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Domination on cactus chains of pentagons

ДОМИНИРОВАНИЕ НА ПЯТИУГОЛЬНЫХ КАКТУС-ЦЕПЯХ

ДОМИНАЦИЈА НА ПЕТОУГОЊИМ КАКТУС-ЛАНЦИМА

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ABSTRACT:

Introduction/purpose: A graph as a mathematical object occupies a special place in science. Graph theory is increasingly used in many spheres of business and scientific fields. This paper analyzes pentagonal cactus chains, a special type of graphs composed of pentagonal cycles in which two adjacent cycles have only one node in common. The aim of the research is to determine the dominant set and the dominance number on ortho and meta pentagonal cactus chains.

Methods: When the corresponding destinations are treated as graph nodes and the connections between them as branches in the graph, the complete structure of the graph is obtained, to which the laws of graph theory are applied. The vertices of the pentagon are treated as nodes of the graph and the sides as branches in the graph. By applying mathematical methods, the dominance was determined on one pentagon, then on two pentagons with a common node, and then on ortho and meta pentagonal cactus chains.

Results: The research has shown that the dominance number on the ortho chain of the length $h \geq 2$ is equal to the value of the expression while on the meta chain it is equal to the value of the expression $h+1$, which was proven in this paper.

Conclusion: The results show that the dominant sets and the dominance numbers on ortho and meta pentagonal cactus chains are determined and explicitly expressed by mathematical expressions. They also point to the possibility of their application in the fields of science as well as in the spheres of business in which these structures appear.

KEYWORDS: graph, pentagonal cactus-chain, dominant set, dominance number.

Р е з ю м е :

Введение/цель: Граф как математический объект занимает особое место в науке. Теория графов все чаще используется во многих видах деятельности и различных научных областях. В данной статье анализируются пятиугольные кактус-цепочки, как особый вид графов, состоящих из пятиугольных циклов, в которых два соседних цикла имеют только один общий узел. Цель исследования заключалась в определении доминирующего множества и доминирующего числа в орто- и мета-пятиугольных кактус-цепочках.

Методы: Когда соответствующие положения рассматриваются как узлы графа, а связи между ними – как ветви графа, получается полная структура графа, к которой применяются законы теории графов. Вершины пятиугольника рассматриваются как узлы графа, а стороны – как ветви графа. С помощью математических методов, было определено доминирование на одном пятиугольнике, затем на двух пятиугольниках с общим узлом, а затем на орто- и мета-пятиугольных кактус-цепочек.

Результаты: Исследование показало, что число доминирования на орто-цепи с длиной $h \geq 2$ равно значению выражения в то время как на мета-цепи оно равно значению выражения $h+1$, что и следовало доказать в данной статье.

Выводы: Результаты исследования показали, что доминирующие множества и числа доминирования в орто- и мета-пятиугольных кактус-цепочках определяются и эксплицитно исчисляются математическими выражениями. Они также указывают на возможность их применения как в области науки, так и в сферах бизнеса, в которых присутствуют эти структуры.

К л ю ч е в ы е с л о в а : граф, пятиугольная кактус-цепочка, доминирующее множество, число доминирования.

ABSTRACT:

Увод/цель: Граф као математички објекат заузима посебно место у науци. Теорија графова налази све већу примену у многобројним сферама пословања, као и научним областима. У овом раду анализирани су петугаони кактус-ланци који представљају посебну врсту графа састављеног од петугаоних циклуса у којима два суседна циклуса имају заједнички само

један чвор. Циљ истраживања јесте одређивање доминантног скупа и доминацијског броја на орто и мета петоугаоним кактус-ланцима.

Методе: Када се одговарајућа одредишта третирају као чворови графа, а везе међу њима као гране у графу, добија се потпуна структура графа на коју се примењују законитости теорије графова. Темена петоугла су третирана као чворови графа, а странице као гране у графу. Применом математичких метода одређена је доминација на једном петоуглу, затим на два петоугла са заједничким чвором, а након тога на орто и мета петоугаоним кактус-ланцима.

Резултати: Истраживања су показала да је доминацијски број на орто ланцу дужине $h + 2$ једнак вредности израза $h + 1$, док је на мета ланцу једнак вредности израза $h + 1$, што је доказано у раду.

Закључак: Резултати показују да су доминантни скупови и доминацијски бројеви на орто и мета петоугаоним кактус-ланцима одређени и експлицитно исказани математичким изразима. Такође, упућују на могућност њихове примене у областима науке, као и у сферама пословања у којима се појављују ове структуре.

KEYWORDS: граф, петоугаони кактус-ланац, доминантни скуп, доминацијски број.

INTRODUCTION

Mathematical apparatus and mathematical methods are used in almost all fields of science, both natural (Ghergu & Radulescu, 2011; Veličković et al, 2020) and social (Vladimirovich & Vasilyevich-Chernyaev, 2021). A graph as a mathematical object occupies a special place in science (Bakhshesh, 2022; Hajian & Rad, 2021; Hernández Mira et al, 2021). It is used in medicine, genetics, chemistry, etc. All structural formulas of covalently bound compounds are graphs. Chemical elements are represented by graphs where atoms are vertices and chemical bonds are lines in the graph (Balaban, 1985). A graphical representation of chemical structures provides a visual insight into molecular bonds and chemical properties of molecules. The QSPR study has shown that many of chemical properties of molecules are closely related to theoretical graphical invariants called molecular descriptors (Mihalić & Trinajstić, 1992). The theoretical graphical invariant is also the dominance number, which is the simplest variant of the k -dominance number that is used many times in mathematics (Zmazek & Žerovnik, 2003).

A graph is usually denoted by G , a set of its vertices (nodes) by $V(G)$ and a set of its branches (lines) by $E(G)$.

A set D that is a subset of the set $V(G)$ is called a k -dominant set in the graph G if for each vertex outside the set D there is at least one vertex in the set D such that the distance between them is less than or equal to k . The number of elements of the smallest k -dominant set is called the k -dominance number and is denoted by γ_k . If $k = 1$, the 1-dominance number is called the dominance number and is denoted by γ and the 1-dominant set is called the dominant set.

A cactus graph is a connected graph in which no line (branch) is in more than one cycle. The study of cactus graphs began in the middle of the 20th century. In his work (Husimi, 1950) Husimi uses these graphs in studies of cluster integrals. Riddell (Riddell, 1951) uses them in the theory of condensation. They were later used in the theory of electrical and communication networks (Zmazek & Žerovnik, 2005) as well as in chemistry (Sharma et al, 1997; Gupta et al, 2001; Gupta et al, 2002).

It is known that many chemical compounds have a pentagonal shape in their configuration. Among them are cycloalkanes, which are very common compounds in the nature. The five-membered and six-membered cycloalkanes, cyclopentane (Figure 1) and cyclohexane, which contain 5 and 6 ring carbon atoms, respectively, are very stable and their structures appear in many biological molecules.

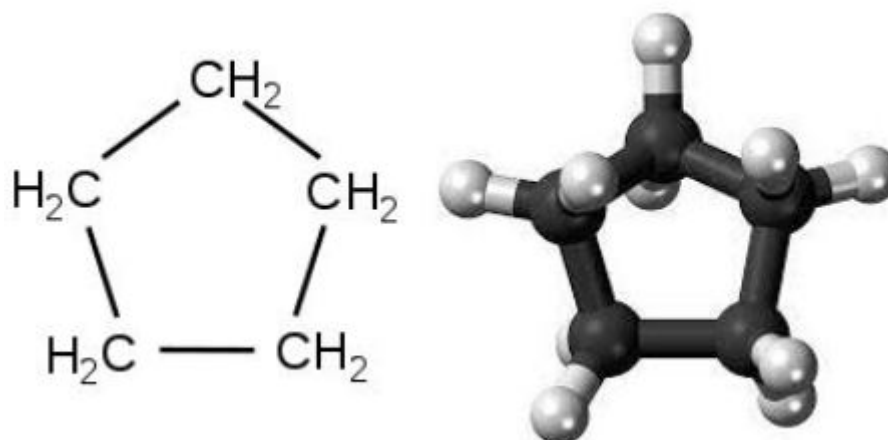


FIGURE 1

Cyclopentane

Рис. 1 – Циклопентан

Слика 1 – Циклопентан

Their ring structures are also included in the composition of steroids. A large number of steroids are synthesized in laboratories and used in the treatment of cancer, arthritis, various allergies and other diseases (Balaban & Zeljković, 2021). Pentagonal forms in combination with hexagonal forms are present in many compounds, among which are heterocyclic compounds: morphine, benzofuran, dibenzothiophene and others.

In this paper, we analyze the k -dominance of pentagonal cactus chains. Hexagonal cactus chains were investigated in papers (Farrell, 1987; Vukičević & Klobučar, 2007). Afterwards, the papers (Majstorovic et al, 2012; Klobučar & Klobučar, 2019) determined the dominance number on a uniform hexagonal cactus chain, the dominance number on an arbitrary hexagonal network, and the total and double total dominance number on a hexagonal network. The K -dominance on rhomboidal cactus chains (Carević et al, 2020) as well as on the icosahedral-hexagonal network (Carević, 2021) was also investigated.

PENTAGONAL CACTUS-CHAINS

The pentagonal cactus-chain G is a graph consisting of a cycle with 5 vertices. A vertex that is common to two or three pentagons is called a cut-vertex. If each pentagon in the graph G has at most 2 cut-vertices and each cut-vertex is divided between exactly 2 pentagons, the graph G is called a pentagonal cactus-chain.

With $\#^h$ we will denote a pentagonal cactus-chain of the length h and $\#^h = \#_1 \#_2 \dots \#_h$ where $\#^h$ are successive pentagons in the chain (Figure 2).

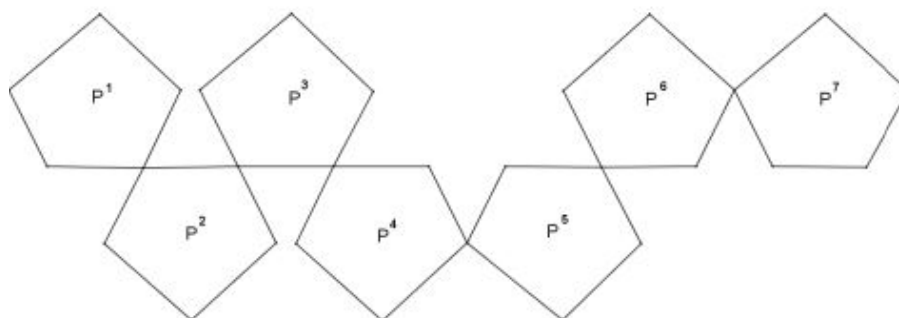


FIGURE 2
Pentagonal cactus-chain of the length 7
Рис. 2 – Пятиугольная кактус-цепочка длиной 7
Слика 2 – Петоугаони кактус-ланац дужине 7

Denote by x and y the vertices in the graph G and by $d(x, y)$ the distance between them, where the distance between two vertices is equal to the number of branches located from one vertex to another. Denote by $\#_{\#}$ the minimum distance between the pentagons $\#^{\#}$ and $\#^{\#+2}$:

$$\#_{\#} = \min\{\#(\#, \#): \#^{\#} \#^{\#+2}, \# = 1, 2, \dots, \# - 2\}$$

Then $\#^{\#}$ is the distance between the pentagons $\#^{\#}$ and $\#^{\#+2}$

With the exception of the first and last pentagons in the cactus chain, which have one cut-vertex, all other pentagons have two cut-vertices, and they are called inner pentagons.

In the pentagonal cactus chain $\#_{\#}$, we distinguish between ortho and meta inner pentagons. An inner pentagon is called an ortho pentagon if its cut-vertices are adjacent, and a meta pentagon if the distance between its cut-vertices is $d = 2$.

A pentagonal cactus chain is uniform if all its inner pentagons are of the same type. A chain $\#_{\#}$ is called an ortho-chain, and is denoted by $\#_{\#}$ if all its inner pentagons are ortho-pentagons (Figure 3).

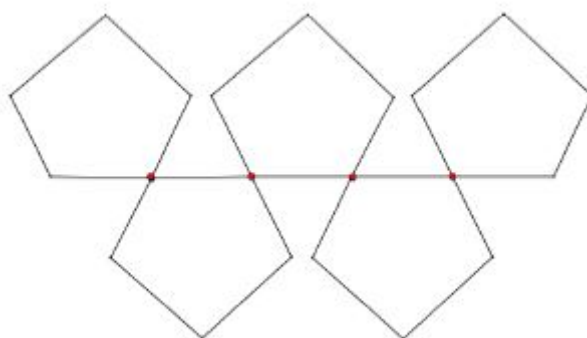


FIGURE 3
Ortho cactus-chain
Рис. 3 – Орто-кактус-цепочка
Слика 3 – Орто кактус-ланац

Analogously, a chain $\#_{\#}$ is called a meta-chain, and is denoted by $\#_{\#}$ if all its inner pentagons are meta-pentagons (Figure 4).

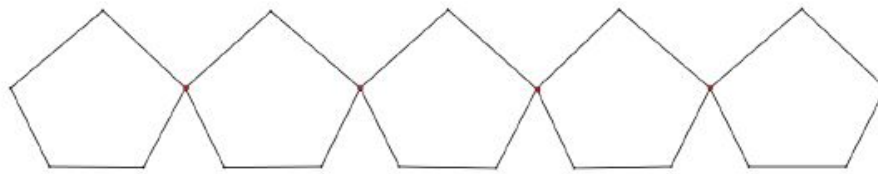


FIGURE 4
Meta cactus-chain

Рис. 4 – Мета кактус-цепочка
Слика 4 – Мета кактус-ланац

To determine the dominant set on the uniform pentagonal cactus chains $\#_{\#}$ and $\#_{\#}$, it will be necessary to point out certain vertices in the cactus chain. That is why it is necessary to mark them. In the orthopentagon $\#_{\#}$ the cut-vertices are adjacent and we will denote them by $\#_{\#}$ and $\#_{\#+1}$. The other vertices in $\#_{\#}$ it will be denoted by $\#_1^{\#}$, $\#_2^{\#}$ and $\#_3^{\#}$ (Figure 5):

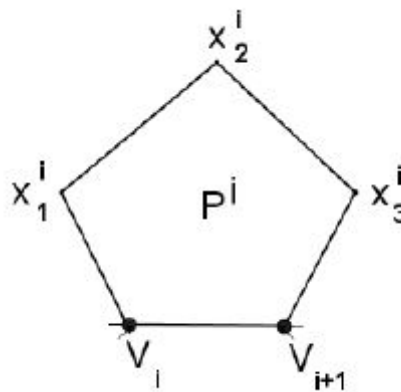


FIGURE 5
Marking vertices in the ortho pentagon

Рис. 5 – Обозначение вершин в ортогональном пятиугольнике
Слика 5 – Означавање чворова у орто петоуглу

In the meta pentagon $\#_{\#}$ the cut-vertices are at a distance $d = 2$ and we will denote them by $\#_{2\#-1}$ and $\#_{2\#+1}$. With $\#_{2\#}$ we will denote the vertex to which it applies $d(\#_{2\#-1}, \#_{2\#}) = d(\#_{2\#}, \#_{2\#+1}) = 1$. The other two nodes in the pentagon $\#_{\#}$ will be denoted $\#_1^{\#}$ and $\#_2^{\#}$ (Figure 6):

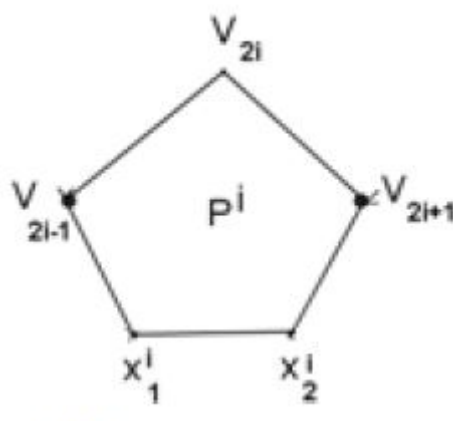


FIGURE 6

Marking vertices in the meta pentagon

Рис. 6 – Обозначение вершин в мета-пятиугольнике

Слика 6 – Означавање чворова у мета петоуглу

RESEARCH RESULTS

In this section, we consider 1-dominance on ortho and meta pentagonal cactus chains. We will first consider the dominance of one pentagon and two adjacent pentagons in the ortho and meta chain of cacti.

Lemma 3.1. The dominance number for the pentagon is $\gamma = 2$.

Proof: Let us denote the vertices of the pentagon by $\#_1, \#_2, \#_3, \#_4, \#_5$ (Figure 7):

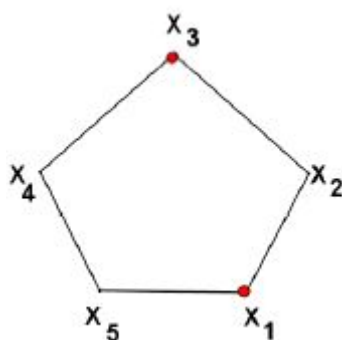


FIGURE 7

Dominant set on a pentagon

Рис. 7 – Доминирующее множество на пятиугольнике

Слика 7 – Доминантни скуп на петоуглу

One pentagon vertex dominates two adjacent vertices. Let us take the vertex $\#1$. It dominates the vertices $\#2$ and $\#5$. As the pentagon has 5 vertices, domination over the other two vertices $\#3$ and $\#4$ is necessary. We conclude that one of the remaining two vertices must belong to the dominant set on the pentagon. Let it be the vertex $\#3$. Thus, the set $D = \{\#1, \#3\}$ is the dominant set for a given pentagon but it is not the only dominant set for a given pentagon but it is not the only dominant set whose cardinality is equal to 2. They are also sets that contain any two non-adjacent pentagon vertices. Let us prove that any of the mentioned two-membered sets is the minimum dominant set on the pentagon. Assuming that there is a dominant set of less cardinality D' , it would have to contain only one vertex and one vertex cannot dominate the remaining

4 vertices of the pentagon. Thus, the minimum dominant set on a pentagon is a two-membered set, so the dominance number for the pentagon is $\# = 2$.

Lemma 3.2. The dominance number for two pentagons with one cut-vertex is $\# = 3$.

Proof: Let us denote the vertices of two pentagons by one common vertex with $\#_1, \#_2, \dots, \#_9$ (Figure 8):

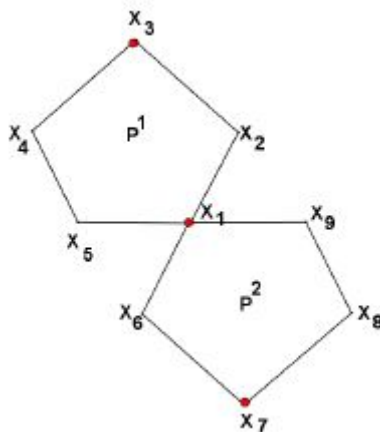


FIGURE 8

Dominant set for two pentagons with a cut-vertex

Рис. 8 – Доминирующее множество для двух пятиугольников с пересекающейся вершиной

Слика 8 – Доминантни скуп за два петоугла са пресецим чвором

Let $\#_1$ be the cut-vertex of the given pentagons $\#^1$ and $\#^2$. Based on Lemma 3.1. the pentagon $\#^1$ excluding the vertex $\#_1$ must have another dominant vertex that is not adjacent to the vertex $\#_1$. Let it be the vertex $\#_3$. Also by applying Lemma 3.1. the pentagon excluding the vertex must have another dominant vertex that is not adjacent to the vertex $\#_1$. Let it be the vertex $\#_7$. Thus the nodes $\#_1, \#_3$, and $\#_7$, dominate over the nodes $\#_2, \#_4, \#_5, \#_6, \#_8$ and $\#_9$ so the dominant set for the pentagons $\#^1\#^2$ is the set $D = \{\#_1, \#_3, \#_7\}$. Analogous to the consideration in Lemma 3.1. the set D is not the only three-membered set that is dominant $\#^1\#^2$ on but there is no dominant set of less cardinality. Suppose that there is a dominant set D' whose cardinality is equal to 2. Let D' contain one vertex from each pentagon, for example $\#' = \{\#_1, \#_3\}$. The vertices $\#_1$ and $\#_3$ would then dominate over the remaining 7 vertices in $\#^1\#^2$ and this is impossible. The vertex $\#_1$ as a common vertex for both pentagons dominates over two neighboring vertices in both pentagons, so it dominates over 4 vertices in $\#^1\#^2$. The vertex $\#_3$, or any other vertex not adjacent to the vertex $\#_1$ dominates two adjacent vertices. So, the total sum of vertices covered by dominance is $4 + 2 = 6$ and that is less than 7. Thus, 2 vertices cannot dominate the remaining 7 vertices in $\#^1\#^2$. We conclude that the minimum dominant set for $\#^1\#^2$ is a three-membered set and $\# = 3$.

Let us consider the dominance on pentagonal ortho and meta cactus chains of arbitrary length.

Theorem 3.1. $\#(\#_h) = \left\lceil \frac{3h}{2} \right\rceil$ for each $h \geq 2$ $\#_h \in \mathbb{N}$

Proof: We observe a pentagonal ortho cactus-chain $\#_h = \#^1\#^2\dots\#^h$ (Figure 9) and a set:

$$D_{0h} = \{x_2^i, i = 1, h\} \cup \{v_{2i}^i, i = 1, \left\lceil \frac{h}{2} \right\rceil\}$$

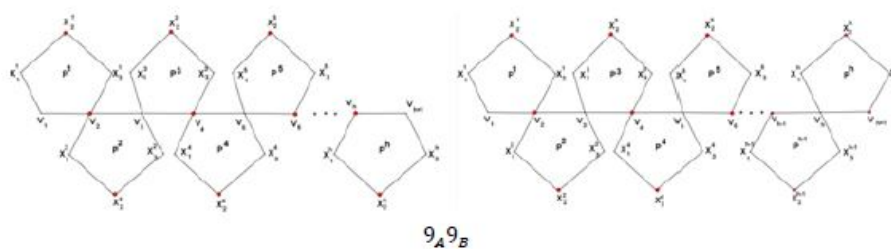


FIGURE 9

Minimum dominant set for

Рис. 9 – Минимально доминирующее множество для

Слика 9 – Минимални доминантни скуп за

Let us prove that $\#_{\# \#}$ is the dominant set of minimum cardinality for apentagonal ortho cactus-chain $\#_{\#} = \#^1 \#^2 \dots \#^{\#}$.

Let us divide the ortho-chain $\#_{\#}$ into subchains $\#^{2\#-1} \#^{2\#}, i = 1, 2, \dots, \left\lfloor \frac{h}{2} \right\rfloor$ (Figure 10) and the last pentagon $\#_{\#}$ if h is an odd number.

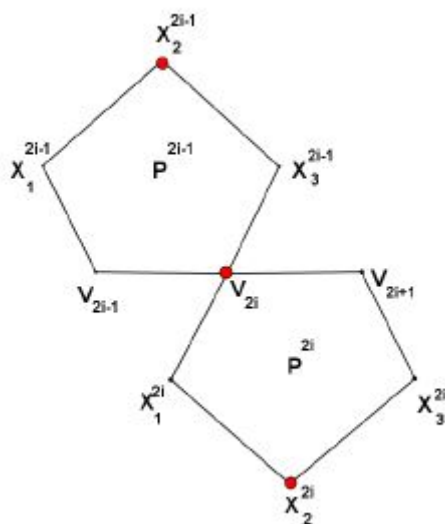


FIGURE 10

Subchain of the ortho-chain

Рис. 10 – Подцепочка орто-цепочки

Слика 10 – Подланца орто ланца

Based on Lemma 3.2. the set $A_i = \{x_2^{2i-1}, x_2^{2i}, v_{2i}\}$ for $i = 1, 2, \dots, \left\lfloor \frac{h}{2} \right\rfloor$ is the dominant set of minimum cardinality for the subchain $\#^{2\#-1} \#^{2\#}$. An ortho-chain of the length h for $h = 2k, k \in \mathbb{N}$ is composed of $\frac{h}{2}$ subchains $\#^{2\#-1} \#^{2\#}, i = 1, 2, \dots, \frac{h}{2}$ (Figure 9A), so the set

$$D_1 = \bigcup_{i=1}^k A_i \text{ for } k = \frac{h}{2}$$

is a dominant set for the ortho-chain $\#_{\#}$. Therefore, it is

$$\gamma(0_h) \leq \text{card}(D_1) = \frac{h}{2} \cdot 3 = \frac{3h}{2}$$

where we have marked the cardinality of the set $\#_1$ with $\text{card}(\#_1)$. If h is an odd number (Figure 9_B), then the set

$$D_2 = U_{i=1}^k A_i U \{x_2^h, V_{h+1}\} \text{ for } k = \left\lfloor \frac{h}{2} \right\rfloor$$

is a dominant set for the ortho-chain $\#_h$ and then is

$$\gamma(O_h) \leq \text{card}(D_2) = \left\lfloor \frac{h}{2} \right\rfloor \cdot 3 + 2 = \left\lceil \frac{3h}{2} \right\rceil.$$

$\cdot 3 + 2 =$

Note that the set

for $k =$

if $\cdot$ an even number is equal to the following expression:

$\cdot$

Also for $k =$

and $\cdot$ is an odd number, the set

is equal to the following expression:

In case h is an even number,

then we conclude that it is

So, the set

$\cdot, i = 1, h\}$

$\cdot, i = 1,$

$\cdot$ is the dominant set for the ortho-chain

when h is even or odd number.

Also, in the case where h is an even number,

$\cdot$ So,

when h is even or odd number. Prove that the set

is the dominant set of minimal cardinality. Each subchain

contains 3 dominant nodes based on Lemma 3.2. Based on this, we conclude that each dominant set on the chain

contains more than 3 or exactly 3 dominant nodes in each subchain

and more than 2 or exactly 2 dominant nodes in the last pentagon if h is an odd number, based on Lemma

3.1. So, we conclude that it is

$\cdot 3$ in case h is an even number, and

$\cdot 3 + 2$ in case h is an odd number. When we combine both cases, we get that

It follows from

and

that it is

Corollary 3.1.

for each.

$2 \# h \# N$.

Theorem 3.2.

$\cdot) = h + 1$ for each $\cdot$.

$2 \# h \# N$.

Proof: We observe a pentagonal meta cactus-chain

(Figure 11) and set:

, $i = 1, h + 1\}$

Figure 11 – Minimum dominant set for

Рис. 11 – Минимально доминирующее множество для

Слика 11 – Минимални доминантни скуп за

Let us prove that

is the dominant set of minimum cardinality for a pentagonal meta cactus-chain

.

. Based on Lemma 3.1. each pentagon has a dominant set made up of two non-adjacent vertices. Thus, the set .

. is dominant for the pentagon

for each $i = 1, h$. By merging the dominant sets of all pentagons in the chain, we get a set that is dominant for the whole chain. But, each pentagon

has a common vertex with the pentagon

for each $i = 1, h - 1$. Common vertices should not be repeated in the dominant set. So, the set

is the dominant set for the meta-chain

Note that it is

, $i = 1, h + 1\}$

Thus, the set

, $i = 1, h + 1\}$ is the dominant set for the meta-chain

for each $h \in \mathbb{N}$ and $h \geq 2$. Let us prove that

is the dominant set of minimal cardinality. Suppose that there is a set S of less cardinality that is dominant on the meta-chain

. The set S would then have one node less than the set

. Let it be a vertex

for any $i = 1, h$. Then the pentagon

would have only one dominant node

. Based on Lemma 3.1. that is not possible. We conclude that

is the minimum dominant set for

so it is

$) = h + 1$.

Corollary 3.2.

for each.

$2 \leq h \in \mathbb{N}$.

Conclusion

In this paper, we have shown the arrangement of vertices in dominant sets on uniform ortho and meta pentagonal cactus chains that appear in molecule structures of numerous compounds. We also proved that the dominance number for a pentagonal ortho-chain of the length h is equal to the value of the expression while for a pentagonal meta-chain it is equal to $h + 1$.

REFERENCES

- Bakhshesh, D. 2022. Isolate Roman domination in graphs. *Discrete Mathematics, Algorithms and Applications*, 14(3), art.number:2150131. Available at: <https://doi.org/10.1142/S1793830921501317>.
- Balaban, A.T. 1985. Applications of graph theory in chemistry. *Journal of chemical information and computer sciences*, 25(3), pp.334-343. Available at: <https://doi.org/10.1021/ci00047a033>

- Balaban, M. & Zeljković, S. 2021. *HEMIJA Teorija i eksperimenti*. Banja Luka, Republic of Srpska, Bosnia and Herzegovina: University of Banja Luka, Faculty of natural sciences and mathematics [online]. Available at: https://hemija.pmf.unibl.org/wp-content/uploads/2021/07/Balaban_Zeljkovic_Hemija_Teorija-i-eksperimenti.pdf (in Serbian) [Accessed: 20 February 2022]. ISBN: 978-99955-21-91-2.
- Carević M.M. 2021. Dominating Number on Icosahedral-Hexagonal Network. *Mathematical Problems in Engineering*, art.ID:6663389. Available at: <https://doi.org/10.1155/2021/6663389>.
- Carević, M.M., Petrović, M. & Denić, N. 2020. Dominating sets on the rhomboidal cactus chains and the icosahedral network. In: *19th International Symposium INFOTEH-Jahorina*, Jahorina, pp.152-157, March 18-20 [online]. Available at: <https://infotech.etf.ues.rs.ba/zbornik/2020/radovi/P-4/P-4-2.pdf> [Accessed: 20 February 2022].
- Farrell, E.J. 1987. Matchings in hexagonal cacti. *International Journal of Mathematics and Mathematical Sciences*, 10(art.ID:234184), pp.321-338. Available at: <https://doi.org/10.1155/S0161171287000395>.
- Ghergu, M. & Radulescu, V. 2012. *Nonlinear PDEs: Mathematical Models in Biology, Chemistry and Population Genetics*. Berlin Heidelberg: Springer-Verlag. ISBN 13: 9783642226632.
- Gupta, S., Singh, M. & Madan, A.K. 2001. Applications of graph theory: Relationship of molecular connectivity index and atomic molecular connectivity index with anti-HSV activity. *Journal of Molecular Structure: THEOCHEM*, 571(1-3), pp.147-152. Available at: [https://doi.org/10.1016/S0166-1280\(01\)00560-7](https://doi.org/10.1016/S0166-1280(01)00560-7).
- Gupta, S., Singh, M. & Madan, A.K. 2002. Application of Graph Theory: Relationship of Eccentric Connectivity Index and Wiener's Index with Anti-inflammatory Activity. *Journal of Mathematical Analysis and Applications*, 266(2), pp.259-268. Available at: <https://doi.org/10.1006/jmaa.2000.7243>.
- Hajian, M. & Rad, N.J. 2021. Fair Total Domination Number in Cactus Graphs. *Discussiones Mathematicae Graph Theory*, 41, pp.647-664. Available at: <https://doi.org/10.7151/DMGT.2225>.
- Hernández Mira, F.A., Parra Inza, E., Almira, J.M. S. & Vakhania, N. 2021. Properties of the Global Total k-Domination Number. *Mathematics*, 9(5), art.ID:480. Available at: <https://doi.org/10.3390/math9050480>.
- Husimi, K. 1950. Note on Mayers' theory of cluster integrals. *The Journal of Chemical Physics*, 18(5), pp.682-684. Available at: <https://doi.org/10.1063/1.1747725>.
- Klobučar, A. & Klobučar, A. 2019. Total and Double Total Domination Number on Hexagonal Grid. *Mathematics*, 7(11), art.number:1110. Available at: <https://doi.org/10.3390/math7111110>.
- Majstorovic, S., Doslic, T. & Klobucar, A. 2012. -Domination on hexagonal cactus chains. *Kragujevac Journal of Mathematics*, 36(2), pp.335-347 [online] Available at: https://imi.pmf.kg.ac.rs/kjm/pub/13569261514726_kjom3602-17.pdf [Accessed: 20 February 2022]
- Mihalić, Z. & Trinajstić, N. 1992. A graph-theoretical approach to structure-property relationships. *Journal of Chemical Education*, 69(9), art.ID:701. Available at: <https://doi.org/10.1021/ed069p701>.
- Riddell, R.J. 1951. *Contributions to the theory of condensation*. Ph.D. thesis. University of Michigan ProQuest Dissertations Publishing [online]. Available at: <https://www.proquest.com/openview/4c69a76aaebdf43a91617e8dc2be8fe6/1?pq-origsite=gscholar&cbl=18750&diss=y> [Accessed: 20 February 2022].
- Sharma, V., Goswami, R. & Madan, A. K. 1997. Eccentric connectivity index: A novel highly discriminating topological descriptor for structure-property and structure-activity studies. *Journal of chemical information and computer sciences*, 37(2), pp.273-282. Available at: <https://doi.org/10.1021/ci960049h>.
- Veličković, J., Arsić, N.B. & Stošić, L.T. 2020. The Efficiency of Galvanic Wastewater Treatment Facility 'Frad' in Aleksinac. *Trendovi u poslovanju*, 8(2), pp.78-85. Available at: <https://doi.org/10.5937/trendpos2002078V>.
- Vladimirovich, G.S. & Vasilyevich-Chernyaev, M. 2021. The experience of applying mathematical methods for analysis of the microgeneration sector in Russia. *International Review*, (1-2), pp.153-160. Available at: <https://doi.org/10.5937/intrev2102156V>.
- Vukičević, D. & Klobučar, A. 2007. -Dominating sets on linear benzenoids and on the infinite hexagonal grid. *Croatica Chemica Acta*, 80(2), pp.187-191 [online]. Available at: <https://hrcak.srce.hr/12849> [Accessed: 20 February 2022].

- Zmazek, B. & Žerovnik, J. 2005. Estimating the traffic on weighted cactus networks in linear time. In: *Ninth International Conference on Information Visualisation (IV'05)*, London, UK, pp.536-541, July 6-7. Available at: <https://doi.org/10.1109/IV.2005.48>.
- Zmazek, B. & Žerovnik, J. 2003. Computing the weighted Wiener and Szeged number on weighted cactus graphs in linear time. *Croatica Chemica Acta*, pp.137-143 [online]. Available at: <https://hrcak.srce.hr/103089> [Accessed: 20 February 2022].