dispersion, i. e.

$$\frac{dC}{dF} > 0 \tag{2}$$

with the additional condition

$$\frac{dC}{dF} > \frac{C}{F}\ , \tag{3}$$

which is a differential inequality.

The dispersion curves in Fig. 1 of Malischewsky *et al.* (2017) concern the first higher Rayleigh mode in the simple LFB (Layer with Fixed Bottom) – model for the Poisson ratios ν = 0.37 and 0.44, respectively. It can be demonstrated that both conditions (2) and (3) are fulfilled for specific frequency ranges, and the occurrence of negative group velocities is obvious. By using eq. (1) and numerical differentiation, the group velocity U is obtained and is presented together with the former phase velocity curves C in Fig. 1. The negative branches of group velocity occur in the frequency ranges

$$F \in (0.53,\ 0.55) \text{ for } \nu = 0.37 \text{ and}$$
$$F \in (0.64,\ 0.75) \text{ for } \nu = 0.44\ .$$

These pictures are very similar to those ones obtained by Negishi (1987) in Lamb-wave context.

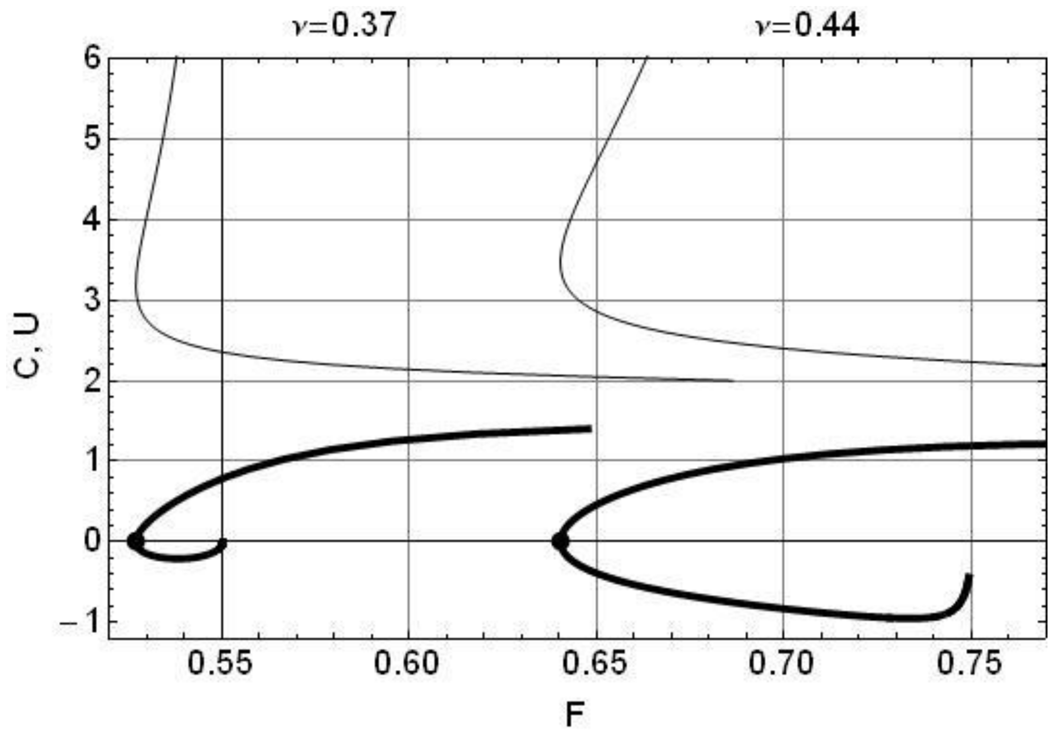

Figure 1. Phase (thin) and group (thick) velocity curves for Poisson ratios $\nu = 0.37$ and $\nu = 0.44$, respectively; the ZGV-points with lower frequencies are marked with black points

Obviously, a necessary precondition for negative group velocities is the existence of so-called ZGV – points (Zero Group Velocity points, see Fig. 1). The derivative dC/dF becomes infinite for the phase-velocity curve at these points. A marginal note: Condition (2) can be easily fulfilled for the fundamental Rayleigh mode of the LOH-model (Layer Over Halfspace) by using the results of Tuan et al. (2008) and specifying the parameters for which the dispersion curve starts with $dC/dF > 0$ for zero frequency. However, in absence of a ZGV-point, condition (3) is not fulfilled, and a negative group velocity does not occur.

Following Forbriger *et al.* (2020) these effects described here for the first higher mode occur for the fundamental mode in certain more complex models as well.

It is yet unclear, what the role of these special waves could be in the seismic regime of Mexico City's valley, but their pure possible existence is scientifically interesting enough to be mentioned and to position it into a larger context.

Let us come back to the title question by affirming it: If any facts are theoretically possible, the nature usually realizes them. Successful predictions is what we expect from a consistent theory. The most prominent example of recent years is the confirmation of the existence of gravitational waves after Einstein's prediction from 1916. For the phenomenon discussed here, the seismological observation has not been made yet. However, on laboratory scale the phenomenon is confirmed by experiment [Wolf *et al*. (1988), Cès *et al*. (2011)].

Acknowledgements

PM devotes this comment to the memory of his good friend and former colleague Dr. Dieter Freund (1943-2018), who taught him to look cheerfully at the life in good and bad times.

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