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Original scientific papers

Portfolio investment based on probabilistic multi-objective optimization and uniform design for experiments with mixtures

Инвестиционный портфель, основанный на вероятностной многоцелевой оптимизации и унифицированной модели с вариационными экспериментами

Инвестициони портфолио заснован на пробабилистичкој вишекритеријумској оптимизацији и униформном дизајну за експерименте са варијацијама

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Acceso abierto diamante

Abstract

Introduction/purpose: In this paper, a new approach to solving the portfolio investment problem is formulated to handle simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return. Probability - based multi - objective optimization is combined with uniform design for experiments with mixtures to conduct processing.

Methods: Preliminarily, probability - based multi - objective optimization is employed to synthesize the bi-objective problem of simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return into a single objective one of the total preferable probability of each alternative scenario. The total preferable probability is the product of all partial preferable probabilities of each performance utility; subsequently, the method of uniform design for experiments with mixtures is used to create a set of effective sampling points for the portfolio investment problem to provide discretization in data processing and simplifying treatment, of which the proportion x_i follows the constraint condition of $x_1 + x_2 + x_3 + \dots + x_s = 1$ with the total number of variables s for x_i .

Results: The new approach is used to deal with the portfolio Investment problem that is, in essence, simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return, which leads to reasonable consequences. The results are with the quality of rationality from the respect of the probability theory for simultaneous optimization of multiple objectives.

Conclusion: This method naturally reflects the essence of the portfolio investment problem and opens a new way of solving the relevant problem.

Keywords: portfolio investment problem, multi-objective optimization, preferable probability, discrete sampling, probability theory.

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Резюме

Введение/цель: В данной статье представлен новый подход к решению задач инвестиционного портфеля путем одновременной оптимизации максимизации ставки доходности и минимизации дисперсии ставки доходности. Обработка выполняется путем сочетания вероятностной многокритериальной оптимизации с унифицированной моделью вариационных экспериментов.

Методы: Вероятностная многокритериальная оптимизация в основном используется при сочетании двухкритериальной задачи одновременной оптимизации максимизации ставки доходности и минимизации дисперсии ставки доходности в однокритериальной задаче общей предпочтительной вероятности по каждому альтернативному сценарию. Общая предпочтительная вероятность является произведением всех частных вероятностей предпочтительной полезности производительности, поэтому метод унифицированной модели вариационных экспериментов используется при создании множества эффективных точек выборки для решения задачи инвестиционного портфеля с целью достижения дискретизации при обработке данных и упрощения процедуры, в которой пропорция x_i подчиняется условию ограничения $x_1 + x_2 + x_3 + \dots + x_s = 1$ с общим числом переменных s для x_i .

Результаты: В решении задач инвестиционного портфеля используется новый подход, который по сути представляет собой одновременную оптимизацию максимизации ставки доходности и минимизации дисперсии ставки доходности, что приводит к разумным результатам. Результаты обладают качеством рациональности с точки зрения теории вероятностей для одновременной оптимизации нескольких целей.

Выводы: Настоящий метод естественным образом отражает суть проблемы инвестиционного портфеля и дает новый способ решения важнейшей задачи.

Ключевые слова: задача инвестиционного портфеля, многокритериальная оптимизация, предпочтительная вероятность, дискретная выборка, теория вероятностей.

Abstract

Увод/циљ: Формулисан је нови приступ решавању проблема инвестиционог портфолија помоћу истовременог оптимизовања максимизације стопе приноса и минимизације варијансе стопе приноса. Процесирање се изводи комбинацијом вишекритеријумске оптимизације засноване на вероватноћи са униформним дизајном за експерименте са варијацијама.

Метод: Вишекритеријумска оптимизација заснована на вероватноћи превасходно се користи да се двокритеријумски проблем истовременог оптимизовања максимизације стопе приноса и минимизације варијансе стопе приноса синтетиче у једнокритеријумски проблем укупне пожељне вероватноће сваког алтернативног сценарија. Укупна пожељна вероватноћа је производ свих парцијалних пожељних вероватноћа корисности сваке перформансе. Дакле, метод униформног дизајна за експерименте са варијацијама користи се за креирање скупа ефективних тачака узорковања за проблем инвестиционог портфолија како би се постигла дискретизација у процесирању података и поједноставио поступак у којем пропорција x_i следи услов ограничења $x_1 + x_2 + x_3 + \dots + x_s = 1$ са укупним бројем варијабли s за x_i .

Резултати: Нови приступ користи се за решавање проблема инвестиционог портфолија који, у суштини, представља истовремено оптимизовање максимизације стопе приноса и минимизације варијансе стопе приноса, што води до разумљивих последица. Резултати имају квалитет рационалности са становишта теорије вероватноће за истовремену оптимизацију вишеструких циљева.

Закључак: Овај метод природно одсликава суштину проблема инвестиционог портфолија и пружа нов начин за његово решавање.

Keywords: проблем инвестиционог портфолија, вишекритеријумска оптимизација, пожељна вероватноћа, дискретно узорковање, теорија вероватноће.

Introduction

Portfolio investment aims to diversify investment risks in an effective way. Markowitz proposed a decision - making model of portfolio investment in 1952 which is seen as the foundation of the modern portfolio theory (Wang, 2022).

In Markowitz's treatment, the expected rate of return and the variance of the rate of return are employed to evaluate risky securities, with the latter used to reflect risk. The significant consequence of Markowitz's investigation is that investors should invest their funds in several securities instead in only one, so that investment risk could be reduced and appropriate investment returns obtained.

However, Markowitz's algorithm could only deal either with maximizing the expected rate of return and setting the variance of the rate of return as a restraint condition or with minimizing the variance of the rate of return and letting the expected rate of return be a constraint condition once at a time. In other words, such an approach could not handle simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return rationally due to the lack of appropriate methodology for dealing with multi-objective optimization (Sarmas et al, 2020; Oberoi et al, 2020; Nisani & Shelef, 2021).

Recently, Zheng et al (2022a) proposed probability - based multi - objective optimization in viewpoint of system theory, which created a brand new concept of "preferable probability"; furthermore, assessments for probability - based multi - objective optimization were put forward from the respects of the probability theory and the set theory. As a rationally novel approach concerning multiple objectives, it could be used in many fields, including energy planning, programming problems, operation research, financial affairs, management programs, material selection, mechanical design, engineering design, etc.

In this article, probability - based multi - objective optimization is combined with uniform design for experiments with mixtures to deal with the portfolio problem, so as to deal with the problem of simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return rationally.

Solution of the portfolio problem in the light of probability - based multi - objective optimization methodology and uniform design for experiments with mixtures

In this section, probability - based multi - objective optimization and uniform design for experiments with mixtures are organically combined, which establishes a rational method for solving the portfolio investment problem of simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return, i.e, a bi-objective problem. The probability - based multi - objective optimization method is used to transfer a bi - objective optimization problem into a single - objective optimization problem from the perspective of the probability theory naturally; the discretization of uniform design for experiments with mixtures provides an effective discrete sampling to simplify mathematical processing.

The systematic implementation is demonstrated by sub - sections A), B), and C).

A) Fundamental spirit of probabilistic multi - objective optimization

In the spirit of probability - based multi - objective optimization, each objective can be analogically represented as a single event in a system (Zheng et al, 2022a) and the whole event of multi - objective simultaneous optimization corresponds to the product of all single objectives (events). All performance utility indexes of a candidate are preliminarily classified into two types: i.e., beneficial type and unbeneficial type, in accordance with the role and preference of a candidate in the optimization, respectively. Specifically, the assessment of the preferable probability P_{ij} of both beneficial indicators and unbeneficial indicators can be carried out according to the evaluation procedure in Fig. 1 (Zheng et al, 2022a).

The meanings of the quantities and the factors in Fig. 1 are as follows:

P_{ij} indicates the partial preferable probability of the j -th performance utility indicator of the i -th alternative scenario, X_{ij} ; n expresses the total number of the alternative scenario; m reflects the total number of the performance (objective); $\bar{X}_j$ represents the arithmetic value of the j -th performance utility indicator; $X_{j\max}$ and $X_{j\min}$ show the maximum and minimum values of the j -th performance utility indicator, respectively; α_j and β_j express the normalized factors of the j -th performance utility indicator X_{ij} in the beneficial status and in the unbeneficial status, individually; the beneficial status or the unbeneficial status of the j -th performance utility indicator X_{ij} is determined according to its specific role or preference in the instant problem; and P_i represents the total (overall) preferable probability of the i -th alternative scenario (Zheng et al, 2022a).

Here, as to the portfolio investment problem, a simultaneous maximization of the rate of return and minimization of the variance of the rate of return is a typical bi-objective problem which contains a beneficial indicator and an unbeneficial indicator, respectively.

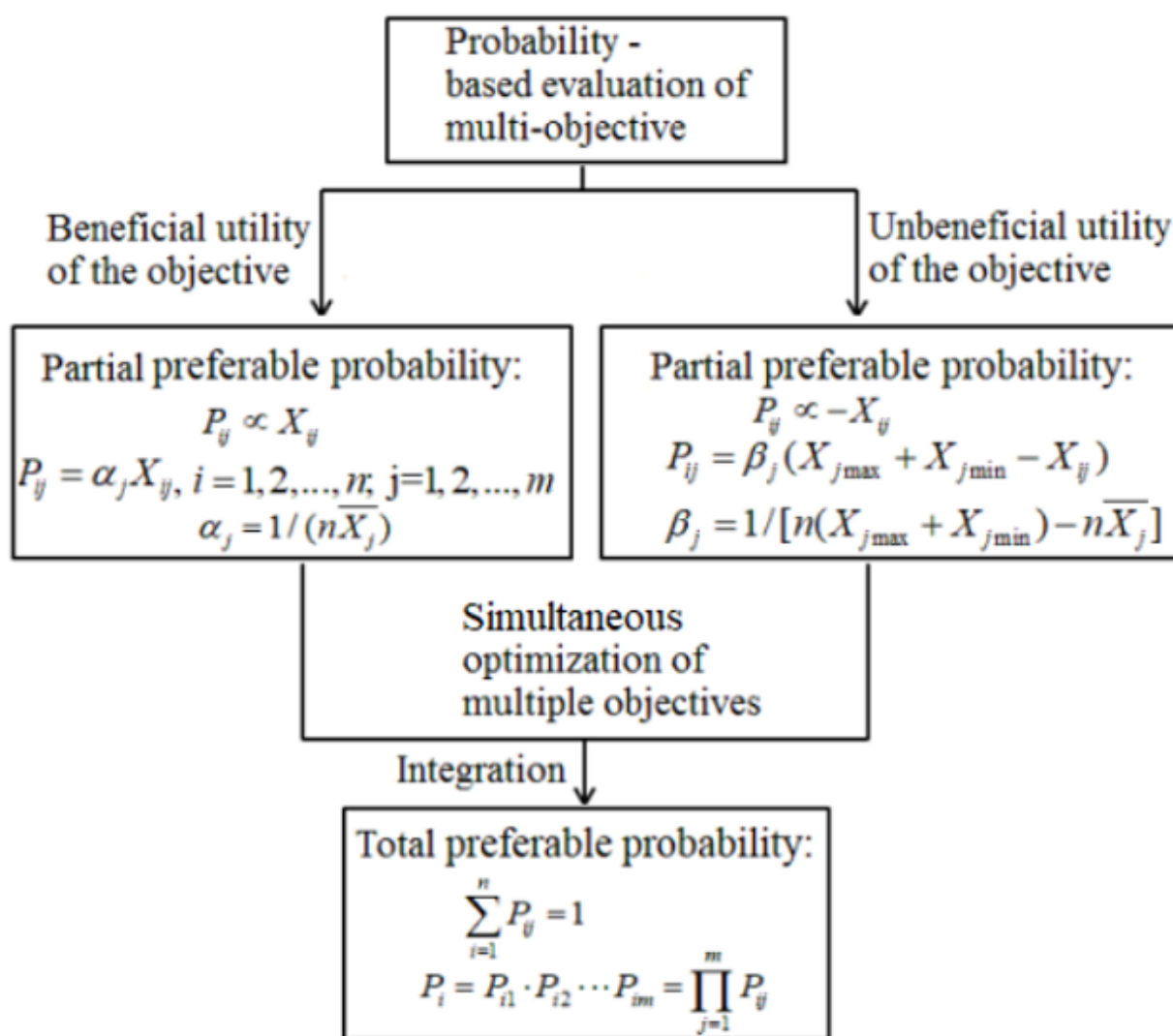


Figure 1

Evaluation procedure of the PMOO method

Рис. 1 – Процедура оценки метода многокритериальной оптимизации, основанной на вероятности

Слика 1 – Поступак евалуације метода вишекритеријумске оптимизације засноване на вероватноћи

B) Discrete sampling in the spirit of uniform design for experiments with mixtures

In this bi - objective portfolio investment problem, the total preferable probability is the decisive objective function which needs to be maximized in a high-dimensional space, so the complex data treatment might be involved; thus uniform design for experiments with mixtures (UDEM) can be employed to simplify data processing rationally.

Uniform design for experiments with mixtures (UDEM), based on the good lattice point (GLP), was proposed by Fang et al (2018). The method of UDEM can be used to create a set of effective sampling points for experimental design with the restraint of $x_1 + x_2 + x_3 + \dots + x_s = 1$ for the proportion x_i with the total number of s (Fang et al, 2018); therefore, it can be used as an efficient sampling method for the portfolio investment problem here to conduct the simplification of data processing with discretization.

In addition, Fang especially developed uniform design tables and their usage tables for proper application (Fang et al, 2018).

According to Fang et al (2018), the concrete steps of uniform design for experiments with mixtures (UDEM) are generally as follows:

I. Selection of the uniform design table

Given the number of mixtures s and the number of sampling points n , select the corresponding table $U_n^*(n^t)$ or $U_n(n^t)$ and the usage table from the uniform design table provided by Fang et al (2018), and the number of columns of the usage table is selected as $s-1$ at this time. Give a mark of the original elements in the uniform design table $U_n^*(n^t)$ or $U_n(n^t)$ with $\{q_{ik}\}$.

II. Constructing a new element c_{ki}

For each i , construct its new element c_{ki} according to the following formula,

$$c_{ki} = (2q_{ki} - 1)/(2n). \quad (1)$$

III. Constructing uniform sampling points for the mixtures, x_{ki}

$$x_{ki} = (1 - c_{ki}^{\frac{1}{s-i}}) \prod_{j=1}^{i-1} c_{kj}^{\frac{1}{s-j}}, \quad i=1, \dots, s-1. \quad x_{ks} = \prod_{j=1}^{s-1} c_{kj}^{\frac{1}{s-j}}, \quad k=1, \dots, n. \quad (2)$$

Thus, $\{x_{ki}\}$ gives the corresponding uniform design table $UM_n(n^s)$ of the mixture under the conditions of s and n .

C) Portfolio investment problem

According to Markowitz's study, the return rate function f_1 and the risk function f_2 could be introduced, and their expressions are, respectively,

$$f_1 = E(R) = \sum_{i=1}^s x_i \mu_i, \quad (3)$$

$$f_2 = [(X_1\sigma_1)^2 + (X_2\sigma_2)^2 + (X_3\sigma_3)^2 + \dots + (X_n\sigma_n)^2 + \gamma_{1,2}(X_1\sigma_1)(X_2\sigma_2) + \gamma_{1,3}(X_1\sigma_1)(X_3\sigma_3) + \gamma_{1,4}(X_1\sigma_1)(X_4\sigma_4) + \dots + \gamma_{i,j}(X_i\sigma_i)(X_j\sigma_j) + \dots + \gamma_{s-1,s}(X_{s-1}\sigma_{s-1})(X_s\sigma_s)]^{0.5}. \quad (4)$$

In Eq. (3), μ_i is the rate of return of the i -th security and s is the total number of the securities. In Eq. (4), γ_{ij} is the correlation coefficient between the i -th security and the j -th security; σ_i is the risk of the i -th security.

According to the objective evaluation in probability - based multi - objective optimization methodology (Zheng et al, 2022a), f_1 exhibits the bigger the better, which therefore belongs to the beneficial objective, and f_2 manifests the smaller the better, which thus belongs to the unbeneficial objective, respectively.

Therefore, the answer to the portfolio problem is the optimization of a bi-objective problem. Furthermore, probability - based multi - objective optimization methodology can be used to assess it reasonably. Of course, all the evaluations in probability - based multi - objective optimization methodology can be applied rationally.

Applications

In this section, an example illustrates the use of the above steps for solving the portfolio investment problem.

Take a combination of four securities, i.e., securities A , B , C , and D as a typical example. The specific optimization process is explained in detail.

Let the expected rate of return of Security A be $\mu_1 = 11.29\%$, and let its standard deviation of return be $\sigma_1 = 24.53\%$; let the expected rate of return of Security B be $\mu_2 = 18.10\%$, and let its standard deviation of return be $\sigma_2 = 19.94\%$; let the expected rate of return of Security C be $\mu_3 = 8.29\%$, and let its standard deviation of return be $\sigma_3 = 11.80\%$; and let the expected rate of return of Security D be $\mu_4 = 11.52\%$, with its standard deviation of return of $\sigma_4 = 12.75\%$. Furthermore, it is assumed that the following correlation coefficients between two of the above securities are $\gamma_{1,2} = 0$, $\gamma_{1,3} = 0.68$, $\gamma_{1,4} = 0$, $\gamma_{2,3} = 0$, $\gamma_{2,4} = 0$, $\gamma_{3,4} = 0$.

Now we need to make a decision of simultaneous optimization of both the maximization of the rate of return and minimization of the variance of the rate of return on this portfolio investment.

Solution

In this section, the problem of "portfolio" is analyzed based on probability - based multi - objective optimization methodology. This is a typical bi-objective optimization problem.

Let x_1 , x_2 , x_3 and x_4 be the investment percentages of four securities, A , B , C and D , respectively. There is actually a constraint condition for this problem, i.e., $x_1 + x_2 + x_3 + x_4 = 1$; therefore, it has actually three independent variables, namely x_1 , x_2 and x_3 .

Since the sampling points of this bi-objective optimization problem are positioned in the 4 - dimensional space, it is necessary to include at least 23 sampling points with the characteristics of the "good lattice point" in the effective region for the discretization of data processing (Yu et al, 2022; Zheng et al, 2022a; Zheng et al, 2022b).

According to Fang et al (2018), this is a "uniform design for experiments with mixtures" problem due to the constraint condition of the four variables. Let us take the uniform table $U^*_{23}(23^7)$ as the initial table to construct a uniform test design table $UM_{23}(23^4)$ with mixtures, as shown in Table 1.

The uniform test table $UM_{23}(23^4)$ with the mixtures of Table 1 is based on the uniform design table $U^*_{23}(23^7)$. Because here the number of variables s equals to 4, and n equals to 23, from the above rules, $x_{k1} = 1 - c_{k1}^{1/3}$, $x_{k2} = c_{k1}^{1/3} \times (1 - c_{k2}^{1/2})$, $x_{k3} = c_{k1}^{1/3} \times c_{k2}^{1/2} \times (1 - c_{k3})$, $x_{k4} = c_{k1}^{1/3} \times c_{k2}^{1/2} \times c_{k3}$ (Fang et al, 2018).

Furthermore, we can get the values of the rate of return function f_1 and the risk function f_2 , the distribution of their partial preferable probability and their total preferable probability, as well as the ranking at the sampling points (alternative scenario), which are shown in Table 2.

Fig. 2 shows the variation of the return rate with respect to risk at the discrete sampling points. The results reflect that the 2nd discrete sampling point gives the maximum total preferable probability closely followed by the 6th sampling point; therefore, they could be taken as the optimal solution to this portfolio problem.

Regarding the 2nd sampling point, the corresponding investment ratio is $x_1'' = 0.0222$, $x_2'' = 0.3494$, $x_3'' = 0.2596$, $x_4'' = 0.3688$, which leads to the rate of return of 12.98% and the risk of 9.09%.

As for the 6th sampling point, its investment ratio is at $x_1' = 0.0871$, $x_2' = 0.4665$, $x_3' = 0.1068$, $x_4' = 0.3397$, and its obtained rate of return is 14.23% with the risk of 10.73%.

Table 1

Uniform test table $UM_{23}(23^4)$ with the mixtures based on the uniform design table $U^*_{23}(23^7)$

No.	q_1 0	q_2 0	q_3 0	c_1	c_2	c_3	x_1	x_2	x_3	x_4
1	11	17	19	0.4565	0.7174	0.8044	0.2300	0.1178	0.1276	0.5246
2	22	10	14	0.9348	0.4130	0.5870	0.0222	0.3494	0.2596	0.3688
3	9	3	9	0.3696	0.1087	0.3696	0.2824	0.4810	0.1492	0.0874
4	20	20	4	0.8478	0.8478	0.1522	0.0535	0.0750	0.7389	0.1326
5	7	13	23	0.2826	0.5435	0.9783	0.3438	0.1725	0.0105	0.4733
6	18	6	18	0.7609	0.2391	0.7609	0.0871	0.4665	0.1068	0.3397
7	5	23	13	0.1957	0.9783	0.5435	0.4195	0.0063	0.2621	0.3121
8	16	16	8	0.6739	0.6739	0.3261	0.1233	0.1570	0.4850	0.2347
9	3	9	3	0.1087	0.3696	0.1087	0.5228	0.1871	0.2586	0.0315
10	14	2	22	0.5870	0.0652	0.9348	0.1627	0.6235	0.0139	0.1999
11	1	19	17	0.0217	0.8043	0.7174	0.7209	0.0288	0.0707	0.1796
12	12	12	12	0.5	0.5	0.5	0.2063	0.2325	0.2806	0.2806
13	23	5	7	0.9783	0.1957	0.2826	0.0073	0.5536	0.3150	0.1241
14	10	22	2	0.4130	0.9348	0.0652	0.2553	0.0247	0.6731	0.0470
15	21	15	21	0.8913	0.6304	0.8913	0.0376	0.1982	0.0831	0.6811
16	8	8	16	0.3261	0.3261	0.6739	0.3117	0.2953	0.1282	0.2649
17	19	1	11	0.8043	0.0217	0.4565	0.0700	0.7929	0.0745	0.0626
18	6	18	6	0.2391	0.7609	0.2391	0.3793	0.0793	0.4119	0.1295
19	17	11	1	0.7174	0.4565	0.0217	0.1048	0.2903	0.5917	0.0131
20	4	4	20	0.1522	0.1522	0.8478	0.4661	0.3256	0.0317	0.1766
21	15	21	15	0.6304	0.8913	0.6304	0.1425	0.0479	0.2992	0.5104
22	2	14	10	0.0652	0.5870	0.4130	0.5975	0.0941	0.1810	0.1274
23	13	7	5	0.5435	0.2826	0.1957	0.1839	0.3822	0.3490	0.0849

Таблица 1 – Таблица унифицированного теста $UM_{23}(23^4)$ с вариантами, основанными на таблице унифицированной модели $U^*_{23}(23^7)$

Табела 1 – Табела униформног теста $UM_{23}(23^4)$ са варијацијама заснованим на табели униформног дизајна $U^*_{23}(23^7)$

Table 2

Evaluation results of the rate of return f_1 the risk f_2 the preferable probability and the ranking at the sampling points

No.	q_1 o	q_2 o	q_3 o	c_1	c_2	c_3	x_1	x_2	x_3	x_4
1	11	17	19	0.4565	0.7174	0.8044	0.2300	0.1178	0.1276	0.5246
2	22	10	14	0.9348	0.4130	0.5870	0.0222	0.3494	0.2596	0.3688
3	9	3	9	0.3696	0.1087	0.3696	0.2824	0.4810	0.1492	0.0874
4	20	20	4	0.8478	0.8478	0.1522	0.0535	0.0750	0.7389	0.1326
5	7	13	23	0.2826	0.5435	0.9783	0.3438	0.1725	0.0105	0.4733
6	18	6	18	0.7609	0.2391	0.7609	0.0871	0.4665	0.1068	0.3397
7	5	23	13	0.1957	0.9783	0.5435	0.4195	0.0063	0.2621	0.3121
8	16	16	8	0.6739	0.6739	0.3261	0.1233	0.1570	0.4850	0.2347
9	3	9	3	0.1087	0.3696	0.1087	0.5228	0.1871	0.2586	0.0315
10	14	2	22	0.5870	0.0652	0.9348	0.1627	0.6235	0.0139	0.1999
11	1	19	17	0.0217	0.8043	0.7174	0.7209	0.0288	0.0707	0.1796
12	12	12	12	0.5	0.5	0.5	0.2063	0.2325	0.2806	0.2806
13	23	5	7	0.9783	0.1957	0.2826	0.0073	0.5536	0.3150	0.1241
14	10	22	2	0.4130	0.9348	0.0652	0.2553	0.0247	0.6731	0.0470
15	21	15	21	0.8913	0.6304	0.8913	0.0376	0.1982	0.0831	0.6811
16	8	8	16	0.3261	0.3261	0.6739	0.3117	0.2953	0.1282	0.2649
17	19	1	11	0.8043	0.0217	0.4565	0.0700	0.7929	0.0745	0.0626
18	6	18	6	0.2391	0.7609	0.2391	0.3793	0.0793	0.4119	0.1295
19	17	11	1	0.7174	0.4565	0.0217	0.1048	0.2903	0.5917	0.0131
20	4	4	20	0.1522	0.1522	0.8478	0.4661	0.3256	0.0317	0.1766
21	15	21	15	0.6304	0.8913	0.6304	0.1425	0.0479	0.2992	0.5104
22	2	14	10	0.0652	0.5870	0.4130	0.5975	0.0941	0.1810	0.1274
23	13	7	5	0.5435	0.2826	0.1957	0.1839	0.3822	0.3490	0.0849

Таблица 2 – Результаты оценки ставки доходности f_1 , риска f_2 , предпочтительной вероятности и ранжирования по точкам выборки

Табела 2 – Резултати процене стопе приноса f_1 , ризика f_2 , пожељне вероватноће и рангирање по тачкама узорковања

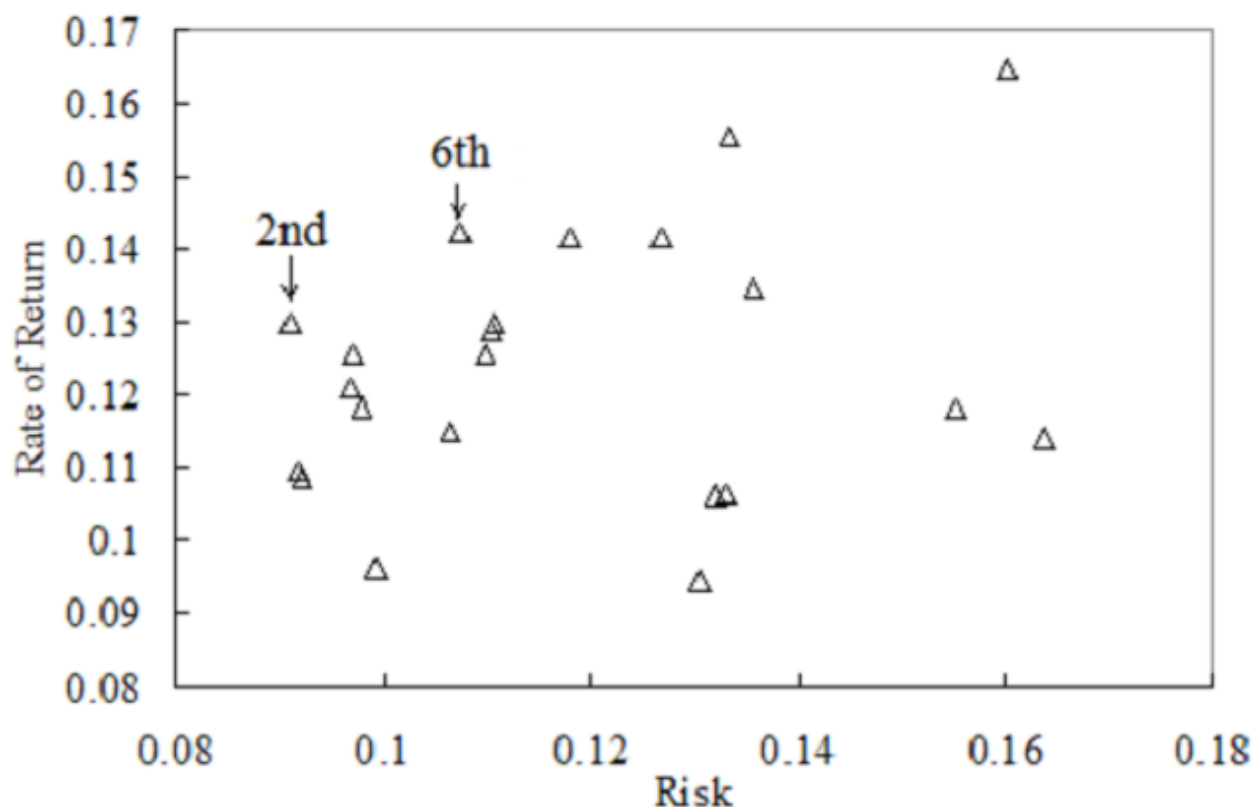


Figure 2

Variation of the return rate with respect to the risk at the discrete sampling points

Рис. 2 – Варианты ставки доходности по сравнению с риском в дискретных точках выборки

Слика 2 – Варијација стопе приноса у односу на ризик у тачкама дискретног узорковања

Conclusion

In this paper, the probability-based multi-objective optimization method is combined with uniform design for experiments with mixtures to study the portfolio investment problem, which aims to give a rational approach to handling the problem of simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return; the analysis shows that probability - based multi - objective optimization methodology could provide the optimal solution with the characteristics of simultaneous optimization of both maximizing the rate of return and minimizing the variance of the rate of return; uniform design for experiments with mixtures could be used to properly conduct discretization for data processing and simplification.

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