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A study on integral transforms of the generalized Lommel-Wright function

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Original scientific papers

A study on integral transforms of the generalized Lommel-Wright function

Исследование интегральных преобразований обобщенных функций Ломмеля-Райта

Студија о интегралним трансформацијама генерализоване функције Ломела и Рајта

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ABSTRACT:

Introduction/purpose: The aim of this article is to establish integral transforms of the generalized Lommel-Wright function.
Methods: These transforms are expressed in terms of the Wright Hypergeometric function
Results: Integrals involving the trigonometric, generalized Bessel function and the Struve functions are obtained.
Conclusions: Various interesting transforms as the consequence of this method are obtained.

KEYWORDS: Generalized Lommel-Wright functions J(z), Hankel transform, K-transform, Wright function, Whittaker function.

Резюме:

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Введение/цель: Целью данной статьи является установление интегральных преобразований обобщенной функции Ломмеля-Райта.
Методы: Эти преобразования выражаются в терминах гипергеометрической функции Райта.
Результаты: В результате получены интегралы с тригонометрическими, обобщенными функциями Бесселя и Струве.
Выводы: Вследствие применения данного метода получаются различные интересные преобразования.
Ключевые слова: обобщенные функции Ломмеля-Райта J(z), преобразование Ханкеля, K-преобразование, функция Райта, функция Уиттекера.

Abstract:

Увод/циљ: Циљ овог рада јесте успостављање интегралних трансформација генерализоване функције Ломела и Рајта.
Методе: Интегралне трансформације изражене су помоћу Рајтове хипергеометријске функције.
Резултати: Добијени су интеграли који укључују тригонометријске, генерализоване Беселове и Струвеове функције.
Закључак: Као последице ове методе добијају се разне занимљиве трансформације.
Keywords: генерализоване функције Ломела и Рајта J(z), Ханкелова трансформација, К-трансформација, Рајтова функција, Витакерова функција.

Introduction

The transform defined by the following integral equation

$$R_\nu\{f(x);p\} = g(p,\nu) = \int_0^{+\infty} (px)^{1/2} K_\nu(px) f(x)dx \tag{1}$$

is called the k transform with p as a complex parameter and $K\nu(px)$ is called the Modified Bessel function of the third kind or the Macdonald function, see (Mathai et al, 2010, p.53). The Hankel transform of a function f(x), denoted by g(p, ν) is defined as

$$g(p,\nu) = \int_0^{+\infty} (px)^{1/2} J_\nu(px) f(x)dx, \quad p > 0 \tag{2}$$

where $J\nu(px)$ is called the Bessel-Maitland function or the Maitland-Bessel function (Mathai et al, 2010, p.22 and p.56).

The Wright hypergeometric function defined by the series (Srivastava & Manocha, 1984):

$${}_p\psi_q\left[\begin{array}{ll}(\alpha_1, A_1), \,..,(\alpha_p, A_p); & \\ & z\\ (\beta_1, B_1)..., (\beta_q, B_q) & \end{array}\right] = \sum_{k=0}^{+\infty} \frac{\prod_{j=1}^{p}\Gamma(\alpha_j + A_j k) z^k}{\prod_{j=1}^{q}\Gamma(\beta_j + B_j k) k!}, \tag{3}$$

where the coefficients A_1, ...A_p and B_1, ...B_q are positive real numbers such that

$$1+\sum_{j=1}^{q} B_j-\sum_{j=1}^{p} A_j \geq 0, \tag{4}$$

can be slightly generalized (3) as given below.

$${}_p\psi_q\left[\begin{array}{cc}(\alpha_1,1),..,(\alpha_p,1); & \\ & z\\ (\beta_1,1)...,(\beta_q,1) & \end{array}\right]=\frac{\prod_{j=1}^{p}\Gamma(\alpha_j)}{\prod_{j=1}^{q}\Gamma(\beta_j)}{}_pF_q\left[\begin{array}{cc}\alpha_1,..,\alpha_p; & \\ & z\\ \beta_1,...,\beta_q & \end{array}\right], \tag{5}$$

where pFq is the generalized hypergeometric function defined by (Srivastava & Manocha, 1984; Rainville, 1960)

$${}_pF_q\left[\begin{array}{cc}\alpha_1,..,\alpha_p; & \\ & z\\ \beta_1,...,\beta_q & \end{array}\right]=\sum_{k=0}^{+\infty}\frac{(\alpha_1)_n,....,(\alpha_p)_n z^n}{(\beta_1)_n,....,(\beta_q)_n n!}= \\ {}_pF_q(\alpha_1,...,\alpha_p;\beta_1,....,\beta_q;z), \tag{6}$$

where (λ)n is the well known Pochhammer symbol (Srivastava & Manocha, 1984).

The series representation of the generalized Lommel Wright function as (Kachhia & Prajapati, 2016);

$$J_{\nu,\lambda}^{\mu,m}(z)=\sum_{k=0}^{+\infty}\frac{(-1)^k\Gamma(k+1)(\frac{z}{2})^{2k+\nu+2\lambda}}{\Gamma(\lambda+k+1)^m\Gamma(\nu+k\mu+\lambda+1)k!}, \\ (z\in\mathbb{C}/(-\infty,0],\ m\in\mathbb{N},\ \nu,\lambda\in\mathbb{C},\mu>0). \tag{7}$$

Also, we have the following relations of the generalized Lommel Wright functions with trigonometric functions and the generalized Bessel function $J_{\nu,\lambda}^{\mu}(z)$ and the Struve function as follows:

$$J_{1/2,0}^{1,1}(z)=\sqrt{\frac{2}{\pi z}}\sin(z) \tag{8}$$

$$J^{1,1}_{-1/2,0}(z) = \sqrt{\frac{2}{\pi z}}\cos(z) \tag{9}$$

$$J^{\mu,1}_{\nu,\lambda}(z) = \mathbb{J}^{\mu}_{\nu,\lambda}(z) \tag{10}$$

$$J^{1,1}_{\nu,1/2}(z) = H_\nu(z). \tag{11}$$

The following known results of Mathai and Saxena (Mathai & Saxena, 1973):

$$\int_0^{+\infty} x^{\delta-1} J_\eta(ax)dx = \frac{2^{\delta-1}a^{-\delta}\Gamma(\frac{\delta+\eta}{2})}{\Gamma(1+\frac{\eta-\delta}{2})}, \quad \Re(\eta) < \Re(\delta) < 3/2, \;\; a > 0 \tag{12}$$

$$\int_0^{+\infty} x^{\delta-1} K_\eta(ax)dx = 2^{\delta-2}a^{-\delta}\Gamma(\delta\pm\eta)/2, \tag{13}$$

$$\int_0^{+\infty} x^{\delta-1}\exp(-at)K_\eta(ax)dx = \frac{\Gamma(\delta\pm\eta)/2}{(2a)^{\delta-1}\Gamma(\delta+1/2)}, \tag{14}$$

$$\int_0^{+\infty} x^{\delta-1}\exp(1/2\;x)W_{(\eta,\alpha)}(x)dx = \frac{\Gamma(1/2\pm\alpha+\delta)\Gamma(-\eta-\delta)}{\Gamma(1/2\pm\alpha-\eta)}, \tag{15}$$

$$\int_0^{+\infty} x^{\delta-1}\exp(-1/2\;x)M_{(\eta,m)}(x)dx = \frac{\Gamma(2m+1)\Gamma(m+\delta+1/2)\Gamma(\eta-\delta)}{\Gamma(m-\delta+1/2)\Gamma(m+\eta+1/2)}, \tag{16}$$

$$\int_0^{+\infty} x^{\delta-1} W_{(\eta,\alpha)}(x) W_{(-\eta,\alpha)}(x) dx = \frac{\Gamma((\delta+1)/2 \pm \alpha)\Gamma(\delta+1)}{2\Gamma(1+\delta/2 \pm \eta)}. \tag{17}$$

Various generalizations and cases of the Lommel-Wright function have been investigated. For details, see (Paneva-Konovska, 2007; Menaria et al, 2016; Mondal & Nisar, 2017; Srivastava & Daoust, 1969; Kiryakova, 2000).

Integral formulas involving the Lommel-Wright functions have been developed by many authors. See e.g., (Choi & Agarwal, 2013; Choi et al, 2014; Jain et al, 2016; Chaurasia & Pandey, 2010). In this sequel, here, we aim at establishing a certain new generalized integral formula involving the generalized Lommel-Wright function $J_{\nu,\lambda}^{\mu,m}(z)$ interesting integral formulas which are derived as special cases.

Main results

This section deals with the evaluation of integrals formulas involving the Lommel-Wright function defined in (7) and the integrals involving the product of the Bessel function of first kind, Kelvin's function and the Whittaker function (Whittaker & Watson, 2013) with the generalized Lommel-Wright function.

THEOREM 1. *Let* $z \in \mathbb{C}/(-\infty, 0]$, $m \in \mathbb{N}$, $\nu, \lambda \in \mathbb{C}, \mu > 0$. *Then the Hankel transform of the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1} J_\eta(az) J_{\nu,\lambda}^{\mu,m}(b\, z^w) dz = \frac{1}{2}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)} \times$$

$$ {}_2\psi_{m+2}\left[\begin{array}{c} (1,1), (\frac{\eta+\rho+w\nu+2w\lambda}{2}, w); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu), ((\frac{2+\eta-(\rho+w\nu+2w\lambda}{2}), -w); \\ \left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w \end{array}\right]. \tag{18}$$

Proof. On using (7) in the integrand of (1) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1} J_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w) dz =$$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n \Gamma(n+1)(b/2)^{2n+\nu+2\lambda}}{\Gamma(\lambda+n+1)^m \Gamma(\nu+\lambda+n\mu+1) n!} \int_0^{+\infty} z^{\rho+w(2n+\nu+2\lambda)-1} J_\eta(az) dz.$$

Now using (12) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1} J_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w)dz = \Big(1/2\Big)\Big(b/2\Big)^{\nu+2\lambda}\Big(2/a\Big)^{\rho+w(\nu+2\lambda)} \times$$
$$\sum_{n=0}^{+\infty} \frac{\Gamma(n+1)\Gamma(\eta+\rho+w\nu+2w\lambda+2wn)/2\Big(-b^2/4\Big)^n\Big(4/a^2\Big)^{wn}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)\Gamma(2+\eta-\rho-w\nu-2w\lambda-2wn)/2\, n!}$$
$$= \frac{1}{2}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)} \times$$
$${}_2\psi_{m+2}\left[\begin{matrix} (1,1), \left(\frac{\eta+\rho+w\nu+2w\lambda}{2}, w\right); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu), \left(\left(\frac{2+\eta-(\rho+w\nu+2w\lambda)}{2}\right), -w\right); \end{matrix} \left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{19}$$

THEOREM 2. *Let* $z \in \mathbb{C}/(-\infty,0],\ m \in \mathbb{N},\ \nu,\lambda \in \mathbb{C}, \mu > 0.$ *Then the K-Transform of the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1} K_\eta(az) J_{\nu,\lambda}^{\mu,m}(b\, z^w)dz = \frac{1}{4}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)} \times$$
$${}_2\psi_{m+1}\left[\begin{matrix} (1,1), (\frac{\rho+w\nu+2w\lambda\pm\eta}{2}, w); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu); \end{matrix} \left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{20}$$

Proof. On using (7) in the integrand of (2) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1} K_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w)dz =$$
$$\sum_{n=0}^{+\infty} \frac{(-1)^n\Gamma(n+1)(b/2)^{2n+\nu+2\lambda}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!} \int_0^{+\infty} z^{\rho+w(2n+\nu+2\lambda)-1} K_\eta(az)dz.$$

Now using (13) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1} K_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w)dz = \Big(1/4\Big)\Big(b/2\Big)^{\nu+2\lambda}\Big(2/a\Big)^{\rho+w(\nu+2\lambda)} \times$$
$$\sum_{n=0}^{+\infty} \frac{\Gamma(n+1)\Gamma(\rho+w\nu+2w\lambda\pm\eta+2wn)/2\Big(-b^2/4\Big)^n\Big(4/a^2\Big)^{nw}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}$$
$$= \frac{1}{4}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)}$$
$${}_2\psi_{m+1}\left[\begin{matrix} (1,1), (\frac{\rho+w\nu+2w\lambda\pm\eta}{2}, w); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu); \end{matrix} \left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{21}$$

THEOREM 3. Let $z \in \mathbb{C}/(-\infty,0]$, $m \in \mathbb{N}$, $\nu,\lambda \in \mathbb{C}, \mu > 0$. *Then the K-Transform of the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az)K_\eta(az)J_{\nu,\lambda}^{\mu,m}(b\,z^w)dz = \frac{2a\sqrt{\pi}\left(b/2\right)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\times$$
$${}_2\psi_{m+2}\left[\begin{array}{c}(1,1),(\rho+w\nu+2w\lambda\pm\eta,2w);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\rho+w\nu+2w\lambda+1/2,2w);\end{array}\right. \qquad [22a]$$

$$\left.\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \qquad [22b]$$

Proof. On using (7) in the integrand of (3) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az)K_\eta(az)J_{\nu,\lambda}^{\mu,m}(bz^w)dz =$$
$$\sum_{n=0}^{+\infty}\frac{(-1)^n\Gamma(n+1)(b/2)^{2n+\nu+2\lambda}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\times$$
$$\int_0^{+\infty} z^{\rho+w(2n+\nu+2\lambda)-1}\exp(-az)K_\eta(az)dz. \qquad (23)$$

Now using (14) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az)K_\eta(az)J_{\nu,\lambda}^{\mu,m}(bz^w)dz = \frac{2a\sqrt{\pi}\left(b/2\right)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\times$$
$$\sum_{n=0}^{+\infty}\frac{\Gamma(n+1)\Gamma(\rho+w\nu+2w\lambda\pm\eta+2wn)(\frac{-b^2}{4(4a)^w})^n}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)(\rho+w\nu+2w\lambda+1/2,2w)n!}$$
$$=\frac{2a\sqrt{\pi}(b/2)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\times$$
$${}_2\psi_{m+2}\left[\begin{array}{c}(1,1),(\rho+w\nu+2w\lambda\pm\eta,2w);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\rho+w\nu+2w\lambda+1/2,2w);\\\left(\frac{-b^2}{4(4a^2)^w}\right)\end{array}\right]. \qquad (24)$$

THEOREM 4. Let $z \in \mathbb{C}/(-\infty,0]$, $m \in \mathbb{N}$, $\nu,\lambda \in \mathbb{C}, \mu > 0$. *Then the product of the Whittaker function and the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(w\,z^{\theta})dz = \frac{\left(w/2\right)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}\Gamma(1/2\pm\alpha-\eta)}$$
$$_3\psi_{m+1}\left[\begin{array}{c}(1,1),(1/2\pm\alpha+\rho+\nu\theta+2\lambda\theta,2\theta),(-\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\ (\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu);\end{array}\right. \tag{25a}$$

$$\left.\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{25b}$$

Proof. Putting az = x, adz = dx as z → 0, x → 0 and z → +∞, x → +∞ and using (7) in the integrand of (4) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(wz^{\theta})dz =$$
$$\frac{\left(w/2\right)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}}\sum_{n=0}^{+\infty}\frac{\Gamma(n+1)\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\times$$
$$\int_0^{+\infty} x^{\rho+\theta(2n+\nu+2\lambda)-1}\exp(x/2)W_{\eta,\alpha}(x)dx.$$

Now using (15) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(wz^{\theta})dz = \frac{(w/2)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}\Gamma(1/2\pm\alpha-\eta)}\times$$
$$\sum_{n=0}^{+\infty}\left[\frac{\Gamma(n+1)\Gamma(1/2\pm\alpha+\rho+\theta\nu+2\theta\lambda+2\theta\,n)\Gamma(-\eta-\rho-\theta\nu-2\theta\lambda-2\theta\,n)}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\right.$$
$$\left.\times\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n\right] = \frac{\left(w/2\right)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}\Gamma(1/2\pm\alpha-\eta)}\times$$
$$_3\psi_{m+1}\left[\begin{array}{c}(1,1),(1/2\pm\alpha+\rho+\theta\nu+2\theta\lambda,2\theta),(-\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\ (\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu);\\ \left.\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right].\end{array}\right. \tag{26}$$

THEOREM 5. *Let* $z \in \mathbb{C}/(-\infty,0]$, $m \in \mathbb{N}$, $\nu,\lambda \in \mathbb{C}, \mu > 0$. *Then the product of the Whittaker function and the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J^{\mu,m}_{\nu,\lambda}(w\ z^{\theta})dz = \tag{27a}$$

$$\frac{\left(w/2\right)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^{\rho}\Gamma(\alpha+\eta+1/2)}\times$$
$$ {}_3\psi_{m+2}\left[\begin{array}{c}(1,1),(\alpha+\rho+1/2+\nu\theta+2\lambda\theta,2\theta),(\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\alpha-\rho-\theta\nu-2\theta\lambda+1/2,-2\theta)\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{27b}$$

Proof. Putting az = x, adz = dx as z → 0, x → 0 and z → +∞, x → +∞ and using (7) in the integrand of (5) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J^{\mu,m}_{\nu,\lambda}(wz^{\theta})dz = \frac{\left(w/2\right)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}}\times$$
$$\sum_{n=0}^{+\infty}\frac{\Gamma(n+1)\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\times$$
$$\int_0^{+\infty} x^{\rho+\theta(2n+\nu+2\lambda)-1}\exp(-x/2)M_{\eta,\alpha}(x)dx.$$

Now using (16) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J^{\mu,m}_{\nu,\lambda}(wz^{\theta})dz =$$
$$\frac{(w/2)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^{\rho}\Gamma(\alpha+\eta+1/2)}\times$$
$$\sum_{n=0}^{+\infty}\left[\frac{\Gamma(n+1)\Gamma(\alpha+\rho+\theta\nu+2\theta\lambda+1/2+2\theta\ n)\Gamma(\eta-\rho-\theta\nu-2\theta\lambda-2\theta\ n)}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)\Gamma(\alpha-\rho-\theta\nu-2\theta\lambda-2n\theta+1/2)n!}\times\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n\right] = \frac{\left(w/2\right)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^{\rho}\Gamma(\alpha+\eta+1/2)}\times$$
$$ {}_3\psi_{m+2}\left[\begin{array}{c}(1,1),(\alpha+\rho+1/2+\nu\theta+2\lambda\theta,2\theta),(\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\alpha-\rho-\theta\nu-2\theta\lambda+1/2,-2\theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{28}$$

THEOREM 6. *Let* $z\in\mathbb{C}/(-\infty,0],\ m\in\mathbb{N},\ \nu,\lambda\in\mathbb{C},\mu>0.$ *Then the product of the Whittaker function and the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1} W_{\eta,\alpha}(az) W_{-\eta,\alpha}(az) J_{\nu,\lambda}^{\mu,m}(w\, z^{\theta}) dz = \frac{\left(w/2\right)^{\nu+2\lambda} (1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}}$$
$$_3\psi_{m+2}\left[\begin{array}{c} (1,1), (\frac{\rho+\theta(\nu+2\lambda)+1}{2} \pm \alpha, \theta), (\rho+\theta(\nu+2\lambda)+1, 2\theta); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu), 2(1+\frac{\rho+\theta(\nu+2\lambda)}{2} \pm \eta, \theta); \end{array} \left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{29}$$

Proof. Putting az = x, adz = dx as z → 0, x → 0 and z → +∞, x → +∞ and using (7) in the integrand of (6) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1} W_{-\eta,\alpha}(az) W_{\eta,\alpha}(az) J_{\nu,\lambda}^{\mu,m}(wz^{\theta}) dz = \frac{\left(w/2\right)^{\nu+2\lambda} (1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}} \times$$
$$\sum_{n=0}^{+\infty} \frac{\Gamma(n+1)\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n}{\Gamma(\lambda+n+1)^m \Gamma(\nu+\lambda+n\mu+1) n!} \times$$
$$\int_0^{+\infty} x^{\rho+\theta(2n+\nu+2\lambda)-1} W_{-\eta,\alpha}(x) W_{\eta,\alpha}(x) dx.$$

Now using (17) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1} W_{-\eta,\alpha}(az) W_{\eta,\alpha}(az) J_{\nu,\lambda}^{\mu,m}(wz^{\theta}) dz = \frac{(w/2)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}} \times$$
$$\sum_{n=0}^{+\infty} \frac{\Gamma(n+1)\Gamma(\frac{\rho+\theta(\nu+2\lambda+2n+1)}{2} \pm \alpha)\Gamma(\rho+\theta\nu+2\theta\lambda+2\theta\, n)(\frac{-w^2}{4(a^2)^{\theta}})^n}{\Gamma(\lambda+n+1)^m \Gamma(\nu+\lambda+n\mu+1) 2\Gamma(1+\frac{\rho+\theta(\nu+2\lambda+2n)}{2} \pm \eta) n!}$$
$$= \frac{\left(w/2\right)^{\nu+2\lambda} (1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}} \times \tag{30a}$$

$$_3\psi_{m+2}\left[\begin{array}{c} (1,1), (\frac{\rho+\theta(\nu+2\lambda)+1}{2} \pm \alpha, \theta), (\rho+\theta(\nu+2\lambda)+1, 2\theta); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu), 2(1+\frac{\rho+\theta(\nu+2\lambda)}{2} \pm \eta, \theta); \end{array} \left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{30b}$$

SPECIAL CASES

In this section, we get some integral formulas involving a trigonometric function and the generalized Lommel-Wright function as follows:

COROLLARY 1. *If we take m = 1, μ = 1, λ = 0 and ν = 1/2 in (1) and then by using (8), we derive the following integral formula:*

$$\int_0^{+\infty} z^{\rho-w/2-1} J_\eta(az)\sin(b\,z^w)dz = \left(b/4\right)\sqrt{\pi}\left(\frac{2}{a}\right)^{\rho+w/2} \times$$
$${}_1\psi_2\left[\begin{array}{c}(\frac{\eta+\rho+w/2}{2}, w);\\ (3/2,1),((\frac{2+\eta-(\rho+w/2)}{2}),-w);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{31}$$

COROLLARY 2. *If we take m = 1, μ = 1, λ = 0 and ν = 1/2 in (2) and then by using (8), we obtain:*

$$\int_0^{+\infty} z^{\rho-w/2-1} K_\eta(az)\sin(b\,z^w)dz = \left(b/8\right)\sqrt{\pi}\left(\frac{2}{a}\right)^{\rho+w/2} \times$$
$${}_1\psi_1\left[\begin{array}{c}(\frac{\rho+w/2\pm\eta}{2}, w);\\ (3/2,1);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{32}$$

COROLLARY 3. *If we take m = 1, μ = 1, λ = 0 and ν = 1/2 in (3) and then by using (8), we obtain:*

$$\int_0^{+\infty} z^{\rho-w/2-1} \exp(-az)K_\eta(az)\sin(b\,z^w)dz = \left(\frac{\pi\; ab}{(2a)^{\rho+w/2}}\right) \times$$
$${}_1\psi_2\left[\begin{array}{c}(\rho+w/2\pm\eta, 2w);\\ (3/2,1),(\rho+w/2+1/2,2w);\end{array}\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \tag{33}$$

COROLLARY 4. *If we take m = 1, μ = 1, λ = 0 and ν = 1/2 in (4) and then by using (8), we obtain*:

$$\int_0^{+\infty} z^{\rho-\theta/2-1} \exp(az/2)W_{\eta,\alpha}(az)\sin(w\,z^\theta)dz =$$
$$\left(\frac{w/2\sqrt{\pi}}{(a)^{\rho+w/2}(\Gamma(1/2\pm\alpha-\eta))}\right)\times$$
$${}_2\psi_1\left[\begin{array}{c}(1/2\pm\alpha+\rho+\theta/2,2\theta),(\eta-\rho-\theta/2,-2\theta);\\ (3/2,1);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{34}$$

COROLLARY 5. *If we take m = 1, μ = 1, λ = 0 and ν = 1/2 in (5) and then by using (8), we obtain*:

$$\int_0^{+\infty} z^{\rho-\theta/2-1} \exp(-az/2) M_{\eta,\alpha}(az) \sin(w\ z^{\theta}) dz =$$
$$\left(\frac{w/2\sqrt{\pi}(1/a)^{\theta/2}\Gamma(2\alpha+1)}{(a)^{\rho}(\Gamma(\alpha+\eta+1/2))}\right)\times$$
$${}_2\psi_2\left[\begin{array}{c}(\alpha+\rho+\theta/2, 2\theta), (\eta-\rho-\theta/2, -2\theta);\\ (3/2, 1), (\alpha-\theta+1/2, -2\theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{35}$$

COROLLARY 6. *If we take $m = 1$, $\mu = 1$, $\lambda = 0$ and $\nu = 1/2$ in (6) and then by using (8), we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1} W_{\eta,\alpha}(az) W_{-\eta,\alpha}(az) \sin(w\ z^{\theta}) dz = \left(\frac{w/2\sqrt{\pi}(1/a)^{\theta/2}}{(a)^{\rho}}\right)$$
$${}_2\psi_2\left[\begin{array}{c}(\frac{\rho+\theta/2+1}{2}\pm\alpha, \theta), (\rho+\theta/2+1, 2\theta); \times\\ (3/2, 1), 2(1+\frac{\rho+\theta/2}{2}\pm\eta, \theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{36}$$

COROLLARY 7. *If we take $m = 1$, $\mu = 1$, $\lambda = 0$ and $\nu = -1/2$ in (1) and then by using (9), we derive the following integral formula*:

$$\int_0^{+\infty} z^{\rho-w/2-1} J_{\eta}(az) \cos(z) dz = \sqrt{\left(\frac{\pi}{4}\right)}\left(2/a\right)^{\rho-w/2}\times$$
$${}_1\psi_2\left[\begin{array}{c}(\frac{\eta+\rho-w/2}{2}, w);\\ (1/2, 1), ((\frac{2+\eta-(\rho-w/2)}{2}), -w);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right]. \tag{37}$$

COROLLARY 8. *If we take $m = 1$, $\mu = 1$, $\lambda = 0$ and $\nu = -1/2$ in (2) and then by using (9), we obtain:*

$$\int_0^{+\infty} z^{\rho-w/2-1} K_{\eta}(az) \cos(z) dz = 1/4\sqrt{\pi}\left(\frac{2}{a}\right)^{\rho-w/2}\times$$
$${}_1\psi_1\left[\begin{array}{c}(\frac{\rho-w/2\pm\eta}{2}, w);\\ (1/2, 1);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right]. \tag{38}$$

COROLLARY 9. *If we take $m = 1$, $\mu = 1$, $\lambda = 0$ and $\nu = -1/2$ in (3) and then by using (9), we obtain:*

$$\int_0^{+\infty} z^{\rho-w/2-1}\exp(-az)K_\eta(az)\cos(b\, z^w)dz = \left(\frac{2a\pi}{(2a)^{\rho-w/2}}\right)\times$$
$${}_1\psi_2\left[\begin{array}{c}(\rho-w/2\pm\eta,2w);\\(1/2,1),(\rho-w/2+1/2,2w);\end{array}\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \tag{39}$$

COROLLARY 10. *If we take m = 1, μ = 1, λ = 0 and ν = −1/2 in (4) and then by using (9), we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1}\exp(az/2)W_{\eta,\alpha}(az)\cos(w\, z^\theta)dz =$$
$$\left(\frac{\sqrt{\pi}}{(a)^{\rho-\theta/2}(\Gamma(1/2\pm\alpha-\eta))}\right)\times$$
$${}_2\psi_1\left[\begin{array}{c}(1/2\pm\alpha+\rho-\theta/2,2\theta),(\eta-\rho+\theta/2,-2\theta);\\(1/2,1);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{40}$$

COROLLARY 11. *If we take m = 1, μ = 1, λ = 0 and ν = −1/2 in (5) and then by using (9), we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1}\exp(-az/2)M_{\eta,\alpha}(az)\cos(w\, z^\theta)dz =$$
$$\left(\frac{\sqrt{\pi}(1/a)^{-\theta/2}\Gamma(2\alpha+1)}{(a)^\rho(\Gamma(\alpha+\eta+1/2))}\right)\times$$
$${}_2\psi_2\left[\begin{array}{c}(\alpha+\rho-\theta/2+1/2,2\theta),(\eta-\rho+\theta/2,-2\theta);\\(1/2,1),(\alpha-\rho+\theta+1/2,-2\theta);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{41}$$

COROLLARY 12. *If we take m = 1, μ = 1, λ = 0 and ν = −1/2 in (6) and then by using (9), we obtain*:

$$\int_0^{+\infty} z^{\rho-\theta/2-1}W_{\eta,\alpha}(az)W_{-\eta,\alpha}(az)\cos(w\, z^\theta)dz = \left(\frac{\sqrt{\pi}(1/a)^{-\theta/2}}{(a)^\rho}\right)\times$$
$${}_2\psi_2\left[\begin{array}{c}(\frac{\rho-\theta/2+1}{2}\pm\alpha,\theta),(\rho-\theta/2+1,2\theta);\\(1/2,1),2(1+\frac{\rho-\theta/2}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{42}$$

COROLLARY 13. *If we take m = 1 in (1) and then by using (10), we derive the following integral formula:*

$$\int_0^{+\infty} z^{\rho-1} J_\eta(az) J^{\mu,1}_{\nu,\lambda}(b\, z^w)dz = \Big(1/2\Big)\Big(b/2\Big)^{\nu+2\lambda}\Big(2/a\Big)^{\rho+w(\nu+2\lambda)} \times$$
$${}_2\psi_3\left[\begin{array}{c}(1,1)(\frac{\eta+\rho+w\nu+2w\lambda}{2}, w);\\ (\lambda+1,1),(\nu+\lambda+1,\mu),((\frac{2+\eta-(\rho+w\nu+2w\lambda)}{2}),-w);\end{array}\right.$$
$$\left.\Big(\frac{-b^2}{4}\Big)\Big(\frac{4}{a^2}\Big)^{w}\right]. \tag{43}$$

COROLLARY 14. *If we take m = 1 in (2) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1} K_\eta(az) J^{\mu,1}_{\nu,\lambda}(b\, z^w)dz = \Big(1/4\Big)\Big(b/2\Big)^{\nu+2\lambda}\Big(2/a\Big)^{\rho+w(\nu+2\lambda)} \times$$
$${}_2\psi_2\left[\begin{array}{c}(1,1),(\frac{\rho+w\nu+2w\lambda\pm\eta}{2}, w);\\ (\lambda+1,1),(\nu+\lambda+1,\mu);\end{array}\Big(\frac{-b^2}{4}\Big)\Big(\frac{4}{a^2}\Big)^{w}\right]. \tag{44}$$

COROLLARY 15. *If we take m = 1 in (3) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az) K_\eta(az) J^{\mu,1}_{\nu,\lambda}(b\, z^w)dz = \left(\frac{2a\sqrt{\pi}(b/2)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\right)\times$$
$${}_2\psi_3\left[\begin{array}{c}(1,1),(\rho+w\nu+2w\lambda\pm\eta,2w);\\ (\lambda+1,1),(\nu+\lambda+1,\mu),(\rho+w(\nu+2\lambda)+1/2,2w);\end{array}\right.$$
$$\left.\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \tag{45}$$

COROLLARY 16. *If we take m = 1 in (4) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2) W_{\eta,\alpha}(az) J^{\mu,1}_{\nu,\lambda}(w\, z^\theta)dz = \tag{46a}$$

$$\left(\frac{(w/2)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}(\Gamma(1/2\pm\alpha-\eta))}\right)\times$$
$${}_3\psi_2\left[\begin{array}{c}(1,1)(1/2\pm\alpha+\rho+\theta\nu+2\theta\lambda,2\theta),(\eta-\rho-\theta\nu-2\theta\lambda,-2\theta);\\ (\lambda+1,1),(\nu+\lambda+1,\mu);\end{array}\right.$$
$$\left.\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{46b}$$

COROLLARY 17. If we take m = 1 in (5) and then by using (10), we obtain:

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J^{\mu,1}_{\nu,\lambda}(w\ z^{\theta})dz =$$
$$\left(\frac{(w/1)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^{\rho}(\Gamma(\alpha+\eta+1/2))}\right)\times$$
$${}_3\psi_3\left[\begin{array}{c}(1,1),(\alpha+\rho+\theta\nu+2\theta\lambda+1/2,2\theta),(\eta-\rho-\theta\nu-2\theta\lambda,-2\theta);\\(\lambda+1,1),(\nu+\lambda+1,\mu),(\alpha-\rho-\theta\nu-2\theta\lambda+1/2,-2\theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{47}$$

COROLLARY 18. If we take m = 1 in (6) and then by using (10), we obtain:

$$\int_0^{+\infty} z^{\rho-1}W_{\eta,\alpha}(az)W_{-\eta,\alpha}(az)J^{\mu,1}_{\nu,\lambda}(w\ z^{\theta})dz = \left(\frac{(w/2)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}}\right)\times$$
$${}_3\psi_3\left[\begin{array}{c}(1,1),(\frac{\rho+\theta(\nu+2\lambda)+1}{2}\pm\alpha,\theta),(\rho+\theta(\nu+2\lambda)+1,2\theta);\\(\lambda+1,1),(\nu+\lambda+1,\mu),2(1+\frac{\rho+\theta(\nu+2\lambda)}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{48}$$

COROLLARY 19. *If we take m = 1, μ = 1 and λ = 1/2 in (1) and then by using (11), we derive the following integral formula:*

$$\int_0^{+\infty} z^{\rho-1}J_{\eta}(az)H_{\nu}(b\ z^{w})dz = \left(1/2\right)\left(b/2\right)^{\nu+1}\left(2/a\right)^{\rho+w(\nu+1)}\times$$
$${}_2\psi_3\left[\begin{array}{c}(1,1)(\frac{\eta+\rho+w\nu+w}{2},w);\\(3/2,1),(\nu+3/2,1),((\frac{2+\eta-(\rho+w\nu+w)}{2}),-w);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right] \tag{49}$$

COROLLARY 20. *If we take m = 1, μ = 1 and λ = 1/2 in (2) and then by using (11), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}K_{\eta}(az)H_{\nu}(b\ z^{w})dz = \left(1/4\right)\left(b/2\right)^{\nu+1}\left(2/a\right)^{\rho+w(\nu+1)}\times \tag{50a}$$

$${}_2\psi_2\left[\begin{array}{c}(1,1),(\frac{\rho+w\nu+w\pm\eta}{2},w);\\(3/2,1),(\nu+3/2,\mu);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right]. \tag{50b}$$

COROLLARY 21. *If we take m = 1, μ = 1 and λ = 1/2 in (3) and then by using (11), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az)K_\eta(az)H_\nu(b\,z^w)dz = \left(\frac{2a\sqrt{\pi}(b/2)^{\nu+1}}{(2a)^{\rho+w(\nu+1)}}\right)\times$$
$$ {}_2\psi_3\left[\begin{array}{c}(1,1),(\rho+w\nu+w\pm\eta,2w);\\(3/2,1),(\nu+3/2,1),(\rho+w(\nu+1)+1/2,2w);\end{array}\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \tag{51}$$

COROLLARY 22. *If we take m = 1, μ = 1 and λ = 1/2 in (4) and then by using (11), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)H_\nu(w\,z^\theta)dz = \left(\frac{(w/2)^{\nu+1}}{(a)^{\rho+\theta(\nu+1)}(\Gamma(1/2\pm\alpha-\eta))}\right)$$
$$ {}_3\psi_2\left[\begin{array}{c}(1,1)(1/2\pm\alpha+\rho+\theta\nu+\theta,2\theta),(\eta-\rho-\theta\nu-\theta,-2\theta);\\(3/2,1),(\nu+3/2,1);\\\left(\frac{-w^2}{4(a^2)^\theta}\right)\end{array}\right]. \tag{52}$$

COROLLARY 23. *If we take m = 1, μ = 1 and λ = 1/2 in (5) and then by using (11), we obtain*:

$$\int_1^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)H_\nu(w\,z^\theta)dz =$$
$$\left(\frac{(w/2)^{\nu+1}(1/a)^{\theta(\nu+1)}\Gamma(2\alpha+1)}{(a)^\rho(\Gamma(\alpha+\eta+1/2))}\right)\times$$
$$ {}_3\psi_3\left[\begin{array}{c}(1,1),(\alpha+\rho+\theta\nu+\theta+1/2,2\theta),(\eta-\rho-\theta\nu-\theta,-2\theta);\\(3/2,1),(\nu+3/2,1),(\alpha-\rho-\theta\nu-\theta+1/2,-2\theta);\\\left(\frac{-w^2}{4(a^2)^\theta}\right)\end{array}\right]. \tag{53}$$

COROLLARY 24. *If we take m = 1, μ = 1 and λ = 1/2 in (6) and then by using (11), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}W_{\eta,\alpha}(az)W_{-\eta,\alpha}(az)H_\nu(w\,z^\theta)dz = \left(\frac{(w/2)^{\nu+1}(1/a)^{\theta(\nu+1)}}{(a)^\rho}\right)\times \tag{54a}$$

$$ {}_3\psi_3\left[\begin{array}{c}(1,1),(\frac{\rho+\theta(\nu+1)+1}{2}\pm\alpha,\theta),(\rho+\theta(\nu+1)+1,2\theta);\\(3/2,1),(\nu+3/2,1),2(1+\frac{\rho+\theta(\nu+1)}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{54b}$$

References

Chaurasia, V.B.L. & Pandey, S.C. 2010. On the fractional calculus of generalized Mittag-Leffler function. *SCIENTIA Series A: Mathematical Sciences*, 20, pp.113-122 [online]. Available at: https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.399.5089&rep=rep1&type=pdf [Accessed: 9 February 2022].

Choi, J. & Agarwal, P. 2013. Certain unified integrals associated with Bessel functions. *Boundary Value Problems*, art.number:95. Available at: https://doi.org/10.1186/1687-2770-2013-95

Choi, J., Mathur, S. & Purohit, S.D. 2014. Certain new integral formulas involving the generalized Bessel functions. *Bulletin of the Korean Mathematical Society*, 51(4), pp.995-1003. Available at: https://doi.org/10.4134/BKMS.2014.51.4.995.

Jain, S., Choi, J., Agarwal, P. & Nisar, K.S. 2016. Integrals involving Laguerre type plynomials and Bessel functions. *Far East Journal of Mathematical Sciences (FJMS)*, 100(6), pp.965-976. Available at: https://doi.org/10.17654/MS100060965.

Kachhia, K.B. & Prajapati, J.C. 2016. On generalized fractional kinetic equations involving generalized Lommel-Wright functions. *Alexandria Engineering Journal*, 55(3), pp.2953-2957. Available at: https://doi.org/10.1016/j.aej.2016.04.038.

Kiryakova, V.S. 2000. Multiple (multiindex) Mittag-Leffler functions and relations to generalized fractional calculus. *Journal of Computational and Applied Mathematics*, 118(1-2), pp.241-259. Available at: https://doi.org/10.1016/S0377-0427(00)00292-2.

Mathai, A.M., Saxena, R.K. & Haubold, H.J. 2010. *The H-function: Theory and Applications*. New York, NY: Springer. Aavailable at: https://doi.org/10.1007/978-1-4419-0916-9.

Mathai, A.M. & Saxena, R.K. 1973. *Generalized Hypergeometric Functions with Applications in Statistics and Physical Sciences*. Berlin Heidelberg: Springer- Verlag. Available at: https://doi.org/10.1007/BFb0060468.

Menaria, N., Nisar, K.S. & Purohit, S.D. 2016. On a new class of integrals involving product of generalized Bessel function of the first kind and general class of polynomials. *Acta Universitatis Apulensis*, 46, pp.97-105 [online]. Available at: https://www.emis.de/journals/AUA/pdf/74_1366_aua_2841701.pdf [Accessed: 9

Mondal, S.R. & Nisar, K.S. 2017. Certain unified integral formulas involving the generalized modified k-Bessel function of first kind. *Communications of the Korean Mathematical Society*, 32(1), pp.47-53. Available at: https://doi.org/10.4134/CKMS.c160017.

Paneva-Konovska, J. 2007. Theorems on the convergence of series in generalized Lommel-Wright functions. *Fractional Calculus and Applied Analysis*, 10(1), pp.59-74 [online]. Available at: http://eudml.org/doc/11298 [Accessed: 9 February 2022].

Rainville, E.D. 1960. *Special Functions*. New York: The Macmillan Company.

Srivastava, H.M. & Daoust, M.C. 1969. Certain generalized Neumann expansions associated with Kampe-de-Feriet function. *Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen Series A-Mathematical Sciences*, 72(5), pp.449-457.

Srivastava, H.M. & Manocha, H.L. 1984. *A treatise on generating functions*. Chichester, West Sussex, England: E. Horwood & New York: Halsted Press. ISBN: 9780853125082.

Whittaker, E.T. & Watson, G.N. 2013. *A Course of Modern Analysis, reprint of the fourth* (1927) *edition*. Cambridge University Press, Cambridge Mathematical Library. Available at: https://doi.org/10.1017/CBO9780511608759.

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