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ПРИМЕНЕНИЕ КОНЕЧНЫХ ТОЧЕК ВЫБОРКИ В МНОГОЦЕЛЕВОЙ ОПТИМИЗАЦИИ, ОСНОВАННОЙ НА ВЕРОЯТНОСТИ С ПОМОЩЬЮ ЕДИНОЙ ЭКСПЕРИМЕНТАЛЬНОЙ РАЗРАБОТКИ

ПРИМЕНА КОНАЧНИХ ТАЧАКА УЗОРКОВАЊА У ВИШЕКРИТЕРИЈУМСКОЈ ОПТИМИЗАЦИЈИ ЗАСНОВАНОЈ НА ВЕРОВАТНОЋИ ПОМОЋУ УНИФОРМНОГ ЕКСПЕРИМЕНТАЛНОГ ДИЗАЈНА

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Abstract:

Introduction/purpose: An approximation for assessing a definite integral is continuously an attractive topic owing to its practical needs in scientific and engineering areas. An efficient approach for preliminarily calculating a definite integral with a small number of sampling points was newly developed to get an approximate value for a numerical integral with a complicated integrand. In the present paper, an efficient approach with a small number of sampling points is combined to the novel probability– based multi–objective optimization (PMOO) by means of uniform experimental design so as to simplify the complicated definite integral in the PMOO preliminarily.
Methods: The distribution of sampling points within its single peak domain is deterministic and uniform, which follows the rules of the uniform design method and good lattice points; the total preferable probability is the unique and deterministic index in the PMOO.
Results: The applications of the efficient approach with finite sampling points in solving typical problems of PMOO indicate its rationality and convenience in the operation.
Conclusion: The efficient approach with finite sampling points for assessing a definite integral is successfully combined with PMOO by means of the uniform design method and good lattice points.

Keywords: preferable probability, multi–objective optimization, finite sampling points, simplifying evaluation, uniform design method..

Резюме:

Введение/цель: Аппроксимация для оценки определенного интеграла не перестает привлекать внимание ученых, ввиду своего практического применения в различных областях инженерных наук. Недавно был разработан эффективный подход к вычислению определенного интеграла с небольшим числом точек выборки для получения приблизительного значения численного интеграла со сложным подынтегральным выражением. В данной работе в целях упрощения сложного определенного интеграла в МООВ был применен эффективный подход с небольшим числом точек выборки, объединенный с новой многоцелевой оптимизацией, основанной на вероятности (МООВ) с помощью единой экспериментальной разработки.

Методы: Распределение точек выборки в пределах области с одним пиком является детерминированным и равномерным, что соответствует правилам метода единой разработки и точек идеальной решетки; общая предпочтительная вероятность является уникальным и детерминированным индексом в МООВ.

Результаты: Применение эффективного подхода с конечными точками выборки при решении типовых проблем в МООВ указывает на его рациональность и удобство в эксплуатации.

Выводы: Эффективный подход с конечными точками выборки для оценки определенного интеграла успешно комбинируется с МООВ с помощью метода единой разработки и точек идеальной решетки.

Ключевые слова: предпочтительная вероятность, многоцелевая оптимизация, конечные точки выборки, упрощение оценки, единый метод разработки.

Abstract:

Увод/циљ: Апроксимација процене коначног интеграла не престаје да буде привлачна тема захваљујући својој практичној примени у научним и инжењерским областима. Недовно је развијен ефикасан

приступ израчунавању одређеног интеграла с малим бројем тачака узорковања како би се добила приближна вредност за нумерички интеграл са компликованим интеграндом. У овом раду ефикасан приступ с малим бројем тачака узорковања комбинован је са новом вишекритеријумском оптимизацијом заснованом на вероватноћи (ПМОО) помоћу униформног експерименталног дизајна с циљем да се поједностави компликовани одређени интеграл у ПМОО.

Методе: Дистрибуција тачака узорковања унутар подручја издвојеног врха детерминистичка је и униформна, што следи из правила метода униформног дизајна и тачака добре решетке. Укупна пожељна вероватноћа је јединствени и детерминистички индекс у ПМОО.

Резултати: Примене ефикасног приступа с коначним тачкама узорковања за решавање типичних проблема у ПМОО указују на његову рационалност и погодност при операцијама.

Закључак: Ефикасан приступ с коначним тачкама узорковања за оцену одређеног интеграла успешно се комбинује са ПМОО помоћу метода униформног дизајна и тачака добре решетке.

Keywords: пожељна вероватноћа, вишекритеријумска оптимизација, коначне тачке узорковања, поједностављивање евалуације, метод униформног дизајна.

Introduction

Recently, an efficient approach for assessing a definite integral with a small number of sampling points has been proposed based on the uniform experimental design method and the good lattice point from the viewpoint of practical application (Yu et al, 2022) preliminarily. It indicated that the efficient evaluation of a definite integral for a periodical function in its single peak domain can be obtained by using 11 sampling points in one dimension, 17 sampling points in two dimensions, and 19 sampling points in three dimensions with a small relative error preliminarily. The fundamental of the finite sampling points (FSPs) for assessing a definite integral was the rules of uniform and deterministic distribution of the FSPs according to the good lattice point (Hua & Wang, 1981; Fang, 1980; Fang, et al, 1994, 2018; Ripley, 1981; Wang & Fang, 2010), or the so-called "quasi – Monte Carlo method" (QMC).

The so–called "curse of dimensionality" problem was broken in the publication of the calculating results of Paskov & Traub (1995) by using Halton sequences and Sobol sequences for accounting a ten – tranche CMO (Collateralized Mortgage Obligation) in high dimensions, reaching even 360 dimensions. Their findings were that QMC methods performed very well as compared to simple MC methods, as well as to antithetic

MC methods (Tezuka, 1998, 2002; Paskov & Traub, 1995; Paskov, 1996; Sloan & Wozniakowski, 1998). Afterwards, a lot of similar phenomena were found in different evaluations for pricing problems by using different types of low–discrepancy sequences (Tezuka,1998). All these consequences provide a powerful support to using the QMC with finite sampling points to conduct a definite integral numerically.

In the present paper, the newly developed efficient approach for assessing a definite integral with a small number of sampling points is combined to the novel probability – based multi – objective optimization (PMOO) so as to simplify the complicated definite integral in PMOO. The novel PMOO aims to overcome the shortcomings of personal and subjective factors in the previous multi – object optimizations, so a novel concept of preferable probability and the corresponding assessment are developed (Zheng, 2022; Zheng et al, 2021, 2022). The preferable probability is used to reflect the preferablity degree of the candidate in the optimization, all performance utility indicators of candidates are divided into beneficial or unbeneficial types according to their features in the selection, and each performance utility indicator contributes to one partial preferable probability quantitatively. The total preferable probability is the product of all partial preferable probabilities in the viewpoint of probability theory, which is the overall consideration of various response variables simultaneously so as to reach a compromised optimization. The total preferable probability is the unique deterministic index in the optimal process comparatively. Appropriate achievements have been obtained.

Essence of the uniform experimental design method

The uniform experimental design method (UED) was proposed by Fang & Wang (1994, 2018) and the essence of the UED contains:

A) **Uniformity.** The sampling points for an experiment are evenly distributed in the input variable (parameter) space, so the term "space filling design" is widely used in the literature. The UED arranges the test design (test point, sampling points in space) through a uniform design table, which is deterministic without any randomness.
B) **Overall Mean Model.** The UED is to hope that the test point can give the minimum deviation of the total mean value of the output (response) variable from the actual total mean value.
C) **Robust.** The UED design can be applied to a variety of situations and is robust to model changes.
D) **Following basic procedures are involved in the UED**:

1) Total Mean Model

It assumes that there exists a deterministic relationship between the input independent variables $x_1, x_2, x_3, \ldots, x_s$ and the response y by

$$y = f(x_1, x_2, x_3, \ldots x_r), \;\; X = \{x_1,\ x_2,\ x_3,\ \ldots,\ x_r\} \in C^r. \tag{1}$$

Furthermore, it supposes that the experiment domain is the unit cube $C^r = [0, 1]^r$, the total mean value the response y on C^r, is,

$$E(y) = \int_{C^r} f(x_1, x_2, x_3, \ldots x_r) \cdot dx_1 \cdot dx_1 \cdot dx_3 \cdots dx_r \,,$$

(2)

If m sampling points $p_1, p_2, p_3, \ldots, p_m$ are taken on C^r, then the mean value of y on these m sampling points is

$$\overline{y(D_m)} = \frac{1}{m}\sum_{j=1}^{m} f(p_j)\,.$$ (3)

In Eq.(3), $D_m = \{p_1, p_2, p_3, \ldots, p_m\}$ represents a design of these m sampling points.

Fang & Wang (1994, 2018) proved that if the sampling points $p_1, p_2, p_3, \ldots, p_m$, pm are uniformly distributed on the domain C^r, the deviation $E(y) - y(D_m)$ of the sampling point set on C^r, and D_m is the smallest approximately.

2) Uniform Design Table

Fang & Wang (1994, 2018) and Wang & Fang (2010) developed a Uniform Design Table for the proper utilization of the UED which can be employed by anyone to arrange their sampling points. However, the preliminarily necessary number of sampling points was not clarified by Fang in their UED. Here in this paper, the number of sampling points suggested in the article of Yu et al (2022) is adopted for our utilization.

3)Regression

Regression is the next procedure to complete the optimum.

For our purpose, the total preferable probability and the approximate expression for the response $y' = f'(x_1, x_2, x_3, \ldots, x_r)$ can be obtained through data fitting, which is close to the true model (Fang & Wang, 1994, 2018).

The application of uniform design is becoming more and more extensive these years, including a successful application of the uniform experimental design in the Chinese Missile Design and Ford Motor Company of the USA, and the number of successful cases is increasing.

Combination of finite sampling points with the probability–based multi–objective optimization by means of the uniform experimental design

The above statements indicate the remarkable features of the UED, i.e., the uniform distribution of experiment / sampling points within the test domain and the small number of tests, fully representative of each point, and an easy to perform regression analysis. So here the Finite Sampling Points method is combined with the novel probability–based multi–objective optimization by means of the uniform experimental design and the good lattice point (GLP) to simplify the complicated data processing preliminarily in the following section.

In order to demonstrate the combination of finite sampling points with the probability–based multi–objective optimization, some typical examples are given in the following sections in detail.

1)Multi–objective optimization of tower crane boom tie rods

Qu et al (2004) conducted the multi – objective optimization of tower crane boom tie rods by the fuzzy optimization model.

Through a careful analysis, they set the minimum mass $W(X)$ of the boom tie rod and the minimum angular displacement $\theta(X)$ of the boom as the multiple objectives, and obtained the following model,

$$W(X) = 208.323x_1 + 433.868x_2, \tag{4}$$

$$\theta(X) = \frac{2.0288 \times 10^{-4}}{9.8621x_1 + 5.3471x_2}. \tag{5}$$

The constraint conditions are,

$$0.003379 < x_1 < 0.005805, \tag{6}$$

$$0.003379 < x_2 < 0.005468. \tag{7}$$

According to the optimal requirements of $W(X)$ and $\theta(X)$, both $W(X)$ and $\theta(X)$ are unbeneficial indexes (Qu et al, 2004) which have “the smaller the better” features in the optimization.

Thus, according to the probability–based multi–objective optimization (Zheng, 2022; Zheng et al, 2021, 2022), the partial preferable probabilities of $W(X)$ and $\theta(X)$ are expressed as

$$P_W = \beta_W \cdot [W_{max} + W_{min} - W(X)], \tag{8}$$

$$P_\theta = \beta_\theta \cdot [\theta_{max} + \theta_{min} - \theta(X)], \tag{9}$$

In Eqs. (8) and (9), β_W, W_{min}, and W_{max} express the normalization factor, the minimum and maximum values of the index $W(X)$, respectively; β_θ, θ_{min}, and θ_{max} indicate the normalization factor, the minimum and maximum values of the index $\theta(X)$, individually.

Simultaneously,

$$\beta_W = \frac{1}{\int_{x_{1l},x_{2u}}^{x_{1u},x_{2u}} [W_{\max} + W_{\min} - W(X)] dx_1 \cdot dx_2} \tag{10}$$

$$\beta_\theta = \frac{1}{\int_{x_{1l},x_{2u}}^{x_{1u},x_{2u}} [\theta_{\max} + \theta_{\min} - \theta(X)] \cdot dx_1 \cdot dx_2} \tag{11}$$

In Eqs. (8) and (9), x_{1L}, x_{1U}, x_{2L} and x_{2U} express the lower limit and the upper limit of x_1 and x_2 in their domain, respectively.

According to the common procedure, the subsequent thing is to substitute Eqs. (4) and (5) into Eqs. (8) through (11) with the constraints of Eqs. (6) and (7) to conduct the evaluations. It can be seen that the assessments are tediously long and complicated due to the sophisticated integration. However, if we use the finite sampling points algorithm proposed by Yu et al (2022), the approximate assessments of the definite integral in Eqs. (10) and (11) can be simplified with the finite numbers of discrete sampling points.

According to Yu et al (2022), 17 discrete sampling points are suggested for the two independent variablesx_1 and x_2 preliminarily. So the Uniform Design Table of $U^*_{17}(17^5)$ is taken to conduct the approximate assessment. The designed results for the 17 discrete sampling points are shown in Table 1 together with the calculated consequences of $W(X)$ and $\theta(X)$, in which x_{10} and x_{20} indicate the original positions from the Uniform Design Table $U^*_{17}(17^5)$ for the $[1, 17] \times [1, 17]$ domain.

Table 2 shows the evaluation results of this problem.

TABLE 1
Designed results U*17(175) together with the calculated consequences of W(X) and θ(X)

No.	x10	x20	x1 / m^2	x2 / m^2	W / T	θ/□
1	1	7	0.003450	0.004178	2.5314	0.0036
2	2	14	0.003593	0.005038	2.9343	0.0033
3	3	3	0.003736	0.003686	2.3776	0.0036
4	4	10	0.003879	0.004546	2.7805	0.0032
5	5	17	0.004021	0.005407	3.1834	0.0030
6	6	6	0.004164	0.004055	2.6267	0.0032
7	7	13	0.004307	0.004915	3.0296	0.0030
8	8	2	0.004449	0.003563	2.4729	0.0032
9	9	9	0.004592	0.004424	2.8758	0.0029
10	10	16	0.004735	0.005284	3.2788	0.0027
11	11	5	0.004877	0.003932	2.7220	0.0029
No.	x10	x20	x1 / m^2	x2 / m^2	W / T	θ/□
12	12	12	0.005020	0.004792	3.1250	0.0027
13	13	1	0.005163	0.003440	2.5682	0.0029
14	14	8	0.005306	0.004301	2.9712	0.0027
15	15	15	0.005448	0.005161	3.3741	0.0025
16	16	4	0.005591	0.003809	2.8174	0.0027
17	17	11	0.005734	0.004669	3.2203	0.0025

Таблица 1 – Полученные результаты U*17(175) вместе с рассчитанными последствиями W(X) и θ(X)
Табела 1 – Пројектовани резултати U*17(175) заједно са израчунатим последицама W(X) и θ(X)

TABLE 2
Evaluation results of this problem

	Partial preferable probability		Total	
No.	$P_{W(x)}$	$P_{\theta(X)}$	$P_t \times 10^3$	Rank
1	0.0659	0.0471	3.1006	16
2	0.0576	0.0536	3.0905	17
3	0.0690	0.0473	3.2642	13
4	0.0608	0.0538	3.2703	12
5	0.0525	0.0592	3.1091	15
6	0.0639	0.0540	3.4512	8
7	0.0557	0.0593	3.3037	11
8	0.0671	0.0542	3.6333	5
9	0.0588	0.0595	3.4993	7
10	0.0506	0.0639	3.2346	14
11	0.0620	0.0596	3.6957	3
12	0.0537	0.0641	3.4426	9
13	0.0651	0.0598	3.8930	1
14	0.0569	0.0642	3.6514	4
15	0.0486	0.0680	3.3052	10
16	0.0600	0.0643	3.8609	2
17	0.0518	0.0681	3.5246	6

Таблица 2 – Результаты оценки данной проблемы
Табела 2 – Резултати процене овог проблема

Table 2 shows that the preliminarily assessed result of the total preferable probability of sampling point *No. 13* exhibits the maximum in the first glance, so the optimal configuration could be around sampling point *No. 13*.

As to sampling point *No. 13*, the optimal mass $W_{optim.}$ of the boom tie rod and the optimal angular displacement $\theta_{optim.}$ of the boom are *2.5682 tons* and *0.0029°* at $x_1 = 0.0052\ m^2$ and $x_2 = 0.0034\ m^2$, which are better than those of Qu's (2004) results of *2.8580 tons*, and $0.0026°$ at $x_1 = 0.0058\ m^2$ and $x_2 = 0.0038\ m^2$, comprehensively.

Moreover, regression can be applied for further optimization. The regressed result of the total probability P_t with respect to x_1 and x_2 is

$$P_t \times 10^3 = 8.2971 - 249.4110x_1 - 304.5570x_2 - 0.0978 \times 10^{-1} x_1^{-1} - 0.0083 \times 10^{-1} x_2^{-1},$$

(12)

$$R^2 = 0.9362.$$ (13)

The regressed result of the W with respect to x_1 and x_2 is

$$W = 2.89\times10^{-15} + 208.3230x_1 + 433.8680x_2,$$ (14)

$$R^2 = 1.$$ (15)

The regressed result of the total probability θ with respect to x_1 and x_2 is

$$\theta = 0.0035 - 0.1459x_1 - 0.2412x_2 - 5.7700\times10^{-6}x_1^{-1} - 1.4000\times10^{-7}x_2^{-1},$$ (16)

$$R^2 = 0.9941.$$ (17)

The optimal result of the regressed formula of Eq. (12) being maximum is $P_t^*\times10^3 = 3.8890$ at $x_1 = 0.0058\ m^2$ and $x_2 = 0.0034\ m^2$; the corresponding values for optimal W and θ are, $W^* = 2.6754$ *tons,* $\theta^* = 0.0028°$ which are much better than those of Qu's results as well.

2) Multi–objective optimization with a single input variable

It is certain that multi–objective optimization with a single input variable is a very simple problem and direct assessment can be conducted.

The simple example is that the optimal solution of the min$f_1(x) = x^2$ together with $\min f_2(x) = (x-2)^2$ simultaneously within the range of $x \in [-5, 7]$, which was discussed by Huang & Chen (2009) with tediously long and complex evolutionary computations of Pareto optimization.

Here, by using the probability–based multi–objective optimization, the problem can be reanalyzed and the partial preferable probability for $f_1(x)$ and $f_2(x)$ can be expressed as,

$$P_{f1} = (49 - x^2)/432, \quad P_{f2} = [49 - (x-2)^2]/432.$$ (14)

Thus, the total preferable probability $P_t = P_{f1} \cdot P_{f2}$ takes its maximum value at $x = 1$ distinctly; therefore, the simultaneous minimum values of $f_1(x)$ and $f_2(x)$ are compromisingly equaled to 1. Obviously, the assessing process is much simpler than that of complex evolutionary computations of Pareto optimization (Huang & Chen, 2009).

Furthermore, if the sampling point method is used, 11 sampling points can be employed for the assessment preliminarily (Yu, et al, 2022). The uniformly distributed sampling points are shown in Table 3 in their domain $x \in [-5, 7]$ together with the value of P_t and their ranking.

TABLE 3
The positions of the distribution of the sampling points in the integral domain 5 7 together with the value of Pt and their ranking

No	Location of point	Pt□10^2	Rank
1	-4.45455	0.114658	6
2	-3.36364	0.408543	5
3	-2.27273	0.722118	4
4	-1.18182	0.991634	3
5	-0.09091	1.171558	2
6	1.00000	1.234568	1
7	2.09091	1.171558	2
8	3.18182	0.991634	3
9	4.27273	0.722118	4
10	5.36364	0.408543	5
11	6.45455	0.114658	6

Таблица 3 – Положения распределения точек выборки в интегральной области [-5, 7] вместе со значением Pt и их ранжированием
Табела 3 – Позиције дистрибуције тачака узорковања у домену интеграла [-5, 7] заједно са вредношћу Pt и њихово рангирање

Again, the maximum value for P_t is located at $x = 1$ exactly.

Discussion

1) On the number of the discrete sampling points in the evaluation

In the literature of Yu et al (2022), it is suggested roughly but not proven mathematically that 17 and 19 sampling points are proper preliminarily for evaluating a complicated integral.

Here, we would stress the following. In accordance wih Hua and Wang (1081) and Fang and Wang (1994), as to the GLP, the discrepancy of the low–discrepancy point set is $O(p^{-1}(\log p)^{s-1})$ for the s – dimension with the prime number p, so if we take 11 GLPs for a 1 – dimensional problem, the value of $O(1/11) \approx 0.0909$, i.e., less than 10%; analogically, for a 2 – dimensional problem, if we adopt to use 17 GLPs, the value of $O(p^{-1}(\log p)^{s-1})$ is approximately $O(17^{-1}(\log 17)^1) \approx 0.0724$, which is near to the situation of 1 – dimensional problem; while for a 3 – dimensional problem, if we take 19 GLPs, the approximate result of $O(p^{-1}(\log p)^s)$ is $O(19^{-1}(\log 19)^2) \approx 0.0861$, which is close to the situation of a 1 – dimensional problem as well. However, if we accept 23, 29, 31 or even 41 GLPs for 3-d, the consequences for $O(p^{-1}(\log p)^{s-1})$ are 0.0806, 0.0737, 0.0717, or 0.0634, respectively, which are nearly the same as that of 19 GLPs basically.

The successful results of assessing complicated definite integrations realize the applicability of the approximation from the point of view of engineering practice. Perhaps the abstruse physical detail is related to the spatial correlation of spatial sampling points, which was pointed by Ripley (1981) and worth to be further explored by mathematicians.

2) On the combination of the finite sampling points in probability- based multi–objective optimization by means of the Uniform Experimental Design

The newly developed efficient approach for preliminarily assessing a definite integral with a small number of sampling points can be combined with the novel probability–based multi–objective optimization (PMOO), provided the discrete specimen points are uniformly and deterministically distributed within the domain according to the rules of the GLP and the UED. The optimal results in the present paper for typical examples indicate the advantages of this treatment. However, further applications and mathematical intensions of the appropriate algorithm for assessing numerical integration developed newly are needed to be deeply explored in future.

Besides, in order to improve the precision of approximate maximum by using discrete sampling point method, sequential algorithm for optimization can be combined with the probability – based multi – objective optimization in its discreterization (Zheng et al, 2022).

Conclusion

From the above discussion, the efficient approach for preliminarily calculating a definite integral with a small number of sampling points is successfully combined with the novel probability–based multi–objective optimization (PMOO) so as to simplify the complicated calculation of a definite integral in PMOO. The Uniform Experimental Design method and the good lattice point are involved in the combination, thus significantly simplifying complicated data processing by approximation.

References

Fang, K. 1980. Uniform design — Application of Number Theory Method in Experimental Design. *Acta Mathematicae Applicatea Sinica,* 3(4), pp.363-272..

Fang, K-T., Liu, M-Q., Qin, H. & Zhou, Y-D. 2018. *Theory and Application of Uniform Experimental Designs*. Beijing: Science Press & Singapore: Springer Nature. Available at: https://doi.org/10.1007/978-981-13-2041-5.

Fang, K-T. & Wang, Y. 1994. *Number-theoretic Methods in Statistics*. London, UK: Chapman & Hall. ISBN: 0-412-46520-5.

Hua, L-K. & Wang, Y. 1981. *Applications of Number Theory to Numerical Analysis*. Berlin & New York: Springer-Verlag & Beijing: Science Press. ISBN: 9783540103820.

Huang, B. & Chen, D. 2009. Effective Pareto Optimal Set of Multi-objective Optimization Problems. *Computer & Digital Engineering, 37*(2), pp.28-34 [online]. Available at: https://caod.oriprobe.com/articles/17362139/Effe ctive_Pareto_Optimal_Set_of_ Multi_Objective_Op.htm [Accessed: 20 March 2022].

Paskov, S.H. 1996. New methodologies for valuing derivatives. In: Pliska, S. & Dempster, M. & (Eds.) *Mathematics of Derivative Securities*, pp.545-582. Cambridge: Isaac Newton Institute & Cambridge University Press. Available at: https://doi.org/10.7916/D8TB1FRJ.

Paskov, S.H. & Traub, J.F. 1995. Faster valuation of financial derivatives. Journal of Portfolio Management 22(1), pp.113-120. Available at: https://doi.org/10.3905/jpm.1995.409541.

Qu, X., Lu, N., & Meng, X. 2004. Multi-objective Fuzzy Optimization of Tower Crane Boom Tie Rods. *Journal of Mechanical Transmission,* 28(3), pp.38-40 [online]. Available at: https://caod.oriprobe.com/articles/7413876 /Fuzzy_Optimization_of_Arm_Link_R od_in_Tower_Crane.htm [Accessed: 20 March 2022].

Ripley, B.D. 1981. *Spatial Statistics.* Hoboken, NJ: John Wiley & Sons. ISBN: 0-47169116-X.

Sloan, I.H. & Wozniakowski, H. 1998. When Are Quasi-Monte Carlo Algorithms Efficient for High Dimensional Integrals?. *Journal of Complexity*, 14(1), pp.1-33. Available at: https://doi.org/10.1006/jcom.1997.0463.

Tezuka, S. 1998. Financial applications of Monte Carlo and Quasi-Monte Carlo methods. In: Hellekalek, P. & Larcher, G. (Eds.) *Random and Quasi- Random Point Sets. Lecture Notes in Statistics,* 138, pp.303-332. New York: Springer. Available at: https://doi.org/10.1007/978-1-4612-1702-2_7.

Tezuka, S. 2002. Quasi-Monte Carlo - Discrepancy between theory and practice. In: Fang, K.T., Niederreiter, H. & Hickernell, F.J. (Eds.) *Monte Carlo and Quasi-Monte Carlo Methods 2000*, pp.124-140. Heidelberg: Springer-Verlag. Available at: https://doi.org/10.1007/978-3-642-56046-0_8.

Wang, Y. & Fang, K. 2010. On number-theoretic method in statistics simulation. *Science in China Series A: Mathematics,* 53, pp.179-186. Available at: https://doi.org/10.1007/s11425-009-0126-3.

Yu, J., Zheng, M., Wang, Y. & Teng, H. 2022. An efficient approach for calculating a definite integral with about a dozen of sampling points. *Vojnotehnički glasnik/Military Technical Courier*, 70(2), pp. 340-356. Available at: h ttps://doi.org/10.5937/vojtehg70-36029.

Zheng, M. 2022. Application of probability-based multi–objective optimization in material engineering. *Vojnotehnički glasnik/Military Technical Courier*, 70(1), pp.1-12. Available at: https://doi.org/10.5937/vojtehg70-35366.

Zheng, M., Teng, H., Yu, J., Cui, Y. & Wang, Y. 2022. *Probability-Based Multi-objective Optimization for Material Selection*. Singapore: Springer. ISBN: 978-981-19-3350-9.

Zheng, M., Wang, Y. & Teng, H. 2021. A New "Intersection" Method for Multi-objective Optimization in Material Selection. *Tehnički glasnik,* 15(4), pp.562-568. Available at: https://doi.org/10.31803/tg-20210901142449.

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