Vojnotehnicki glasnik/Military Technical Courier
ISSN: 0042-8469
ISSN: 2217-4753
vojnotehnicki.glasnik@mod.gov.rs
University of Defence
Serbia

Regularization in quantum field theories

Fabiano, Nicola

Regularization in quantum field theories
Vojnotehnicki glasnik/Military Technical Courier, vol. 70, no. 3, 2022
University of Defence, Serbia
Available in: https://www.redalyc.org/articulo.oa?id=661772310012
DOI: https://doi.org/10.5937/vojtehg70-34284

Review papers

Regularization in quantum field theories

РЕГУЛЯРИЗАЦИЯ В КВАНТОВЫХ ТЕОРИЯХ ПОЛЯ

РЕГУЛАРИЗАЦИЈА У ТЕОРИЈАМА КВАНТНОГ ПОЉА

Nicola Fabiano
University of Belgrade, Serbia
nicola.fabiano@gmail.com
https://orcid.org/0000-0003-1645-2071

DOI: https://doi.org/10.5937/vojtehg70-34284
Redalyc: https://www.redalyc.org/articulo.oa?id=661772310012

Received: 10 May 2022
Accepted: 24 June 2022

Abstract:

Introduction/purpose: The principal techniques of regularization schemes and their validity for gauge field theories are discussed.
Methods: Schemes of dimensional regularization, Pauli–Villars and lat- tice regularization are discussed.
Results: The Coleman–Mandula theorem shows which gauge theories are renormalizable.
Conclusion: Some gauge field theories are renormalizable, the Standard Model in particular.

Keywords: regularization, renormalization, Gauge Field Theory, Coleman–Mandula Theorem..

Резюме:

Введение/цель: В данной статье рассматриваются основные методы схем регуляризации и их применимость в калибровочных теориях полей.
Методы: В статье применены схемы размерной регуляризации, Паули - Вилларса и регуляризации решетки. обсуждаются регуляризация.
Результаты: Теорема Коулмана-Мандулы показывает какие калибровочные теории подлежат ренормализации.
Выводы: В ходе исследования выявлено, что некоторые теории калибровочного поля перенормируемы, в частности – стандартная модель.

Ключевые слова: регуляризация, перенормировка, теория калибровочного поля, теорема Колемана - Мандулы.

Abstract:

Увод/циљ: Разматрају се основне технике шема регулари- зације као и њихова ваљаност за теорије калибрационих поља.
Методе: Примењују се шеме димензионалне регулариза- ције, Паули-Виларсова регуларизација као и регуларизације решетке.
Резултати: Колеман-Мандула теорема показује које кали- брационе теорије се могу ренормализовати.
Закључак: Неке теорије калибрационог поља се могу ре- нормализовати, специфично стандардни модел.

Keywords: регуларизација, ренормализација, теорија калибрационог поља, Колеман–Мандула теорема.

Regularization schemes

Up to now, we have encountered quantum electrodynamics and other theories such as the scalar potential ϕ^4 and the Standard Model (Fabiano, 2021a,b). In QED, we have seen in some detail how to get rid of infinities coming from loop integrations and obtain meaningful results for physical quantities with renormalization. For this purpose, we have used dimensional regularization, but there are other regularization schemes with different properties.

Dimensional regularization

This is the scheme we have already used in (Fabiano, 2021a,b), per- haps the most versatile one (Bollini & Giambiagi, 1972; 't Hooft & Veltman, 1972). First, a Wick rotation (Wick, 1954) is performed to an Euclidean space. Then the action is extended to an arbitrary dimension D that becomes a complex number. In these regions, all Feynman diagrams are finite. All integrals are analytically continued for $D \to 4$, and the resul- ting simple poles due to Gamma functions are to be reabsorbed into the physical parameters. This scheme, beyond its simplicity, has the great advantage of preserving all symmetries of the theory that do not depend on dimensionality such as gauge symmetry, Poincaré symmetry etc., as well as the Ward–Takahashi identities (Ward, 1950; Takahashi, 1957). A remark on the notation. We have already encountered the minimal subtraction scheme MS ('t Hooft, 1973; Weinberg, 1973), where the counterterms computed with dimensional regularization have no finite part. There is another widely used scheme, the *modified minimal subtraction scheme*, or the $\overline{\text{MS}}$ (Bardeen et al, 1978), where the finite part is a constant by means of the substitution

$$\frac{1}{D-4} \to \frac{1}{D-4} + \frac{\gamma}{2} - \frac{1}{2}\log 4\pi \ , \tag{1}$$

where, as usual, $\gamma \approx 0.57721$ is the Euler–Mascheroni constant.

Pauli–Villars regularization

In this procedure of 1949 (Pauli & Villars, 1949) the propagator is modified as:

$$\frac{1}{p^2-m^2} \to \frac{1}{p^2-m^2} - \frac{1}{p^2-M^2} = \frac{m^2-M^2}{p^4} + \frac{m^2-M^2}{p^6} + \mathcal{O}\left(\frac{1}{p^8}\right) \ , \tag{2}$$

where the fictitious mass is chosen $M \gg m$. The propagator behaviour for large momenta $\sim 1/p^4$ is usually enough to render finite all Feynman graphs. Eventually, the $M^2 \to +\infty$ limit is taken to decouple the unphysical particle. This technique has the advantage of preserving local gauge invariance in QED, as well as Ward identities.

Lattice regularization

Another popular scheme is the lattice regularization, where the theory is defined on a four–dimensional Euclidean lattice with the finite spacing a (Wilson, 1975; Kadanoff, 1966). This spacing serves as a cutoff $\Lambda = 1/a$ for the Feynman integrals, rendering the results finite. This approach is mostly used for QCD, and results are extrapolated to the continuum limit for $a \to 0$comparing different lattice spacings. Almost invariably, this method is used to simulate QCD on computers using Monte Carlo methods. The symmetry on the lattice is of course lost as Lorentz invariance is broken. There is also the problem of fermion doubling, with the appearance of more particles for each original fermion. This approach is also very computationally intensive with large memory bandwidth requirements.

Overview of renormalization

The divergences are given by graphs with loops. To determine the degree of divergence of any graph we need to know the dimensions of various fields, coupling constants and the behaviour of propagators at large momenta. As the action is given by

$$S = \int \mathrm{d}^D x \; \mathcal{L}(\phi, \partial\phi) \tag{3}$$

and has the dimensions of #, that is zero dimensions in our units, $[S]=0$, then the Lagrangian has the dimensions in length units (for energy units just reverse the sign)

$$[\mathcal{L}] = -D \; . \tag{4}$$

From the free action for a generic bosonic field ϕ and for a 1/2 spin fermion ψ, we readily obtain

$$[\phi] = -\frac{D-2}{2} \tag{5}$$

and

$$[\psi] = -\frac{D-1}{2} \; . \tag{6}$$

The dimensions of the coupling constants are then easily computed, for instance in the Higgs potential with $g\phi^4/4!$ interaction, $[g]=D-4$, so in 4 dimension g is dimensionless. We will now calculate the *superficial degree of divergence* D of a Feynman diagram. Any diagram with loops could be represented by

$$\int \mathrm{d}^D p \; f(p) = \int \mathrm{d}p \; F(p) \; ,$$

(7)

(f is made out of different propagators in general) and the behaviour of F when all internal momenta go to infinity gives the superficial degree of convergence D

$$F(p) \sim p^{D-1} \text{ for } p \to +\infty \ . \tag{8}$$

When $D > 0$, the diagram diverges like a power

$$\int^{\Lambda} \mathrm{d}p \, p^{D-1} \sim \Lambda^{D} \ , \tag{9}$$

while if $D = 0$ implies a logarithmic divergence, $\log \Lambda$, and the integrals with $D < 0$ are convergent.

The asymptotic behaviour for large momenta of various propagators are well known: for bosonic scalar fields ϕ and vector fields A_μ . it is $1/p^2$, while for electron (lepton) fields ψ is $1/p$. In general, the asymptotic behaviour for a propagator $\Delta_f(p)$ of a field f is given by

$$\Delta_f(p) \sim p^{-2+2s_f} \ , \tag{10}$$

and it can be shown that for a massive field f that transforms under Lorentz group as (A, B) one has $s_f = A + B$, so loosely speaking s_f is the "spin" of field. For massless bosonic fields, $s_f = 0$. The photon $(\text{spin}=1)$ propagator and also the graviton field $g_{\mu\nu}$ (with $\text{spin}=2$) behave like $1/p^2$.

By power counting, one could calculate the superficial degree of convergence D. Each fermion propagator contributes to p^{-1}, each boson propagator gives a p^{-2} term, each loop from integration contributes with a p^4 term, and each vertex with n derivatives contributes at most with a p^4 term. We will see the superficial degree of divergence for QED graphs in some detail. Define

$$\begin{aligned} L &= \text{number of loops}, \\ V &= \text{number of vertices}, \\ E_\psi &= \text{number of external electron legs}, \\ I_\psi &= \text{number of internal electron legs}, \\ E_A &= \text{number of external photon legs}, and \\ I_A &= \text{number of internal photon legs}\ , \end{aligned} \tag{11}$$

$$D = 4L - 2I_A - I_\psi\ . \tag{12}$$

We want to rewrite this relation as a function of external legs only, no matter how many internal legs or loops the graph may have.

Consider electrons. Each vertex connects to one end of an internal electron leg. For external legs, only one end connects onto a vertex, thus:

$$V = I_\psi + \frac{1}{2}E_\psi \text{ implies } I_\psi = V - \frac{1}{2}E_\psi\ . \tag{13}$$

For photons, each vertex connects to one end of an internal photon line, unless it is external, that is

$$V = 2I_A + E_A \text{ implies } I_A = \frac{1}{2}(V - E_A)\ . \tag{14}$$

We know that the total number of independent momenta is equal to L, which in turn equals the total number of internal lines in the graph minus the number of vertices, because of moment conservation at each vertex, plus one, as we have overall momentum conservation as well. So:

$$L = I_\psi + I_A - V + 1\ . \tag{15}$$

By substituting for I_ψ, I_A, L the expressions found in eqs. (13)–(15) into eq. (12), we obtain

$$D = 4 - \frac{3}{2}E_\psi - E_A \ . \tag{16}$$

What is renormalizable?

The procedure of renormalization we have met in QED is not substantially different from any other theory. When calculating Feynman diagrams one encounters diagrams with momenta integration inside loops. These integrals diverge, and have to be regularized in some manner, that is, their divergencies should be isolated. Then these infinities are reabsorbed by a set of bare physical parameters, such as coupling constants and masses. These parameters have divergencies that cancel out the ultraviolet infinities coming from loops in Feynman diagrams. Eventually, we are left with the physical (or "renormalized" or "dressed") parameters, that are the actual parameters one could measure in an experiment.

Since there is only a finite number of such parameters in a Lagrangian, one can make only a finite number of such redefinitions. In other words, it is possible to renormalize only a theory with a finite number of fundamentally divergent diagrams that are the building blocks of all divergent diagrams of the theory. For instance, QED is such a theory, and we have encountered those kinds of diagrams in (Fabiano, 2021a,b).

Of course, all this procedure has to be built on solid grounds, requiring a sound mathematical proof that this can be actually done. It is usually done by an induction argument, that is, if one proves that the n th order of a theory is finite, and the $n+1$th order is finite in terms of the n th order, then the theory is renormalizable. The induction proof uses Weinberg's theorem, which essentially states that a Feynman graph converges if the superficial degree of the divergence D of the graph and all its subgraphs is negative. We will now find out whether a particular theory is renormalizable. Consider its Lagrangian and compute the dimensions of the coupling g starting from eqs. (4)–(6). Let d be the length dimension of g, that is

$$[g] = d \ , \tag{17}$$

and from the scaling of the Lagrangian parameters we have met in (Fabiano, 2021b), eq. (16) in particular, for which $e = e_0\mu^{-(4-D)/2}\ldots$, we could deduce the scaling

$$g \sim g_0 L^{-d} \ \text{ or } \ g \sim g_0 E^d \ , \tag{18}$$

L being a length scale, E an energy scale, and g_0 the bare coupling constant. Suppose now that $d > 0$, then we see that with decreasing distance, or increasing energy, the coupling constant g increases indefinitely:

$$g = +\infty \text{ for } \ L \to 0, \text{ or } \ E \to +\infty \ . \tag{19}$$

As the coupling constant increases, perturbation theory will fail; therefore, it will not be renormalizable.

So, we have obtained the important result: if the length dimension of the coupling constant is positive, then the theory is *non renormalizable*. On the other hand, if d is negative, $g \to 0$ for increasing energy, then perturbation theory is applicable. In this case, the theory is called *super renormalizable*. If the coupling constant is adimensional, then the theory is *renormalizable*.

Non renormalizable theories

Non renormalizable theories have coupling constants with negative energy dimensions: for instance, any theory with the interaction $g\phi^n$ with $n > 4$ in four dimensions. Such theories have infinite divergent Feynman diagrams of infinite different kinds. The proliferation of different types of divergencies cannot be controlled by redefinition of a finite number of physical parameters.

Some examples of such theories are:

Any nonpolinomial action: an action that has an infinite number of terms like $\sum_{n=3}^{+\infty} g_n c_n \phi^n$. Independently of the dimension there will be an (infinite) number of dimensionful coupling constants with negative energy dimensions.

Fermi's interaction: the four fermion interactions proposed by Fermi in 1934 (Fermi, 1934a,b) much before the electroweak theory, $G_F(\overline{\psi}\psi)^2$. As it is well known, $G_F \sim 1/m_W^2$, so the coupling has the energy dimension of -2.

Massive vector boson with a non Abelian gauge group: a vector field with mass M has a propagator such as

$$\frac{g_{\mu\nu} - p_\mu p_\nu / M^2}{p^2 - M^2 + i\epsilon}$$

that goes like a constant $-1/M^2$ at infinity. No integral of a loop diagram could converge with such behaviour.

Gravitation: Newtonian potential is Gm_1m_2/r. So G has negative energy dimensions.

Theories with anomalies: symmetries of the original classical Lagrangian could be broken by quantum effects and are called *anomalies*. They in turn spoil Ward–Takahashi identities, essential for proving that a theory could be renormalizable.

Renormalizable theories

These theories are of course the most important ones. They have only a finite numbers of necessary counterterms, and their coupling constant is adimensional. Some examples follow.

ϕ^4 **in four dimensions:** a scalar field with such interaction, like the Higgs potential, has a dimensionless coupling constant g for $D = 4$. From hints by the ϵ–expansion method, this theory is also probably free in four dimensions.

QED: we already discussed quantum electrodynamics in (Fabiano, 2021a,b), and explicitly wrote the counterterms. Historically, it was the first theory to be proven renormalizable.

Standard Model: the SM of particles with a gauge group $SU_{col}(3) \times SU_L(2) \times U_Y(1)$ broken to $SU_{col}(3) \times U_{em}(1)$ has three adimensional coupling constants (Glashow, 1959; Salam & Ward,1959; Weinberg, 1967). Notice, however, that electroweak model alone, $SU_L(2) \times U_Y(1)$, is not renormalizable. The further presence of quarks is needed in order to cancel all anomalies and render the SM anomaly free.

Yukawa theory: it is also part of the SM. It describes a coupling between fermions and scalars given by

$$g\phi\overline{\psi}\psi$$

the coupling constant g is, as usual, dimensionless (Yukawa, 1935).

Spontaneously broken non Abelian gauge theories: although we have seen that a massive vector boson is non renormalizable, spontaneously broken massless non Abelian gauge symmetries are actually renormalizable. These are spontaneously broken Yang–Mills theories. The proof was given by 't Hooft and Veltman in 1972 ('t Hooft & Veltman, 1972), and only after that the usage of gauge theories was fully justified. It is important to notice that unbroken Yang–Mills theories are renormalizable only in four dimensions.

Two dimensional fermion theory: for $D = 2$, a term $(\overline{\psi}\psi)^2$ of Fermi's theory is renormalizable there.

Super renormalizable theories

They converge very rapidly, only a finite number of graphs is divergent. Actually, the degree of divergence decreases when the number of loops increases.

ϕ^3 **:** in three dimensions, this bosonic theory is super renormalizable. However, this theory is ill–defined because the potential is unbounded from below, so the vacuum is unstable.

ϕ^4 **:**in three dimensions, this theory is super renormalizable as its coupling is such that $[g] = D - 4$, negative for$D < 4$.

Two dimensional boson theory:for $D = 2$, that is, only time and a space coordinate, there is a sort of magic. Any theory of bosonic field is super renormalizable, because the field itself is dimensionless, and $[g] = -2$.

Two dimensional theory: combining the results previously obtained, in two dimensions a theory

$$P(\phi)\overline{\psi}\psi$$

where P is an arbitrary polynomial is super renormalizable.

Why gauge theory?

We have followed the full path starting from the Lagrangian to a measurable physical quantity. On our walk, we have encountered infinite quantities and rigorous results that allow us to get rid of them. All the time we have dealt with gauge theories that combine Poincaré group invariance (that is the Lorentz plus translation group) and some internal symmetry groups, the gauge group, for instance $U(1)$ for QED or $SU(3)$ for QCD.

A question naturally arises whether it is possible to have theories with different kinds of symmetries than those previously described, which are able to give physically meaningful results?

This question has been answered by the Coleman–Mandula *no–go theorem* of 1967 (Coleman & Mandula, 1967) and, to a certain extent, the short answer is "no".

We recall that the Lorentz group preserves the distance with Minkowski metric $s^2 = x_\mu g^{\mu\nu} x_\nu$. It has $L_{\mu\nu}$ generators of rotations, boosts and inversions that obey the $SO(3, 1)$ Lie algebra

$$[L_{\mu\nu}, L_{\rho\sigma}] = ig_{\mu\sigma}L_{\nu\rho} + ig_{\nu\rho}L_{\mu\sigma} - ig_{\mu\rho}L_{\nu\sigma} - ig_{\nu\sigma}L_{\mu\rho} \ . \tag{20}$$

Remember that the Lie algebra is defined by its generators T^a with com- muting properties

$$[T^a, T^b] = if^{abc}T^c \ , \tag{21}$$

where f^{abc} is the structure constant. The Lie algebra is obtained from the Lie group by taking the logarithm of group elements G.

The generators $L_{\mu\nu}$ together with the generators of translations P^μ form the Poincaré algebra. While the translations commute among them

$$[P^\mu, P^\nu] = 0 \ , \tag{22}$$

they do not commute with the Lorentz generator, because the latter has two indices opposed to only one:

$$[L^{\mu\nu}, P^\rho] = ig^{\mu\rho}P^\nu - ig^{\mu\nu}P^\rho \ . \tag{23}$$

Wigner (Wigner, 1939) gave all possible classifications for real particles from the Poincaré group, where states are labelled by the invariant mass $P^2 = m^2$, the spin s and the helicity h.

1. $P^2 = m^2 > 0$ and the spin s is discrete, then the state is $|m, s\rangle$, $s = 0, 1/2, 1, 3/2, \ldots$.
2. $P^2 = m^2$, and the state is determined by its helicity, $|h\rangle$, where $h = \pm s$, $s = 0, 1/2, 1, 3/2, \ldots$.
3. $P^2 = m^2 = 0$, and the spin is continuous, so h. is continuous. These states do not seem to be realized in nature.

Coleman–Mandula theorem

It states that, given some reasonable physical assumptions we will discuss later, the only possible Lie algebra of symmetry generators consist of the generators of the Poincaré group and of some other symmetry generators of the gauge group that commute between them. Let $\mathcal{P}$ be the Poincaré group, P its algebra, and the symmetry group, $\mathcal{G}$ its algebra. Then the only possible algebra CM of allowed symmetry group $\mathcal{CM}$ is given by the direct product of those two, that is

$$CM = P \otimes G \ . \tag{24}$$

In plain language, it means these two groups never mix, the Lorentz indices do not affect the group indices and vice versa. For instance, in QED, an $U(1)$ rotation will not affect electron energy, likewise a Lorentz boost is unable to flip electron charge.

The assumptions of this theorem are very reasonable. Consider the scattering matrix S, and its symmetry group $\mathcal{CM}$ with the following assumptions

- **Mass gap:** for any given mass $m > 0$ there is only a finite number of particles with mass less than m. No continuous spectrum is allowed.
- **Scattering:** it occurs at almost all energies except maybe for some discrete set of energies.
- **Analyticity:** the S matrix for two body scattering is an analytic function of angle, energy and momentum, except maybe for some discrete set of energies.
- **"Ugly technical assumption":** stating that the matrix elements of the group generators are distributions in momentum space.

Under these assumptions, the only allowed algebra for the symmetry group $\mathcal{CM}$ of the S matrix is given by eq. (24).

There is actually a possible way out of this theorem. If one considers a symmetry that exchanges bosons with fermions, so called *supersymmetry*, then it is possible to extend this particular symmetry to the allowed symmetries of the S matrix without breaking the Coleman–Mandula theorem, which is known as the Haag-Łopuszański-Sohnius theorem (Haag et al, 1975).

It must be stressed, however, that up to this date supersymmetric particles are yet to be discovered.

References

Bardeen, W.A., Buras, A.J., Duke, D.W. & Muta, T. 1978. Deep-inelastic scattering beyond the leading order in asymptotically free gauge theories. *Physical Review D*, 18(11), pp.3998-4017. Available at: https://doi.org/10.1103/PhysRevD.18.3998.

Bollini, C.C. & Giambiagi, J.J. 1972. Dimensional renorinalization : The number of dimensions as a regularizing parameter. *Il Nuovo Cimento B (1971-1996)*, 12(1), pp.20–26. Available at: https://doi.org/10.1007/BF02895558.

Coleman, S. & Mandula, J. 1967. All Possible Symmetries of the . Matrix. P*hysical Review*, 159(5), pp.1251-1256. Available at: https://doi.org/10.1103/PhysRev.159.1251.

Fabiano, N. 2021a. Quantum electrodynamics divergencies. *Vojnotehnički glasnik/Military Technical Courier*, 69(3), pp.656-675. Available at: https://doi.org/10.5937/vojtehg69-30366.

Fabiano, N. 2021b. Corrections to propagators of quantum electrodynamics. *Vojnotehničkiglasnik/Military Technical Courier*, 69(4), pp.930-940. Available at: https://doi.org/10.5937/vojtehg69-30604.

Fermi, E. 1934a. Tentativo di una teoria dei raggi β. *Il Nuovo Cimento (1924- 1942)*, 11, art.number:1 (in Italian). Available at: https://doi.org/10.1007/BF02959820.

Fermi, E. 1934b. Versuch einer Theorie der β-Strahlen. I. *Zeitschrift für Physik*, 88(3-4), pp.161-177 (in German). Available at: https://doi.org/10.1007/BF01351864.

Glashow, S. 1959. The renormalizability of vector meson interactions. *Nuclear Physics*, 10(February–May), pp.107-117. Available at: https://doi.org/10.1016/0029-5582(59)90196-8.

Haag, R.J., Łopuszański, J.T. & Sohnius M. 1975. All possible generators of supersymmetries of the .-matrix. *Nuclear Physics B*, 88(2), pp.257-274. Available at: https://doi.org/10.1016/0550-3213(75)90279-5.

't Hooft, G. 1973. Dimensional regularization and the renormalization group. Nuclear Physics B, 61, pp.455-468. Available at: https://doi.org/10.1016/0550-3213(73)90376-3.

't 'Hooft, G. & Veltman, M. 1972. Regularization and renormalization of gauge fields. *Nuclear Physics B*, 44(1), pp.189–213. Available at: https://doi.org/10.1016/0550-3213(72)90279-9.

Kadanoff, L.P. 1966. Scaling laws for Ising models near *T.*. *Physics Physique Fizika*, 2(6), pp.263-272. Available at: https://doi.org/10.1103/PhysicsPhysiqueFizika.2.263.

Pauli, W. & Villars F. 1949. On the Invariant Regularization in Relativistic Quantum Theory. *Reviews of Modern Physics*, 21(3), pp.434-444. Available at: https://doi.org/10.1103/RevModPhys.21.434.

Salam, A. & Ward, J.C. 1959. Weak and electromagnetic interactions. *Il Nuovo Cimento (1955-1965)*, 11(4), pp.568-577. Available at: https://doi.org/10.1007/BF02726525.

Takahashi, Y. 1957. On the generalized ward identity. *Il Nuovo Cimento (1955- 1965)*, 6(2), pp.371–375. Available at: https://doi.org/10.1007/BF02832514.

Ward, J.C. 1950. An Identity in Quantum Electrodynamics. *Physical Review*, 78(2), p.182. Available at: https://doi.org/10.1103/PhysRev.78.182.

Wick, G.C. 1954. Properties of Bethe-Salpeter Wave Functions. *Physical Re- view*, 96(4), pp.1124–1134. Available at: https://doi.org/10.1103/PhysRev.96.1124.

Weinberg, S. 1967. A Model of Leptons. *Physical Review Letters*, 19(21), pp.1264-1266. Available at: https://doi.org/10.1103/PhysRevLett.19.1264.

Weinberg, S. 1973. New Approach to the Renormalization Group. *Physical Re- view D*, 8(10), pp.3497-3509. Available at: https://doi.org/10.1103/PhysRevD.8.3497.

Wigner, E. 1939. On unitary representations of the inhomogeneous Lorentz group. *Annals of Mathematics*, 40(1), pp.149–204. Available at: https://doi.org/10.2307/1968551.

Wilson, K.G. 1975. The renormalization group: Critical phenomena and the Kondo problem. *Reviews of Modern Physics*, 47(4), pp.773–840. Available at: https://doi.org/10.1103/revmodphys.47.773.

Yukawa, H. 1935. On the interaction of elementary particles. *Proceedings of the Physico-Mathematical Society of Japan. 3rd Series*, 17, pp.48-75. Available at: https://doi.org/10.11429/ppmsj1919.17.0_48.

Additional information

FIELD: Mathematics

ARTICLE TYPE: Review paper

Alternative link

https://scindeks.ceon.rs/article.aspx?artid=0042-84692203720F (html)
https://aseestant.ceon.rs/index.php/vtg/article/view/34284 (pdf)
https://doaj.org/article/46edef0d31fb4982988f8a7d9c2d7ae3 (pdf)
https://elibrary.ru/item.asp?id=48719264 (pdf)
http://www.vtg.mod.gov.rs/archive/2022/military-technical-courier-3-2022.pdf (pdf)