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Articles

A mathematical view towards improving undergraduate student performance and mitigating dropout risks.

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Abstract: In this paper, we assess the relevance of social and cognitive factors such as self-efficacy, locus of control and exposure to negative social influence in relation to undergraduate student dropout. To this purpose, we analyze a compartmental model involving a system of nonlinear ODEs, which is loosely based upon the SIR model of mathematical epidemiology and describes the academic performance of the student population. We examine threshold values that govern the stability of the equilibria and can be viewed as target values to be reached in order to alleviate undergraduate students dropout. A backward bifurcation is observed to occur, analytically and numerically, provided that certain conditions are satisfied.
A sensitivity analysis is then performed to find how the threshold values respond to changes in the parameters, a procedure for estimating these parameters being also proposed. Concrete values are then computed using survey data from a Ghanaian university. The impact of parameter variation upon the dynamics of the system, particularly on certain population sizes and on threshold values, is also numerically illustrated. Our findings are then interpreted from a social cognitive perspective, realistic policy changes being proposed along with appropriate teaching and coaching strategies.
Keywords: locus of control, self-efficacy, social influence, threshold parameters, dropout, stability, bifurcation.

1 Introduction

As the stress and anxiety levels of undergraduate students are rising higher and higher, there are many factors, which are prone to influencing their dropout intentions and their resilience when facing adverse circumstances or balancing academic and personal life challenges. Often enough, dropout intentions are a result of unsatisfactory academic performance culminating in repeated course failure. The students, who fail multiple examinations face emotional upset, are burdened with a sizable amount of additional work and are starting to face the looming spectre of expulsion due to not meeting the required academic standards.

There is ample evidence suggesting that locus of control (LOC), generally defined as the degree to which people believe that they can control the outcomes of events, which occur in their lives [19], influences how students react to course failure and their persistence towards achieving academic goals [8]. In this regard, LOC distinguishes whether the outcomes of their actions are contingent on their own efforts and abilities (internal control) or on external factors such as luck, fate or the behaviors of others (external control) [15]. Locus of control is linked with attribution theory [27], which asserts that individuals are hard-wired to seek causes or reasons for negative or unexpected consequences in their lives such as failure in important examinations [23]. These reasons are grouped into locus of causality, stability and controllability.

Locus of causality attributes the causes of examination failure to personal (internal) or to external factors. For instance, ability and effort are internal factors, while luck and task difficulty are external factors. Stability distinguishes between causes that change over time (unstable) and causes that do not (stable), while controllability differentiates between causes that can be controlled (controllable) and causes that cannot (uncontrollable) [23]. Ability is regarded as stable factor, albeit uncontrollable, while effort, on the other hand, is considered to be an unstable but controllable factor. The students with high internal LOC attribute course failure to lack of effort, while those with low internal LOC ascribe it to low ability [23]. Thus, students with high internal LOC have a greater sense of control over their academic performance and are more motivated to succeed, whereas students with low internal LOC have less motivation, believing that they, in fact, have little control over their academic performance [27].

Another factor that may strongly influence the motivation of students to persist in pursuing their academic programs is self-efficacy (S-E) defined as the belief of an individual in his or her capacity to achieve a desired outcome [2]. In an educational setting, of interest is academic self-efficacy considered as being students' perception of their ability to organize and complete specific courses of action in order to achieve specific learning outcomes in a particular situation [9, 22].

Self-efficacy influences the amount of effort students put in their academic work, their persistence when persevering through challenges and their resilience in the face of academic failure [9, 20]. Students with lower S-E are more likely to avoid difficult tasks and have a lesser commitment to achieving goals [22]. This means that students resitting failed courses need elevated levels of S-E to persist in their undergraduate studies.

The social cognitive theory of psychological functioning [2] emphasizes learning from social environments, that is, the fact that people learn desirable and undesirable behaviors chiefly by interacting with others. Accordingly, the decisions of students about whether to perseverate in their studies or to dropout are heavily influenced by their peers [1].

Improving the academic performance of undergraduate students, which has the potential to greatly reduce dropout risks, is a crucial issue in higher education [17]. Several studies on student persistence and dropout in higher education have focused on exploring the institutional and individual characteristics, which are associated with students' decision to dropout [3, 6]. Other studies, based upon statistical and data mining techniques, are dedicated to the early detection and prediction of dropout cases [12, 28]. Only a minority of studies, however, are concerned with how students react to repeated examination failure, which is one of the leading causes of dropout. From a social cognitive perspective [2], the beliefs of students regarding their LOC and S-E affect decisions in regard to persistence or dropout, aspect which deserves further investigation. Additionally, prior research has only rarely used a mathematical modelling approach to account for the LOC and S-E of students, along with social influences, as precursors to dropout decisions in undergraduate education

This paper investigates a mathematical model that relies on a system of nonlinear ODEs in order to analyze the extent to which the LOC and S-E of college students, along with exposure to negative social influences, influence student dropout. There are three main purposes of this paper, listed as follows.

1. To analyze a mathematical model that assesses the relevance of social and cognitive factors in relation to student dropout.
2. To examine threshold values, not unlike the basic reproduction number of mathematical epidemiology, that govern the dynamics of the model and can be viewed as target values to be reached in order to alleviate the problem of students dropout from undergraduate studies, from a sensitivity viewpoint.
3. To find realistic policy changes in order to address instances of repeated course failure and low academic performance as well as high dropout rates in higher education from a social cognitive perspective.

Consequently, the findings of this study can be used for policy review in order to reduce dropout risk, specifically for addressing the coping strategies of students as a response to examination failure, in relation to their LOC and S-E, as well as to the negative social influences they are subjected to.

The remaining part of this paper is organized as follows. In Section 2, we state our main assumptions along with further considerations that lead us towards employing our specific mathematical model. In Section 3, we review prior related work, establish the importance of certain threshold values and investigate the existence of a backward bifurcation from both an analytical and a numerical viewpoint. Section 4 is concerned with proposing a procedure for parameter estimations, then followed for survey data from a Ghanaian university. Section 5 is dedicated to a sensitivity analysis performed in order to find how the threshold values governing the stability of the equilibria respond to changes in the parameters. In Section 6, we numerically illustrate the impact of parameter variation on the dynamics of the system, particularly on the sizes of the populations of resit and dropout students, respectively. Finally, a discussion of our results from a social cognitive perspective is given in Section 7, along with several concluding remarks.

2 The model

In most universities, students, which are subject to compelling circumstances, are entitled to a make-up examination after following proper notification procedures to the appropriate university board. While the students who pass the make-up examination can continue pursuing their program unabated, those who fail are usually required to resit the course, no further make-up examinations being scheduled. Resitting courses may lead to delayed graduation, increased tuition and lost wages as the additional time spent on coursework could have been used for earning wages [23]. Furthermore, failing the mandated number of resit attempts for a given course constitutes grounds for expulsion from the university. Consequently, understanding how students cope with examination failure is crucial in convincing students to persist in their studies, rather than to dropout.

To investigate the impact of S-E and LOC upon undergraduate students dropout, we shall employ a model, which is loosely based on the standard SIR (susceptible-infective- recovered) model of mathematical epidemiology and has been introduced in [29]. This model reads as

$$\begin{aligned}\dot{p} &= \mu + (\mu_1 - \mu)d + \eta r + \sigma m - \kappa p - \mu p,\\ \dot{m} &= \kappa p - m(\beta_1 r + \beta_2 d) - \sigma m - \mu m,\\ \dot{r} &= m(\beta_1 r + \beta_2 d) - \eta r - \beta_3 r d - \mu r,\\ \dot{d} &= \beta_3 r d - \mu_1 d,\end{aligned}$$

where p represents the percentage of passing students (from the total student population, including dropout students), m represents the percentage of students, who fail at least one course and have to take make-up examinations, r represents the percentage of students, who fail make-up examinations and have to resit courses, and d represents the percentage of dropout students.

The model accounts for several distinct mechanisms of (negative) social influence between make-up, resit and dropout students (quantified by the term $m(\beta_1 r + \beta_2 d)$)and between resit and dropout students (quantified by the term $\beta_3 r d$)). Make-up students are assumed to re-enter the pass population at a rate . upon passing all their make-up examinations as a result of their high internal LOC. Also, resit students are assumed to re- enter the pass population at a rate n upon successfully resitting all their failed courses as a result of their S-E. The meaning of the other parameters, which appear in (1), is explained in Table 1. See also [29] for further details and [13] for a related, lower dimensional model, which does not keep track of the influence of the dropout students. Similar types of models have been successfully used to discuss other educational and social matters [1,16].

Table 1.

The meaning of the parameters appearing in model (1).

Table 1. The meaning of the parameters appearing in model (1).

Parameter	Description
$1/\mu$	average number of years spent in college by a typical student
μ_1	dropout rate
κ	rate of movement from the passing compartment to the make-up compartment
σ	rate of movement from the make-up compartment to the passing compartment
η	rate of movement from the resit compartment to the passing compartment
β_1	average negative influence of resit students on make-up students
β_2	average negative influence of dropout students on make-up students
β_3	average negative influence of dropout students on resit students

3 Model analysis

Model (1) has already been investigated in [29] from a stability viewpoint, two threshold parameters being found to govern the stability of the equilibria. Specifically, the model has a resit and dropout-free equilibrium (understood as the "ideal" equilibrium) given by

$$I_0 = \left(\frac{\mu+\sigma}{\kappa+\mu+\sigma}, \frac{\kappa}{\kappa+\mu+\sigma}, 0, 0\right).$$

Its stability was found to depend upon the values of the resit reproduction number of the model denoted by $\mathcal{R}_{0R}$ and defined as the average number of make-up students that a single member of the resit group will influence to resit a course. It has been determined in [29] that

$$\mathcal{R}_{0R} = \frac{\beta_1\kappa}{(\kappa+\mu+\sigma)(\eta+\mu)}$$

and that the resit and dropout-free equilibrium I_0 is locally asymptotically stable if $\mathcal{R}_{0R} < 1$ and unstable if $\mathcal{R}_{0D} > 1$. Further, model (1) has a dropout-free equilibrium (understood as the "subideal" equilibrium) given by

$$I_\vartheta = \left(\frac{(\mu+\eta)(\sigma+\beta_1-\eta)}{\beta_1(\mu+\kappa+\eta)}, \frac{\eta+\mu}{\beta_1}, \frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{\beta_1(\mu+\kappa+\eta)}, 0\right)$$

It has been found in [29] that the dropout-free equilibrium I_v. is locally asymptotically stable if $\mathcal{R}_{0D} < 1$ and unstable if $\mathcal{R}_{0D} > 1$, where R_{0D} is the dropout reproduction number, defined as the average number of resit students that a single member of the dropout group will influence to leave the university, and given by

$$\mathcal{R}_{0D} = \frac{\beta_3(\mu+\kappa+\sigma)(\eta+\mu)}{\mu_1\beta_1(\eta+\kappa+\mu)}(\mathcal{R}_{0R}-1).$$

The existence and multiplicity of the resit and dropout-persistent equilibrium (shortened from now on as RDPE and understood as the "realistic" equilibrium) has been characterized in [29] via a comparatively more involved result (Theorem 3.3) formulated chiefly in terms of R_{0D}.. It makes then sense to further discuss the exchanges of stability, which occur between the equilibria of (1), and to this purpose, we shall perform a bi- furcation analysis based on a general result given in [4]. To keep notational consistency with [4], we make the following notations: $p = x_1$, $m = x_2$, $r = x_3$ and $d = x_4$. Consequently, (1) can then be restated as

$$\begin{aligned}
\frac{\mathrm{d}x_1}{\mathrm{d}t} &= \mu + (\mu_1 - \mu)x_4 + \eta x_3 + \sigma x_2 - \kappa x_1 - \mu x_1 := h_1, \\
\frac{\mathrm{d}x_2}{\mathrm{d}t} &= \kappa x_1 - x_2(\beta_1 x_3 + \beta_2 x_4) - \sigma x_2 - \mu x_2 := h_2, \\
\frac{\mathrm{d}x_3}{\mathrm{d}t} &= x_2(\beta_1 x_3 + \beta_2 x_4) - \eta x_3 - \beta_3 x_3 x_4 - \mu x_3 := h_3, \\
\frac{\mathrm{d}x_4}{\mathrm{d}t} &= \beta_3 x_3 x_4 - \mu_1 x_4 := h_4
\end{aligned}$$

or, in vector form, $\mathrm{d}X/\mathrm{d}t = H(X)$ with

$$X = (x_1, x_2, x_3, x_4)^{\mathrm{T}} \quad \text{and} \quad H = (h_1, h_2, h_3, h_4)^{\mathrm{T}}$$

Let β_3 be the bifurcation parameter. Solving equation (2) for $\mathcal{R}_{0D} = 1$ gives us the critical value of β_3 as being

$$\beta_3^* = \frac{\mu_1\beta_1(\eta+\kappa+\mu)}{\kappa\beta_1 - (\eta+\mu)(\mu+\kappa+\sigma)}.$$

Note that $\mathcal{R}_{0D} = 1$ implies that $\kappa\beta_1 - (\eta+\mu)(\mu+\kappa+\sigma) > 0$. The explicit expression of the Jacobian matrix evaluated at I_v for $\beta_3 = \beta_3^*$ can be simplified as

$$\begin{pmatrix} -(\kappa+\mu) & \sigma & \eta & \mu_1-\mu \\ \kappa & -(\frac{\beta_1\mu_1}{\beta_3^*}+\mu+\sigma) & -(\eta+\mu) & -\frac{\beta_2(\eta+\mu)}{\beta_1} \\ 0 & \frac{\beta_1\mu_1}{\beta_3^*} & 0 & \frac{\beta_2(\eta+\mu)}{\beta_1}-\mu_1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We observe that $J(I_\vartheta)$ has a right eigenvector $w=(w_1,w_2,w_3,w_4)^{\mathrm{T}}$ and a left eigenvector $v=(v_1,v_2,v_3,v_4)^{\mathrm{T}}$ associated with the eigenvalue 0 at $\beta_3=\beta_3^*$ with

w1 = $-\frac{\beta_3^*\beta_1\mu_1(\eta-\sigma)-\beta_3^*\beta_2(\eta-\sigma)(\eta+\mu)+{\beta_1}^2\mu_1(\eta+\mu-\mu_1)}{{\beta_1}^2\mu_1(\eta+\kappa+\mu)}$,

w2 = $\frac{\beta_3^*(\beta_1\mu_1-\beta_2\eta-\beta_2\mu)}{{\beta_1}^2\mu_1}$, $w_4=1$

w3 = $-\frac{\beta_3^*\beta_1\mu_1(\mu+\kappa+\sigma)-\beta_3^*\beta_2(\mu+\kappa+\sigma)(\eta+\mu)+\mu_1{\beta_1}^2(\kappa+\mu_1)}{{\beta_1}^2\mu_1(\eta+\kappa+\mu)}$,

v1 = $0,\quad v_2=0,\quad v_3=0,\quad v_4=1.$

To apply the bifurcation result presented in [4], we first determine the values of the expression . and . indicated therein, which are now given by

$$a=\sum_{k,i,j=1}^{4} v_k w_i w_j \frac{\partial^2 h_k}{\partial x_i \partial x_j}(I_\vartheta,\beta_3^*), \qquad b=\sum_{k,i=1}^{4} v_k w_i \frac{\partial^2 h_k}{\partial x_i \partial \beta_3^*}(I_\vartheta,\beta_3^*).$$

Since $v_k=0$ for $k=1,2,3$ and the only nonzero partial derivatives of h_4 at I_v for $\beta_3=\beta_3^*$ are

$$\frac{\partial^2 h_4}{\partial x_3 \partial x_4}(I_\vartheta,\beta_3^*)=\frac{\partial^2 h_4}{\partial x_4 \partial x_3}(I_\vartheta,\beta_3^*)=\beta_3^*,$$
$$\frac{\partial^2 h_4}{\partial x_4 \partial \beta_3^*}(I_\vartheta,\beta_3^*)=\frac{\kappa\beta_1-(\mu+\eta)(\mu+\kappa+\sigma)}{\beta_1(\mu+\kappa+\eta)}=\frac{\mu_1}{\beta_3^*},$$

one sees that

$$\begin{aligned} a &= v_4 w_3 w_4 \frac{\partial^2 h_4}{\partial x_3 \partial x_4}(I_\vartheta,\beta_3^*)+v_4 w_4 w_3 \frac{\partial^2 h_4}{\partial x_4 \partial x_3}(I_\vartheta,\beta_3^*) \\ &= \frac{2\beta_3^*}{{\beta_1}^2\mu_1(\eta+\kappa+\mu)}\{\beta_3^*(\mu+\kappa+\sigma)[-\beta_1\mu_1+\beta_2(\eta+\mu)]-\mu_1\beta_1^2(\kappa+\mu_1)\}. \end{aligned}$$

From (3) it follows that

$$a = \frac{2\beta_3^*}{\beta_1(\eta+\kappa+\mu)[\kappa\beta_1-(\eta+\mu)(\mu+\kappa+\sigma)]} \times \left\{(\eta+\kappa+\mu)(\mu+\kappa+\sigma)\left[-\beta_1\mu_1+\beta_2(\eta+\mu)\right] - \beta_1(\kappa+\mu_1)\left[\kappa\beta_1-(\eta+\mu)(\mu+\kappa+\sigma)\right]\right\},$$

$$b = v_4 w_4 \frac{\partial^2 h_4}{\partial x_4 \partial \beta_3^*}(I_\vartheta, \beta_3^*) = \frac{\mu_1}{\beta_3^*}.$$

It is then seen that $b > 0$, while the sign of .is variable and can be discussed in terms of B_1 By using Theorem 4.1 of [4] we are now able to characterize the stability switches, which occur at $\mathcal{R}_{0D} = 1$

Theorem 1. *The RDPE of system* (1) *is locally asymptotically stable for* $\mathcal{R}_{0D} > 1$1 (*but close to* 1), *and a backward bifurcation occurs at* $\mathcal{R}_{0D} = 1$, *provided that one of the following equivalent conditions holds*:

(i) The dropout rate remains under a critical value with

$$\mu_1 < \mu_1^{**} = \frac{\beta_2(\eta+\mu)(\eta+\kappa+\mu)(\mu+\kappa+\sigma)-\beta_1\kappa[\kappa\beta_1-(\eta+\mu)(\mu+\kappa+\sigma)]}{\kappa\beta_1(\mu+\kappa+\sigma+\beta_1)}.$$

(ii) The level of negative social influence of resit on make-up students remains under a critical value with

$$\beta_1 < \beta_1^{**} = \frac{-b_1+\sqrt{b_1^2-4a_1c_1}}{2a_1},$$

Where

$$a_1 = \kappa(\mu_1+\kappa), \qquad b_1 = \kappa(\mu+\kappa+\sigma)(-\mu-\eta+\mu_1)$$

$$c_1 = -(\eta+\mu)(\mu+\kappa+\sigma)\beta_2(\mu+\eta+\kappa)$$

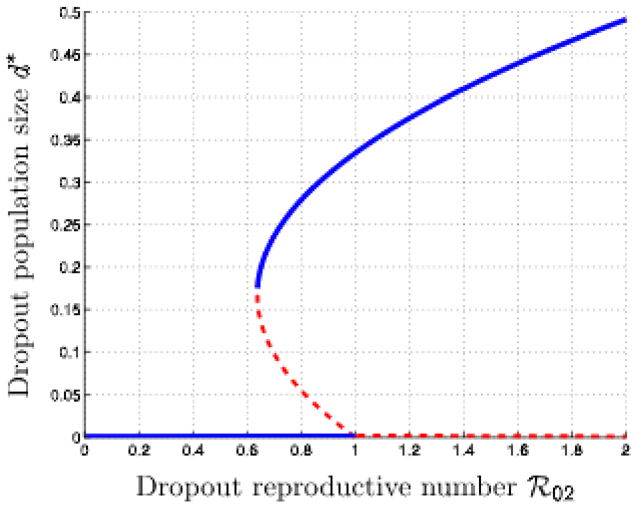

Figure 1.

The threshold R_{0D} is plotted on the horizontal axis, and the corresponding $d^{\#}$ components of the equilibria are plotted on the vertical axis. The solid lines denote stable equilibria and the dotted lines denote unstable equilibria.

In epidemiology, the occurrence of a backward bifurcation indicates that it is way more difficult to eliminate a disease only by acting towards reducing its basic reproduction number R_0 since it may be necessary to bring R_0 well below 1 to achieve disease eradication. Here, a backward bifurcation occurs at $\mathcal{R}_{0D} = 1$ If $\mathcal{R}_{0D} < 1$ (but close to 1), there is a small realistic equilibrium, which is unstable, while the dropout-free equilibrium and a larger positive realistic equilibrium are locally asymptotically stable. If $\mathcal{R}_{0D} > 1$ (but close to 1), the small positive realistic equilibrium disappears, while the dropout-free equilibrium loses its stability.

The existence of a backward bifurcation is illustrated in Fig. 1 through a numerical example by drawing a bifurcation diagram around $\mathcal{R}_{0D} = 1$ To draw a bifurcation curve (the graph of $d^{\#}$ as a function of R_{0D}), we choose $\mu = 0.0268$, $\mu_1 = 0.0062$, $\varkappa = 0.11$, $\eta = 0.048$, $\sigma = 0.051$, $\beta_1 = 0.247$, $\beta_2 = 0.180$ and $\beta 3 = 0.033$, and then we obtain $\mu^{\#}{}_1 \approx 0.0095$ and R#0D approximately equals to 0.63716. The backward bifurcation occurs, provided that R_{0D} # (0.63716, 1).

4 Parameter estimation

To find concrete estimations for the values of parameters used in our model, we carried out a questionnaire survey for which the subjects were undergraduate students from the University of Education, Winneba, Ghana. In that aspect, our findings might be very much context-specific, economically and culturally.

4.1 Instrument

The survey instrument consists in demographic variables (age and university grade) and items related to social influence, internal LOC and S-E beliefs (see Table 2). The items related to internal LOC beliefs are adapted from [18], and the items related to S-E beliefs are adapted from [11]. A five-point Likert scale is used to measure internal LOC and S-E. The scores on the Likert scale are 1 = strongly disagree (SD), 2 = disagree (D)

Figure 2.
Distribution of students responses to internal control and selfefficacy constructs

3 = neutral (N), 4 = agree (A) and 5 = strongly agree (SA). Internal LOC and S-E con- structs both have five items with Cronbach's Alpha reliability statistics of $\alpha = 0.896$ and $\alpha = 0.935$, respectively. These values are in the acceptable range of reliability [10]. Some of the items on the instruments are negatively worded to prevent students from choosing only satisfying responses. These items are recoded for the analysis (see

Table 2). The distribution of student responses to internal LOC and S-E items is illustrated in Fig. 2.

Table 2

Variables and items used for parameter estimation

Table 2. Variables and items used for parameter estimation.

Variable	Item
Demographics	Age and level (year)
Social influence	How many of your friends are resitting a course?
	How many of your friends have dropout from the university?
Internal control beliefs	The more effort I put into my study, the better I do in it.
	No matter what I do, I cannot seem to do well in my courses*.
	I see myself as largely responsible for my academic performance.
	How well I do in my exams at the university is often due to "luck"*.
	There is little I can do about my performance in the university*.
Self-efficacy beliefs	If I practice every day, I can pass all my courses.
	I am confident that I will achieve academic goals that I set for myself.
	I keep trying to accomplish my goals even if it is harder than I thought.
	I will succeed in the university course that I am doing.
	I can change my level of academic performance considerably with effort.

Table 2 (*Continued from previous page*)

4.2 Sample statistics

A total of 385 undergraduate students from five faculties are surveyed for the study. We use purposive sampling techniques to select these students so that we can get pass, make- up and resit students in our sample. The distribution of students within the compartments of the model is given in Table 3. The majority (46%, N = 178) of the participants are first year students. This choice has been made because many studies have found that a substantial proportion of undergraduate dropouts are first year students (see, e.g., [26]). There are 85 second year students, 87 third year students and 35 fourth year students. A chi-square test reveals a proportionate distribution of the students in the compartments according to year group ($\chi 2 = 6.643, p > 0.05$).

Table 3.
Distribution of students within the compartments of the model.

Table 3. Distribution of students within the compartments of the model.

	Freshman	Sophomore	Junior	Senior	Total
pass	143	60	64	24	291
make-up	24	15	14	5	58
resit	11	10	9	6	36
total	178	85	87	35	385

The students in the pass compartment had the highest level of internal LOC (Mean = 3.99, SD = 0.75) and S-E (Mean = 4.29, SD = 0.78). They were followed by the make- up group with levels of internal LOC given as (Mean = 2.97, SD = 0.54) and levels of self efficacy given as (Mean = 2.70, SD = 0.56). The students in the resit compartment had the lowest level of internal LOC (Mean = 1.67, SD = 0.53) and the lowest level of S-E (Mean = 1.73, SD = 0.58). Analysis of variance (ANOVA) test shows that there is a significant difference in the mean level of internal LOC ($F_{(2,382)} = 199.54$, $p < 0.05$) and S-E ($F(2,382) = 272.01$, $p < 0.05$) of the total student population.

4.3 Estimation of parameters

Estimations for the parameters of the model can then be obtained using cross-sectional data from our survey, similar approaches being previously used in related studies [1, 14]. This provides crude estimates of these parameters in order to demonstrate the potential applicability of our model. See also [13] for partial results in this direction based on a somewhat less nuanced methodology.

Table 4.
Number of resit and dropout friends.

Table 4. Number of resit and dropout friends.

	N	Number of resit friends	Number of dropout friends
Make-up	58	43	31
Resit	36	–	7

Estimate of μ. The length of the undergraduate program is 4 years, the (minority) of students, who fail to graduate within the stipulated period being given a maximal extension of 2 years. It is then assumed that the average graduation time is 4 years, which implies that $\mu = 1/(4 \# 365) =$ 0.068% per day.

Estimate of $\varkappa$. The rate at which otherwise passing undergraduate students fail examinations is estimated as being $\varkappa = 0.111$ by using the corresponding average percentage of students, who fail end of semester examinations.

Estimate for μ_1. Using enrollment and graduation statistics made available by the University of Education, Winneba, in April 2016, which cover up to 20 previous years [30], the average dropout rate is estimated as being 22.63% per year, which is 0.062% per day.

Estimate of σ. In our survey sample, there are 58 students in the make-up compartment. The level of internal LOC of make-up students is 2.97. The estimate of σ is obtained under the assumption that the transition of students from the make-up (M) to the passing (P) compartments owes to the students' level of internal LOC. Since a cross-sectional survey is used for data collection, we compute the average level of internal LOC as being σ = 2.97/58 = 0.051 per day.

Estimate of η. The parameter η is estimated under the assumption that the transition rate of college students from the resit *(R)* to the passing *(P)* compartments owes to the stu- dents' level of S-E. In our survey samples, there are 36 students in the resit compartment. The average level of S-E of resit students is 1.73, which amounts to η = 1.73/36 = 0.048 per day.

Estimates of β_1, β_2 and β_3. These parameters, understood to measure average (negative) influence, are the most difficult to estimate. To find appropriate estimations, we shall resort to finding the average number of negative contacts via using the average number of resit and dropout friends. Table 4 shows the number of resit and dropout friends reported by each group.

Normally, undergraduate courses are scheduled on weekdays from 8:00 AM to 5:00 PM with a two-hour lunch break. On average, undergraduate students are then expected to spend on the campus 8 hours per day. The contact rate between make-up and resit individuals can then be computed as the product of average number of contacts and the hours of contact per day. This gives β_1 = 43/58 8/24 = 0.247 per day. For the contact rate between the make-up, resit and dropout students, we estimate that make-up and resit students will contact their dropout friends after school hours, including break time, which amounts for 6 hours assuming students will go to bed at 9:00 pm. This gives β_2 = 31/58 6/24 = 0.134 and β_3 = 7/36 6/24 = 0.049 per day.

Table 5 shows the estimate for each parameter in the model leading to R_{0D} = 3.462>1 and R_{0D} = 39.07>1, which indicates that our system has a dropout-endemic equilibrium. The results are illustrated in Fig. 3.

Table 5.

Values of parameters.

Table 5. Values of parameters.

Parameter	Value	Unit	Parameter	Value	Unit
μ	0.00068	day^{-1}	μ_1	0.00062	day^{-1}
κ	0.111	day^{-1}	σ	0.051	day^{-1}
η	0.048	day^{-1}	β_1	0.247	day^{-1}
β_2	0.134	day^{-1}	β_3	0.049	day^{-1}

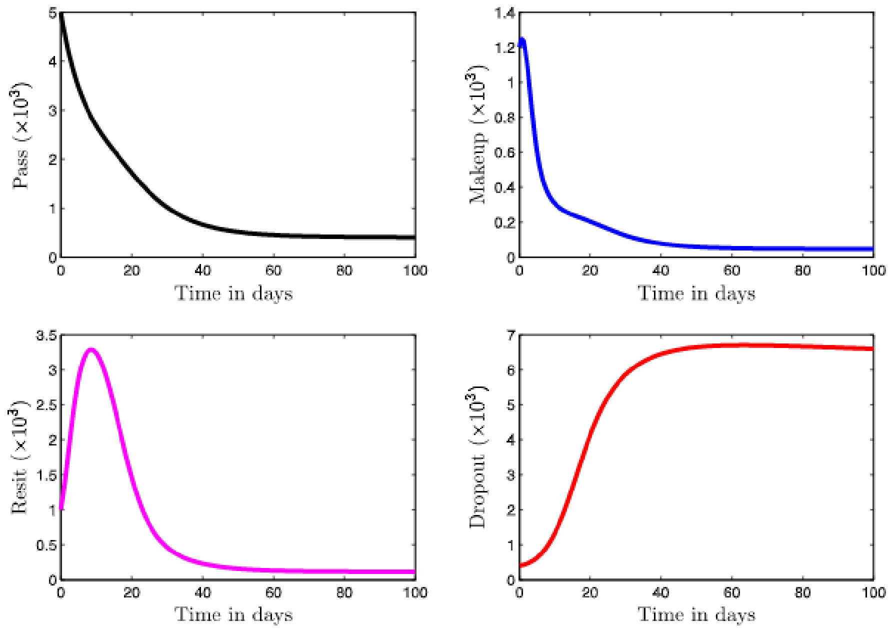

Figure 3.
Time series plots of state variables reaching RDPE

5 A sensitivity analysis

In order to find appropriate policies to reduce dropout rates, we shall investigate the sensitivity of R_{0R} and R_{0D} with respect to model parameters, that is, how the values of R_{0R} and R_{0D} increase or decrease after parameter changes. To this purpose, we use the following normalized forward-sensitivity index, which measures the elasticity of a variable Q with respect to a parameter p in the form

$$\zeta_p^Q = \frac{p}{Q} \cdot \frac{\partial Q}{\partial p}.$$

This index shows how sensitive Q is to changes of p. Specifically, a positive or negative index means that an increase in the value of the parameter leads to an increase or decrease of the variable [7], while giving also information about the order of (relative) growth or decrease. Concrete expressions for the sensitivity indices of R_{0R} and R_{0D} with respect to model parameters are given in Appendix, the estimated value of each parameter employed being given in Table 5. Table 6 gives the sensitivity of R_{0R} with respect to each parameter.

Table 6

Sensitivity indices for R_{0R} with respect to model parameters. Parameter κ

Table 6. Sensitivity indices for $\mathcal{R}_{0R}$ with respect to model parameters.

Parameter	κ	β_1	μ	η	σ
Sensitivity	0.318	1	−0.018	−0.986	−0.313

Table 7.

Sensitivity indices for R_{0D} with respect to model parameters. Parameter κ

Table 7. Sensitivity indices for $\mathcal{R}_{0D}$ with respect to model parameters.

Parameter	κ	μ_1	β_1	β_3	μ	η	σ
Sensitivity	0.434	−1	−0.1278	1	−0.012	−0.701	−0.127

As shown in Table 6,$\varsigma_{\beta_1}^{R_{0R}} = 1$, which means that decreasing the negative influence of resit students on make-up students by 10% would lead to a corresponding 10% de- crease in the number of resit students. The positivity of $\varsigma_{\kappa}^{R_{0R}}$. indicates that increasing the rate at which passing students fail examinations increases the number of resit students.

For parameters leading to negative sensitivity indices, an increase (or decrease) in these parameters will result in a corresponding decrease (or increase) in the number of resit students. Among the negative sensitivity indices, $\varsigma_{\eta}^{R_{0R}}$. had the highest valued followed by $\varsigma_{\sigma}^{R_{0R}}$. and $\varsigma_{\mu}^{R_{0R}}$. This suggests that interventions that seek to reduce the number of resit students should include strategies that will improve their level of S-E and internal LOC.

Similarly, the sensitivity indices of R_{0D} . with respect to parameters, computed for the parameters having the estimated values given in Table 5, are shown in Table 7. Also, this table gives, in some sense, a preferential ranking of the influencing parameters (social influence of peers, typified by B_1 and B_3, S-E, typified by ., internal LOC beliefs, typified by .) on the basis of the degree of influence on the dropout reproduction number R_{0D}.

Comparing the resulting values of sensitivity indices, it follows that R_{0D} is the most sensitive to β_3 and μ_1. Apart from those, R_{0D} is also sensitive to η. The value of $\varsigma_{\eta}^{R_{0D}}$ implies that a 10% increase in the S-E of resit students will result in a 7% decrease in the number of dropouts.

6 Numerical simulations

n this section, numerical simulations are given in order to illustrate the impact of parameter variation on the dynamics of the system and on the values of the reproductive numbers. Figure 4 shows that there is a linear increase in R_{0R} as the rate at which the negative social influence of resit students on make-up students (β_1) increases. Also, R_{0R} decreases with increases in the internal LOC (σ). This shows that students with low level of internal LOC are more likely to be negatively influenced to resit courses, increased efforts being needed to reduce negative social influences on make-up students with low level of internal LOC.

We also investigate the impact of internal LOC, negative social influence of resit students on make-up students and S-E (η) on the size of the population of resit students. Figure 5 shows the effects of increasing σ on the population of resit students. We increase the negative social influence of resit on make-up students (β_1 = 0.74) and observe an increase in the population of resit students regardless of students' level of internal LOC. Low negative social influence (β_1 = 0.007) and low level of internal LOC (σ = 0.048)

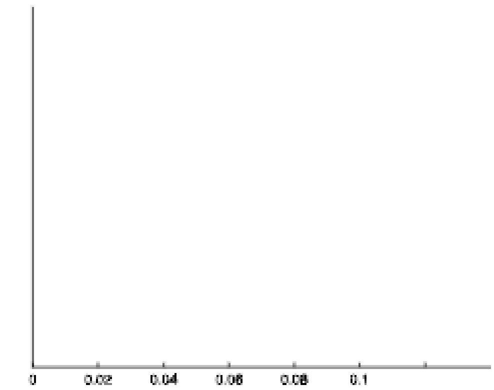

Figure 4.
Impact of internal LOC on social influence.

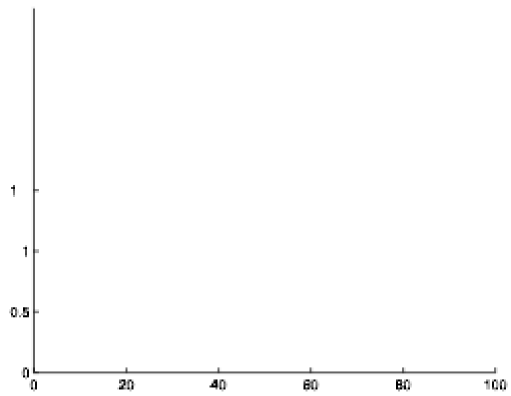

Figure 5.
Impact of internal LOC, negative social peer influence and S-E on the population of resit students.

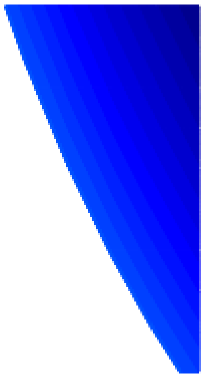

Figure 6.
The combined effect of internal LOC (σ) and S-E (η) on R_{0R} .

reduced the population of resit students, but not as much as a high level of internal LOC ($\sigma = 0.75$). High levels of internal LOC ($\sigma = 0.75$) and of S-E ($\eta = 0.78$) coupled with low negative social influence ($\beta_1 = 0.0074$) reduced the population of resit students by about 50%.

The impact of the S-E of resit students (η) and of the negative social influence of dropout students on resit students (β_3) is then investigated. We start by increasing the negative social influence ($\beta_3 = 0.077$) of dropout students on resit students. This increases the population of dropout students. However, a high level of S-E ($\eta = 0.75$) reduces the dropout population by 16.7%.

To investigate the combined effect of internal LOC and S-E on the number of resit and dropout students, a contour plotting (see Fig. 6) is used to illustrate how those parameters affect R_{0R} . Our results show that increasing σ alone does not reduce R_{0R} . However, when σ is held constant, increasing η reduces R_{0R} . This means that in order to reduce the number of resit students who eventually become dropouts, it is necessary to implement intervention policies that will increase the S-E of students who fail examinations.

Also, since our stability analysis is local, rather than global, it does not *a priori* preclude the occurrence of chaotic behavior. However, our (admittedly limited) numerical simulations have found no evidence of such behavior for reasonable initial data

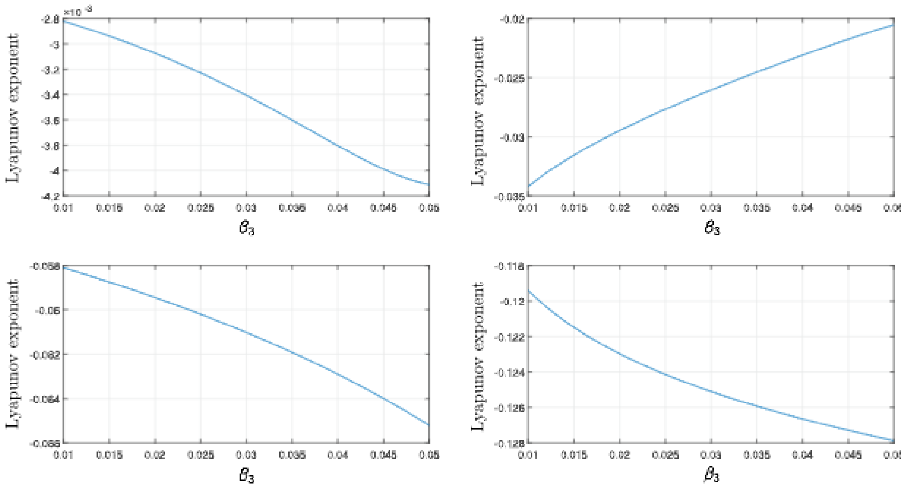

Figure 7.
Lyapunov exponents via Wolf's algorithm for initial percentages $(p_0, m_0, r_0, d_0) = (0.718, 0.143, 0.089, 0.050)$.

7 Discussion and concluding remarks

How students cope with course failure is an important issue to be considered when trying to understand the rationale behind the persistence and dropout intentions of undergraduate students. Their motivation, effort and persistence towards passing failed courses is rooted in their level of S-E [2, 9], internal LOC beliefs [8], as well as in their interactions with peers [1, 2, 24, 25].

We investigate how S-E, LOC and social influence of peers affect the academic performance and dropout intentions of undergraduate students via an analytic approach, which involves investigating the behavior of solutions and the stability of the equilibria for a mathematical model consisting in a system of nonlinear ODEs. A sensitivity analysis and a few numerical simulations also provide insights with immediate practical applicability. We observe that the negative social influence of resit students on make-up students increases the number of resit students. Also, high levels of internal LOC beliefs and of S- E decrease the number of resit and dropout students. In order to reduce the number of resit students, who eventually become dropouts, it is then necessary to implement intervention policies that will increase the S-E of students who fail courses.

It has been suggested that students develop a sense of their S-E from their own evaluation of previous performances and from observing similar actions performed by others. Prominent contributing factors are also appraising verbal judgments made by others such as feedback from teachers [5]. Accordingly, special attention should be paid to these aspects of social cognitive development when teaching or coaching students, who have to resit courses. Meaningful and encouraging feedback should be given to them, and the assignments should be shorter, more frequent and followed by abundant feedback instead of being longer and in smaller numbers as usually done for regular courses [22]. This happens since students can build their S-E by setting difficult, but attainable goals and assessing their progress towards those goals [21].

Thus, students, who resit courses, must be encouraged to set their own learning goals and continuously self-evaluate their progress. Further, observing other students succeed in their academic studies can raise S-E and motivate them to do their best to achieve the task of concern because students are likely to believe that if others can achieve it, they can do it as well [2]. Hence, encouraging resit students to interact with past resit students, who have passed their resit courses, can motivate the former to pass as well. Students should be encouraged to reduce the number of dropout friends they have or at least to limit their interactions with them as much as possible since their negative influence can lead to students' own dropout. Particular attention should be given to finding ways of building students' internal LOC and S-E, which will help them become academically motivated and engaged in learning. This will hopefully alleviate the pervasive problem of course failure and dropout in undergraduate studies.

In this paper, we have discussed the dynamics of a mathematical model, which investi- gates the effects of failure coping strategies and of negative social influences on students' decision to dropout. A framework originating in mathematical epidemiology has been used to model and measure how students' levels of internal LOC and S-E, together with negative social influences, affect their dropout decision. The level of internal LOC has been assumed to influence students' ability to pass make-up examinations, while the level of S-E has been assumed to influence the ability to pass resit courses.

The dynamics of the number of resit students is observed to be driven by a threshold parameter $R_{0R.}$ called the resit reproduction number. Similarly, the dynamics of the number of dropout students is found to be driven by another parameter $R_{0D.}$ called the dropout reproduction number, both being closely related in their scopes to the basic reproduction number of mathematical epidemiology. We showed that a backward bifurcation occurs if certain conditions are met, fact which was further confirmed via numerical simulations. This means that eradicating dropout might be very difficult. In this case, decreasing $R_{0D.}$ below 1 will not necessarily eliminate the dropout student population even in the long run. In this setting, an effective way of avoiding dropout is to adopt new measures and regulations for motivating resit students to study harder and attend lectures.

Detailed knowledge of the influence of the parameters is essential in order to develop effective policies to reduce the number of students, who dropout from undergraduate programs. In this regard, we performed a sensitivity analysis to evaluate the sensitivity indices of R_{0R} and of R_{0D} , respectively, with respect to model parameters. It has observed that R_{0R} has a direct relationship with $\varkappa$ (the rate of examination failure) and β_1 (the negative social influence of resit students on make-up students) and an inverse relationship with the η (the level of S-E) and σ

(the level of internal LOC). This means that an increase in σ or η will reduce the resit reproduction number, while an increase in κ or β_1 will increase the resit reproduction number, which will subsequently result in more resit students. The sensitivity indices of 0D show that 0D has an inverse relationship with σ and η. It has also been noted that, in the given concrete context, β_3 ranks first in a preferential ranking of the influencing parameters on the basis of the degree of influence on the dropout reproduction number R_{0D} . We have also illustrated the roles of internal LOC, S-E and negative social influences via graphical plots.

Our results highlight the importance of students' strategies to cope with examination failure, which is related to their levels of internal LOC and of S-E and are an important factor in the decision to persist in their studies or to dropout from undergraduate programs. The analysis of our data driven model then yields results with practical relevance that can be of help to policy makers when addressing the ever-actual problem of undergraduate student dropout.

Like any other model, ours is not without limitations either. Specifically, the model of concern in this paper is based upon the intrinsic assumption that the internal LOC and S-E are the most reliable predictors of academic performance and persistence in undergraduate studies along with the (negative) influence of peers. However, students may dropout from undergraduate programs for reasons, which are entirely different from those considered in this paper. Regardless of these limitations, our model presents a distinct view, based on a social cognitive perspective and on mathematical modelling, towards improving the academic performance of undergraduate students and reducing dropout risks. Further improvements and augmentations can be made in order to incorporate the influence of other factors such as demographic characteristics or learning habits to mitigate these limitations.

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Appendix

Sensitivity indices of R0R with respect to ϰ, β1, β2, μ, η and σ:

$$\zeta_{\kappa}^{\mathcal{R}_{0R}} = \frac{\mu+\sigma}{\kappa+\mu+\sigma}, \qquad \zeta_{\beta_1}^{\mathcal{R}_{0R}} = 1, \qquad \zeta_{\mu}^{\mathcal{R}_{0R}} = -\frac{\mu(\kappa+2\mu+\sigma+\eta)}{(\kappa+\mu+\sigma)(\eta+\mu)},$$
$$\zeta_{\eta}^{\mathcal{R}_{0R}} = -\frac{\eta}{\eta+\mu}, \qquad \zeta_{\sigma}^{\mathcal{R}_{0R}} = -\frac{\sigma}{\kappa+\mu+\sigma}.$$

Sensitivity indices of R0D with respect to κ, β1, μ, η, β3, μ1 and σ:

$$\zeta_{\kappa}^{\mathcal{R}_{0D}} = \frac{\kappa(-\beta_1+\eta-\sigma)(\eta+\mu)}{(\eta+\kappa+\mu)((\mu+\kappa+\sigma)\eta+\mu^2+(\kappa+\sigma)\mu-\kappa\beta_1)},$$

$$\zeta_{\mu}^{\mathcal{R}_{0D}} = \frac{(\kappa^2+(2\mu+\eta+\sigma+\beta_1)\kappa+(\eta+\mu)^2)\mu}{(\eta+\kappa+\mu)((-\beta_1+\eta+\mu)\kappa+(\mu+\sigma)(\eta+\mu))}$$

$$\zeta_{\eta}^{\mathcal{R}_{0D}} = \frac{\eta\kappa(\mu+\kappa+\sigma+\beta_1)}{(\eta+\kappa+\mu)((-\beta_1+\eta+\mu)\kappa+(\mu+\sigma)(\eta+\mu))},$$

$$\zeta_{\sigma}^{\mathcal{R}_{0D}} = \frac{\sigma(-\eta-\mu)}{\kappa\beta_1-(\eta+\mu)(\mu+\kappa+\sigma)}, \qquad \zeta_{\beta_3}^{\mathcal{R}_{0D}} = 1, \qquad \zeta_{\mu_1}^{\mathcal{R}_{0D}} = -1,$$

$$\zeta_{\beta_1}^{\mathcal{R}_{0D}} = -\frac{(\eta+\mu)(\mu+\kappa+\sigma)}{\mu^2+(\eta+\kappa+\sigma)\mu+\eta(\kappa+\sigma)-\kappa\beta_1}.$$