Predefined-time synchronization of 5D Hindmarsh–Rose neuron networks via backstepping design and application in secure communication

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Predefined-time synchronization of 5D Hindmarsh–Rose neuron networks via backstepping design and application in secure communication

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Abstract: In this paper, the fast synchronization problem of 5D Hindmarsh–Rose neuron networks is studied. Firstly, the global predefined-time stability of a class of nonlinear dynamical systems is investigated under the complete beta function. Then an active controller via backstepping design is proposed to achieve predefined-time synchronization of two 5D Hindmarsh–Rose neuron networks in which the synchronization time of each state variable of the master-slave 5D Hindmarsh–Rose neuron networks is different and can be defined in advance, respectively. To show the applicability of the obtained theoretical results, the designed predefined-time backstepping controller is applied to secure communication to realize asynchronous communication of multiple different messages. Three numerical simulations are provided to validate the theoretical results.
Keywords: 5D Hindmarsh–Rose neuron networks, predefined-time stability, complete beta function, secure communication.

Introduction

The fast synchronization problem of nonlinear systems has attracted much attention in recent years [1, 2, 4, 32, 33], e.g., finite-time synchronization problem for two nonlinear systems. The primary purpose of finite-time synchronization is to design an appropri- ate controller to achieve the master-slave systems coupled within a finite time interval [3]. Ahmad et al. [4] proposed an active controller to realize finite-time multi-switching synchronization of chaotic systems. The initial condition of the nonlinear systems will affect the settling time of finite-time synchronization. If the initial condition of the sys- tems is unknown, the settling time of finite-time synchronization cannot be obtained in advance. To solve the above problem, fixed-time stability was proposed by [25] and applied to fixed-time synchronization. Different from finite-time synchronization, fixed- time synchronization has a definite synchronization settling time [8,14–16]. Hu et al. [12] proposed a fixed-time stability theorem. The proposed approach shows better performance than [25]. Kong et al. [16] investigated the fixed-time synchronization of discontinuous fuzzy inertial neural networks with parameter uncertainties. Then Kong et al. [14] fur- ther investigated the fixed-time synchronization of discontinuous fuzzy inertial neural

networks with time varying delays. Chen et al. [7] reconstructed the Lyapunov function and reproved the fixed-time stability theorem through inequalities, and its settling time was further improved. Lin et al. [19] proposed a new fixed-time stability theorem and proved that the settling time is more accurate estimation by segmenting the Lyapunov function. Many studies on the settling time estimation are based on inequality theories. How to improve the accuracy of the settling time estimation of fixed-time stability is still a direction for further research.

In practical applications, such as secure communications or multiple agents control systems, it is hoped that in the controller design stage, the least upper bound of the settling time can be set as a tuning parameter of the system. Fixed-time stability is difficult to establish a direct relationship between the upper bound of the settling time and the parameters of nonlinear system. In order to solve the above-mentioned difficulties, a new kind of time stability, named predefined-time stability, was introduced in [27]. The research on predefined-time stability is still in the initial stage [13, 22, 26]. Muoz-Vázquez et al. [24] proposed an active predefined-time controller to achieve synchronization between two coupled Lorenz systems and applied to secure communication. To enable fully exact tracking of actuated mechanical systems, an predefined-time controller was proposed to second-order systems in [23]. Predefined-time stability theorems can realize that all variables of the nonlinear system are stable within a predefined time. In secure communication, different messages are of different importance, and the transmission sequence is also different. Therefore, it is very necessary to design an active control algorithm to achieve different settling time for different variables, so as to achieve asynchronous time synchronization of multiple messages communications.

In data encryption and secure communication, there are three very important factors:

(i) complexity of the dynamical nonlinear system; (ii) short transmission response time; fast synchronization. These factors can increase the difficulty of hackers to crack. The one-dimensional chaotic system has simple model, easy circuit implementation, and relatively simple synchronization, but its output only has one state, which is not conducive to confidentiality. Therefore, chaotic systems with relatively high dimension are generally considered in secure communication. The 2D Hindmarsh–Rose (HR) chaotic system has attracted much attention because of its fast computational speed [10], which is more than ten times faster than the Hodgkin and Huxley model [11], and complex dynamical behaviors, such as bursting and chaos, observed in real biological neurons. Many researches have been conducted on the HR neural chaotic systems [21, 28, 29, 31], and the HR neural chaotic systems have been extended to 3D HR neural chaotic systems [28], 4D HR neural chaotic systems [21], and even 5D HR neuron networks (5D HRNNs) [31]. The rich dynamical behaviors, including a chaotic super-bursting regime, of the 5D HRNN was shown in [31], and the synchronization of two coupled 5D HR neuron networks was realized. Therefore, the 5D HRNN

is particularly suitable for the field of secure communication. At present, the research on the synchronization of the HR neuron network mainly focuses on asymptotically stable [5,20,30], the research on the fixed-time synchronization of the HRNNs has not been seen yet. Inspired by the above discussions, predefined-time synchronization of two 5D HRNNs via backstepping design is proposed in this paper. The main contribution are the following:

To solve the problem of inaccurate estimation of the settling time, this paper introduces complete beta function to achieve accurate settling time estimation of the predefined-time stability.

An active controller via backstepping design is proposed to achieve the predefined- time synchronization of master-slave 5D HRNNs in which the synchronization time of each state variable of the master-slave 5D HRNNs is different and can be defined in advance, respectively. The designed controller is applied to secure communication to realize asynchronous communication of multiple messages.

(i) To solve the problem of inaccurate estimation of the settling time, this paper introduces complete beta function to achieve accurate settling time estimation of the predefined-time stability.

(ii) An active controller via backstepping design is proposed to achieve the predefined- time synchronization of master-slave 5D HRNNs in which the synchronization time of each state variable of the master-slave 5D HRNNs is different and can be defined in advance, respectively. The designed controller is applied to secure communication to realize asynchronous communication of multiple messages.

The remainder of this paper is structured as follows. Some preliminaries are included in Section 2. In Section 3, a new predefined-time stability of a class of nonlinear systems is investigated under the complete beta function. With the help of predefined-time stability, the predefined-time synchronization of master-slave 5D HRNNs via backstepping design is investigated in Section 4. Then the designed predefined-time backstepping controller is applied to secure communication in Section 5. The conclusion is given in Section 6.

2 Preliminaries

Consider a nonlinear system described by the following [18]:

$$\dot{x} = f(x; r),$$

where $x \in \mathbb{R}^n$ is the state vector of system (1). x_0 is the initial condition. $r \in \mathbb{R}^b$ with $r \in \mathbb{R}^b$ is the parameters $f : R \to \mathbb{R}^n$ is a nonlinear function the process of deriving fixed-time stability, the complete beta function and complete gamma function will play a key role. The definition of these functions will be provided the following.

Definition 1. (See [9].) Let $B(\sigma,\theta)$. The complete beta function, denoted by B(σ, θ), is defined by the Euler integral and the complete gamma function through

$$B(\sigma, \theta) = \int_{0}^{1} z^{\sigma-1}|(1 - z)^{\theta-1}\, \mathrm{d}z = \frac{\Gamma(\sigma)\Gamma(\theta)}{\Gamma(\sigma + \theta)},$$

where $\Gamma(\cdot)$ is the complete gamma function, which is defined by the Euler integral $\Gamma(z) = \int_{0}^{\infty} e^{-t} t^{z-1}\, \mathrm{d}t$.

The complete beta function is mainly used in statistics, but it is also used in other fields, e.g., actuarial science, economics or telecommunications. In this paper, we apply it to fixed-time stability.

Definition 2. (See [25].) The origin of system (1) is globally fixed-time stable if it is globally finite-time stable and the settling time is bounded, i.e., there exists $T_{max} > 0$ such that, for all $x_0 \in \mathbb{R}^n$, $T(x_0) \leqslant T_{max}$.

Definition 3. (See [19].) The origin of system (1) is said to be predefined-time stable if it is globally fixed-time stable and the settling time) $T(x_0)$ is

$$T(x_0) \leqslant T_c \quad \forall x_0 \in \mathbb{R}^n,$$

where T_c is a tuning constant parameter and called a predefined time.

Lemma 1. (See [6].) For system (1), let there exist a continuous radially unbounded and positive definite function $V(x): \mathbb{R}^n \to \mathbb{R}$ and $\delta > 0, 0 < \kappa < 1$ such that $\dot{V}(x) \leqslant -\delta V^{\kappa}(x) \quad \forall t \geqslant t_0$ and $V(x(t_0)) \geqslant 0$,

where t_0 is the any initial time; $x(t_0)$ is the any initial value. Then the relationship between time t and $V(x(t))$ is described as

$$t = \frac{V^{1-\kappa}(x(t_0))}{\delta(1 - \kappa)} + t_0,$$

and system (1) is finite-time stable

Lemma 2. (See [12].) For system (1), let there exist a continuous radially unbounded and positive definite function $V(x): \mathbb{R}^n \to \mathbb{R}$ and $\alpha, \beta, \gamma, \eta > 0$ satisfying γη > 1 such that

$$\dot{V}(x) \leqslant -\left(\alpha V^{\gamma}(x) + \beta\right)^{\eta}, \quad x(t) \in \mathbb{R}^n \setminus \{0\}.$$

Then system (1) can converge to the zero in the settling time T_{max}^1, and T_{max}^1 is described

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$$T(x_0) \leqslant T_{\max}^1 \triangleq \frac{1}{\beta^\eta}\left(\frac{\beta}{\alpha}\right)^{1/\gamma}\left(1 + \frac{1}{\gamma\eta - 1}\right).$$

Lemma 3. (See [24].) For system (1), let there exist a continuous radially unbounded and positive definite function $V(x): \mathbb{R}^n \to \mathbb{R}$ and $0 < \mu < 1$ such that

$$\dot{V}(x) \leqslant -\frac{\pi}{2\mu T_c}\left(V^{1-\mu}(x) + V^{1+\mu}(x)\right).$$

Then system (1) is globally predefined-time stable, and the predefined time is T_c.

Lemma 4. (See [17].) If conditions satisfy $x_1, x_2, \ldots, x_n \geqslant 0, 0 < \epsilon_1 \leqslant 1, \epsilon_2 > 1$, then

$$\sum_{i=1}^n x_i^{\epsilon_1} \geqslant \left(\sum_{i=1}^n x_i\right)^{\epsilon_1}, \qquad \sum_{i=1}^n x_i^{\epsilon_2} \geqslant n^{1-\epsilon_2}\left(\sum_{i=1}^n x_i\right)^{\epsilon_2}.$$

3 Predefined-time stability

The goal of this section is to propose a new fixed-time stability proof method for system (1). By making some modification of the fixed-time system, a new predefined-time stability theorem is proposed.

Theorem 1. For system (1), if there exist a continuous radially unbounded and positive definite function $V(x): \mathbb{R}^n \to \mathbb{R}$ and $\alpha, \beta, \gamma, \eta > 0$ α, satisfying $\gamma\eta > 1$ such that

$$\dot{V}(x) \leqslant -\left(\alpha V^\gamma(x)^\gamma + \beta\right)^\eta,$$

then system (1) is globally fixed-time stable with the settling time

$$T(x_0) \leqslant T_{\max}^2 = \frac{\beta^{1/\gamma - \eta}}{\alpha^{1/\gamma}\gamma} B(\sigma, \theta),$$

where $B(\sigma, \theta)$ is the complete beta function.

Proof. Since $\alpha, \beta > 0$, then

$$\frac{\mathrm{d}V(x)}{\mathrm{d}t} \leqslant -\left(\alpha V^\gamma(x)^\gamma + \beta\right)^\eta \leqslant -\beta^\eta < 0.$$

Since $\mathrm{d}V(x)/\mathrm{d}t \leqslant -\beta^\eta$, there exists a constant $T(x_0) = V(x_0)/\beta^\eta$ such that $\lim_{t \to T(x_0)} V(x) = 0$ and $V(x) = 0$ for all $t > T(x_0)$. It follows that $\mathrm{d}V(x) \leq -\mathrm{d}t$, then $\frac{\mathrm{d}V(x)}{(\alpha V(x)^\gamma + \beta)^\eta} \leq -\mathrm{d}t$, then $-\frac{\mathrm{d}V(x)}{(\alpha V(x)^\gamma + \beta)^\eta} > \mathrm{d}t$.

We have

$$\nabla^{n-1}\sigma(k) = f\big(x(k), u(k)\big),$$

$$\nabla z(\omega, k) = -\omega z(\omega, k) + \sigma(k),$$

$$x(k) = \int_0^{+\infty} \mu_{\alpha-n+1}(\omega) z(\omega, k)\, d\omega,$$

$$y(k) = g\big(x(k), u(k)\big),$$

Let $z = ((\alpha/\beta)V^\gamma + 1)^{-1}$ then z goes to 1 when and to 0 then $V \to +\infty$,

$$V = \left(\frac{\beta}{\alpha}\left(\frac{1}{z} - 1\right)\right)^{1/\gamma}$$

and

$$dV = -\frac{(\frac{\beta}{\alpha})^{1/\gamma}}{z^2\gamma}\left(\frac{1}{z} - 1\right)^{1/\gamma-1} dz.$$

Thus formula (4) can be written as

$$\int_0^{+\infty} \frac{\beta^{-\eta}}{(\frac{\alpha}{\beta}V^\gamma + 1)^\eta}\, dV = -\int_1^0 \frac{\beta^{-\eta}}{(\frac{\alpha}{\beta}\frac{\beta}{\alpha}(\frac{1}{z}-1) + 1)^\eta} \cdot \frac{(\frac{\beta}{\alpha})^{1/\gamma}}{z^2\gamma} \cdot \left(\frac{1}{z} - 1\right)^{1/\gamma-1} dz$$

$$= -\int_1^0 \beta^{-\eta} z^\eta \cdot \frac{(\frac{\beta}{\alpha})^{1/\gamma}(\frac{1}{z}-1)^{1/\gamma-1}}{z^2\gamma}\, dz$$

$$= -\frac{(\frac{\beta}{\alpha})^{1/\gamma}}{\beta^\eta\gamma} \cdot \int_1^0 z^{\eta-2} \cdot \frac{(1-z)^{1/\gamma-1}}{z^{1/\gamma-1}}\, dz$$

$$= \frac{(\frac{\beta}{\alpha})^{1/\gamma}}{\beta^\eta\gamma} \cdot \int_0^1 z^{\eta-1-1/\gamma} \cdot (1-z)^{1/\gamma-1}\, dz.$$

Assume $\sigma = 1/\gamma$ and $\theta - 1 = \eta - 1 - 1/\gamma$, then

$$B(\sigma, \theta) = \int_0^1 z^{\sigma-1}(1-z)^{\theta-1}\, dz = \frac{\Gamma(\sigma)\Gamma(\theta)}{\Gamma(\sigma+\theta)},$$

Assume $\sigma = 1/\gamma$ and $\theta - 1 = \eta - 1 - 1/\gamma$, then

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$$B(\sigma,\theta) = \int_0^1 z^{\sigma-1}(1-z)^{\theta-1}\,\mathrm{d}z = \frac{\Gamma(\sigma)\Gamma(\theta)}{\Gamma(\sigma+\theta)},$$

and formula (4) can be written as

$$T(x_0) \leqslant \int_0^{+\infty} \frac{\beta^{-\eta}\,\mathrm{d}V}{(\frac{\alpha}{\beta}V^\gamma+1)^\eta} = \frac{(\frac{\beta}{\alpha})^{1/\gamma}}{\beta^\eta\gamma}\cdot B(\sigma,\theta) = T_{\max}^2.$$

$$(5)$$

By Definition 2, systems (1) is fixed-time stable, and the settling time $T_{\max}^2$ is bounded Importar imagen for any, which completes the proof

Remark 1. Obviously, Theorem 1 provides a new proof process of fixed-time stability. Although Lyapunov function (3) is the same as that in [12] and [19], different settling time is obtained in Theorem 1. In the proof of Theorem 1, complete beta function is applied for the first time to realize the accurate settling time estimation, and a small upper bound of the settling time is obtained, closer to the real value.

Theorem 2. *The settling time $T_{\max}^2$ in Theorem 1 is more accurate than the settling time $T_{\max}^1$ in Lemma*

Proof. From the derivation process of Lemma 2 in [12] we have

$$T_{\max}^1 - T(x_0) \geqslant T_{\max}^1 - \int_0^{+\infty} \frac{1}{(\alpha V^\gamma+\beta)^\eta}\,\mathrm{d}V \geqslant 0,$$

and from formula (5) we have

$$T_{\max}^2 = \int_0^{+\infty} \frac{1}{(\alpha V^\gamma+\beta)^\eta}\,\mathrm{d}V,$$

then

$$T_{\max}^1 \geqslant T_{\max}^2.$$

That means the settling time $T_{\max}^2$ is closer to the real convergence time than $T_{\max}^1$. This proof is completed.

Next, using the results obtained in Theorem 1, a new predefined-time stability isderived for a Lyapunov like condition.

Theorem 3. For system (1), let there exists a continuous radially unbounded and positive definite function $V:\mathbb{R}^n\to\mathbb{R}$ such that any solution $x(t,x_0)$ of system (1) satisfies

$$\dot{V} \leqslant -\frac{T_{\max}^2}{T_c}\left(\alpha V^\gamma + \beta\right)^\eta,$$

(6)

where $\alpha, \beta, \gamma, \eta > 0$, $\gamma\eta > 1$, and $T_{\max}^2$ are given in Theorem 1. Then system (1) is globally predefined-time stable, and the strong predefined time is T_c.

Proof. According to the supposing of Theorem 3, one has that

$$T(x_0) = \int_0^{T(x_0)} dt \leqslant -\int_{+\infty}^0 \frac{T_c}{T_{\max}^2}\frac{1}{(\alpha V^\gamma + \beta)^\eta} dV$$

$$= \int_0^{+\infty} \frac{T_c}{T_{\max}^2}\frac{1}{(\alpha V^\gamma + \beta)^\eta} dV = \int_0^{+\infty} \frac{T_c}{T_{\max}^2}\frac{\beta^{-\eta}}{(\frac{\alpha}{\beta}V^\gamma + 1)^\eta} dV$$

$$= \frac{T_c}{T_{\max}^2}\int_0^{+\infty} \frac{\beta^{-\eta}\,dV}{(\frac{\alpha}{\beta}V^\gamma + 1)^\eta} = \frac{T_c}{T_{\max}^2}\frac{(\frac{\beta}{\alpha})^{1/\gamma}}{\beta^\eta\gamma}\cdot B(\delta,\theta) = T_c,$$

where T_c is a tunable parameter of system (1). Consequently, system (1) is predefined- time stable, and the predefined time is T_c.

Remark 2. Compared with Lemma 1, Lyapunove function (6) in Theorem 3 has one more constant β and one more adjustable parameter T_c, which turn the asymptotically stability of Lemma 1 into predefined-time stability of Theorem 3.

Remark 3. $T_{\max}^2$ is related to α, β, and $B(\delta,\theta)$. If $T_{\max}^2$ is to be adjusted, complicated calculation is needed. Compared with Theorem 1, Lyapunov function (6) in Theorem3 has one more adjustable parameter T_c. The complex relationship between the system parameters and the settling time is transformed into an one-to-one correspondence between the settling time and the parameter T_c. By tuning T_c, system (1) can be stabilized at different predefined time, which is simpler and more effective than the fixed-time stability.

Example 1. Consider the following system:

$$\dot{y}(t) = -\operatorname{sign}\big(y(t)\big)\cdot\big(\alpha|y(t)|^\gamma + \beta\big)^\eta,$$

where $y(t)$ is a state variable, and the constants $\alpha, \beta, \gamma, \eta > 0$ and $\gamma\eta > 1$ satisfy the requirements of Theorem 1. Hence, system (7) is fixed-time stability by Theorem 1. In order to verify the validity of the Theorem 1, several sets of different parameters are simulated for system (7). The simulation results are given in Fig. 1 in which two sets of different parameters are chosen: (i) $\alpha = 1.2, \beta = 0.9, \gamma = 4/7, \eta = 5/2$; (ii) $\alpha = 5, \beta = 2, \gamma = 2, \eta = 0.9$. By computation $T_{\max}^1 = 2.6220$ s, $T_{\max}^2 = 1.1662$ s, under the first set of parameters, and $T_{\max}^1 = 0.7626$ s, $T_{\max}^2 = 0.6235$ s under the second set of parameters, which shows that the estimation of the settling time

in Theorem 1 are moreaccurate compared with Lemma 2 and proves Theorem 2. Consider the following system:

$$\dot{y}(t) = -\frac{C_v}{T_c} \cdot \text{sign}(y(t)) \cdot (\alpha |y(t)|^{\gamma} + \beta_1 + d_{\text{dis}})^{\eta},$$

where $|d_{\text{dis}}| < D$ with According to Theorem 3, system (8)is predefined-time stable. In Fig. 2, system (8) converges to zero before a predefinedtime T_c for several different initial conditions, which is explicitly defined in advance; thesimulation results are shown in Fig. 2(*a*) with = 0:8 s and Fig. 2(b) with = 0:2 s.Consider the following system:

$$\dot{y}(t) = -\frac{C_v}{T_c} \cdot \text{sign}(y(t)) \cdot (\alpha |y(t)|^{\gamma} + \beta_1 + d_{\text{dis}})^{\eta},$$

(8)

According to Lemma 1, system (9) is finite-time stable. Figure 3 shows the comparativeresults of systems (8) and (9). From Fig. 3(a) the convergence time of systems (8)and (9) changes with different initial conditions. When the initial value is y(0) = 80, theconvergence time of system (9) is 12 s, which exceeds the predefined time T_c = 10 s.The convergence time of system (8) is less than T_c = 10 s under any initial condition.Compared with system (9), the convergence time of system (8) can be estimated inadvance without initial condition known. If the initial conditions are known, by settingdifferent parameters, the convergence time of system (9) can be estimated in advance. Thesimulation results are shown in Fig. 3(b) in which the parameters are chosen:$y(0) = 4,$ $\delta = 21.1336$ with$T_c = 0.2$ s, $\delta = 8.4534$ with $T_c = 0.5$ s, $\delta = 5.2834$ with $T_c = 0.8$ s. .From Fig. 3(b) system (9) can achieve more accurate settling time estimation if the initialconditions are known. Consider the following system:

$$\dot{y}(t) = -\frac{\pi}{2\mu T_c} \cdot \text{sign}(y(t)) \cdot (\alpha |y(t)|^{1-\mu} + \beta |y(t)|^{1+\mu} + d_{\text{dis}}).$$
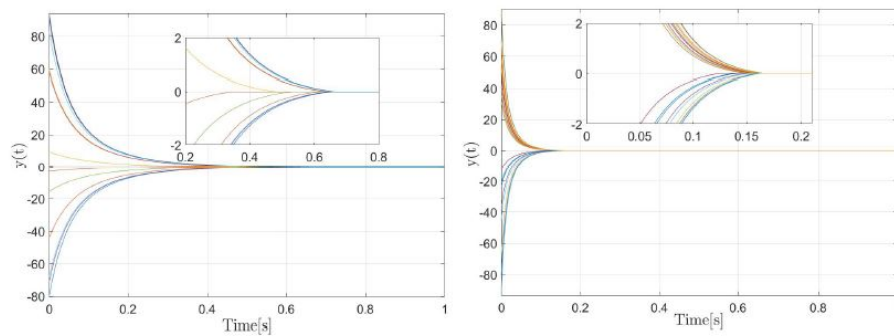
[10]

According to Lemma 3, system (10) is predefined-time stabile. Figure 4 shows the comparative results of systems (8) and (10) with disturbance occurring. It can be seen from Fig. 4 that Theorem 3 can effectively suppress disturbance and has better robustness than Lemma 3.
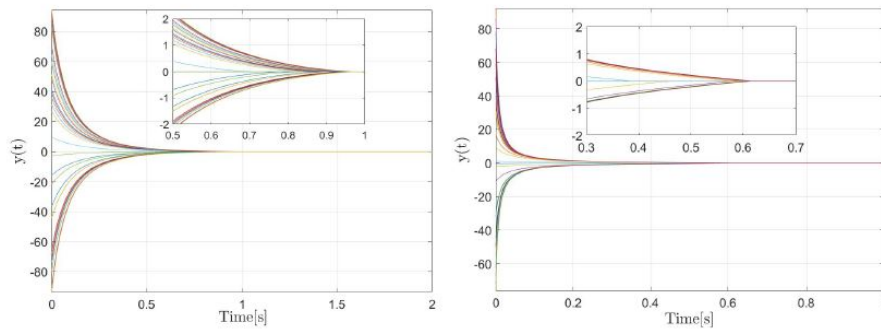
Figure 1. Trajectories of (7) for different initial conditions.

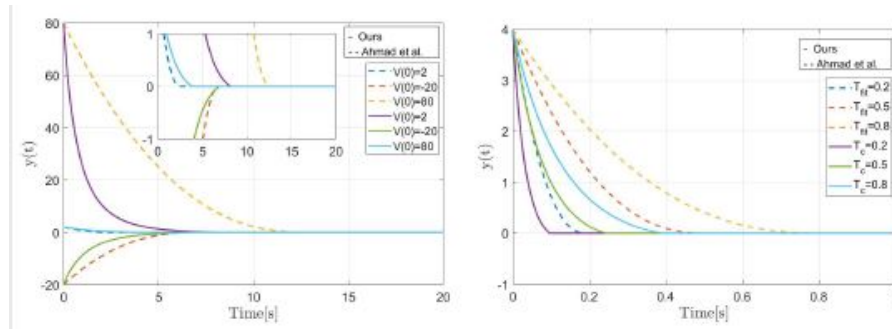(a) $\alpha = 1.2$, $\beta = 0.9$, $\gamma = 4/7$, $\eta = 5/2$ (b) $\alpha = 5$, $\beta = 2$, $\gamma = 2$, $\eta = 0.9$

Figure 2. Trajectories of (8) for different initial conditions and the parameters

(a) $T_c = 0.8$ s   (b) $T_c = 0.2$ s

Figure 3. Comparative results of systems (8) and (9).

(a) $T_c = 10$ s   (b) Different T_c

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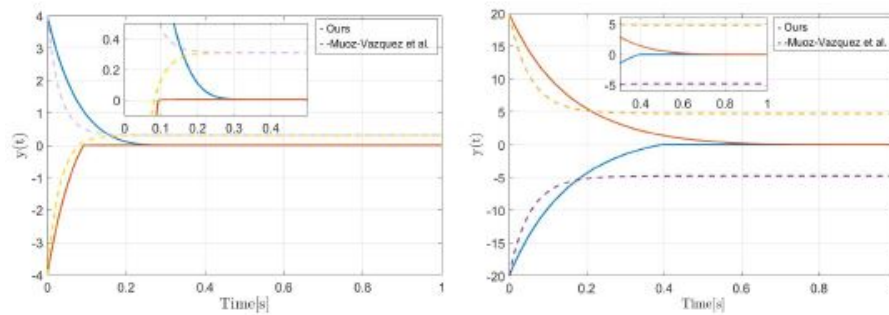
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Figure 4. Comparative results of systems (8) and (10).

(a) $y(0) = \pm 4$, $d_{\mathrm{dis}} = 0.9$ (b) $y(0) = \pm 20$, $d_{\mathrm{dis}} = 0.9$

4 Predefined-time synchronization controller via backstepping design

In this section, we will apply the theorems obtained in Section 3 and apply them to thepredefined-time synchronization of master-slave 5D HRNNs via backstepping design. The 5D HRNN is described by

$$
\begin{aligned}
\dot{x}_1 &= -ax_1^3 + bx_1^2 + x_2 - cx_3 \\
&\quad + I_0 \cos(\Omega t - \psi) - k_1 \left(d + 3sx_5^2\right)x_1, \\
\dot{x}_2 &= f - gx_1^2 - |x_2 - hx_4 + d_{\mathrm{dis}}, \\
\dot{x}_3 &= l\left[m(x_1 + x_{10}) - x_3\right], \\
\dot{x}_4 &= n\left[p(x_2 + x_{20}) - qx_4\right], \\
\dot{x}_5 &= x_1 - k_2 x_5,
\end{aligned}
\tag{11}
$$

where $z(\omega, a) = 0$ is the membrane potential variable $0 < \alpha < 1$ to $n - 1 < \bar{\alpha} < n,$, is the recovery current variable,which also is called spiking variable associated with fast ions, $0 < \alpha < 1$ to $n - 1 < \bar{\alpha} < n,$, is the adaptation variable associated with slow ions, $0 < \alpha < 1$ to $n - 1 < \bar{\alpha} < n,$ is an even slower process. $0 < \alpha < 1$ to $n - 1 < \bar{\alpha} < n,$ is themagnetic flux across the neuron's cell membrane. $0 < \alpha < 1$ to $n - 1 < \bar{\alpha} < n,$ is the amplitude of a harmonicstimulus with frequency and phase $0 < \alpha < 1$ to $n - 1 < \bar{\alpha} < n,$. are the constantparameters. In neuron activity, the parameters $n < l \ll 1$ play a very important role; l is the ratio of time scales between fast and slow fluxes across the neuron's membrane. n controls the speed change of a slower dynamical process , in particular, the calciumexchange between intracellular warehouse and the cytoplasm [21].j$^{|d_{\mathrm{dis}}|}$< D is thedisturbance.Let (11)

Let (11) be the master 5D HRNN, and the slave 5D HRNN is given by

$$\dot{\hat{x}}_1 = -a\hat{x}_1^3 + b\hat{x}_1^2 + \hat{x}_2 - c\hat{x}_3$$
$$+ I_0 \cos(\Omega t - \psi) - k_1\big(d + 3s\hat{x}_5^2\big)\hat{x}_1 + u_1,$$
$$\dot{\hat{x}}_2 = f - g\hat{x}_1^2 - \hat{x}_2 - h\hat{x}_4 + u_2,$$

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$$\dot{\hat{x}}_3 = l\big[m(\hat{x}_1 + x_{10}) - \hat{x}_3\big] + u_3,$$
$$\dot{\hat{x}}_4 = n\big[p(\hat{x}_2 + x_{20}) - q\hat{x}_4\big] + u_4,$$
$$\dot{\hat{x}}_5 = \hat{x}_1 - k_2\hat{x}_5 + u_5,$$

(12)2

$i = 1; 2; : : : ; 5$, where $\hat{x}_i \in \mathbb{R}$ represents the membrane potential variable; **ui** is an activecontroller to be designed to achieve the predefined-time synchronization of master-slave5D HRNNs.

Theorem 4. Suppose that Theorem 3 holds. The slave 5D HRNN (12) can achieve predefined-time synchronization with the master 5D HRNN (11) via the following controllaw:

$$u_1 = a\hat{x}_1^3 - b\hat{x}_1^2 - ax_1^3 + bx_1^2 - e_2 + ce_3 + 3k_1 s\hat{x}_5^2\hat{x}_1 - 3k_1 sx_5^2 x_1$$
$$- \frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\beta^\eta \operatorname{sign}(e_1(t)) - \frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\alpha \operatorname{sign}\big((e_1(t))\big|e_1(t)\big|^{\gamma\eta},$$

[13]

$$u_2 = -he_4 - \left(\frac{2^{\eta-1}T_{\max}^2}{T_{c2}}\beta^\eta + D\right)\operatorname{sign}(e_2(t))$$

$$- \frac{2^{\eta-1}T_{\max}^2}{T_{c2}}\alpha \operatorname{sign}(e_2(t))\big|e_2(t)\big|^{\gamma\eta},$$

[14]

$$u_3 = -\frac{2^{\eta-1}T_{\max}^2}{T_{c3}}\beta^\eta \operatorname{sign}(e_3(t)) - \frac{2^{\eta-1}T_{\max}^2}{T_{c3}}\alpha \operatorname{sign}(e_3(t))\big|e_3(t)\big|^{\gamma\eta},$$

[15]

$$u_4 = -\frac{2^{\eta-1}T_{\max}^2}{T_{c4}}\beta^\eta \operatorname{sign}(e_4(t)) - \frac{2^{\eta-1}T_{\max}^2}{T_{c4}}\alpha \operatorname{sign}(e_4(t))\big|e_4(t)\big|^{\gamma\eta},$$

[16]

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$$u_5 = -\frac{2^{\eta-1}T_{\max}^2}{T_{c5}}\beta^\eta \, \text{sign}\big(e_5(t)\big) - \frac{2^{\eta-1}T_{\max}^2}{T_{c5}}\alpha \, \text{sign}\big(e_5(t)\big)\big|e_5(t)\big|^{\gamma\eta},$$

[17]

$i = 1; 2; :::;5$, where T_{ci} represents the predefined synchronization time of each statevariable. Then the master-slave 5D HRNNs can realize predefined-time synchronization, and the predefined time is given as

$$T_c = T_{c1} + \max\{T_{c3}, T_{c2} + T_{c4}, T_{c5}\}.$$

Proof. Define the following error system of master-slave 5D HRNNs (11) and (12): $e_i = \hat{x}_i - x_i,$, where $i = 1,2,\ldots,5.$ Then we have

$$\begin{aligned}
\dot{e}_1 &= -a\hat{x}_1^3 + b\hat{x}_1^2 + ax_1^3 - bx_1^2 + e_2 - ce_3 \\
&\quad - k_1 de_1 - 3k_1 s\hat{x}_5^2\hat{x}_1 + 3k_1 sx_5^2 x_1 + u_1, \\
\dot{e}_2 &= -g\hat{x}_1^2 + gx_1^2 - e_2 - he_4 + u_2 - d_{\text{dis}}, \\
\dot{e}_3 &= l\big[me_1 - e_3\big] + u_3, \\
\dot{e}_4 &= n\big[pe_2 - qe_4\big] + u_4, \\
\dot{e}_5 &= e_1 - k_2 e_5 + u_5.
\end{aligned}$$

(18)

Firstly, we consider the predefined-time synchronization between $\hat{x}_1$ and x_1, i.e., e_1 converges to zero in predefined-time T_{c1}. Considering the candidate Lyapunov function $V_{e_1} = \|e_1\|_1,$ then the derivative of the candidate Lyapunov function is

$$\begin{aligned}
\dot{V}_{e_1} &= \text{sign}\big(e_1(t)\big)\dot{e}_1(t) \\
&= \text{sign}\big(e_1(t)\big)\big(-a\hat{x}_1^3 + b\hat{x}_1^2 + ax_1^3 - bx_1^2 + e_2 - ce_3 \\
&\quad - k_1 de_1 - 3k_1 s\hat{x}_5^2\hat{x}_1 + 3k_1 sx_5^2 x_1 + u_1\big) \\
&= \text{sign}\big(e_1(t)\big)\left(-k_1 de_1 - \frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\beta^\eta \, \text{sign}\big(e_1(t)\big)\right. \\
&\quad \left. - \frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\alpha \, \text{sign}\big(e_i(t)\big)\big|e_1(t)\big|^{\gamma\eta}\right) \\
&= -k_1 d\big|e_1(t)\big| - \frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\beta^\eta - \frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\alpha\big|e_1(t)\big|^{\gamma\eta}.
\end{aligned}$$

By Lemma 4, one can obtain that

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$$
\begin{aligned}
\dot{V}_{e1} &\leqslant -\frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\beta^\eta - \frac{2^{\eta-1}T_{\max}^2}{T_{c1}}\alpha\big|e_1(t)\big|^{\gamma\eta} \\
&\leqslant -\frac{T_{\max}^2}{T_{c1}}\left(2^{\eta-1}\beta^\eta + 2^{\eta-1}\alpha\big|e_1(t)\big|^{\gamma\eta}\right) \\
&\leqslant -\frac{T_{\max}^2}{T_{c1}}\left(\beta + \alpha\big|e_1(t)\big|^\gamma\right)^\eta.
\end{aligned}
$$

Thus, in accordance to Theorem 3, the variable $\hat{x}_1$ and x_1 can achieve predefined-timesynchronization under the controller (13) and the predefined-time is T_{c1}. Then if we plug $e_1 = 0$ into the fifth formula of error system (18), we have

$$
\dot{e}_5 = -k_2 e_5 + u_5.
$$

Then we are going to implement the predefined-time synchronization between $\hat{x}_5$ and x_5, i.e., e_5 converges to zero in predefined-time T_{c5}. Considering the candidate Lyapunov function $V_{e_5} = \|e_5\|_1$, then

$$
\begin{aligned}
\dot{V}_{e5} &= \mathrm{sign}\big(e_5(t)\big)\dot{e}_5(t) \\
&= \mathrm{sign}\big(e_5(t)\big)\left(-k_2 e_5 - \frac{2^{\eta-1}T_{\max}^2}{T_{c5}}\beta^\eta\,\mathrm{sign}\big(e_5(t)\big)\right. \\
&\quad \left. -\frac{2^{\eta-1}T_{\max}^2}{T_{c5}}\alpha\,\mathrm{sign}\big(e_5(t)\big)\big|e_5(t)\big|^{\gamma\eta}\right) \\
&= -k_2|e_5| - \frac{2^{\eta-1}T_{\max}^2}{T_{c5}}\beta^\eta - \frac{2^{\eta-1}T_{\max}^2}{T_{c5}}\alpha\big|e_5(t)\big|^{\gamma\eta} \\
&\leqslant -\frac{T_{\max}^2}{T_{c5}}\left(\beta + \alpha\big|e_5(t)\big|^\alpha\right)^\eta.
\end{aligned}
$$

Thus, in accordance to Theorem 3, the variables $\hat{x}_5$ and x_5 i.e., e_5 can achieve predefined-timesynchronization under the controller (17), and the predefined-time is T_{c5}. Similarly, we have

$$
\dot{e}_3 = -lme_3 + u_3.
$$

Considering the candidate Lyapunov function $(V_{e_3} = \|e_3\|_1$.

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$$
\begin{aligned}
\dot{V}_{e_3} &= \operatorname{sign}\big(e_3(t)\big)\dot{e}_3(t) = \operatorname{sign}\big(e_3(t)\big)(-lme_3 - u_3) \\
&= -lm|e_3| - \frac{2^{\eta-1}T_{\max}^2}{T_{c3}}\beta^\eta - \frac{2^{\eta-1}T_{\max}^2}{T_{c3}}\alpha\big|e_3(t)\big|^{\gamma\eta} \\
&\leqslant -\frac{T_{\max}^2}{T_{c3}}\big(\beta + \alpha\big|e_3(t)\big|^\gamma\big)^\eta .
\end{aligned}
$$

In accordance to Theorem 3, the same result can be obtained, i.e., the variables $\hat{x}_3$ and x_3 can achieve predefined-time synchronization under the controller (15), and the predefinedtimeis . Similarly, we have

$$
\dot{e}_2 = -e_2 - he_4 + u_2 - d_{\mathrm{dis}} .
$$

The Lyapunov function is designed as $V_{e_2} = \|e_2\|_1$, then

$$
\begin{aligned}
\dot{V}_{e_2} &= \operatorname{sign}\big(e_2(t)\big)\dot{e}_2(t) \\
&= \operatorname{sign}\big(e_2(t)\big)\bigg(-e_2 - he_4 + he_4 - \bigg(\frac{2^{\eta-1}T_{\max}^2}{T_{c2}}\beta^\eta + D\bigg)\operatorname{sign}\big(e_2(t)\big) \\
&\quad - \frac{2^{\eta-1}T_{\max}^2}{T_{c2}}\alpha\operatorname{sign}\big(e_2(t)\big)\big|e_2(t)\big|^{\gamma\eta}\bigg) - d_{\mathrm{dis}} \\
&= -|e_2| - \frac{2^{\eta-1}T_{\max}^2}{T_{c2}}\beta^\eta - \frac{2^{\eta-1}T_{\max}^2}{T_{c2}}\alpha\big|e_2(t)\big|^{\gamma\eta} - (D + d_{\mathrm{dis}}) \\
&\leqslant -\frac{T_{\max}^2}{T_{c2}}\big(\beta + \alpha\big|e_2(t)\big|^\gamma\big)^\eta .
\end{aligned}
$$

Thus, in accordance to Theorem 3, the variables $\hat{x}_2$ and x_2 can achieve predefined-timesynchronization under the controller (14), and the predefined-time is T_{c2}. Then we pluge into the forth formula of error system (18), and we have

$$
\dot{e}_4 = -nqe_4 + u_4 .
$$

The Lyapunov function is designed as $V_{e_4} = \|e_4\|_1$, then

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$$\dot{V}_{e4} = \text{sign}\big(e_4(t)\big)\dot{e}_4(t)$$

$$= \text{sign}\big(e_4(t)\big)\left(-nqe_4 - \frac{2^{\eta-1}T_{\max}^2}{T_{c4}}\beta^\eta \,\text{sign}\big(e_4(t)\big)\right.$$

$$\left. - \frac{2^{\eta-1}T_{\max}^2}{T_{c4}}\alpha\,\text{sign}\big(e_4(t)\big)\big|e_4(t)\big|^{\gamma\eta}\right)$$

$$= -nq|e_4| - \frac{2^{\eta-1}T_{\max}^2}{T_{c4}}\beta^\eta - \frac{2^{\eta-1}T_{\max}^2}{T_{c4}}\alpha\big|e_4(t)\big|^{\gamma\eta}$$

$$\leqslant -\frac{T_{\max}^2}{T_{c4}}\big(\beta + \alpha\big|e_4(t)\big|^\gamma\big)^\eta.$$

Thus, in accordance to Theorem 3, the variables $\hat{x}_4$ and x_4 can achieve predefined-time synchronization under the controller (16), and the predefined-time is Tc4$_{T_{c4}}$. Then the five state variables of the 5D HRNNs all achieve predefined-time synchronization, and the synchronization time of each variable is different, i.e., the predefined synchronization time of is $\hat{x}_1$ is T_{c1}, $\hat{x}_2$ is $T_{c1} + T_{c2}$, $\hat{x}_3$ is $T_{c1} + T_{c3}$, $\hat{x}_4$ is $T_{c1} + T_{c2} + T_{c4}$, and $\hat{x}$^x5 is Tc1 + Tc5.$_{T_{c1}+T_{c5}}$ Thus, the synchronization time of the master-slave 5D HRNNs is

$$T_c = T_{c1} + \max\{T_{c3}, T_{c2} + T_{c4}, T_{c5}\}.$$

This proof is completed.

Remark 4. By designing the controller of each state variable of the system via backstepping, the design process can be made more simpler and convenient. We can also realize the synchronization of some of state variables of the system according to the needs of actual applications by designing the controller via backstepping.

Remark 5. State variables of the system can realize synchronization at different predefined time, which increases the complexity of the system. In this way, important message can be transmitted first in secure communication and different messages has its own synchronous transmission time, which increases the complexity of transmission and improves the security communication.

Example 2. Figures 5(a)–5(c) show the chaotic behaviors of the 5D HR neuron network(11) in a super-bursting regime projected onto the 3D subspaces of the 5D phase space: x_1, x_2, x_5-space, x_1, x_3, x_5-space x_1, x_4, x_5, space, respectively. The constant parameters have standard values: $a = 1.0, b = 3.0, c = 0.99, f = 1.01, g = 5.0128,$

$h = 0.0278, l = 0.00215, m = 3.966, x_0 = 1.605, n = 0.0009, p = 3.0, y_0 = 1.619,$

$q = 0.9573.$.

By simple computation, $\alpha = 1.2$, $\beta = 1.4$, $\gamma = 5/6$, $\eta = 2.7$, Tc1 = 0.2, Tc2 = 0.2, Tc3 = 0.4, Tc4 = 0.4, and Tc5 = 0.8, then the conditions of Theorem 3 hold. Parameter q = 0.17, then the

conditions of Lemma 3 hold. The master-slave 5D HRNNs are step-by-step predefined-time synchronization. For different state variables, the predefined synchronization time is different. Among them, the predefined synchronization time of $\hat{x}_1$
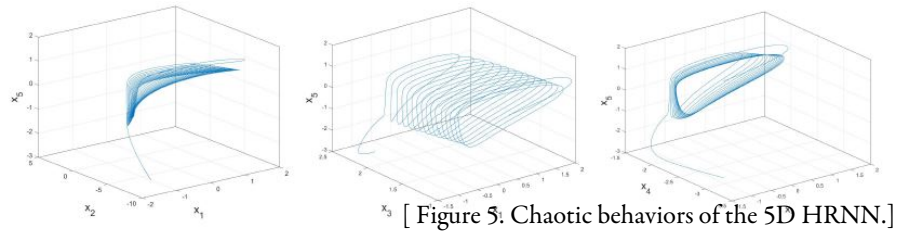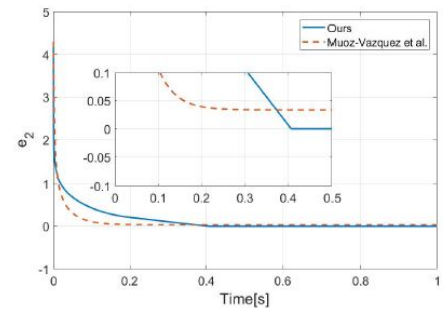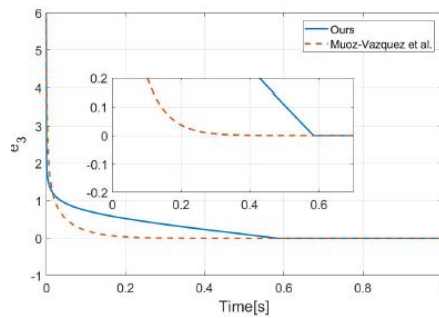

[Figure 5. Chaotic behaviors of the 5D HRNN.]

(a) x_1, x_2, x_5-space (b) x_1, x_3, x_5-space (c) x_1, x_4, x_5-space

[Figure 9. Decrypted messages mo1 and mo2. Figure 10. Synchronization errors em1 and em2 .]
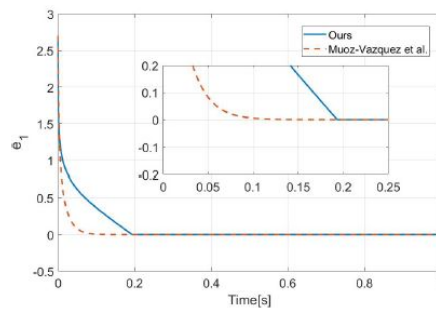
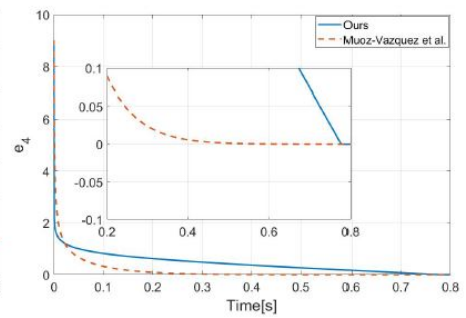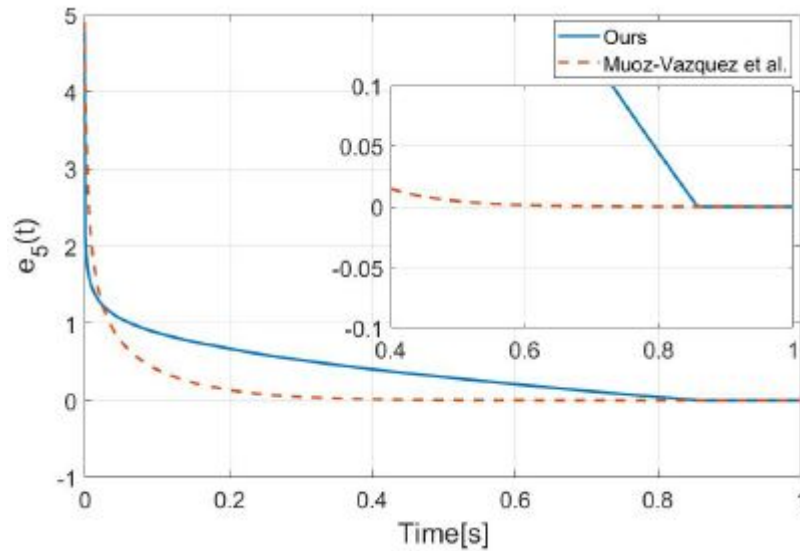

(a) $T_{c1} = 0.2$

(b) $T_{c2} = 0.2$

(c) $T_{c3} = 0.4$

(d) $T_{c4} = 0.4$

(e) $T_{c5} = 0.8$

is $T_{c1} = 0.2$, $\hat{x}_2$ is $T_{c1} + T_{c2} = 0.4$, $\hat{x}_3$ is $T_{c1} + T_{c3} = 0.6$, $\hat{x}_4$ is $T_{c1} + T_{c2} + T_{c4} = 0.8$,

and $\hat{x}_5$ is $T_{c1} + T_{c5} = 1.0$. The simulation results are shown in Fig. 6(a) $(e_1 = \hat{x}_1 - x_1)$, Fig 6(b) $(e_2 = \hat{x}_2 - x_2)$. Fig 6(c) $(e_3 = \hat{x}_3 - x_3)$, Fig 6(d) $(e_4 = \hat{x}_4 - x_4)$, Fig 68(e) $(e_5 = \hat{x}_5 - x_5)$. As can be seen from Fig. 6(b), Theorem has better robustness than Lemma 3 and can effectively suppress disturbance.

5 Secure communication

This section proposes a secure communication algorithm based on the predefined-time synchronization of 5D HRNNs.

Example 3. By using plaintext signal, sender generates the following signals:

$$m_1 = \begin{cases} r_1(t), & 0 \leqslant t < 1, \\ 1.6 \sin t + 0.7 \cos(0.3t), & t \geqslant 1, \end{cases}$$

$$m_2 = \begin{cases} r_1(t), & 0 \leqslant t < 1, \\ -\sin(1.2t) + 2\cos(3t), & t \geqslant 1, \end{cases}$$

where *r1(t)* and *r2(t)* denote random signals. Sender calculates the encrypted signal

$$C_1(t) = m_1(t) + x_1(t), \qquad C_2(t) = m_2(t) + x_5(t).$$

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Figures 7 and 8 illustrate the state trajectories of m_i and C_i, where $i = 1, 2$. The initialconditions of 5D HRNNs (11) and (12), $r_1(t)$, $r_2(t)$, $m_1(t)$, and $m_2(t)$ can be knownonly to the sender.

Receiver receives the secret keys *a, b, c, d, f, g, h, l, n, p, q* and the encryptedsignals $C_1(t), C_2(t)$. Then receiver generates the slave 5D HRNN (12). Receiver choosesthe controller (13), (14), (15), (16), and (17); $\alpha = 1.2$, $\beta = 1.4$, $\gamma = 5/6$, and $\eta = 2.7$. According to Theorem 3, the state variables x_1 and $\hat{x}_1$, x_5 and $\hat{x}_5$ of master-slave5D HRNNs can realize their own predefined-time synchronization, and their predefinedsynchronization time is $T_{c1} = 2$ and $T_{c5} = 8$, respectively. Figures 9 and 10 illustrate thereceiver decrypted the encrypted messages by calculating $m_{oi} = C_i - \hat{x}_i$ and the message error $e_{mi} = m_{oi} - m_i$. As can be seen from Fig. 10, message m1 achieved accuratetrans mission within $T_{c1} = 2$ s., and message m^2 achieved accurate transmission within $T_{c1} + T_{c5} = 10$ s.

. In this way, different messages has its own synchronous transmissiontime, and the receiver can get accurate message after the predefined time, which increasesthe complexity of transmission and improves the security communication.
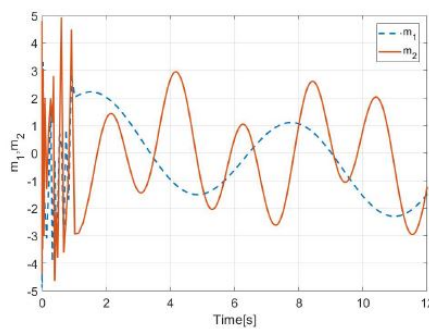

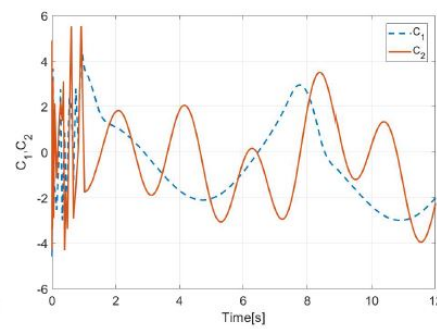**Figure 7.** Plaintext messages m_1 and m_2.

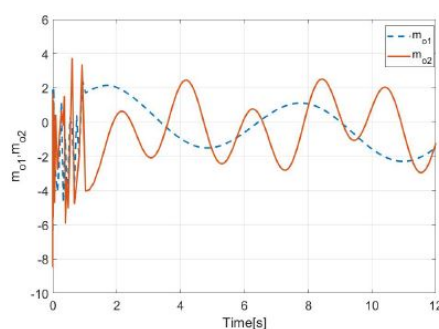**Figure 8.** Encrypted signals C_1 and C_2.

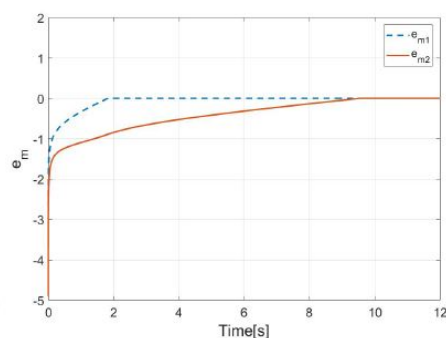**Figure 9.** Decrypted messages m_{o1} and m_{o2}.

Figure 10. Synchronization errors e_{m1} and e_{m2}.

6 Conclusion

the predefined-time synchronization of 5D HRNNs in which the synchronization time of each state variable of the master-slave 5D HRNNs is different and can be defined in advance. To show the

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applicability of the theoretical results obtained, the designed predefined-time controller has been applied to secure communication to realize asynchronous communication of multiple messages. In the future research work, we will continue to study the predefined- time secure communication under the influence of disturbances and apply it to other fields, such as discrete-time Boolean control networks, fuzzy sets, and fractional order neural networks.

Acknowledgments

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