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A COMPLETE CLASSIFICATION OF THE MERSENNE'S PRIMES AND ITS IMPLICATIONS FOR COMPUTING

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ABSTRACT

A study of Mersenne's primes is carried out using the multiplicative group modulo 360 and a complete classification is obtained by its residual classes. This allows the search for Mersenne's primes to be classified into four subgroups mutually exclusive (disjoint) and contributes to the ordered selection of exponents to be computationally tested. According to this idea, Mersenne's trapeze is presented with the purpose of giving a geometric representation for this classification. Finally, from one of the theorems presented and proven for primes in modulo 360, a conjecture is established that could be solved by computing.

Keywords: *Mersenne's prime; residual classes; Mersenne's trapeze.*

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UNA CLASIFICACIÓN COMPLETA DE LOS NÚMEROS PRIMOS DE MERSENNE Y SUS IMPLICACIONES PARA LA COMPUTACIÓN

RESUMEN

Se realiza un estudio de los números primos de Mersenne utilizando el grupo multiplicativo módulo 360 y se obtiene una clasificación completa mediante sus clases residuales. Esto permite clasificar la búsqueda de los números primos de Mersenne en cuatro subgrupos mutuamente excluyentes (disjuntos) y contribuye a la selección ordenada de exponentes a probar computacionalmente. Acorde a esta idea, el trapecio de Mersenne se presenta con el propósito de dar una representación geométrica para esta clasificación. Finalmente, a partir de uno de los teoremas presentado y demostrado para primos en módulo 360, se establece una conjetura que podría resolverse mediante verificación computacional.

Palabras Clave: *Primos de Merssene; Clases residuales; Trapecio de Mersenne.*

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1. INTRODUCTION

A selection of problems in the theory of numbers focuses on mathematical problems within the boundaries of geometry and arithmetic, among these the Mersenne's primes stand out, due to their computational search and incomprehensible randomness and the finding of even perfect number and it connection to perfect numbers and cryptography [1, 2, 3, 4].

A Mersenne's prime M_ρ is a prime number of the form $M_\rho = 2^\rho - 1$, where ρ is also a prime number. The current search for Mersenne's primes M_ρ, is led by the GIMPS computational project (www.mersenne.org) and to date, only 51 of these numbers have been found. In this work, a study for the Mersenne's primes under the multiplicative group modulo 360 is presented with the objective of contributing to this search by improving the selection processes of the prime exponents and provide a complete classification of these numbers. The methods used below are proper and basic under a certain level of number theory [5].

2. CONTEXTUALIZATION

Let be M_ρ a Mersenne's prime.

Definition 1 (Ova-angular residue de M_ρ). Let be M_ρ and ρ prime numbers. The solution for x, y of the equation $\rho \cong x \pmod{360}$ and $M_\rho \cong y \pmod{360}$, it will be called Ova-angular of ρ or M_ρ respectively and will be denoted by $\Gamma_\rho = x$, $\Gamma_{M_\rho} = y$, such that:

$$\Gamma_\rho = \rho - 360 \left\lfloor \frac{\rho}{360} \right\rfloor, \qquad \Gamma_{M_\rho} = M_\rho - 360 \left\lfloor \frac{M_\rho}{360} \right\rfloor,$$

Equivalent

$$\rho \cong \Gamma_\rho \pmod{360}, \quad M_\rho \cong \Gamma_{M_\rho} \pmod{360}.$$

Notice that Γ, is the residue that leaves a prime number when it is divided by some integer, in this case by 360.

Definition 2 (ρ frequency rotation). Let be $\mathbb{P}$ the set of prime numbers. If $\rho \in \mathbb{P}$, then is said that its frequency of rotation denoted $\mathcal{G}_\rho$, is given by the integer part of ρ when divided by 360.

$$\mathcal{G}_\rho = \left\lfloor \frac{\rho}{360} \right\rfloor.$$

From Definitions 1 y 2, it is true that

$$\forall \rho \in \mathbb{P}, \qquad \rho = \Gamma_\rho + 360(\mathcal{G}_\rho).$$

Definition 3 (Ova-angular function). Let $f_{(\mathrm{mod}\ 360)}: \mathbb{P} \to \mathbb{N}$ be such that if $\rho \in \mathbb{P}$ then $f(\rho) = \Gamma_\rho + 360(\mathcal{G}_\rho)$ and $0 < \Gamma_\rho < 360$.

It is clear that the function f is well defined, in particular f is surjective.

2.1 Complete Classification

Theorem 1 (Γ_ρ Mersenne's Primes). Let ρ be a prime number and $M_\rho = 2^\rho - 1$ *a* Mersenne's prime. If $C_{Mersenne}$ is the set formed by the Ova-angular Γ of the Mersenne primes $2^\rho - 1$. Then:

$$C_{Mersenne} = \{3,7,31,127,247,271\}$$

Proof. Let $M_\rho = 2^\rho - 1$ be a Mersenne's prime, then there exists $\Gamma \in C$, such that $M_\rho = \Gamma + 360(\mathcal{G})$. Next we analyze which of the 99 elements $\Gamma \in C$ are possible to be Mersenne's primes, prior to this the following criteria will be taken into account:

Criterion A. Given the expression $\Gamma + 360(\mathcal{G}) = 2^{\rho} - 1$ it has to $\Gamma + 1 = 2^{\rho} - 360(\mathcal{G})$, which indicates for $\rho > 3$ that $\Gamma + 1$ must be divisible by 2, 4 and 8.

Criterion B. Given the expression $(\Gamma + 1) + 360(\mathcal{G}) = 2^{\rho}$ it is possible in certain cases that $\Gamma + 1$ is a multiple of 3 then it has the expression $3m + 360(\mathcal{G}) = 2^{\rho}$. Taking common factor 3 we have that $3k = 2^{\rho}$, which is not possible since it contradicts the fundamental theorem of arithmetic and also 3 does not divide any power of 2. Consequently, this criterion affirms that $\Gamma + 1$ cannot be a multiple of 3.

Criterion C. Given the expression $(\Gamma + 1) + 360(\mathcal{G}) = 2^{\rho}$ it is possible in certain cases that $\Gamma + 1$ is a multiple of 5 then it has the expression $5n + 360(\mathcal{G}) = 2^{\rho}$. Taking common factor 5 we have that $5r = 2^{\rho}$, which is not possible since $5r$ ends in 5 or 0, and the powers of 2 end in 2, 4, 8, 6, that is; according to the fundamental theorem of arithmetic, 5 does not divide any power of 2. Consequently, this criterion states that $\Gamma + 1$ cannot be a multiple of 5.

Criterion D. Let $\Gamma \in \{103{,}223, 343\}$ be, so for these values in the expression $\Gamma + 1 = 2^{\rho} - 360(\mathcal{G})$ it is conditioned that 2^{ρ} must end in the digit 4. Then $\rho \cong 2 \pmod 4$, this is $\rho = 4m + 2$, then ρ is even which would be contradictory since ρ is odd prime. Consequently, this criterion affirms that $\Gamma \neq 103, 223, 343$.

Criterion E. Let $\Gamma = 151$ be, so for this value in equality $(\Gamma + 1) + 360(\mathcal{G}) = 2^{\rho}$, we have infinite solutions to $\rho = 3(4n - 1)$ with $n \in \mathbb{N}$. Which is contradictory to the fact that ρ must be prime. Consequently, this criterion affirms that $\Gamma \neq 151$.

According to these criteria, we get: From Criterion A, the following are discarded
$\{1, 2, 5, 11, 13, 17, 19, 29, 37, 41{,}43, 49, 53, 59, 61, 67, 73, 77, 83, 89, 91, 97, 101, 107, 109, 113, 121, 131, 133, 137,$
$139, 149{,}157, 161, 163, 167, 169, 173, 179, 181, 187, 191, 193, 197, 203, 209, 211{,}217, 221, 227, 229, 233, 241,$
$251, 253, 257, 259, 263, 269, 277, 281, 288, 289, 293, 299, 301, 307, 313, 317, 323, 329, 331, 337,$
$341, 347, 349, 353\}$.

From *Criterion B,* the following are discarded $\{23, 47, 71, 119, 143, 287, 311\}$.

From *Criterion C* the following are discarded $\{79, 199, 239, 319, 359\}$.

From *Criterion D* the following are discarded $\{103, 223, 343\}$.

From *Criterion E* it is discarded $\{151\}$.

Then the only possible $\Gamma \in C$ for a Mersenne's prime to be formed must be $\Gamma \in \{3, 7, 31, 127, 247, 271\}$. ■

Theorem 2 (Singular Mersenne). The number of Mersenne primes in class Γ_3 and in class Γ_7 equals 1, *i.e.* the only Mersenne's primes with residues 3 and 7 are $2^2 - 1$ and in $2^3 - 1$ respectively.

Proof. For the Γ_3 of the Mersenne primes you get that $3 + 360(k) = 2^{\rho} - 1$ for some k integer. Suppose that there is another prime $M_{\rho} \neq 2^2 - 1$ with the same residue and $\rho > 2$. Then this prime must satisfy for k integer:

$$k = \frac{2^{\rho} - 4}{360} = \frac{2^{(\rho-2)} - 1}{90} \rightarrow k \text{ a pure fractional.}$$

Note. k is a pure fractional since $2^{\rho} - 1$ ends in $1, 3, 5, 7$, so it would never be a multiple of 90.

Then k is an integer and k is a pure fractional →←. Then the only Mersenne's prime in the class Γ_3 is $2^2 - 1 = 3$.

Now, for the Γ_7 class of Mersenne primes, we have to: $7 + 360(k) = 2^{\rho} - 1$. Suppose that there is another prime number $M_{\rho} = 2^3 - 1$ with the same residue and $\rho > 3$. Then this prime must satisfy for k integer:

$$k = \frac{2^{\rho} - 8}{360} = \frac{2^{(\rho-8)} - 1}{45},$$

Since k is an integer then $2^{\rho-8}-1$ it must end in 5 or 0, but since it never ends in 0, then the only option is that it must end in 5, so $2^{\rho-8}$ must end in 6. Then $\rho-8 \cong 0 \pmod 4 \rightarrow \rho \cong 0 \pmod 4$, then ρ is an even prime number →←. Then the only Mersenne's prime in the class Γ_7 is $2^3-1=7$. ■

Figure 1 shows all the residuals of the prime numbers greater than 5 in modulo 360 and shows the isosceles trapeze that is formed by joining Mersenne's residues with an exponent greater than 3. Hereafter this trapeze will be called ***Mersenne's trapeze***.

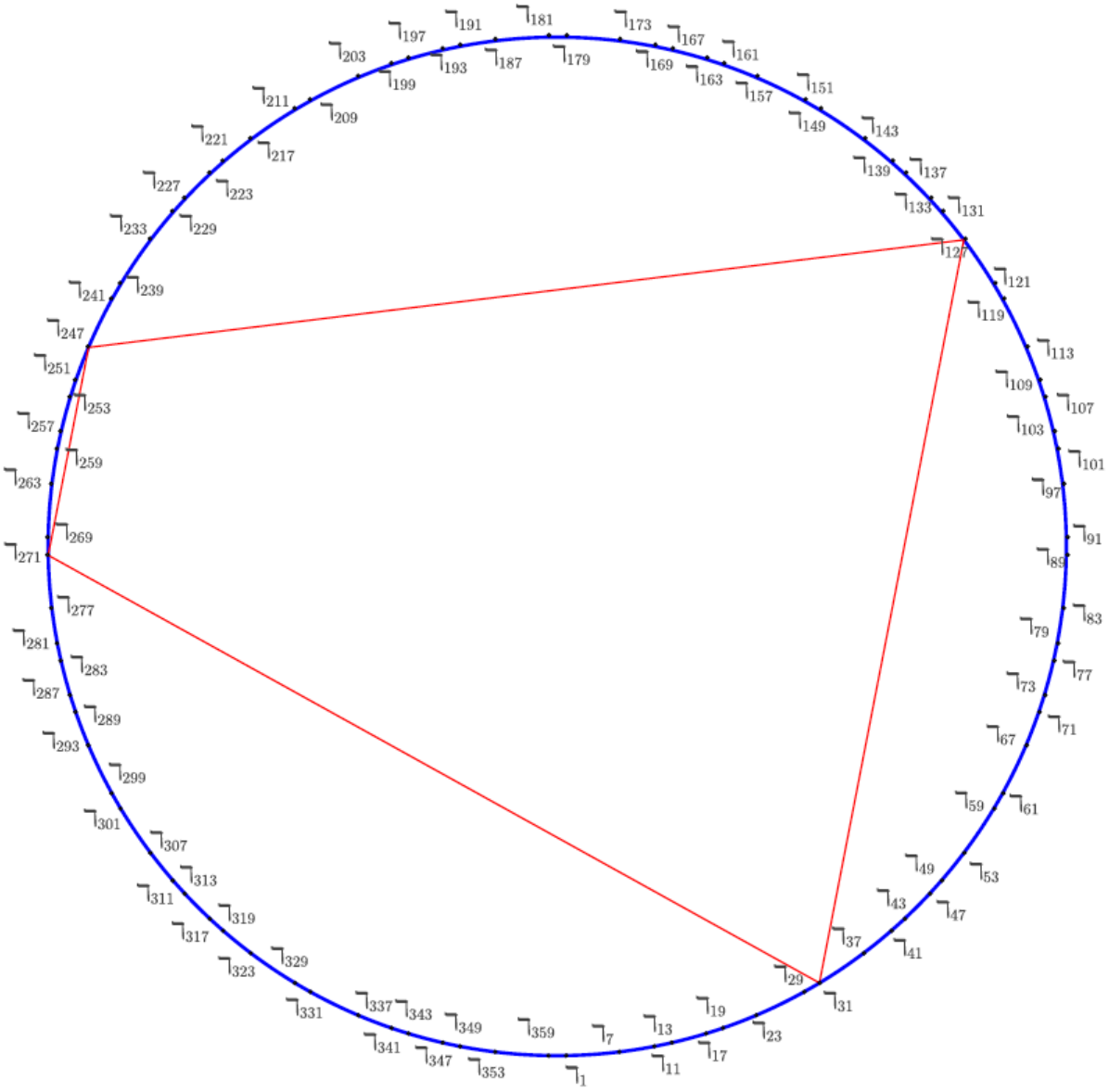

Figure 1. *Mersenne's trapeze*.

Theorem 3. Let $n \in \mathbb{N}$ be, then Mersenne's primes in the class Γ_{31} are in the sequence:

$$31+360K_n = 2^{\rho_n}-1\,,$$

where

$$K_n = \frac{2^{12n-10}-2^2}{3^2 5} \qquad \text{and} \qquad \rho_n = 12n-7 = 12(n-1)+5.$$

Proof. Since $2^{\rho}-1 = 31+360(k)$ then we have that $\rho = \log_2(32+360k)$, now since ρ is a natural number, then it is convenient analyze the values of k for which $\log_2(32+360k)$ intersects with some natural. Let it be

$$k = K_n = \frac{2^{12n-10}-2^2}{3^2 5},$$

then it is clear that

$$\rho_n = \log_2(32 + 360K_n) = \log_2\left(32 + 360\frac{2^{12n-10}-2^2}{3^2 5}\right) = 12n - 7,$$

sequence in which we have the only integer solutions of the required logarithm. ■

Note. It's clear that if $31 + 360K_n$ is prime then it is also a Mersenne's prime and according to Dirichlet's theorem, if there are infinite values of K_n that make prime numbers, then there are infinite Mersenne's primes.

Corollary 1. For the class Γ_{31} of Mersenne's primes, the only Γ_ρ for the prime exponent ρ are: $5, 17, 29, 41, 53, 77, 89, 101, 113, 137, 149, 161, 173, 197, 209, 221, 233, 257, 269, 281, 293, 317, 329, 341, 353$.

Proof. Since $\rho_n = 12n - 7$ then $\Gamma + 360\mathcal{G} = 12n - 7$, also $(\Gamma + 7) = 12(2 - 30\mathcal{G})$. This conditions that $12|\,(k + 7)$ and so the only possible Ova-angular $\Gamma \in C$ for the exponent are: $5, 17, 29, 41, 53, 77, 89, 101, 113, 137, 149, 161, 173, 197, 209, 221, 233, 257, 269, 281, 293, 317, 329, 341, 353$. ■

Theorem 4. *Let* $m \in \mathbb{N}$ be, then Mersenne's primes in the class Γ_{127} are in the sequence:

$$127 + 360K_m = 2^{\rho_m} - 1,$$

where

$$K_m = \frac{2^{12m-8}-2^4}{3^2 5} \quad \text{and} \quad \rho_m = 12m - 5 = 12(m-1) + 7.$$

Proof. Similar to the proof of the previous Theorem (3). ■

Corollary 2. For the class Γ_{127} of Mersenne's primes, the only Γ_ρ for the prime exponent ρ are: $7, 19, 31, 43, 67, 79, 91, 103, 127, 139, 151, 163, 187, 199, 211, 223, 247, 259, 271, 283, 307, 319, 331, 343$.

Proof. Analogous to the previous Corollary (1) proof, only that $12|\,(k + 5)$. ■

Note. It's clear that if $127 + 360K_m$ is prime then it is also a Mersenne's prime, and according to Dirichlet's theorem, if there are infinite values of K_m that make prime numbers, then there are infinite Mersenne's primes.

Theorem 5. Let $r \in \mathbb{N}$ be, the Mersenne's primes in the class Γ_{247} are in the sequence:

$$247 + 360K_r = 2^{\rho_r} - 1,$$

where

$$K_r = \frac{2^{12r-4}-31}{3^2 5} \quad \text{and} \quad \rho_r = 12r - 1 = 12(r-1) + 11.$$

Proof. Similar to the proof of the previous Theorem (3). ■

Corollary 3. For the class Γ_{247} of Mersenne's primes, the only Γ_ρ for the prime exponent ρ are: $11, 23, 47, 59, 71, 83, 107, 119, 131, 143, 167, 179, 191, 203, 227, 239, 251, 263, 287, 299, 311, 323, 347, 359$.

Proof. Analogous to the previous Corollary (1) proof, only that $12|\,(k + 1)$. ■

Note. It's clear that if $247 + 360K_r$ is prime then it is also a Mersenne's prime, and according to Dirichlet's theorem, if there are infinite values of K_r that make prime numbers, then there are infinite Mersenne's primes.

Theorem 6. Let $s \in \mathbb{N}$ be, then Mersenne's primes in the class Γ_{271} are in the sequence:

$$271 + 360K_s = 2^{\rho_s} - 1,$$

where

$$K_s = \frac{2^{12s-14}-34}{3^2 5} \qquad \text{and} \qquad \rho_s = 12s - 11 = 12(s-1) + 11.$$

Proof. Similar to the proof of the previous Theorem (3). ■

Corollary 4. For the class Γ_{271} of Mersenne's primes, the only Γ_ρ for the prime exponent ρ are: $1, 13, 37, 49, 61, 73, 97, 109, 121, 133, 157, 169, 181, 193, 217, 229, 241, 253, 277, 289, 301, 313, 337, 349$.

Proof. Analogous to the previous Corollary (1) proof, only that $12|\ (k\ +\ 11)$. ■

Note. It's clear that if $271 + 360K_s$ is prime then it is also a Mersenne's prime, and according to Dirichlet's theorem, if there are infinite values of K_s that make prime numbers, then there are infinite Mersenne's primes.

All established subgroups Γ for each Mersenne's prime family $C_{Mersenne}$ are disjoint. This result is consistent with the one presented in [6].

The reader is invited to articulate the previous result in a computational way, with the *Lucas-Lehmer test* or use the *Elliptic curve test method* presented in [7], to find a new Mersenne prime $M_\rho > M_{43.112.609}$ searching and testing primes in each of the subgroups presented.

2.2 On residues modulo 360

Theorem 7 (Ova-Ova-Prime-Ova). Let be $\rho \in \mathbb{P}$ a prime number, let be C a completed set of residues of $\mathbb{P}$ mod 360, let be $\rho_1 = \Gamma_{\rho_1} + 360(\mathcal{G}_{\rho_1})$ y $\rho_2 = \Gamma_{\rho_2} + 360(\mathcal{G}_{\rho_2})$; ρ_1, ρ_2 arbitrary prime numbers. Be also, $\Gamma_{\rho_a}, \Gamma_{\rho_b}, \Gamma_{\rho_c}, \Gamma_{\rho_d}, \ldots \in C$ and $\rho_3, \rho_4, \rho_5, \rho_5, \ldots$ prime numbers such that

$$\begin{gathered}
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_a} = \rho_3, \\
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_b} = \rho_4, \\
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_c} = \rho_5, \\
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_d} = \rho_6, \\
\ldots + \cdots + 2 - \cdots = \cdots, \\
\ldots + \cdots + 2 - \cdots = \cdots, \\
\ldots + \cdots + 2 - \cdots = \cdots,
\end{gathered}$$

then, at least one of the following is true

$i)$ $\Gamma_{\rho_a} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_a$ is a prime number.
$ii)$ $\Gamma_{\rho_b} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_b$ is a prime number.
$iii)$ $\Gamma_{\rho_c} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_c$ is a prime number.
$iv)$ $\Gamma_{\rho_d} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_d$ is a prime number.
...) ... + ... $= \cdots$ …
...) ... + ... $= \cdots$ …
...) ... + ... $= \cdots$ is a prime number.

Proof. (*by reduction to absurdity*). Let be $\rho \in \mathbb{P}$ a prime number, let be C a completed set of residues of $\mathbb{P}$ mod 360, let be $\rho_1 = \Gamma_{\rho_1} + 360(\mathcal{G}_{\rho_1})$ y $\rho_2 = \Gamma_{\rho_2} + 360(\mathcal{G}_{\rho_2})$; ρ_1, ρ_2 arbitrary prime numbers. Be also, $\Gamma_{\rho_a}, \Gamma_{\rho_b}, \Gamma_{\rho_c}, \Gamma_{\rho_d}, \ldots \in C$ and $\rho_3, \rho_4, \rho_5, \rho_5, \ldots$ prime numbers such that

$$\begin{gathered}
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_a} = \rho_3, \\
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_b} = \rho_4, \\
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_c} = \rho_5, \\
\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_{\rho_d} = \rho_6, \\
\ldots + \cdots + \cdots - \cdots = \cdots, \\
\ldots + \cdots + \cdots - \cdots = \cdots,
\end{gathered}$$

it is clear that $\rho_3, \rho_4, \rho_5, \ldots \in w \subset \mathbb{P}$, where w is a finite set. Suppose that none of the following is true (*Denial of thesis*):

$i)$ $\Gamma_{\rho_a} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_a$ is a prime number.
$ii)$ $\Gamma_{\rho_b} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_b$ is a prime number.
$iii)$ $\Gamma_{\rho_c} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_c$ is a prime number.
$iv)$ $\Gamma_{\rho_d} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_d$ is a prime number.
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ $\ldots$
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ $\ldots$
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ is a prime number.

This is

$i)$ $\Gamma_{\rho_a} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_a$ it is not a prime number.
$ii)$ $\Gamma_{\rho_b} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_b$ it is not a prime number.
$iii)$ $\Gamma_{\rho_c} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_c$ it is not a prime number. (1)
$iv)$ $\Gamma_{\rho_d} + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_d$ it is not a prime number.
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ $\ldots$
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ $\ldots$
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ it is not a prime number.

As $\rho_1 = \Gamma_{\rho_1} + 360\left(\mathcal{G}_{\rho_1}\right)$ and $\rho_2 = \Gamma_{\rho_2} + 360\left(\mathcal{G}_{\rho_2}\right)$, then

$$\Gamma_{\rho_1} = \rho_1 - 360\left(\mathcal{G}_{\rho_1}\right) \quad \text{and} \quad \Gamma_{\rho_2} = \rho_2 - 360\left(\mathcal{G}_{\rho_2}\right), \qquad (2)$$

with ρ_1, ρ_2 arbitrary prime numbers.

On the other hand, it is true that:

$$\begin{aligned}
&\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_3 = \Gamma_{\rho_a},\\
&\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_4 = \Gamma_{\rho_b},\\
&\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_5 = \Gamma_{\rho_c}, \qquad (3)\\
&\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_{6,} = \Gamma_{\rho_d}\\
&\ldots + \cdots + \cdots - \cdots = \cdots,\\
&\ldots + \cdots + \cdots - \cdots = \cdots,\\
&\ldots + \cdots + \cdots - \cdots = \cdots.
\end{aligned}$$

Substituting (3) in (1) we have that

$i)$ $\left(\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_3\right) + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_a$ it is not a prime number.
$ii)$ $\left(\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_4\right) + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_b$ it is not a prime number.
$iii)$ $\left(\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_5\right) + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_c$ it is not a prime number. (4)
$iv)$ $\left(\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \rho_6\right) + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho_d$ it is not a prime number.
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ $\ldots$
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ $\ldots$
$\ldots)$ $\ldots +$ $\ldots$ $= \cdots$ it is not a prime number.

where $2, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \ldots$ are prime numbers and $\rho_3, \rho_4, \rho_5, \ldots \in w$.

So, substituting (2) in (4) and after associating and canceling some terms, we have to

$i)$ $\rho_1 + \rho_2 + 2 - \rho_3 = \rho_a$ it is not a prime number.
$ii)$ $\rho_1 + \rho_2 + 2 - \rho_4 = \rho_b$ it is not a prime number.
$iii)$ $\rho_1 + \rho_2 + 2 - \rho_5 = \rho_c$ it is not a prime number. (5)
$iv)$ $\rho_1 + \rho_2 + 2 - \rho_6 = \rho_d$ it is not a prime number.
...) $\ldots + \cdots + \cdots - \cdots = \cdots$ it is not a prime number.
...) $\ldots + \cdots + \cdots - \cdots = \cdots$ it is not a prime number.
...) $\ldots + \cdots + \cdots - \cdots = \cdots$ it is not a prime number.

$\Longrightarrow\Longleftarrow (Contradiction)$**.**

The expression (5), would be equivalent to affirming that for every combination ρ_1, ρ_2 of arbitrary prime numbers, there is no prime that is the result of $\rho_1 + \rho_2 + 2$ minus another prime number, which is contradictory
with the theorem proven by Harald Helfgott in [8], for example, there are the contradictions: $5 + 7 + 2 - 11 = 3, 127 + 47 + 2 - 37 = 139, 1564643 + 293 + 2 - 971 = 1563967, \ldots$ Thus, it is true that since ρ_1, ρ_2 are arbitrary prime numbers, in one or some cases the primary arithmetic sum $\rho_1 + \rho_2 + 2 - \rho_j = \rho_k$ will also be a prime number.

Thus, we arrive at this contradiction for having denied the thesis, then the assumption initial is false. ■

Example Theorem 7 (Ova-Ova-Prime-Ova)

-- Let be $\rho_1 = 15486059 = 299 + 360(43016)$, $\rho_2 = 32452451 = 251 + 360(90145)$both prime numbers, where $\mathcal{G}_{\rho_1} = 43016$, $\mathcal{G}_{\rho_2} = 90145$, $\Gamma_{\rho_1} = 299$ and $\Gamma_{\rho_2} = 251$, for the latter there are $\Gamma_\rho \in \{5, 11, 31, 43, 49, 53, 61, 73, 89, 91, 103, 109, 113, 119, 121, 131, 133, 143, 151, 163, 169, 173, 179, 193, 203, 221, 239, 241, 259, 269, 271, 281, 283, 289, 301, 311, 313, 319, 323, 329, 341, 353, 359\}$ such that $\Gamma_{\rho_1} + \Gamma_{\rho_2} + 2 - \Gamma_\rho = \rho_n$ is a prime number.

Then by Theorem (7), it happens that at least for one of the Γ_ρ that $\Gamma_\rho + 360\left(\mathcal{G}_{\rho_1} + \mathcal{G}_{\rho_2}\right) = \rho$ is a prime number. Indeed, it happens that:

$103 + 360(43016 + 90145) = 47938063$ is a prime number.
$163 + 360(43016 + 90145) = 47938123$ is a prime number.
$173 + 360(43016 + 90145) = 47938133$ is a prime number.
$179 + 360(43016 + 90145) = 47938139$ is a prime number.
$283 + 360(43016 + 90145) = 47938243$ is a prime number.
$341 + 360(43016 + 90145) = 47938301$ is a prime number.
$353 + 360(43016 + 90145) = 47938313$ is a prime number.

Theorem (7) establishes that at least one occurs, in this case as it was chosen at random, seven numbers have turned out, it is fully fulfilled.

Conjecture 1 (*The number of primes in the Ova-Ova-Prime-Ova combination*). Although the number of prime numbers resulting from theorem (7) is at least one, at most, it should be 13.

The reader is invited to verify or solve the previous conjecture in a computational way.

3. CONCLUSIONS

A complete classification of the Mersenne's primes was presented through the multiplicative group modulo 360. A geometric representation of the residual classes generating these numbers was obtained. It is possible to continue this work analyzing other applications that this theory presents using the geometric properties for different prime numbers, also is possible articulated through computational methods this theory with different primality tests for these numbers finding a new Mersenne's primes (Lucas-Lehmer test or the Elliptic curve

test method). The conjecture about primes in this residual classes was established and the reader is invited to verify or solve the proposed conjecture in a computational way.

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