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Articles

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Rinkos kainos stabilumo tyrimas taikant Lamberto funkciją

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Abstract: In this article we are going to analyze market price stability with different market intensity coefficient and delay argument values. Market price is described as a scalar differential equation with a delay argument. In order to find solutions for the transcendental equation we will use method based on Lambert function. We will present examples of the applications of the method.

Keywords: differential equations, delayed arguments, Lambert function, market price.

Summary: Šiame straipsnyje nagrinėsime rinkos kainos stabilumą, kintant rinkos intensyvumo rodikliui ir vėlav- imo argumentui. Rinkos kaina aprašome tiesinę skaliarinę diferencialinę lygtį su vėluojančiu argu- mentu. Taikysime lygtį atitinkančios transcendentinės charakteristinės lygties šaknų radimo metodą, pagrįsta Lamberto funkcijų panaudojimu. Pateiksime metodo taikymo pavyzdžius.

Keywords: diferencialinė lygtis, vėlavimo argumentas, Lamberto funkcija, rinkos kaina.

Introduction

Recently there has been published many studies on stability of financial markets, power distribution systems [9, 4] and prices of food products, such as meat [7], rice [5], tomatoes [1] or maze [2].

Unstable market prices can have negative consequences for economy, development and health of the people, especially in developing countries [6], where majority of the population has to spend a good fraction of their earnings on essential food products. Therefore increase of product prices or food shortages might severly impact countries economy.

In the articles [3, 8] price stability system is described by Cobweb model, which expresses relation between the supply and demand of products. The system equilibrium is the intersection point between demand and supply curves. The relation between these two curves can indicate whether the price converges to the equilibrium point or it becomes unstable. In the article [5] stability of rice prices in Bangladesh was analyzed using dynamic programming techniques. In the article [2] model of themaze prices in Ghana was described by differential equation

with a delay argumentand the solution was obtained using numerical methods (dde23 from matlab package).

In this article we will analyze market price stability, when the mathematical model is described by linear differential equation with a delay argument. The solution of the transcendental equation will be obtained using Lambert function method.

1 Formulation of the problem

Lets say we have a market price and its equation is described by a mathematical model – differential equation with a delay argument [7, 3].

$$p'(t) = \gamma(D(p(t)) - S p(t - \tau)), \tag{1}$$

where τ is a delay argument, γ market stability coefficient, $D(p(t))$ – demand, $S(p(t))$ – supply, the variable $p(t)$ is a price dependent on time t.

Using demand $D(p(t)) = a + \beta p(t), \beta < 0$ and supply $S(p(t - \tau)) = \lambda + \delta p(t - \tau), \delta < 0$ in equation (1) we get:

$$p'(t) = \gamma(a + \beta p(t) - \lambda - S p(t - \tau)). \tag{2}$$

Denoting variables $v = \gamma\delta, r = -\gamma\beta$, using them in equation (2) and multiplying and dividing $\gamma(a - \lambda)$ by $(\delta - \beta)$ we get:

$$p\bigl(t\bigr) = \gamma\biggl(a - \lambda\biggr)\frac{(\delta - \beta)}{(\delta - \beta)} - r p\bigl(t\bigr) - v p\bigl(t - \tau\bigr). \tag{3}$$

Using market balance price $p\bigl(c\bigr) = \dfrac{(a - \lambda)}{(\delta - \beta)}$ in equation (3) and denoting $z(t) = p(t) - p(e)$ we get a differential equation with a delay argument:

$$z'(t) + rz(t) + vz(t - \tau) = 0. \tag{4}$$

We ar going to find transcendental characteristic equation for the equation (4). Then we denote, that the solution of differential equation can be written as $z(t) = Ce^{st}$, where C and s ar non-zero numbers:

$$Cse^{st} + Cre^{st} + Cve^{s(t-\tau)} = 0. \tag{5}$$

When $C \neq 0$ and $e^{st} \neq 0$, we get a transcendental characteristic equation that describes linear differential equation (1):

$$s + r + ve^{-s\tau} = 0. \tag{6}$$

The function (5) is a solution of the differential equation (1) if and only if the number . in its expression is the root of the transcendental equation (6).

We will find the roots of the characteristic equation (6) using the Lambert function method. Divide both sides of the equation by Ce^{st} and move $ve^{-\tau s}$ to the other side of the equation

$$s + r = -ve^{-\tau s}. \tag{7}$$

Multiplying both sides of the equation by $\tau e^{\tau s + r\tau}$

$$(s+r)\tau e^{(s+r)\tau} = -ve^{\tau s}. \tag{8}$$

Using Lambert function:

$$(s+r)\tau = W(-v\tau e^{\tau s}). \tag{9}$$

Solving the equation for . we get solution of the transcendental characteristic equation (8):

$$sk = \frac{1}{\tau} W_k(-v\tau e^{\tau r}) - r. \tag{10}$$

Using $u = \gamma\delta$ and $r = -\gamma\beta$ we get the solution of transcendental equation of the market price:

$$s_k = \frac{1}{\tau} W\left(-\gamma\delta\tau e^{-\gamma\beta\tau}\right) + \gamma\beta. \tag{11}$$

Since Lambert W function has infinite number of solutions, scalar transcendental characteristic equation will have infinite number of solutions as well, they can be written as:

$$s_k = \frac{1}{\tau} W_k\left(-\gamma\delta\tau e^{-\gamma\beta\tau}\right) + \gamma\beta, \qquad k = 0, \pm 1, \pm 2, \dots \tag{12}$$

Solution of differential equation with a delay argument (1) is asimptotically stable if all complex numbers s_k, $k = 0, \pm 1, \pm 2, \dots$ have negative real parts.

2 Results of calculation

Using expression (12) we will analyze the dependency of coordinates of solutions of the transcendental equation in complex plane by market stability coefficient parameters. We will be using Lambert function

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branch that is equal to zero. The market stability coefficient was chosen from article [7]. We will analyze stability of poultry market prices, when the mathematical model is scalar linear differential equation with a delay argument:

$$p'(t) = \gamma(375, 2 - 166 - 0.78p(t) - 1, 78p(t - 0.5)), \quad \phi(t) = 0, 181t^2 + 3.4t + 67, \qquad t \in [-0.5, 0] \quad (13)$$

In Fig. 1 we can see that when delay argument is 0.9 and stability coefficient is between 0.1 and 1.406, the system is asymptotically unstable. When the delay argument is 1, the system is stable when stability coefficient is between 0.1 and 1.265 and when delay argument is 2 the system is stable between 0.1 and 0.632. We can draw a conclusion that with a smaller delay the system is stable for a longer period of time.

The second example is from article [6]. We will analyze Kalecki's stationary business cycle model, when *m = 0.95, n = 0.121* .

$$k'(t) = a(k(t) - 1) - b(k(t - \tau) - 1), \qquad \tau > 0, \qquad (14)$$

where $a = \frac{m}{\tau}$, $b = \frac{m}{\tau} + n$. In the second example we analyzed Kalecki's business cycle model with different delay arguments. With chosen values of m and n parameters 0.95 and 0.121 we can draw a conclusion that system is asymptotically stable, when delay argument is between 0.1 and 0.6 and it becomes unstable when delay argument is 0.61 (Fig. 2).
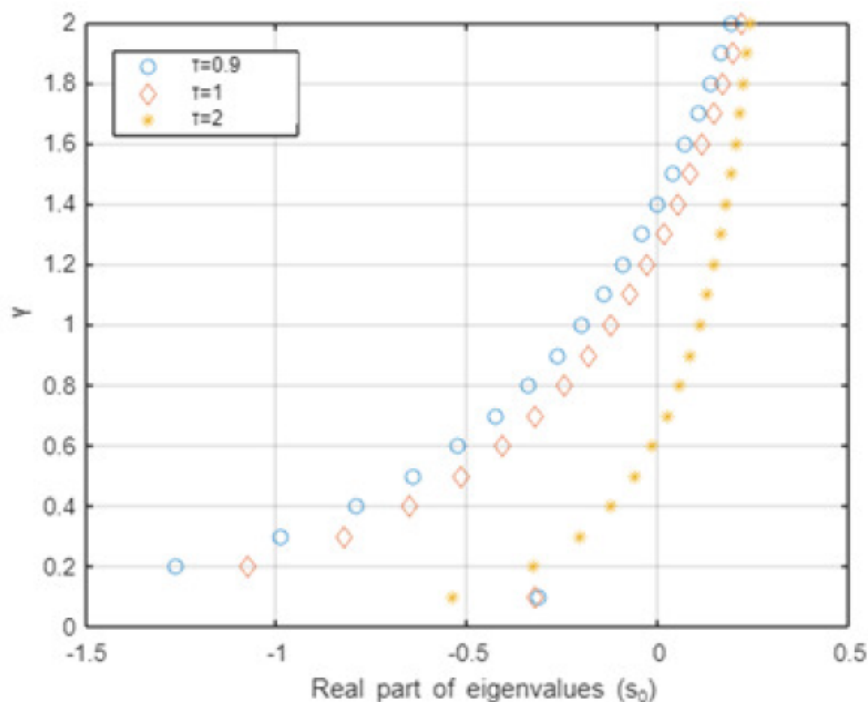

Fig. 1.
Market price stability with different γ market stability coefficient values and delays.

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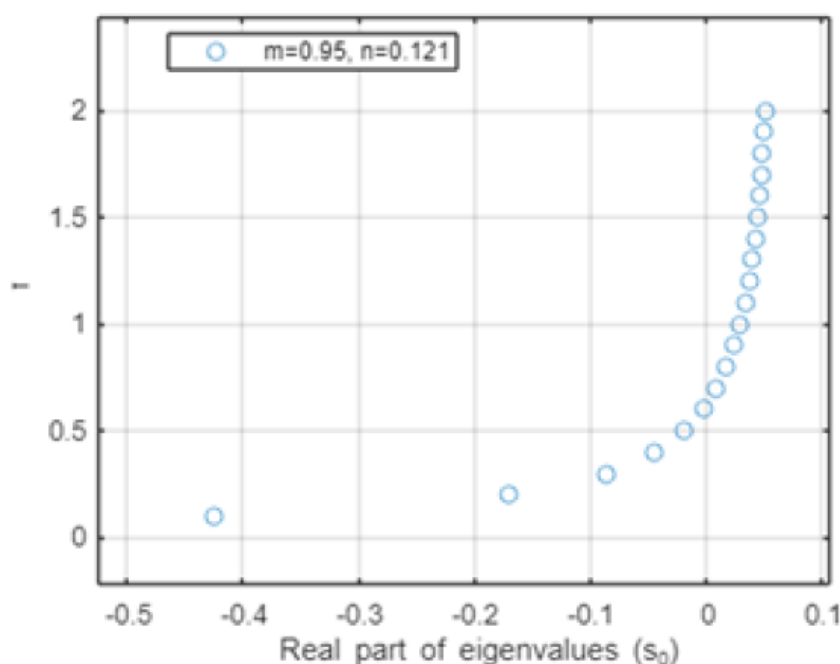
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Fig. 2.
Kalecki's model stability with different delay τ arguments.

3 Conclusions

Lambert function method was used to analyze market price stability. We analyzed roots of the transcendental characteristic equation corresponding to the differential equation with a delay argument. With our method of choice we were able to easily find paramater values when the system is asymptotically stable. Lambert function method is suitable for analyzing stability of various markets.

The lower the value of delay argument, then wider range of market stability coef- ficient values can be selected so that market price is asymptotically stable.

Using different parameters it is possible to analyze stability of Kalecki's business model using Lambert function method.

References

[1] M. Anokye, F.T. Oduro. Cobweb model with buffer stock for the stabilization of tomato prices in Ghana. *J. Manag. Sust.,* .(1), 2013. https ://doi.org/10.5539/jms.v3n1p155.

[2] M. Anokye, F.T. Oduro. Maize price stabilization in Ghana: an application of a continuous-time delay differential equation model with buffer stock. *British J. Math. Comp. Sci.,* .(4):279–296, 2015. https://doi.org/10.9734 /BJMCS/2015/14892.

[3] G. Athanasiou, I. Karafyllis, S. Kotsios. Price stabilization using buffer stocks. *J. Econ. Dyn. Control*, **32**(4):1212–1235, 2008.

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[4] K. Bradbury, L. Pratson, D. Patino-Echeverri. Economic viability of energy storage systems based on price arbitrage potential in real-time U.S. electricity markets. *Appl. Energy*, **114**(C):512–519, 2014.

[5] D. Brennan. Price dynamics in the bangladesh rice market: implications for public intervention. *Agr. Econ.*, **29**(1):15–25, 2003. https://doi.org/10.1111/j.1574-0862.2003.tb00144.x.

[6] R. Franke. Reviving Kalecki's business cycle model in a growth context. *J. Econ. Dyn. Control*, **91**(C):157–171, 2018. https://doi.org/10.1016/j.jedc.2017.12.00.

[7] B. Martina, N. Veronika. The use of functional differential equations in the model of the meat market with supply delay. *Procedia – Soc. Behav. Sci.*, **213**:74–79, 2015. https://doi.org/10.1016/j.sbspro.2015.11.406.

[8] S. Mitra, J.M. Boussard. A simple model of endogenous agricultural commodity price fluctuation with storage. *SSRN Electr. J.*, 2011. https://doi.org/10.2139/ssrn.1908115.

[9] D. Watts, F.L. Alvarado. The influence of futures markets on real time price stabilization in electricity markets. In *Proceedings of the 37th Annual Hawaii International Conference on System Sciences*, 2004. https://doi.org/https://doi.org/10.1109/HICSS.2004.1265167.