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 **Bjelić, Slobodan N.**

 **Marković, Nenad A.**

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Original scientific papers

A more advanced theoretical model of the sphere earth's EM in a foreign homogeneous EM field

Прогрессивная теоретическая модель ЭМ поля земной сферы Земли в чужом однородном ЭМ поле
Напреднији теоријски модел ЕМ поља сфере Земље у страном хомогеном ЕМ пољу

Slobodan N. Bjelić

University of Priština – Kosovska Mitrovica, Serbia

slobodanbjelic49@yahoo.com

 <https://orcid.org/0000-0001-5642-8936>

Nenad A. Marković

Kosovo and Metohija Academy of Applied Studies,
Serbia

nen.mark74@yahoo.com

 <https://orcid.org/0000-0001-6960-1953>

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Abstract

Introduction/purpose: The paper describes a more advanced theoretical model of the Earth's EM field based on two-component hypotheses. A defined mathematical model that shows the rotation of the magnetically conducting sphere of the magnetization M in a foreign magnetic field and the components of the magnetic field that may arise due to the rotation of the Earth around its axis. According to the established model, in relation to the reference values of the planet Earth, the values of the components of the other planets in the solar system were calculated and the results were tabulated.

Methods: The solution to the problem highlighted in the title of the paper was determined using the combined, for that purpose, formalized methods of physics and mathematical analysis, in order to develop a new, more advanced mathematical model. For this purpose, the method of analogy was used, related to the application of similar structural forms and systems for researching electromagnetic processes and planetary rotation. The method of analogy was applied for two interrelated reasons. The first one is that all values that characterize the function of any natural system are subject to change, and the second one is that the applied solutions do not determine the conditions of the structure's function in each specific case.

Results: The solutions in the form of original analytical formulas and numerical values arranged in Table 2, referring to the influence of the rotation of the planets and especially the Earth, will be applied to research the effects of the EM field emitted by the Sun towards the planets, especially the role that the process plays in protecting the planet Earth. The results given in Table 2 are particularly important.

Conclusion: The paper discusses the appearance and effect of the Earth's EM field in a way that is understandable at the current level of scientific development. Scientific findings in science and measurements in geo- and astrophysics indicate the Sun as a possible source of the EM field that extends through interplanetary space and the component of the Earth's magnetic field is only a response to the influence of that source. Natural phenomena and processes on the Earth can be defined in system theory by a model that contains changes in the parameters of the state of the planet.

Keywords: advanced model, theory, planets, rotation, magnetism, magnetic field.

Author notes

a University of Priština – Kosovska Mitrovica, Faculty of Technical Sciences, Kosovska Mitrovica, Republic of Serbia

b Kosovo and Metohija Academy of Applied Studies, Department Uroševac – Leposavić, Leposavić, Republic of Serbia

nen.mark74@yahoo.com

Резюме

Введение/цель: В данной статье описывается прогрессивная теоретическая модель электромагнитного поля Земли, основанная на двухкомпонентных гипотезах. В статье определены математическая модель, показывающая вращение магнитосферы M в чужом магнитном поле и компоненты магнитного поля, которые могут возникнуть вследствие вращения Земли вокруг своей оси. По установленной модели по отношению к эталонным значениям планеты Земля были рассчитаны значения компонентов других планет Солнечной системы. Результаты расчетов представлены в виде таблиц.

Методы: Решение проблемы, обозначенное в заголовке статьи, найдено с помощью комбинированных для этой цели формализованных методов физического и математического анализа с целью разработки новой, прогрессивной математической модели. Для этого использовался метод аналогии, связанный с применением сходных структурных форм и систем для исследования электромагнитных процессов и вращения планет. Метод аналогии применялся по двум взаимосвязанным причинам. Первая заключается в том, что все величины, характеризующие функцию любой естественной системы, подвержены изменению, а вторая – в том, что применяемые решения не определяют условия функционирования структуры для каждого отдельного случая.

Результаты: Решения в виде оригинальных аналитических формул и многочисленных значений, упорядоченных в Таблице 2, относящихся к влиянию вращения планет, в частности Земли, будут применены для исследования воздействия электромагнитного поля, излучаемого Солнцем по отношению к другим планетам. Это особенно важно в изучении роли, которую этот процесс играет в защите планеты Земля. В Таблице 2 приведены особо важные результаты.

Выводы: В статье обсуждаются возникновение и влияние электромагнитного поля Земли в доступной форме для понимания на современном уровне развития науки. Научные исследования и измерения в геофизике и астрофизике указывают на Солнце как на возможный источник ЭМ поля, распространяющийся через межпланетное пространство, а составляющая магнитного поля Земли является лишь откликом на влияние этого источника. Природные явления и процессы на Земле могут быть представлены в теории систем моделью, содержащей изменения параметров состояния планеты.

Ключевые слова: усовершенствованная модель, теория, планеты, вращение, магнетизм, магнитное поле.

Abstract

Увод/циљ: У раду је описан напреднији теоријски модел ЕМ поља Земље заснован на хипотезама о две компоненте. Дефинисан је математички модел који приказује ротацију магнетнопроводне сфере магнетизације M у страном магнетном пољу и компоненте магнетног поља које могу настати услед ротације Земље око своје осе. Према успостављеном моделу, у односу на референтне вредности планете Земље израчунате су вредности компоненти осталих планета у Сунчевом систему, а резултати су приказани табеларно.

Метод: Решење проблема, дефинисано у наслову рада, одређено је помоћу комбинованих, за ту намену формализованих метода физике и математичке анализе, а ради развоја новог напреднијег математичког модела. У ту сврху коришћен је и метод аналогije који се односио на примену сличних структурних форми и система за истраживање електромагнетних процеса и ротације планета. Метода аналогije примењена је због два узајамно повезана разлога. Први је да су све вредности које карактеришу функцију било ког природног система подложне променама, а други да примењивана решења не одређују услове функције структуре у сваком конкретном случају.

Резултати: Решења у виду оригиналних аналитичких формула и бројне вредности наведене у табели 2, референтне у односу на утицај ротације планета а посебно Земље, биће примењена за истраживање утицаја ефеката ЕМ поља које емитује Сунце према планетама нарочито за улогу коју процес има за заштиту Земље. Посебно су важни резултати приказани у табели 2.

Закључак: У раду су размотрени појава и дејство ЕМ поља Земље на начин разумљив садашњем нивоу развоја науке. Научна сазнања и мерења у геофизици и астрофизици наговештавају да је Сунце могући извор ЕМ поља које се простира кроз интерпланетарни простор, а да је компонента магнетног поља Земље само одзив на утицај тог извора. Природни феномени и процеси на Земљи могу се дефинисати у теорији система моделом који садржи промене параметара стања на планети.

Keywords: напреднији модел, теорија, планете, ротација, магнетизам, магнетно поље.

Introduction

The division of the Earth's EM field into electric and magnetic field components is relative. Electrostatic loads create a component of electric field strength and in the vicinity of immobile charge carriers there is only an electric field component while a moving carrier creates a magnetic field component (Bjelić & Marković, 2023). The Earth's magnetic field protects life on the Earth from cosmic particles, and a weakening of the field reduces the protection. In astrophysics, the Earth's magnetic field is defined as a physical field created and maintained by the rotation of charged magma in the core (according to the accepted dynamo theory) (Jacobs, 1987).

Nikola Tesla was the first to realize that the Earth is a good electrical conductor and that the upper layers of the atmosphere have an important role because, according to the measurements, they contain a stable conduction line. Based on Tesla's postulate, EM processes inside the Earth and in its outer layers can be viewed through influence as:

- an electrostatic system with a solid sphere and outer layers carrying charges,
- an elementary electrostatic dipole that creates the electric and magnetic components of the EM field,
- an elementary magnetic dipole that creates the electric and magnetic components of the EM field,
- an elementary resonator of the electric component of the EM field, as suggested by Tesla, and
- a planet that rotates under the action of the forces created by the electric and magnetic field components.

A more advanced model of a dielectric sphere in a foreign homogeneous electric field

To obtain a more advanced model of the dielectric sphere in a foreign homogeneous electric field, the starting relation for the potential is important:

$$\varphi_i = \frac{C_{1i}}{r} + C_{2i} + (C_{3i}r + C_{4i}/r^2)\cos\alpha, \quad (1)$$

$$\varphi_e = \frac{C_{1e}}{r} + C_{2e} + (C_{3e}r + C_{4e}/r^2)\cos\alpha. \quad (2)$$

The potential $\varphi(r)$ for $r \rightarrow \infty$ is $\varphi = \varphi_0 + E_0 r \cos\alpha$. After substitution, we get: $C_{2e} = \varphi_0$, $C_{3e} = E_0$. For a point charge, the electric field $E = q / 4\pi\epsilon_0\epsilon_r r^2$ depends on (r):

$$\varphi_e = -\int E dr = \frac{q}{4\pi\epsilon_0\epsilon_r r} = \frac{C_{1e}}{r}. \quad (3)$$

By analogy, C_{1e}/r is a component of the total charge of the sphere. For the total charge of the sphere $q = Q = 0$ in relation φ_e to this constant is equal to $C_{1e} = 0$. After that, the constant is determined C_{4e} :

$$\varphi_e = \varphi_0 + (E_0 r + C_{4e}/r^2)\cos\alpha. \quad (4)$$

The potential of all points in a spherical dielectric φ_i is a finite value valid only for $C_{1i} = 0$ and $C_{4i} = 0$ (if the $C_{1i} = 0$ component were in the center of the sphere it would be $(C_{1i}/r) \rightarrow \infty$. C_{2i} with sufficient

accuracy to determine the potential in the field of the outer domain C_{2e} is $C_{2i} = C_{2e} = \varphi_0$. Inside the dielectric is: $\varphi = \varphi_0 + C_{3i}r \cos \alpha$.

The constants C_{4e} and C_{3i} are determined from the boundary conditions. From the equations for φ_e , φ_i for $r=R_1$ which the condition equivalent to the condition $E_{it} = E_{et}$, follows: $C_{3i}R_1 = E_0R_1 + C_{4e} / R_1^2$, $C_{4e} = -E_0R_1^3$.

C_{4i} and C_{3i} follow from the equality of the normal components of the induction: $D_{1n} = D_{2n}$ and $E_{it} = E_{et}$ at the limit:

$$-\varepsilon_i \left(\frac{\partial \varphi_i}{\partial r} \right)_{r=R_1} = -\varepsilon_e \left(\frac{\partial \varphi_e}{\partial r} \right)_{r=R_1}, \quad \varepsilon_i (C_{3i})_{r=R_1} = \varepsilon_e \left(E_0 - \frac{2C_{4e}}{R_1^3} \right)_{r=R_1}, \quad (5)$$

$$C_{3i} = E_0 \frac{3\varepsilon_e}{2\varepsilon_e + \varepsilon_i}, \quad C_{4i} = C_{4e} = R_1^3 E_0 \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i}, \quad (6)$$

$$\varepsilon_i E_0 \frac{3\varepsilon_e}{2\varepsilon_e + \varepsilon_i} = \varepsilon_e E_0 \left(1 - 2 \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right). \quad (7)$$

The potential in the dielectric from $z = r \cos \alpha$ is:

$$\varphi_i = \varphi_0 + E_0 r \frac{3\varepsilon_e}{2\varepsilon_e + \varepsilon_i} \cos \alpha = \varphi_0 + E_0 \frac{3\varepsilon_e}{2\varepsilon_e + \varepsilon_i} z. \quad (8)$$

The strength of the electric field in the dielectric is:

$$E_{zi} = -\frac{\partial \varphi_i}{\partial z} = -E_0 \frac{3\varepsilon_e}{2\varepsilon_e + \varepsilon_i}. \quad (9)$$

The strength of the electric field E has the direction of the z axis, does not depend on the coordinate and the field is homogeneous. The potential outside the dielectric is:

$$\varphi_e = \varphi_0 + E_0 \left[r + \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \left(R_1^3 / r^2 \right) \right] \cos \alpha. \quad (10)$$

For the chosen spherical coordinate system, the components of the electric field strength outside the dielectric are:

$$E_{re} = -\frac{\partial \varphi_e}{\partial r} = E_0 \left[1 - \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \left(2R_1^3 / r^3 \right) \right] \cos \alpha. \quad (11)$$

The field strength and potentials of the charged dielectric sphere are calculated according to Gauss's law and the solutions are given in Table 1. The obtained formulas for the sphere are calculated in relation to the parameter α .

On the surface of the sphere for $r=R_1$ is:

$$E_{R1e} = E_0 \left(1 - 2 \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right) \cos \alpha ,$$

$$E_{ae} = -\frac{1}{r} \frac{\partial \varphi}{\partial \alpha} = E_0 \left(1 + \frac{R_1^3}{r^3} \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right) \sin \alpha .$$
(12)

Table 1
Electric field strengths on the sphere

$\alpha = 0$, Field strength, N pole		Equator $\alpha = 90^0$, Field strength		$\alpha = 180^0$, Field strength, S pole	
$E_{R1e} = E_0 \left(1 - 2 \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right)$	$E_\alpha = 0$	$E_{R1e} = 0$	$E_{ae} = E_0 \left(1 + \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right)$	$E_{R1e} = -E_0 \left(1 - 2 \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right)$	$E_\alpha = 0$

Таблица 1 – Напряженность электрического поля в сфере
Табела 1 – Јачине електричног поља у сфери

The components E_r at all points on the sphere are greater than the external field components, and the lines D start from free charges, break at the boundary surface and pass through the sphere without interruption. The resulting field is homogeneous, but the field is also affected by the polarization of the bound charges in the dielectric (Đorđević & Olćan 2012).

$$E_i = E_0 - E_\omega .$$
(13)

The potential in the sphere is determined from the component of the foreign field in the sphere $E_0(z)$ for $r \leq R_1$, $E_{zi} = -\partial \varphi_e / \partial z$, and the potential of the bound charges if $\varphi_i = 0$. For $z = 0$ and $z = r \cos \alpha$ we get:

$$\varphi_i = -(E_0 - E_\omega) r \cos \alpha .$$
(14)

The component $-E_0 r \cos \alpha$ is created by the external field and $E_\omega r \cos \alpha$ is created by the bound charges in the sphere. Part of the potential from the moment of polarization, equivalent to a polarized sphere is

$$\partial \varphi = \frac{p \cos \alpha}{4\pi \varepsilon_e r^2} .$$

The sum of that part and the potential created under the influence of the external field $\varphi_i = \varphi_e + \partial \varphi$ is:

$$\varphi_e = -E_0 r \cos \alpha + \frac{p \cos \alpha}{4\pi \varepsilon_e r^2} .$$
(15)

The boundary conditions $r \leq R_1$ for strengths E_e and inductions D_e outside the sphere are:

$$E_{it} = E_{et}, \left(-\frac{1}{r} \frac{\partial \varphi_e}{\partial \alpha} \right)_{r=R_1} = \left(-\frac{1}{r} \frac{\partial \varphi_i}{\partial \alpha} \right)_{r=R_1}, \quad (16)$$

$$-E_0 \sin \alpha + \frac{p \sin \alpha}{4\pi \varepsilon_e R_1^3} = -(E_0 - E_\omega) \sin \alpha. \quad (17)$$

From the equation $E_n = -\frac{\partial \varphi_e}{\partial r} = E_0 \left(1 - \frac{2R_1^3}{r^3} \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right) \cos \alpha$ and $E_n = -\frac{\partial \varphi_e}{\partial r} = E_0 \left(1 - \frac{2R_1^3}{r^3} \frac{\varepsilon_e - \varepsilon_i}{2\varepsilon_e + \varepsilon_i} \right) \cos \alpha$ is:

$$E_\omega = \frac{p}{4\pi \varepsilon_e R_1^3}. \quad (18)$$

It is also for $r \leq R_1, D_{in} = D_{en}$, that is for $r=R_1$:

$$\varepsilon_e \left(-\frac{\partial \varphi_e}{\partial \alpha} \right)_{r=R_1} = \varepsilon_i \left(\frac{\partial \varphi_i}{\partial \alpha} \right)_{r=R_1},$$

$$\varepsilon_e E_0 \cos \alpha + \frac{p \cos \alpha}{2\pi R_1^3} = \varepsilon_i (E_0 - E_\omega) \cos \alpha. \quad (19)$$

It follows from equation (18):

$$\varepsilon_e E_0 + 2\varepsilon_e E_\omega = \varepsilon_i E_0 - \varepsilon_i E_\omega, \quad E_\omega = \frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + 2\varepsilon_e} E_0. \quad (20)$$

The vector of the polarization moment of the sphere is:

$$p = 4\pi \varepsilon_e R_1^3 E_\omega = 4\pi \varepsilon_e R_1^3 \frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + 2\varepsilon_e} E_0. \quad (21)$$

If the surface load density η is uniformly distributed on the surface of the sphere, it is equal to the normal component of the polarization moment of the sphere $\eta = Pn = P \cos \alpha$, where:

$$P = \frac{p}{V_{sf}} = p / \left(\frac{4}{3} \pi R_1^3 \right) = 3\varepsilon_e \frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + 2\varepsilon_e} E_0 = 3\varepsilon_e E_\omega. \quad (22)$$

A model of an electrically conducting sphere in a foreign homogeneous field

At all points of the plane xOy passing through the center, the conductive and electrostatic unloaded sphere has the same potential $\varphi_0 \neq 0$. At a distance $z = r \cos \alpha$ from the sphere that is very small $q = 0$ according to the radius $R_1 = R_{\text{earth}}$, it is considered that the sphere has no influence on the EM field (if the total load on the sphere is $q = 0$, the influence of the sphere exists if the point load appears as the sum of the free charges on the sphere $Q \neq 0$).

The potential φ at a distance r is:

$$\varphi = \frac{Q}{4\pi\epsilon_a r} + \varphi_0 + E_0 r \cos \alpha. \quad (23)$$

The first member on the right side is the response of the sphere loaded with $Q \neq 0$, and the increase in potential is $E_0 r \cos \alpha$ due to the influence of the foreign field on the right side $z = r \cos \alpha$ (for $r \rightarrow \infty$ the potential is φ_0). The solution of the equation $\varphi = \varphi^M + \varphi^N = (C_1/r) + C_2 + (C_3 r + (C_4/r^2)) \cos \alpha$ is suitable for points infinitely far from the sphere and can be substituted into the equation for the potential (1) and (2), and then there is a unique solution with 4 constants. After substitution, the constants are:

$$C_1 = Q / 4\pi \cdot \epsilon_a, \quad C_2 = \varphi_0, \quad C_3 = E_0, \quad C_4. \quad (24)$$

C_4 cannot be determined by substitution, because in the equations for the potential (1) and (2) there is no component inversely proportional to r^2 , and therefore it is determined from the condition that all points on the boundary surface $r=R_1$ of the sphere have the same potential.

Tangential components at the boundary must be equal to zero. $E_{it} = E_{et} = 0$. Then it is:

$$\varphi = \text{const.} = \frac{Q}{4\pi\epsilon_a R_1} + \varphi_0 + [E_0 R_1 + (C_4 / R_1^2)] \cos \alpha. \quad (25)$$

When changing the angle α , the right side of the equation is constant if:

$$(E_0 R_1 + C_4 / R_1^2) = 0, \quad (26)$$

where: $C_4 = -E_0 R_1^3$.

Therefore, at all points of the dielectric surrounding the conducting sphere, the potential is equal:

$$\varphi = \frac{Q}{4\pi\epsilon_a r} + \varphi_0 + E_0 \left(R_1 - \frac{R_1^3}{r^2} \right) \cos \alpha. \quad (27)$$

For a spherical coordinate system, the two components of the field strength follow from the equations:

$$E_r = -\frac{\partial \varphi}{\partial r}, \quad E_\alpha = -\frac{1}{r} \frac{\partial \varphi}{\partial \alpha}, \quad E_\beta = -\frac{1}{r \sin \alpha} \frac{\partial \varphi}{\partial \beta} = 0, \quad (28)$$

$$E_r = -\frac{\partial \varphi}{\partial r} = \frac{Q}{4\pi\epsilon_a r^2} - E_0 \left[R_1 + \left(2R_1^3 / r^3 \right) \right] \cos \alpha, \quad (29)$$

$$E_\alpha = -\frac{1}{r} \frac{\partial \varphi}{\partial \alpha} = E_0 \left[1 - \left(R_1^3 / r^3 \right) \right] \sin \alpha. \quad (30)$$

For $Q = 0$ on the surface $r=R_1$ is $E_r = -3E_0 \cos \alpha$. The field strength is for $\alpha = 0$, $E_{R1} = -3E_0$, $E\alpha = 0$, $\alpha = 180^\circ$, $E_{R1} = -3E_0$, $E\alpha = 0$. The E_r components at points on the sphere are three times larger than the components of the foreign field.

At the equator for $\alpha = 90^\circ$ and $E_{R1} = 0$ is:

$$E_\alpha = E_0 \left(1 - R_1^3 / R_1^3\right) \sin \alpha = 0. \quad (31)$$

A magnetically conducting sphere in a foreign magnetically homogeneous field

To determine the solution for a magnetic sphere of a radius R_1 in a foreign magnetic field, with given constants inside μ_i and outside μ_e the sphere and boundary conditions, solutions for a dielectric sphere with dielectric constants in a foreign electrostatic field are also used (Dziewonski & Anderson, 1981). In a system of stationary charges and an environment where electrical conductivity is zero, the electrostatic field is a special case of stationary:

$$a) \text{rot} \vec{E} = 0, b) \text{div} \vec{D} = \rho_{\text{conv}}, c) \vec{D} = \varepsilon_a \vec{E}. \quad (32)$$

For $\text{rot} \vec{E} = 0$, the electrostatic field is vortex-free. It $\text{rot} \vec{E} = \text{rot grad} \varphi = 0$ follows $\vec{E} = -\text{grad} \varphi$. So the field is potential because it represents the gradient of the potential function and the sign $(-)$ indicates that the $\vec{E}$ direction is from a point of greater to a point of lesser potential. If the conduction current density is zero, the magnetic field is:

$$a) \text{rot} \vec{H} = 0, b) \text{div} \vec{B} = 0, c) \vec{B} = \mu_a \vec{H}. \quad (33)$$

The following equations of magnetostatics are identical to the equations of electrostatics in the areas with $\rho_{\text{conv}} = 0$, that is $\text{rot} \vec{H} = 0$, $\text{rot} \vec{E} = 0$ and are equivalent to the equations $\vec{H} = -\text{grad} \varphi_m$ and $\vec{E} = -\text{grad} \varphi$.

Boundary conditions on the surface of the magnetic sphere in a foreign magnetic field refer to the equality of the normal components of the magnetic induction and the tangential components of the magnetic field strength in both environments, outside the sphere and inside the sphere (index i for internal space and e for external space).

For a dielectric sphere in a foreign uniform electric field, the boundary condition on the surface is:

$$D_{in} = D_{en}, E_{it} = E_{et}, B_{in} = B_{en}, H_{it} = H_{et}. \quad (34)$$

In the analysis of the influence of the external field $\vec{H}_0$ on the magnetically conducting sphere, the methods of electrostatics $\vec{E} \leftrightarrow \vec{H}$, $\vec{D} \leftrightarrow \vec{B}$, $\vec{P} \leftrightarrow \mu_0 \vec{M}$ are used. Polarization $P = dp / dV$ corresponds to the product of the magnetization $M = dm / dV$ and the constant μ_0 . A dielectric sphere in a foreign electric field is polarized as a homogeneous medium, and the same thing happens with a conducting magnetic sphere $\mu_a > \mu_0$ in a foreign magnetic field. The strength H_0 of the demagnetization magnetic field in the sphere and the direction opposite to the direction of the foreign field defines the magnetization of the sphere. The equation analogous to the field strength E of the dielectric sphere is the magnetic field strength of the magnetization of the magnetoconducting sphere:

$$E_\omega = \frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + 2\varepsilon_e} E_0, \vec{H}_\omega = \frac{\mu - \mu_0}{2\mu_0 + \mu}. \quad (35)$$

Outside the sphere, the component created by the magnetization of the sphere appears as an elementary current field in the center of the sphere, and the magnetic moment m is the sum of the magnetic moments of all elementary currents in the domain. The magnetization of the magnetic field in the sphere is obtained similarly to the polarization moment in the dielectric from equation (31):

$$\begin{aligned} p &= 4\pi\epsilon_e R_1^3 E_\omega = 4\pi\epsilon_e R_1^3 \frac{\epsilon_i - \epsilon_e}{\epsilon_i + 2\epsilon_e} E_0 \Leftrightarrow \\ \Leftrightarrow \mu_e H_m &= 4\pi R_1^3 \mu_e H_\omega = 4\pi R_1^3 \mu_e \frac{\mu_i - \mu_e}{\mu_i + 2\mu_e} E_0 \end{aligned} \quad (36)$$

The product of the magnetization and the constant μ_e are equal to the magnetic moment of the sphere:

$$\mu_e M = \mu_e \frac{dm}{dV} = \frac{\mu_e m}{(4/3)R_1^3} = 3\mu_e \frac{\mu_i - \mu_e}{\mu_i + 2\mu_e} H_0 = 3\mu_e H_\omega. \quad (37)$$

The strength of the magnetic field in the sphere is the difference between the strength of the external H_0 and the field due to the magnetization H_ω and the induction:

$$H = H_0 - H_\omega = \frac{3\mu_e}{\mu_i + 2\mu_e} H_0, \quad B_e = \mu_e H_0, \quad (38)$$

$$B = \mu_i H = \mu_i \frac{3\mu_e}{\mu_i + 2\mu_e} H_0 = \frac{3\mu_i}{\mu_i + 2\mu_e} B_e. \quad (39)$$

If $\mu_i \rightarrow \infty$, $\bar{H}_\omega = H_0$, then $\bar{H} = 0$, $B = 3B_0$. In addition to the speed of light c , in the analysis of EM processes in stationary and/or moving environments, the parameters of those environments are also used.

Basic EM states of the body at rest and in motion (rotation)

In the analysis of the state, one stationary (system 0) is used with a coordinate origin at the point 0, with coordinates in the Cartesian system x, y, z and a time coordinate t and another system connected to a moving medium in relation to the previous one with the a coordinate origin at the point 0_1 and new coordinates in the Cartesian system x_1, y_1, z_1 and the time coordinate t .

If during the time $t = 0$ both coordinate systems coincide and the velocity of the medium in the direction of the axis x is $v(y) \neq 0$ according to the theory of relativity, the Lorentz transformation that connects the space coordinates of both systems and the time t is:

$$x_1 = \frac{x - vt}{\sqrt{1 - \beta^2}}, \quad y_1 = y, \quad z_1 = z, \quad t_1 = \frac{t - (v/c^2)x}{\sqrt{1 - \beta^2}}, \quad \beta = v/c. \quad (40)$$

At some point, the stationary carrier with respect to the system 0 has a field strength E and a magnetic induction B . The field strength E is the force acting on a unit static load q in the 0 coordinate system, and the magnetic induction B means the force acting on a unit current element at rest in the stationary 0 system:

$$\vec{E} = \vec{i}E_x + \vec{j}E_y + \vec{k}E_z, \quad \vec{B} = \vec{i}B_x + \vec{j}B_y + \vec{k}B_z. \quad (41)$$

The strength of the electric field E_1 and the magnetic induction B_1 that the carrier would have without moving in relation to the system with the beginning 0_1 (the system moving at a speed $v \neq 0$) E_1 represent: the effect of the force acting on a stationary unit load q in the system of coordinates with the beginning at 0_1 , and B_1 , the effect of the force on the unit element of the current that is at rest in a moving environment (a moving coordinate system with the beginning 0_1):

$$\vec{E}_1 = \vec{i}E_{x1} + \vec{j}E_{y1} + \vec{k}E_{z1}, \quad \vec{B}_1 = \vec{i}B_{x1} + \vec{j}B_{y1} + \vec{k}B_{z1}. \quad (42)$$

When moving from Maxwell's equations for a stationary medium to the equations of a moving medium, the derivatives $\partial(x,y,z)/\partial t$ (coordinate t) and $\partial(x_1,y_1,z_1)/\partial t_1$ (coordinate t_1) are determined according to equation (40) (Petković, 2016):

$$\begin{aligned} \frac{\partial}{\partial x} &= \alpha \left(\frac{\partial}{\partial x_1} - \frac{v}{c^2} \frac{\partial}{\partial t_1} \right), \quad \frac{\partial}{\partial t} = \alpha \left(-v \frac{\partial}{\partial x_1} + \frac{\partial}{\partial t_1} \right), \\ \frac{\partial}{\partial x_1} &= \alpha \left(\frac{\partial}{\partial x} + \frac{v}{c^2} \frac{\partial}{\partial t} \right), \quad \frac{\partial}{\partial t_1} = \alpha \left(v \frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right), \\ \frac{\partial}{\partial y_1} &= \frac{\partial}{\partial y}, \quad \frac{\partial}{\partial z_1} = \frac{\partial}{\partial z}, \quad \alpha = \frac{1}{\sqrt{1-\beta^2}}. \end{aligned} \quad (43)$$

By developing the operation rot and joining the terms with the same rots in the first Maxwell's equation, $\text{rot}\vec{H} = \vec{J}_{\text{cond}} + \partial\vec{D}/\partial t$ it follows that $\text{rot}\vec{H}_1 = \vec{J}_{\text{cond}1} + \partial\vec{D}_1/\partial t$. The projections of the vectors onto the coordinates in both systems are:

$$\begin{aligned} \vec{H}_1 &= \vec{i}H_{x1} + \vec{j}H_{y1} + \vec{k}H_{z1}, \quad H_{x1} = H_x, \\ H_{y1} &= \alpha(H_y + vD_z), \quad H_{z1} = \alpha(H_z - vD_y), \end{aligned} \quad (44)$$

$$\vec{J}_1 = \vec{i}J_{x1} + \vec{j}J_{y1} + \vec{k}J_{z1}, \quad J_{x1} = \alpha(J_x - v\rho), \quad J_{z1} = J_z, \quad (45)$$

$$\begin{aligned} \vec{D}_1 &= \vec{i}D_{x1} + \vec{j}D_{y1} + \vec{k}D_{z1}, \quad D_{x1} = D_x, \quad D_{x1} = D_x, \\ D_{y1} &= \alpha[D_y - H_z(v/c^2)], \quad D_{z1} = \alpha[D_z + H_y(v/c^2)]. \end{aligned} \quad (46)$$

A similar transformation of the second Maxwell's equation yields:

$$\text{rot}\vec{E} = -\partial\vec{B} / \partial t, \text{ rot}\vec{E}_1 = -\partial\vec{B}_1 / \partial t, \quad (47)$$

$$\vec{E}_1 = \vec{i}E_{x1} + \vec{j}E_{y1} + \vec{k}E_{z1}, E_{x1} = E_x E_{y1} = \alpha(E_y - vB_z), \\ E_{z1} = \alpha(E_z + vB_y), \quad (48)$$

$$\vec{B}_1 = \vec{i}B_{x1} + \vec{j}B_{y1} + \vec{k}B_{z1}, B_{x1} = B_x B_{y1} = \alpha\left(B_y + \frac{v}{c^2}E_z\right), \\ B_{z1} = \alpha\left(B_z - \frac{v}{c^2}vE_y\right). \quad (49)$$

The third and fourth Maxwell's equations in the coordinate system 0_1 have the forms:

$$\text{div}\vec{D}_1 = \rho_1, \text{ div}\vec{B}_1 = 0, \quad (50)$$

and the volume charge density is:

$$\rho_1 = \alpha\left[\rho - \left(v/c^2\right)j_x\right]. \quad (51)$$

In the system 0_1 , differentiation is performed by x_1, z_1, y_1 . For a stationary medium, the conditions of continuity of the tangential components of strength are fulfilled: E_{t1} and H_{t1} and continuity of the normal components D_{n1} and B_{n1} in the system:

$$0_1 \text{ is: } \vec{M}_1 = \vec{B}_1 / \mu_0 - \vec{H}_1, \vec{P}_1 = \vec{D}_1 - \varepsilon_0\vec{E}_1, \quad (52)$$

$$0 \text{ is: } \vec{M} = \vec{B} / \mu_0 - \vec{H}, \vec{P} = \vec{D} - \varepsilon_0\vec{E}. \quad (53)$$

M, M_1 the magnetizations of the medium are the dielectric polarizations P, P_1 , in the systems 0 and 0_1 :

$$\vec{M}_1 = \vec{i}M_{x1} + \vec{j}M_{y1} + \vec{k}M_{z1}, \vec{M} = \vec{i}M_x + \vec{j}M_y + \vec{k}M_z, \quad (54)$$

$$\vec{P}_1 = \vec{i}P_{x1} + \vec{j}P_{y1} + \vec{k}P_{z1}, \vec{P} = \vec{i}P_x + \vec{j}P_y + \vec{k}P_z. \quad (55)$$

Equations (44), (45) and (46) show the dependence of the magnetization projections on the polarization projections in 0 and 0_1 :

$$M_{x1} = M_x, \quad M_{y1} = \alpha(M_y + vP_z), \quad M_{z1} = \alpha(M_z - vP_y), \quad (56)$$

$$P_{x1} = P_x, \quad P_{y1} = \alpha\left(P_y - \frac{v}{c^2}P_z\right), \quad P_{z1} = \alpha\left(P_z + \frac{v}{c^2}P_y\right). \quad (57)$$

From equations (44) and (45), if there is no magnetic field $B = 0$ and there is an electric field $E \neq 0$, there is both an electric and a magnetic field in the system 0_1 . It is also clear that even when there is no electric field in the system 0 , $E = 0$, but there is a magnetic one $B \neq 0$, an electric field is created in the system 0_1 in addition to the magnetic one.

The current density in the system 0_1 is produced not only by the conduction current but also by the volume charge density transfer current component $\alpha\rho \cdot \vec{v}$ (convection current density) according to equation (46):

$$\vec{J}_1 = \vec{i}J_{x1} + \vec{j}J_{y1} + \vec{k}J_{z1}, \quad J_{x1} = \alpha(J_x - v\rho), \quad J_{y1} = J_y, \quad J_{z1} = J_z. \quad (58)$$

According to equation (47), by moving the current from the system 0 with elementary density J_x parallel to itself in the system 0_1 on the load carrier in the coordinate system starting at 0_1 as part of the current density in the charge density $\rho = d(\rho - \frac{v}{c^2}J_x)$, a supplementary component appears $\frac{v}{c^2}J_x$.

From Maxwell's basic equation for EM wave propagation $\mu\mu_0(\partial H / \partial t) = -\partial E / \partial t$ is:

$$\mu\mu_0 \frac{\partial H}{\partial t} = -\partial E \Rightarrow H = \frac{1}{c} \sqrt{\frac{\epsilon}{\mu}} \cdot E, \quad \mu^2 H^2 = \frac{\mu\epsilon}{c^2} \cdot E^2 \Leftrightarrow B^2 = \frac{\mu\epsilon}{c^2} E^2. \quad (59)$$

Due to the movement of the polarized dielectric at the speed v , equations (56) and (57), additional magnetization is created in the coordinate system 0 , and the movement of the magnetized medium at the speed v in system 0 appears as additional polarization.

According to equation (59), for the fields defined in the stationary system 0 and the mobile system 0_1 , the following applies:

$$\frac{E_1^2}{c} - B_1^2 c = \frac{E^2}{c} - B^2 c, \text{ i.e. } \vec{E}_1 \vec{B}_1 = \vec{E} \vec{B}, \quad (60)$$

$$\frac{H_1^2}{c} - D_1^2 c = \frac{H^2}{c} - D^2 c, \text{ i.e. } \vec{D}_1 \vec{H}_1 = \vec{D} \vec{H}. \quad (61)$$

If the speed of movement of the material medium (conductor/ dielectric/ferromagnetic) is low, according to the speed of light $(v/c)^2 \ll 1$, the Lorentz transformations change to Galilean ones, i.e. $x_1 = x - vt$, $y_1 = y$, $z_1 = z$, $t_1 = t$. The mathematical relationships between quantities in the systems 0 and 0_1 are:

$$\begin{aligned}\vec{E}_1 &= \vec{E} + [\vec{v} \times \vec{B}], \quad \vec{J}_1 = \vec{J} - \vec{v}\rho, \quad \vec{D}_1 = \vec{D} + \frac{[\vec{v} \times \vec{H}]}{c^2}, \quad \rho_1 = \rho - \frac{\vec{v} \cdot \vec{J}}{c^2}, \\ \vec{H}_1 &= \vec{H} - [\vec{v} \times \vec{D}], \quad \vec{M}_1 = \vec{M} + [\vec{v} \times \vec{P}], \quad \vec{B}_1 = \vec{B} - \frac{[\vec{v} \times \vec{E}]}{c^2}, \\ \vec{P}_1 &= \vec{P} - \frac{[\vec{v} \cdot \vec{M}]}{c^2}.\end{aligned}\quad (62)$$

Body rotation corresponds to the position of a still body in space in a moving-rotating field created by EM sources by rotation in a time-varying or constant magnetic field. The electric and magnetic components of the field, Figure 1.a,b, can be determined from the magnetic potential vector, i.e. in $\vec{A} = A_r \vec{e}_r + A_\alpha \vec{e}_\alpha + A_\beta \vec{e}_\beta$ the spherical and $\vec{A} = A_x \vec{e}_x + A_y \vec{e}_y + A_z \vec{e}_z$ in the Cartesian systems.

For a body rotating around the z axis, the possible functions are:

$$f(x, y, z, t), \text{ i.e. } A_z(x - ut, y, z), \quad H = H_r(r, \omega \cdot t - \beta). \quad (63)$$

In plane coordinates, rotation is represented by the time-dependent functions:

$$H_x = H_r(r) \cos \omega \cdot t, \quad H_y = H_r(r) \sin \omega \cdot t. \quad (64)$$

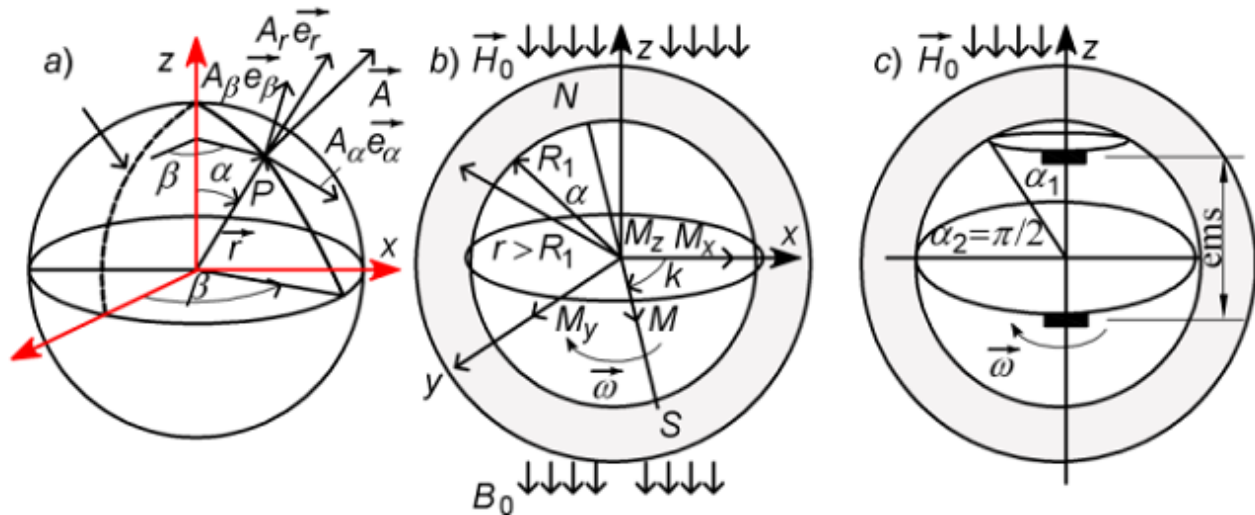


Figure 1

a) Rotation of the magnetic sphere and the appearance of magnetic vector potential, b) reaction of magnetic and conducting spheres in a constant magnetic field, c) emf due to the rotation of the magnetic field (Bjelić, 2021b)

Рис 1 – а) Вращение магнитосферы и возникновение магнитного векторного потенциала, б) реакция магнитосферы и проводящей сферы в постоянном магнитном поле, в) ЭДС вследствие вращения сферы в магнитном поле (Bjelić, 2021b)

Слика 1 – а) Ротација магнетне сфере и појава магнетног векторског потенцијала, б) реакција магнетне и проводне сфере у сталном магнетном пољу, в) ЕМС услед ротације сфере у магнетном пољу (Bjelić, 2021b)

There are several solutions for body rotation in the literature (Davilkovski, Fradkin, Smith, etc.). The EM field in the dielectric layer between the ionosphere and the solid Earth, which rotates together with them in the magnetic field if the magnetic component of the Earth is a response to the EM field of the Sun,

is shaped by the parameters in the Earth and the stable parameters of the electrical conductivity of the ionosphere. The elementary layer-ring of the atmosphere, given its dimensions, is treated as a flat channel $-\infty < x < +\infty$ of width $-b < y < +b$ where the velocity of the dielectric in the foreign field $\vec{v}[v(y), 0, 0]$ is $B[0, 0, B(x)]$ (Mihajlović, 1993).

In an isotropic dielectric and magnetic medium, the conductivities of the layer $\varepsilon(x)$ and $\mu(x)$, Figure 2, change only in the direction of the axis $-\infty < x < +\infty$ by $\mathcal{E}_0$. The field strength in the dielectric layer rotating in a foreign unknown magnetic field (the Sun) is determined from the Lorentz force of the field strength $\vec{E}_1 = \vec{F}/q = \vec{E} + (\vec{v} \times \vec{B})$, stationary in relation to the system at the beginning O_1 (the system rotates at a speed $\vec{v} \neq 0$), while $\vec{E}$ is stationary in relation to the system 0. From equation (53) follows:

$$\begin{aligned} \vec{D} &= \vec{D}_1 - \varepsilon_0 \mu_0 [\vec{v} \times \vec{H}] = \varepsilon_a \vec{E} + \varepsilon_a [\vec{v} \times \vec{B}] - \varepsilon_0 \mu_0 [\vec{v} \times \vec{H}] = \\ &= \varepsilon_a \vec{E} + (\varepsilon_a - \varepsilon_0 / \mu_r) [\vec{v} \times \vec{B}] \end{aligned} \quad (65)$$

If the dielectric layer of the atmosphere between the ionosphere and the ground surface rotates at a speed of $v \approx 465$ m/s, and the speed of light is $c = 3 \cdot 10^8$ m/s, $v^2/c^2 \ll 1$, by the principle of superposition, Figure 2, the vector $\vec{D}$ is defined by:

$$\vec{D} = \varepsilon_a \vec{E} + (\varepsilon_a - \varepsilon_0 / \mu_r) [\vec{v} \times \vec{B}]. \quad (66)$$

The field strength is the gradient of the potential function $\vec{E} = -\text{grad}\varphi$, and the induction projections on the axes x and y are:

$$D_x = -\varepsilon_a \frac{\partial \varphi}{\partial x}, \quad D_y = -\varepsilon_a \frac{\partial \varphi}{\partial y} + \left(\varepsilon_a - \frac{\varepsilon_0}{\mu} \right) [\vec{v} \times \vec{B}]. \quad (67)$$

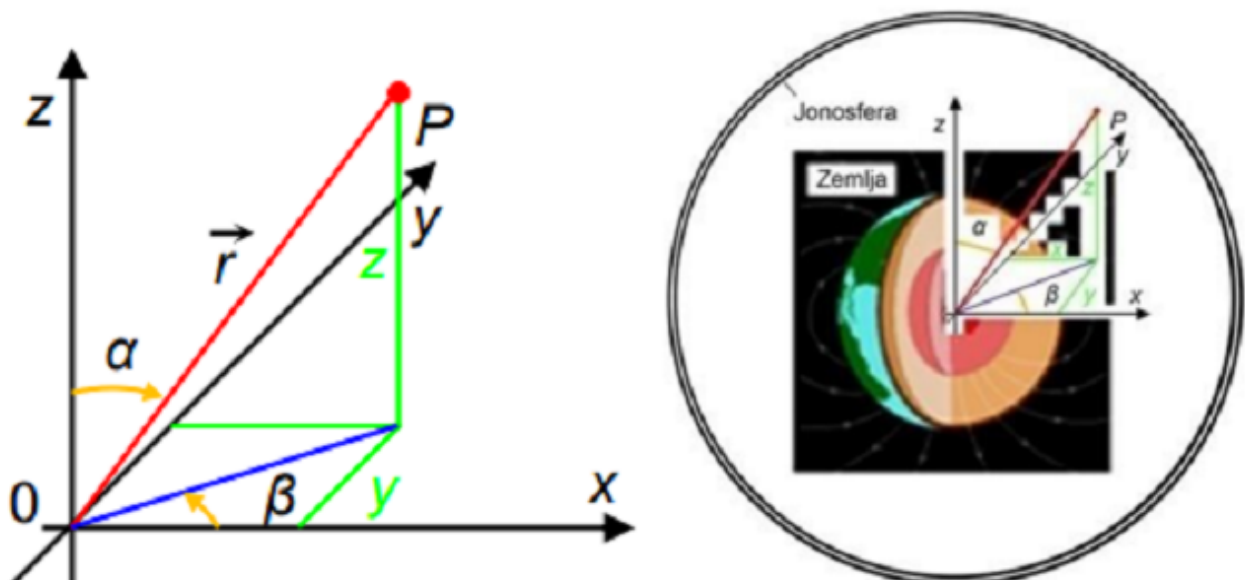


Figure 2

Spherical coordinate system and the position of the solid Earth and the ionosphere (Bjelić, 2021b)

Рис 2 – Сферическая система координат и положение твердой Земли и ионосферы (Bjelić, 2021b)

Слика 2 – Сферни координатни систем и позиција чврсте Земље и јоносфере (Bjelić, 2021b)

Divergence is the ratio of the output vector D of the surface and the volume covered by the surface according to equation (53):

$$\operatorname{div} \vec{D} = \operatorname{div} \left\{ \varepsilon_a \vec{E} + \left(\varepsilon_a - \frac{\varepsilon_0}{\mu_r} \right) [\vec{v} \times \vec{B}] \right\} = 0. \quad (68)$$

By transforming equation (8) into the operator function of the potential, a more suitable form is obtained:

$$\begin{aligned} -\varepsilon_a \Delta \varphi - \frac{\partial \varphi}{\partial x} \frac{d\varepsilon_a}{dx} + \left(\varepsilon_a - \frac{\varepsilon_0}{\mu_r} \right) B \frac{dv}{dy} &= 0, \\ \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + (\ln \varepsilon_r) \frac{\partial \varphi}{\partial x} + \left(1 - \frac{1}{\varepsilon_r \mu_r} \right) \cdot B \frac{dv}{dy} &= 0. \end{aligned} \quad (69)$$

In a spherical system, the potential gradient is:

$$\begin{aligned} \operatorname{grad} \varphi &= \frac{\partial \varphi}{\partial r} \vec{r}_0 + \frac{1}{r} \frac{\partial \varphi}{\partial \alpha} \vec{\alpha}_0 + \frac{1}{r \sin \alpha} \frac{\partial \varphi}{\partial \beta} \vec{\beta}_0, \\ E_r &= -\frac{\partial \varphi}{\partial r}, \quad E_\alpha = \frac{1}{r} \frac{\partial \varphi}{\partial \alpha}, \quad E_\beta = -\frac{1}{r \sin \alpha} \frac{\partial \varphi}{\partial \beta}, \\ \operatorname{div} \vec{V} &= \frac{1}{r} \frac{\partial}{\partial r} (r^2 V_r) + \frac{1}{r \sin \alpha} \frac{\partial}{\partial \alpha} (V_\alpha \sin \alpha) + \frac{1}{r \sin \alpha} \frac{\partial V_\beta}{\partial \beta}, \end{aligned} \quad (70)$$

where $\vec{r}_0$, $\vec{\alpha}_0$, $\vec{\beta}_0$ are the unit vectors of the spherical coordinate system.

The divergence of the field towards the center of the sphere is:

$$\operatorname{div} \vec{r} = 3, \quad \operatorname{div} \varphi(r) \cdot \vec{r} = 3\varphi(r) + r\varphi'(r). \quad (71)$$

Rotation of a magnetic sphere of the magnetization M in a foreign magnetic field

The model of rotation of a conducting sphere in a magnetic field corresponds to the model of rotation of a sphere in a time-varying field. An approximate solution will be determined from the adjusted complex form of the vector potential equation, neglecting wave processes in the domain $R \ll 2\pi/\omega$:

$$\nabla^2 \vec{A} = j\omega\mu_r\mu_0\sigma\vec{A}. \quad (72)$$

The solution of equation (72) has a real and an imaginary component:

$$A_{re} = A_{\beta.re} = \frac{1}{2}(B_0) \left[R + \frac{D}{R^2} \right] \sin \alpha, \quad R \geq R_1 = a, \quad (73)$$

$$A_{im} = A_{\beta.im} = \frac{C}{\xi} \left[\frac{\sin \xi}{\xi} - \cos \xi \right] \sin \alpha, \quad R \leq R_1 = a, \quad (74)$$

$$\xi = R\sqrt{-js}, \quad s = \omega\mu_r\mu_0\sigma, \quad R = a, \quad \xi = a\sqrt{-j\omega\mu_r\mu_0}. \quad (75)$$

The integration constants are:

$$D = a^2 \frac{2(\mu_r + 1)(\sin \xi - \xi \cos \xi) - \xi^2 \sin \xi}{(\mu_r - 1)(\sin \xi - \xi \cos \xi) + \xi^2 \sin \xi}, \quad (76)$$

$$C_1 = \frac{B_0}{2} \frac{a\xi^2}{\sin \xi - \xi \cos \xi} \left(1 + \frac{D}{a^3} \right). \quad (77)$$

For an out-of-sphere domain $R > a$ they are:

$$B_R = B_0 \left(1 + \frac{D}{R^3} \right) \cos \alpha, \quad B_\alpha = -B_0 \left(1 - \frac{1}{2} \frac{D}{R^3} \right) \sin \alpha. \quad (78)$$

In the outer domain, the field is similar to the field of a homogeneously magnetized sphere, or a sphere that is introduced into a homogeneous variable field without eddy currents. At equivalent magnetic permeability is:

$$\mu_{r.re} = \mu'_{r.re} - j\mu''_{r.re} = 2\mu_r \frac{\operatorname{tg} \xi - \xi}{\xi^2 \operatorname{tg} \xi - (\operatorname{tg} \xi - \xi)}. \quad (79)$$

By expanding the tangent in a degree order with the limitation to only two terms of the order (allowed for frequencies), we get:

$$\begin{aligned} \text{Low frequency } \omega \leq 1/\mu_r\mu_0\sigma \cdot a^2, \quad \mu_{r.re} &= \mu_r (1 - 0,1\omega\mu_r\mu_0\sigma \cdot a^2), \\ \text{High frequencies } |\operatorname{tg} \xi| \cong 1, \quad \mu_{r.re} &= 2\mu_r / a\sqrt{j\omega\mu_r\mu_0}. \end{aligned} \quad (80)$$

The magnetization M of a stationary sphere, $\mu_i/\mu_e = \mu_r$ in a constant field $\vec{H}_0$ is determined from equation (37):

$$\mu_e M = 3\mu_e \frac{\mu_i - \mu_e}{\mu_i + 2\mu_e} H_0, \quad M = 3\mu_e \frac{\mu_r - 1}{\mu_r + 2} H_0. \quad (81)$$

With a shift $\mu_r = \mu_{r.re}$ in the harmonic field, equation (37), $\tilde{m}$ is equal to:

$$\begin{aligned} \tilde{m} = V_{sf} \tilde{M} &= \frac{4}{3} \pi a^2 \tilde{M} = 4\pi a^2 \tilde{H}_0 \frac{\mu_{r.re} - 1}{\mu_{r.re} + 2} = \tilde{H}_0 K \angle -k, \\ \tilde{K} &= 4\pi a^2 \frac{\mu_{r.re} - 1}{\mu_{r.re} + 2} \angle -k, \end{aligned} \quad (82)$$

where K is the modulus m for $H_0 = B_0 / \mu_0$, k (phase attitude) of the moment with respect to the external field H_0 .

Electromotive force (emf) can be generated by induction during the rotation of a conductive sphere with a magnetization vector $M = M_z$ of the radius a and the angular velocity ω around the z axis in an external magnetic field of induction B_0 . Maxwell formulated the emf $\mathbf{e} = -d\phi / dt$ of induction where ϕ is covered by a contour for which the direction of the circuit is the same as the direction of the emf $e > 0$ according to the positive direction of the flux $\phi > 0$ in the system where the work W performed is conditioned by the effect of non-Coulomb forces f for the transfer of a unit charge q , $e = W / q$. The work during the transfer of a unit charge achieved by the effect of non-Coulomb forces of EM induction is the integral $\tilde{E}^{ind} = f / q$ of the field strength along a closed contour $\tilde{l}$:

$$\mathbf{e} = \oint_{\tilde{l}} \mathbf{E}^{ind} d\tilde{l} = -\frac{d}{dt} \int_S \mathbf{B} d\tilde{S}. \quad (83)$$

In Faraday's equation, $e = -dN / dt$, N is the number of magnetic lines intersected by the contour and one line corresponds to the unit flux. For some orientation of the flux line cutting speed (or B), the emf on the part $d\tilde{l}$ of the contour $\tilde{l}$ is $d\mathbf{e} = (\tilde{\mathbf{v}} \times \mathbf{B}) d\tilde{l}$. Maxwell's and Faraday's formulas are equivalent - the general Maxwell's equation.

The gradient vanishes on closed-loop integration. If part of the contour is determined by the line ab emf induced in it is:

$$\mathbf{e} = \int_a^b \left(-\nabla \varphi - \partial \tilde{\mathbf{A}} / \partial t + \tilde{\mathbf{v}} \times \mathbf{B} \right) d\tilde{\ell}, \quad \mathbf{e} = \int_a^b \left(-\partial \tilde{\mathbf{A}} / \partial t + \tilde{\mathbf{v}} \times \mathbf{B} \right) d\tilde{\ell}. \quad (84)$$

Due to the symmetry of the sphere towards the plane of the equator, Figure 1.c, from the point corresponding to the position of the N pole, $\alpha_1 = 0$ where one hole is at the position of the equator and the other hole is at $\alpha_2 = \pi / 2$, emf is determined between the circles at those corners. For $\partial \phi / \partial t = \partial A / \partial t = 0$ remains only the component $\tilde{\mathbf{v}} \times \mathbf{B} = \omega a \mathbf{B}_R \mathbf{e}_\alpha$, $d\tilde{\ell} = -a d\alpha \mathbf{e}_\alpha$ where:

$$\mathbf{E}_r = -\frac{\partial \varphi}{\partial r}, \quad \mathbf{E}_\alpha = \frac{1}{r} \frac{\partial \varphi}{\partial \alpha}, \quad \mathbf{E}_\beta = -\frac{1}{r \sin \alpha} \frac{\partial \varphi}{\partial \beta}, \quad (85)$$

$$\operatorname{div} \vec{V} = \frac{1}{r} \frac{\partial}{\partial r} (r^2 V_r) + \frac{1}{r \sin \alpha} \frac{\partial}{\partial \alpha} (V_\alpha \sin \alpha) + \frac{1}{r \sin \alpha} \frac{\partial V_\beta}{\partial \beta}, \quad (86)$$

$$e = \int (\vec{v} \times \vec{B}) d\vec{\ell} = \int_{\alpha_2=\pi/2}^{\alpha_1} \omega [a \sin \alpha \cdot B_R (-a d\alpha)]. \quad (87)$$

For $R \leq a$ it is $B = B_z = 2\mu_0 M / 3$. For $R = a$ you get:

$$B_R = B_z = 2\mu_0 M (\cos \alpha) / 3. \quad (88)$$

The induced emf is:

$$e = \left(\omega \cdot a^2 2\mu_0 M / 3 \right) \int_{\alpha_1}^{\alpha_2} \sin \alpha \cos \alpha \cdot d\alpha = \left(\omega a^2 \mu_0 M / 3 \right) (1 - \sin^2 \alpha_1). \quad (89)$$

Equation (89) shows which quantities determine v B depending on the speed of rotation in space.

Components of the magnetic field that can arise due to the rotation of the planets and the Earth

During the rotation of an electrostatically neutral body (rotor), higher densities of free charges in the area can excite the magnetic field in a similar way to be created by the currents flowing through the rotor, Figure 1.a.

The consequence of the rotation of the body is the creation of a weak magnetic field, and this phenomenon can be used to explain the appearance of the components of the Earth's magnetic field. If the number n of free electrons passing through the unit volume of the conductive part, e is the charge of the electrons, u the resultant chaotic speed of the electrons due to the effect of the field (thermal effects are small due to the small number of collisions of oscillating atoms), the current I through the surface s is:

$$I = Js = dQ / dt = nedV / dt = nesdl / dt = nesv, \quad (90)$$

$$J = I / s \Leftrightarrow I = Js = nev_0 (u / c)^2 s, \quad (91)$$

where n is the volume density of free charges, s the cross-section of the rotor body, v_0 the speed of the rotor, u the speed of the chaotic movement of free charges, and c the speed of light.

Bio-Savar, Ampere and the law of EM induction were created empirically. It was later concluded that these laws can be obtained from Coulomb's law, if the corrections derived from the special theory of relativity (Bjelić, 2019), are applied to the charge interaction forces. Already known physical phenomena can also be considered in two projections and are the result of facts that were not known before. Ampere's law allows the interaction of a conductor with a current to be determined. If the current elements $I_1 dl_1$ and $I_2 dl_2$ are normal to the radius of the sector, $r_{1,2}$ Figure 3.b, the forces of interaction between them are equal.

The assumption is that the positive charge elements dl_1 and dl_2 are also stationary, while the negative charge elements have velocities v_1 and v_2 corresponding to the currents I_1 and I_2 . The EM force, Figure 3.b, is determined by Coulomb's law with the application of the theory of relativity (Bjelić, 2019).

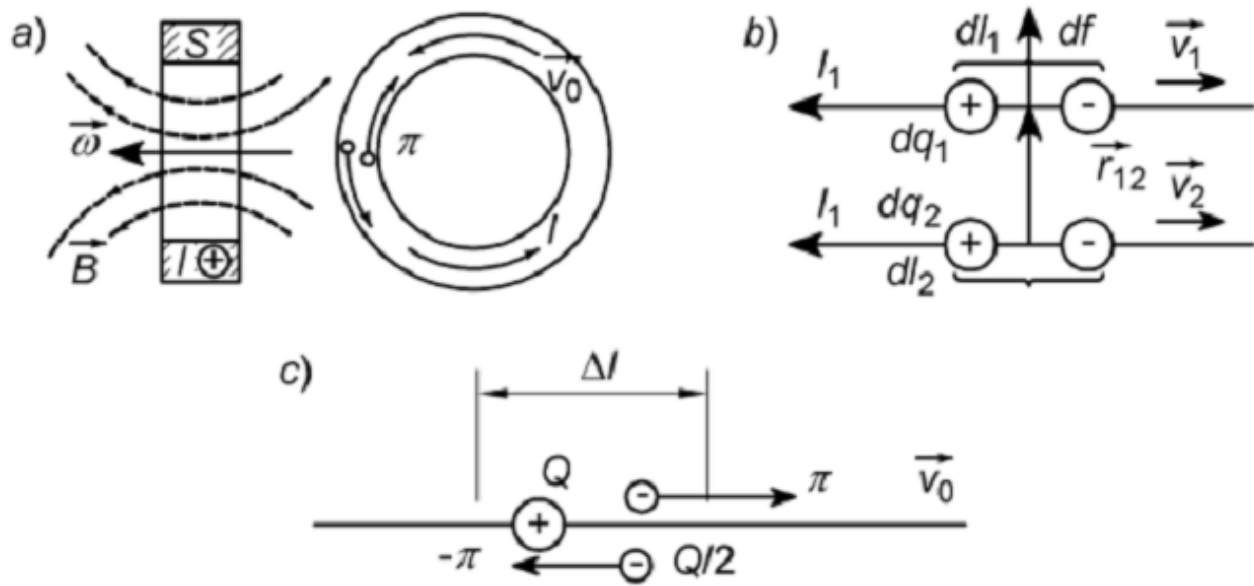


Figure 3

a) Generation of a magnetic field during rotation, b) EM effect of the current elements $I_1 dl_1$ and $I_2 dl_2$, c) The movement of a neutral conductor along its axis at the speed of that chaotic movement v_0 (Bjelić, 2021b)

Рис 3 – а) Создание магнитного поля при вращении, б) ЭМ воздействие элементов тока $I_1 dl_1$ и $I_2 dl_2$, в) движение нейтрального проводника вдоль своей оси со скоростью хаотического движения v_0 (Bjelić, 2021b)

Слика 3 – а) Стварање магнетног поља при ротацији, б) ЕМ ефекат струјних елемената $I_1 dl_1$ и $I_2 dl_2$, в) кретање неутралног проводника дуж своје осе брзином тог хаотичног кретања v_0 (Bjelić, 2021b)

If the requirements of the theory of relativity are taken into account, the forces acting between the charges of the elements dl_1 and dl_2 are:

$$df = -\frac{\mu_0}{4\pi} \frac{I_1 dl_1 I_2 dl_2}{r_{1,2}^2}, \quad (92)$$

$$df = df_0 \left(1 - \frac{v_1 v_2}{c^2} \right) \left[1 - \left(\frac{v_2}{c} \right) \right]^{-1/2}, \quad (93)$$

where the force df_0 is determined by Coulomb's law:

$$df_0 = -\frac{1}{4\pi\epsilon_0} \frac{dq_1 dq_2}{r_{1,2}^2}. \quad (94)$$

Index (1) refers to the rate of charging of a positive or negative element dl_1 , while index (2) refers to the rate of charging, also of a positive or negative element dl_2 .

The force acting from the positive charge of the element dl_2 to the positive charge dl_1 is:

$$df_{++} = df_0 \left(1 - \frac{v_1^+ v_2^+}{c^2} \right) \left[1 - \left(\frac{v_2^+}{c} \right)^2 \right]^{-1/2} = df_0, \quad (95)$$

$$v_1^+ = v_2^+ = 0, \quad df_{-+} = -df_0, \quad (96)$$

$$df_{+-} = df_0 \left[1 - \left(\frac{v_2^-}{c} \right)^2 \right]^{-1/2}, \quad (97)$$

$$df_{--} = df_0 \left(1 - \frac{v_1^- v_2^-}{c^2} \right) \left[1 - \left(\frac{v_2^-}{c} \right)^2 \right]^{-1/2}. \quad (98)$$

In equations (95) to (98), the index (+) refers to the action on the element dl_1 and the index (−) to the action on the element dl_2 .

Adding equations (95) to (98) and omitting the index gives:

$$df = -df_0 \frac{v_1 v_2}{c^2} \left[1 - \left(\frac{v_2}{c} \right)^2 \right]. \quad (99)$$

By expanding the series in middle brackets, the higher-order line terms v/c of equation (99) become:

$$df = -df_0 \frac{v_1 v_2}{c^2} \left[1 - \left(\frac{v_2}{c} \right)^2 + \dots \right]. \quad (100)$$

If in equation (100) the calculation is limited only to the first term of the series (one), then:

$$df = -df_0 \frac{v_1 v_2}{c^2}. \quad (101)$$

If it is known that $\mu_0 \epsilon_0 c^2 = 1$, then:

$$I_1 dl_1 = dq_1 / dt \cdot dl_1 = v_1 dq_1, \quad I_2 dl_2 = dq_2 / dt \cdot dl_2 = v_2 dq_2. \quad (102)$$

According to equations (92) and (102), the interaction forces of the two current elements are equal. When deriving Ampere's law from Coulomb's law, corrections according to the theory of relativity must be taken into account. In equation (100), the higher order terms are discarded because they are very small. If the v^2 relative speed corresponding to the permissible current density, the correction is:

$$\frac{1}{2} \left(\frac{v_2}{c^2} \right)^2 \approx 10^{-23}. \quad (103)$$

The correction deviates from the measurement, although the laws of Bio-Savar, Ampere and EM induction do not raise doubts about their accuracy. In the numerical sequence of equation (100), the first term is the charge rate squared. It is a correction of the force or magnetic field created by the conductor, which depends on the speed of orderly and chaotic movement of charges whose order of magnitude is much higher than the speed of orderly movement. A neutral conductor with no current is observed, which moves along its axis at the speed v_0 , Figure 3.c.

The assumption is that positive charges are stationary in relation to the conducting part, and negative charges are in chaotic motion and equally probable in all directions. The average speed v of that chaotic movement is determined by the Fermi energy and is $v \approx 10^6$ m/s, i.e. significantly exceeds the rate of charging at the permitted current density. The unprincipled assumption is that: the first half of the negative charges move in the direction v_0 and the second half in the opposite direction $-v_0$, Figure 3.a. The movement of charges creates current. For a stationary conductive body, the current as the sum of all charges, Figure 3.c, is equal to zero. The result will not change even when the conductive body is moved because all charges, positive and negative, get the same additional speed Figure 3.a. The situation is different if the sum of velocities is calculated according to the theory of relativity (Bjelić, 2019). The speed of negative charges is determined in relation to the conductive part, which then represents a moving coordinate system. For a stationary observer, the charge rates will be equal to:

$$\text{positive } v_+ = v_0, \quad (104)$$

negative moving in the direction of v_0 :

$$v_-^{(1)} = \frac{v_0 + u}{1 + (v_0 u / c^2)} \approx (v_0 + u) \left(1 - \frac{v_0 u}{c^2} \right), \quad (105)$$

negative if it moves opposite to v_0 :

$$v_-^{(2)} = \frac{v_0 - u}{1 - \frac{v_0 u}{c^2}} \approx (v_0 - u) \left(1 + \frac{v_0 u}{c^2} \right). \quad (106)$$

The total current of all charges can then be determined:

$$I = Q \cdot v, \quad (107)$$

where Q is the charge per unit length (in contrast to equation (102)) for the conductor.

The current created by positive charge is:

$$I_+ = Q \cdot v. \quad (108)$$

The currents created by negative charging are:

$$I_-^{(1)} = -\frac{1}{2}Q(v_0 + u)\left(1 - \frac{v_0 u}{c^2}\right), \quad (109)$$

$$I_-^{(2)} = -\frac{1}{2}Q(v_0 - u)\left(1 + \frac{v_0 u}{c^2}\right). \quad (110)$$

Adding equations (109) and (110) gives the total current:

$$I = Q \cdot v_0 (u/c)^2. \quad (111)$$

$Q = nes$ is determined from the volume charge density n and the cross-sectional area of the conductor s . At the same time, equation (111) takes a form that agrees with equation (91):

$$I = nes \cdot v_0 \left(\frac{u}{c}\right)^2. \quad (112)$$

The effect of moving the neutral conductor along the axis is similar to the effect of the current-carrying conductor. The conductive part can be spaced several times and the current I will not change. It is a consequence of the chaotic movement of charges, because the speed of movement in equation (112) is included in the term with the square. An example estimate is the current for an aluminum rotor: $n = 10^{28} \text{ 1/m}^3$, $e = 1.6 \cdot 10^{-19} \text{ C}$, $s = 0.5 \cdot 10^{-2} \text{ m}^2$, $v_0 = 10 \text{ m/s}$, $u = 10^6 \text{ m/s}$, $c = 3 \cdot 10^8 \text{ m/s}$, $I \approx 10^3 \text{ A}$.

From equation (112), the charge density n and the speed of chaotic movement should be known to estimate the current.

In this example, these values are taken for pure metal. Some impurities can reduce the value n by a higher order of magnitude. Experimental confirmation of formula (112) requires a preliminary analysis of the rotor material to determine n and u . However, there is indirect confirmation of currents and fields generated by cosmic bodies (Bjelić, 2021a; Dziewonski & Anderson, 1981). If equation (112) is applied to analyze the magnetic field of a homogeneous rotating sphere, the total current is:

$$I = \frac{1}{12} \frac{nD^3}{T} e \left(\frac{u_2}{c}\right)^2. \quad (113)$$

The strength of the magnetic field at its poles is approximately:

$$H = I/D, \quad (114)$$

and for magnetic induction is:

$$B = \mu_0 \mu' \cdot H = \mu_0 \mu' \cdot I / D, \text{ i.e. } B = \frac{1}{12} \frac{n D^2}{T} e \left(\frac{u_2}{c} \right)^2 \mu_0 \mu'. \quad (115)$$

Equation (115) can also be obtained from equation (111) $I = e \cdot v_0 (u/c)^2$ (if e is the electron charge) and then the relation is derived from equation (112):

$$B = \left[\frac{1}{12} e \left(\frac{u_2}{c} \right)^2 \mu_0 \cdot \mu \right] = A \frac{1}{T} \frac{(n D^3)}{D} = A \frac{m}{TD}, \quad B = A \frac{m}{TD}, \quad (116)$$

where μ' is relative magnetic conductivity, m is the mass, $D = 2r$ is the body diameter, T is the period of rotation around the axis, A is the constant.

In equation (116), the charge density n is proportional to the mass density $n \approx m/D^3$, and the discharge rate depends on the Fermi energy and is included in the constant A . If the formula for the Earth's magnetic field is written, then it is:

$$B_z = A \frac{m_z}{T_z D_z}. \quad (117)$$

When equation (116) is divided by equation (117) we get:

$$B = \frac{\dot{m}}{\dot{T} \dot{D}} B_z. \quad (118)$$

If the Earth's magnetic field is at the pole $B_z = 0.5 \cdot 10^{-4} \text{ T}$, then the magnetic field of an arbitrary cosmic body is:

$$B = 0.5 \frac{\dot{m}}{\dot{T} \dot{D}} 10^{-4} \text{ T}. \quad (119)$$

Table 2 includes a limited number of objects and allows an assessment of the degree of agreement between computational and experimental data for some space objects. Asterisks mean that the parameters are taken in relative values (to Earth). It is not known at what distance from the object the measurements were made, at the pole or at the equator, while the calculated data refer to the pole on the surface of the object.

Table 2
Comparison of computational and experimental data of some space bodies

Object					Calculated value B 10^{-4} T	Measure value B 10^{-4} T
Earth	1	1	1	1	1	1
Mercury	0.05	0.38	58	$2.3 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$	10^{-3}
Venus	0.81	0.95	243	$3.5 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$	$0.15 \cdot 10^{-3}$
Jupiter	318	11.2	0.4	72	36	4.5
Saturn	95	9.5	0.45	20	10	0.6
Sun	$33 \cdot 10^4$	10^9	25	120	60	(10-100)
White dwarfs	10^6	1.0	1.0	10^6	$0.5 \cdot 10^6$	10^5
Neutron stars	10^3	10^{-3}	10^{-5}	10^{11}	$0.5 \cdot 10^{11}$	10^{12}

Таблица 2 – Сравнение расчетных и экспериментальных данных некоторых небесных тел
Табела 2 – Поређење рачунских и експерименталних података неких свемирских тела

To date, magnetic fields have been measured for thousands of cosmic bodies. Their analysis would allow formula (118) to be evaluated more fully. The approximate coincidence of the calculated data with the experimental data is not considered accidental (Petković, 2016; Prodanović, 2006).

Conclusion

The obtained formulas can predict the order of magnitude of magnetic fields for known cosmic bodies, whose parameters such as masses, diameters and periods of rotation around the axis-differ by tens of thousands of times.

However, these formulas cannot explain many characteristics of magnetic fields, related to various, perhaps still unknown reasons.

The formulas and the results obtained in the paper indicate a possible common origin of the source of the magnetic field on the Earth (the Sun), and their accuracy can only be confirmed by cosmic research, experiment or computer simulation.

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