Nonlinear Analysis: Modelling and Control
ISSN: 1392-5113
ISSN: 2335-8963
nonlinear@mii.vu.lt
Vilniaus Universitetas
Lituania

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Nonlinear Analysis: Modelling and Control, vol. 27, núm. 3, 2022
Vilniaus Universitetas, Lituania
Disponible en: https://www.redalyc.org/articulo.oa?id=694173185011
DOI: https://doi.org/10.15388/namc.2022.27.26638

Articles

Exponential stabilization of fractional-order continuous-time dynamic systems via event-triggered impulsive control*

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Nonlinear Analysis: Modelling and Control, vol. 27, núm. 3, 2022

Vilniaus Universitetas, Lituania

Recepción: 18 Mayo 2021
Revisado: 22 Noviembre 2021
Publicación: 07 Abril 2022

DOI: https://doi.org/10.15388/namc.2022.27.26638

Redalyc: https://www.redalyc.org/articulo.oa?id=694173185011

Abstract: Exponential stabilization of fractional-order continuous-time dynamic systems via eventtriggered impulsive control (EIC) approach is investigated in this paper. Nonlinear and linear fractional-order continuous-time dynamic systems are studied, respectively. The impulsive instants are determined by some given event-triggering function and event-triggering condition, which are dependent on the state of the systems. Sufficient conditions on exponential stabilization for nonlinear and linear cases are presented, respectively. Moreover, the Zeno-behavior of impulsive instants is excluded. Finally, the validity of theoretical results are also illustrated by some numerical simulation examples including the synchronization control of fractional-order jerk chaotic system.

Keywords: event-triggered, impulsive control, fractional order, exponential stabilization, synchronization.

1 Introduction

It is noteworthy that the properties of a number of practical engineering systems, such as electromagnetic waves, chemical physics, and fluid flow, cannot be adequately represented by integer-order dynamic systems but can be well embodied via employing fractional-order models. Due to their wide applications, fractional-order dynamic systems have attracted considerably attention from various fields, such as materials science [27], physics [8], pharmacokinetics [23], mechanics [10], supercapacitors [4], and neural networks [1], just to name a few. Please refer to the monograph [19] for more applications about fractional-order differential systems. Stability analysis [2] is one of the important and interesting topics for fractional-order dynamic systems. Many valuable methods on this issue have been reported, such as robust control [11], fractional-order controller design [28], adaptive control [9], sliding mode control [26], impulsive control [25], and so on.

Impulsive control only introduces transient control at certain discrete moments, which can achieve the control target through minimum control amount [20]. Impulsive control has gradually become a commonly used method in modern control because of its simple structure, lower control cost, and less information transmission [30]. It also has been widely used in coupled systems [6], neural network systems [37], chaotic secure communication [7] and system stabilization [33], etc. In recent years, with the rise of research on fractional-order control systems, many useful results have been proposed, for examples, impulsive synchronization of fractional-order complex networks [17], impulsive stabilization of fractional-order neural networks [31], impulsive control of fractional-order multi-agent systems [18], etc.

It should be noted that most of the results focused on fixed time-triggered impulsive control, that is, the impulse interval is preset. From the perspective of actual effects, impulsive control inputs at some moments are unnecessary, which will lead to the waste of system bandwidth resources [36]. Thanks to the proposal of event-triggered control theory, event-triggered mechanisms invoke data transmissions if predefined conditions on the data are satisfied. As a result, network and energy resources are consumed only when the data is necessary for control, which can achieve the control object with less information exchange. Thus, the design of certain event-triggered strategies have received increasing attention in recent years for integer-order dynamic systems [24, 34, 35, 37] and many references therein. Combining the advantages of impulsive control and eventtriggered control, event-triggered impulsive control (EIC) was proposed in recent years, where the impulsive instants are determined by some designed event-triggering functions and event-triggering conditions. Many scholars have carried out a series of fruitful researches in this field, such as applying the event-triggered impulse control method to synchronization analysis of discrete time-delay complex dynamical networks [12], nonlinear delay systems [15], input-to-state stability for heterogeneous multi-agent systems [13], discrete-time delayed systems and networks [16], Lyapunov stability problem for impulsive systems [14], consensus of multi-agent systems [3], neural networks [22], and so on. Compared with the event-triggered impulsive control for integer-order systems, there are few works on event-triggered impulsive control for fractional-order systems.

Based on the above discussion and inspired by the research in [16,32], this paper will study the exponential stabilization of general fractional-order continuous-time dynamic systems including the nonlinear and linear case via event-triggered impulsive control approach. The main contributions are as follows: (i) The event-triggered impulsive control method is applied to fraction-order continuous time dynamic systems. The impulsive instants are defined by some events depending on the state of the systems. Thus, the impulsive instants are not given in advance, which is different with the time-triggered impulsive control. (ii) The controller does not need to be updated continuously, and the Zeno-behavior of impulsive instants is excluded. Some unnecessary impulsive samples can be avoided by the control strategy and the online resources are saved. (iii) Based on the stability theory and inequality technique, some sufficient conditions on exponential stabilization of nonlinear and linear fractional-order dynamic systems are presented, respectively.

The rest of this paper is organized as follows. In Section 2, problem formulation is introduced. In Section 3, exponential stabilization is studied for nonlinear and linear fractional-order continuous-time dynamic systems via event-triggered impulsive control, respectively. The Zeno-behavior of impulsive instants is also excluded. In Section 4, simulation examples are presented to show the effectiveness of the theoretical results. Conclusions and future study are made in Section 5.

2 Problem formulation

2.1 Caputo fractional derivative

The Caputo and Riemann–Liouville (RL) fractional-order derivatives are the two broadly used to model the fractional-order dynamical systems. Since the initial conditions for fractional-order differential equations with Caputo fractional-order derivative take the same form as for the traditional integer-order differential equations, in this paper, we will adopt the Caputo fractional-order derivative to model the continuous time dynamic systems.

Definition 1. (See [21].) The Caputo fractional-order derivative of $x(t)$ of order $\alpha \in$ (0,1) is defined as follows:

$$ {}_{C}D^{\alpha}_{t_0,t}x(t) = \frac{1}{\Gamma(1-\alpha)} \int\limits_{t_0}^{t} (t-\tau)^{-\alpha} x'(\tau)\,\mathrm{d}\tau, $$

where $\Gamma(\tau)=\int_0^\infty t^{\tau-1}\mathrm{e}^{-t}\,\mathrm{d}t$ is the Gamma function, and $x'(\tau)$ denotes the derivative of $x(\tau)$.

For convenience, in the following, ${}_{C}D^{\alpha}_{t_0,t}x(t)$ will be denoted as $D^{\alpha}_{t_0}x(t)$ if no confusion arisen, where t_0 denotes the initial time.

Note that the differentiability of function is required in the definition of Caputo fractional-order derivative, but a number of function may not be differentiable. The right upper Dini α-order derivative of $x(\tau)$ is introduced.

Definition 2. For $\alpha \in$ (0,1), the upper right Dini fractional-order derivative is defined by

$$ D^{\alpha+}_{t_0}x(t) = \frac{1}{\Gamma(1-\alpha)} \int\limits_{t_0}^{t} (t-\tau)^{-\alpha} D^{+}x(\tau)\,\mathrm{d}\tau, $$

where $D^{+}x(t)=\limsup_{h\to 0^+}(x(t+h)-x(t))/h$ is the right upper Dini derivative of $x(\tau)$.

Definition 3. (See [29].) For a Lebesgue-integrable function $\varphi : [a,b] \to R,$, the fractional-order integral of order $\alpha \in$ (0,1) is defined as follows:

$$D_a^{-\alpha}\varphi(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1}\varphi(\tau)\,\mathrm{d}\tau.$$

Lemma 1. (*See* [21].) *If* $\alpha > 0, \beta > 0, u(t), v(t) \in C^1[a,b],$ *, then*

It is obviously that Lemma 1 also holds for the upper right Dini fractional-order derivative.

Mittag-Leffler function defined as follows is often used to study the dynamic behavior of fractional-order dynamic systems [5]:

$$E_{\alpha,\beta}(z) = \sum_{j=0}^{\infty}\frac{z^j}{\Gamma(\alpha j+\beta)}.$$

Especially, when $\beta = 1,$ Mittag-Leffler function with one parameter is

$$E_\alpha(z) = E_{\alpha,1}(z) = \sum_{j=0}^{\infty}\frac{z^j}{\Gamma(\alpha j+1)}.$$

Lemma 2. (*See* [21].) *Assume that* $\alpha > 0, \beta > 0$ *and* $q \in R.$ *. Then*

$$\int_{t_0}^{t} E_{\alpha,\beta}\left[q\nu^\alpha\right]\nu^{\beta-1}\,\mathrm{d}\nu = (t-t_0)^\beta E_{\alpha,\beta+1}\left[q(t-t_0)^\alpha\right].$$

Lemma 3. (*See* [29].) *Let* $0 < \alpha < 1.$ *Then* $E_\alpha[\mu(t-t_0)^\alpha]$ *is nonnegative, and the following statements are true*:

i. $E_\alpha[\mu(t-t_0)^\alpha]$ *is monotonically nonincreasing and* $0 \leqslant E_\alpha[\mu(t-t_0)^\alpha] \leqslant 1$ *for* $t \geqslant t_0$ *when* $\mu \leqslant 0.$
ii. $E_\alpha[\mu(t-t_0)^\alpha]$ *is monotonically nondecreasing and* $E_\alpha[\mu(t-t_0)^\alpha] \geqslant 1$ *for* $t \geqslant t_0$ *when* $\mu \geqslant 0.$

Lemma 4. *Let* $0 < \alpha < 1, \mu \in R$ *and* $D_{t_0}^{\alpha+}W(t) \leqslant \mu W(t) + \vartheta(t),$ *where* $\vartheta(t)$ *is a given continuous function. Then*

$$W(t) \leqslant W(t_0)E_\alpha\left(\mu(t-t_0)^\alpha\right) + \int_{t_0}^{t}(t-\tau)^{\alpha-1}E_{\alpha,\alpha}\left(\mu(t-\tau)^\alpha\right)\vartheta(\tau)\,\mathrm{d}\tau, \quad t \geqslant t_0.$$

Especially, when $\vartheta(t) = 0,$ *the following inequality holds*:

$$W(t) \leqslant W(t_0)E_\alpha\big[\mu(t-t_0)^\alpha\big], \quad t \geqslant t_0.$$

Proof. Since $D_{t_0}^{\alpha+}W(t) \leqslant \mu W(t) + \vartheta(t),$ *, there exists a nonnegative function* $H(t)$ *satisfying*

$$D_{t_0}^{\alpha+}W(t) = \mu W(t) + \vartheta(t) - H(t),$$

and then

$$W(t) = W(t_0) + \frac{\mu}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}W(\tau)\,\mathrm{d}\tau + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}\big[\vartheta(\tau) - H(\tau)\big]\,\mathrm{d}\tau, \quad t \geqslant t_0.$$

Let $W(t) = p(t-t_0), \vartheta(t) - H(t) = q(t-t_0).$ Then

$$p(t-t_0) = p(0) + \frac{1}{\Gamma(\alpha)}\int_{0}^{t-t_0}(t-t_0-s)^{\alpha-1}\big[\mu p(s) + q(s)\big]\,\mathrm{d}s.$$

Denote $\gamma = t - t_0,$ we have

$$p(\gamma) = p(0) + \mu D_0^{-\alpha}p(\gamma) + D_0^{-\alpha}q(\gamma), \quad \gamma \geqslant 0.$$

Taking the Laplace transform on both sides, we can obtain that

$$P(s) = \frac{p(0)}{s} + \frac{\mu P(s)}{s^\alpha} + \frac{Q(s)}{s^\alpha},$$

where $P(s)$ and $Q(s)$ are the Laplace transforms of $p(\gamma)$ and $q(\gamma).$, respectively. Then

$$P(s) = \frac{s^{\alpha-1}}{s^\alpha - \mu}p(0) + \frac{1}{s^\alpha - \mu}Q(s).$$

Thus, by the inverse Laplace transform we have

$$p(\gamma) = p(0)E_\alpha\big(\mu\gamma^\alpha\big) + q(\gamma) * \gamma^{\alpha-1}E_{\alpha,\alpha}\big(\mu\gamma^\alpha\big).$$

It follows that

$$W(t) \leqslant W(t_0)E_\alpha\big(\mu(t-t_0)^\alpha\big) + \int_{t_0}^{t} (t-\tau)^{\alpha-1}E_{\alpha,\alpha}\big(\mu(t-\tau)^\alpha\big)\vartheta(\tau)\,\mathrm{d}\tau, \quad t \geqslant t_0.$$

The proof is completed.

2.2 System model and problem statement

Consider the following fractional-order continuous-time dynamic control systems with $\alpha \in (0,1)$:

$$D_{t_0}^\alpha x(t) = f\big(x(t), u(t)\big), \quad t \geqslant t_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state and control input, respectively, $f: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ satisfies $f(0,0)=0$, and there is a positive constant l_1 such that

$$\big\|f(x_1,y_1) - f(x_2,y_2)\big\| \leqslant l_1\|x_1-x_2\| + l_1\|y_1-y_2\|.$$

In order to use the benefit of impulsive control and reduce the frequency of the controller update, the following event-triggered impulsive feedback controller (EIC) is used:

$$\begin{aligned} u(t) &= g\big(x(t_k)\big) \quad \text{for } t \in [t_k, t_{k+1}), \\ x(t) &= h\big(x(t^-)\big) \quad \text{for } t = t_k,\ k = 1,2,\dots, \end{aligned} \tag{2}$$

where $x(t_k^-)$ denotes the left limit of function $x(t)$ *at* t_k, $g: \mathbb{R}^n \to \mathbb{R}^m$, $h: \mathbb{R}^n \to \mathbb{R}^n$ satisfy $g(0)=0$, $h(0)=0$, and Lipschitz conditions, i.e., there are positive constants l_2, l_3 such thatk $\|g(x)-g(y)\| \leqslant l_2\|x-y\|$, $\|h(x)-h(y)\| \leqslant l_3\|x-y\|$. The impulsive instants *t*. are defined iteratively by

$$t_{k+1} = \inf\big\{t\colon\ t > t_k \text{ and } F(t) \geqslant 0\big\} \tag{3}$$

in which $F(t)$ is called to be the triggering function defined as

$$F(t) = \big\|e(t)\big\| - \beta\big\|x(t_k)\big\|, \tag{4}$$

where $\beta \in (0,1)$, and $e(t) = x(t_k) - x(t)$ represents measurement error.

Definition 4. The fractional-order continuous-time dynamic system (1) with EIC (2) is said to be event-triggered impulsive exponential stabilization (EIES) if there exit $M>0$, $\lambda>0$ such that

$$\|x(t)\| \leqslant M\|x(t_0)\|e^{-\lambda(t-t_0)}.$$

Definition 5. If there exists a constant $\theta > 0$ such that $\inf_{k\in N}\{t_{k+1} - t_k\} \geqslant \theta > 0$, where $N = \{0, 1, \dots\}$, then the impulsive instants t_k are called to no Zeno-behavior.

3 Exponential stabilization results

In this section, we first discuss that there is no Zeno-behavior for the impulsive instants determined by (3). Then we prove that system (1) is exponential stabilization under the EIC (2) with some given conditions. A corollary for linear case is also presented.

Theorem 1. *There is no Zeno-behavior for the impulsive instants t. determined by* (3).

Proof. For $t \in [t_k, t_{k+1})$, calculating the right upper Dini α-order derivative of $\|e(t)\|$, we have

$$\begin{aligned} D_{t_k}^{\alpha+}\|e(t)\| &\leqslant \|D_{t_k}^{\alpha}e(t)\| = \|D_{t_k}^{\alpha}x(t)\| \\ &= \|f(x(t), u(t))\| = \|f(x(t), u(t)) - f(0,0)\| \\ &\leqslant l_1\|x(t)\| + l_1\|u(t)\| = l_1\|x(t)\| + l_1\|g(x(t_k))\| \\ &\leqslant l_1\|x(t_k) - e(t)\| + l_1 l_2\|x(t_k)\| \\ &\leqslant l_1\|e(t)\| + (l_1 + l_1 l_2)\|x(t_k)\|. \end{aligned}$$

By Lemma 4 one can get

$$\begin{aligned} \beta\|x(t_k)\| &\leqslant \|e(t_{k+1})\| \\ &\leqslant (l_1 + l_1 l_2)\|x(t_k)\|(t_{k+1} - t_k)^{\alpha}E_{\alpha,\alpha+1}[l_1(t_{k+1} - t_k)^{\alpha}]. \end{aligned}$$

Then it follows from $\|e(t_k)\| = 0$ that

$$\begin{aligned} \|e(t)\| &\leqslant (l_1 + l_1 l_2)\|x(t_k)\| \int_{t_k}^{t} (t-\tau)^{\alpha-1}E_{\alpha,\alpha}[l_1(t-\tau)^{\alpha}]\,d\tau \\ &= (l_1 + l_1 l_2)\|x(t_k)\|(t - t_k)^{\alpha}E_{\alpha,\alpha+1}[l_1(t - t_k)^{\alpha}]. \end{aligned}$$

The next event will not be triggered until $F(t_{k+1}) \geqslant 0$, i.e., $\|e(t_{k+1})\| - \beta\|x(t_k)\| \geqslant 0$. Thus,

$$\begin{aligned} \beta\|x(t_k)\| &\leqslant \|e(t_{k+1})\| \\ &\leqslant (l_1 + l_1 l_2)\|x(t_k)\|(t_{k+1} - t_k)^{\alpha}E_{\alpha,\alpha+1}[l_1(t_{k+1} - t_k)^{\alpha}]. \end{aligned} \tag{5}$$

If $\|x(t_k)\| = 0$ for some fixed t_k, then one can get $\|x(t)\| = 0$ for any $t \geqslant t_k$, which implies the stability of system (1) is reached. Thus, without loss of generality, we assume that $\|x(t_k)\| = 0$ Then by (5) we have

$$(l_1 + l_1 l_2)(t_{k+1} - t_k)^\alpha E_{\alpha,\alpha+1}\left[l_1 (t_{k+1} - t_k)^\alpha\right] \geqslant \beta.$$

Let $T \geqslant \sup_{k\in N}\{t_{k+1} - t_k\}$. Then

$$(t_{k+1} - t_k)^\alpha \geqslant \frac{\beta}{(l_1 + l_1 l_2) E_{\alpha,\alpha+1}[l_1 T^\alpha]}.$$

Therefore, there is a $\theta = \exp\{(1/\alpha)\ln(\beta/((l_1 + l_1 l_2)E_{\alpha,\alpha+1}[l_1 T^\alpha]))\} > 0$ such that $t_{k+1} - t_k \geqslant \theta > 0$, that is, there is no Zeno-behavior of impulsive instants.

Theorem 2. *Assume that the impulsive instants* $t_k,\ k = 1, 2, \ldots,$ *are determined by* (3) *and* $\sup_{k\in N}\{t_{k+1} - t_k\} \leqslant T < \infty,\ \inf_{k\in N}\{t_{k+1} - t_k\} \geqslant \theta > 0.$ *Then system (1) with EIC (2) is EIES if the following inequality is satisfied:*

$$\frac{\ln l_3}{T} + \frac{\ln E_\alpha[(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta})T^\alpha]}{\theta} < 0,$$

where $0 < l_3 < 1,\ \beta \in (0,1)$.

Proof. For $t \in [t_k, t_{k+1})$, calculating the right upper Dini .-order derivative along the solution of (1), we have

$$\begin{aligned} D_{t_k}^{\alpha+}\|x(t)\| &\leqslant \|D_{t_k}^{\alpha} x(t)\| = \|f(x(t), u(t))\| \\ &\leqslant l_1\|x(t)\| + l_1\|u(t)\| \\ &\leqslant l_1\|x(t)\| + l_1\|g(x(t_k))\| \\ &= l_1\|x(t)\| + l_1\|g(x(t) + e(t))\| \\ &\leqslant l_1\|x(t)\| + l_1 l_2\|x(t) + e(t)\| \\ &\leqslant (l_1 + l_1 l_2)\|x(t)\| + l_1 l_2\|e(t)\|. \end{aligned} \tag{6}$$

By the definition of impulsive instants t_k and triggering function (4) one can derive that

$$\|e(t)\| \leqslant \beta\|x(t_k)\| = \beta\|x(t) + e(t)\| \leqslant \beta\|x(t)\| + \beta\|e(t)\|,$$

and then

$$\|e(t)\| < \frac{\beta}{1-\beta}\|x(t)\|. \tag{7}$$

In term of (6) and (7), we have

$$D_0^\alpha x(t) = Ax(t) + Bu(t), \quad t \geqslant 0,$$

It follows from Lemma 4 that

$$\|x(t)\| \leqslant \|x(t_k)\| E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t-t_k)^\alpha\right], \tag{8}$$

and then

$$\|x(t_{k+1}^-)\| \leqslant \|x(t_k)\| E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t_{k+1}-t_k)^\alpha\right].$$

Notice that

$$\begin{aligned}\|x(t_k)\| &= \|h(x(t_k^-))\| \leqslant l_3\|x(t_k^-)\| \\ &\leqslant l_3\|x(t_{k-1})\| E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t_k-t_{k-1})^\alpha\right]\end{aligned}$$

$$\begin{aligned}&\leqslant l_3^2\|x(t_{k-1}^-)\| E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t_k-t_{k-1})^\alpha\right] \\ &\leqslant l_3^2\|x(t_{k-2})\| E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t_{k-1}-t_{k-2})^\alpha\right] \\ &\quad \times E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t_k-t_{k-1})^\alpha\right] \\ &\leqslant \cdots \leqslant l_3^k\|x(t_0)\| \prod_{i=1}^k E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t_i-t_{i-1})^\alpha\right].\end{aligned}$$

Recalling (8), we have

$$\begin{aligned}\|x(t)\| \leqslant l_3^k\|x(t_0)\| &\left\{\prod_{i=1}^k E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t_i-t_{i-1})^\alpha\right]\right\} \\ &\times E_\alpha\left[\left(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta}\right)(t-t_k)^\alpha\right].\end{aligned}$$

Since $0 < l_3 < 1, T \geqslant \sup_{k\in N}\{t_{k+1}-t_k\}$, and $\inf_{k\in N}\{t_{k+1}-t_k\} \geqslant \theta > 0$. Then

$$\begin{aligned}
&\|x(t)\| \\
&\leqslant l_3^k\|x(t_0)\|\left\{\prod_{i=1}^{k} E_\alpha\left[\left(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta}\right)T^\alpha\right]\right\}E_\alpha\left[\left(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta}\right)T^\alpha\right] \\
&= l_3^k\|x(t_0)\|\left\{E_\alpha\left[\left(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta}\right)T^\alpha\right]\right\}^k E_\alpha\left[\left(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta}\right)T^\alpha\right] \\
&\leqslant l_3^k\|x(t_0)\|\left\{E_\alpha\left[\left(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta}\right)T^\alpha\right]\right\}^{k+1} \\
&\leqslant l_3^{(t-t_0)/T-1}\left\{E_\alpha\left[\left(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta}\right)T^\alpha\right]\right\}^{(t-t_0)/\theta}+2\|x(t_0)\| \\
&= l_3^{-1}\exp\left\{\frac{\ln l_3}{T}(t-t_0)\right\}\left\{E_\alpha\left[\left(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta}\right)T^\alpha\right]\right\}^2 \\
&\quad\times\exp\left\{\frac{\ln E_\alpha[(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta})T^\alpha]}{\theta}(t-t_0)\right\}\cdot\|x(t_0)\| \\
&= \frac{\{E_\alpha[(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta})T^\alpha]\}^2}{l_3} \\
&\quad\times\exp\left\{\frac{\ln l_3}{T}+\frac{\ln E_\alpha[(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta})T^\alpha]}{\theta}(t-t_0)\right\}\cdot\|x(t_0)\|.
\end{aligned}$$

Denote

$$M=\frac{\{E_\alpha[(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta})T^\alpha]\}^2}{l_3}$$

and

$$\lambda=-\left(\frac{\ln l_3}{T}+\frac{\ln E_\alpha[(l_1+l_1l_2+\frac{\beta l_1l_2}{1-\beta})T^\alpha]}{\theta}\right)>0,$$

we have

$$\|x(t)\|\leqslant M\|x(t_0)\|\mathrm{e}^{-\lambda(t-t_0)},\quad t\geqslant t_0,$$

which implies that the fractional-order continuous-time dynamic system (1) is eventtriggered impulsive stabilization with the EIC (2).

According to Theorems 1 and 2, consider the following linear system:

$$D_{t_0}^\alpha x(t)=Ax(t)+Bu(t),\quad t\geqslant t_0, \tag{9}$$

where $x\in\mathbb{R}^n$, $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, and $u(t)\in\mathbb{R}^m$,, and the event-triggered impulsive feedback controller is designed as follows:

$$\begin{aligned} u(t) &= Kx(t_k) \quad \text{for } t \in [t_k, t_{k+1}), \\ x(t) &= C_k x(t^-) \quad \text{for } t = t_k,\ k = 1, 2, \dots . \end{aligned} \tag{10}$$

where $K \in \mathbb{R}^{m\times n}, C_k \in \mathbb{R}^{n\times n}$ are the control gain matrices determined later, impulsive instants *t.* are also determined by (3). Then we have the following corollary.

Corollary 1. *Assume that the impulsive instants* $t_k, k = 1, 2, \dots,$ *are determined by* (3). *Then system* (9) *with EIC* (10) *is EIES if the following inequality is satisfied*:

$$\frac{\ln\gamma}{T} + \frac{\ln E_\alpha[(\|A+BK\| + \frac{\beta\|BK\|}{1-\beta})T^\alpha]}{\theta} < 0,$$

where $\sup_{k\in N}\{t_{k+1} - t_k\} \leqslant T < \infty, \inf_{k\in N}\{t_{k+1} - t_k\} \geqslant \theta > 0, \gamma = \sup_k\{\|C_k\|\},$ *and* $0 < \gamma < 1, 0 < \beta < 1.$ .. *Furthermore, there is no Zeno-behavior for the impulsive instants determined by* (3).

4 Simulation examples

In this section, numerical examples for linear and nonlinear cases will be presented to illustrate the theoretical results.

Example 1. Consider the following linear fractional-order continuous-time dynamic system:

$$D_0^\alpha x(t) = Ax(t) + Bu(t), \quad t \geqslant 0, \tag{11}$$

where $\alpha = 0.9$, $x \in \mathbb{R}^3$, $A \in \mathbb{R}^{3\times 3}$, $B \in \mathbb{R}^{3\times 3}$, and $u \in \mathbb{R}^3$.

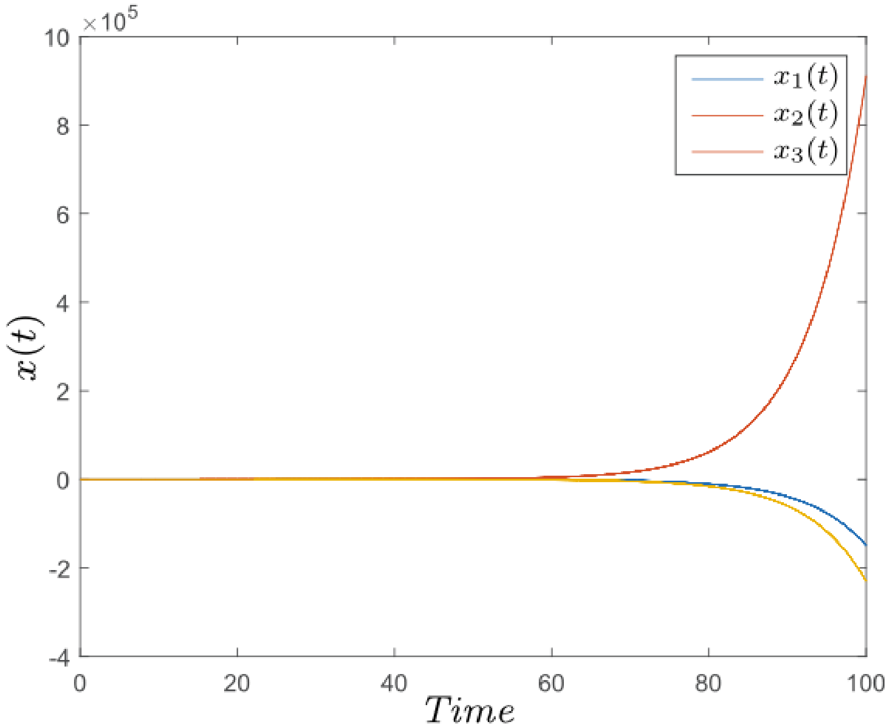

Figure 1.
States response without impulse control.

Assume that

$$A = \begin{bmatrix} -0.3 & -0.12 & -0.15 \\ 0.22 & 0.3 & 0.25 \\ -1.2 & -0.5 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 0.6 \end{bmatrix},$$

and

$$K = \begin{bmatrix} 0.8750 & 0.1000 & -0.1875 \\ 0.0167 & 1.0833 & 0.2083 \\ -2.0000 & -0.8333 & 1.6667 \end{bmatrix}.$$

If there is no impulsive control for system (11), that is, the controller (10) is replaced by $u(t) = Kx(t)$. Then we have

$$D_0^{\alpha} x(t) = (A + BK)x(t), \quad t \geqslant 0. \tag{12}$$

Let the initial condition for system (12) be $x(0) = [-0.8, 1, 0.5]^{\mathrm{T}}$. The state of system (12) is depicted in Fig. 1, which shows system (11) with the continuous control $u(t) = Kx(t)$ is unstable.

Now, we apply the event-triggered impulsive control on system (11). Let

$$C_k = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & -0.15 \end{bmatrix} \quad k = 1, 2, \dots .$$

Assume $\beta = 0.3$, $T = 0.07$, by simple computation we can choose $\theta = 0.0155$ and $\gamma = 0.2$. Thus,

$$\frac{\ln \gamma}{T} + \frac{\ln E_\alpha[(\|A + BK\| + \frac{\beta\|BK\|}{1-\beta})T^\alpha]}{\theta} = -0.7810 < 0.$$

Therefore, by Corollary 1, system (11) is EIES.

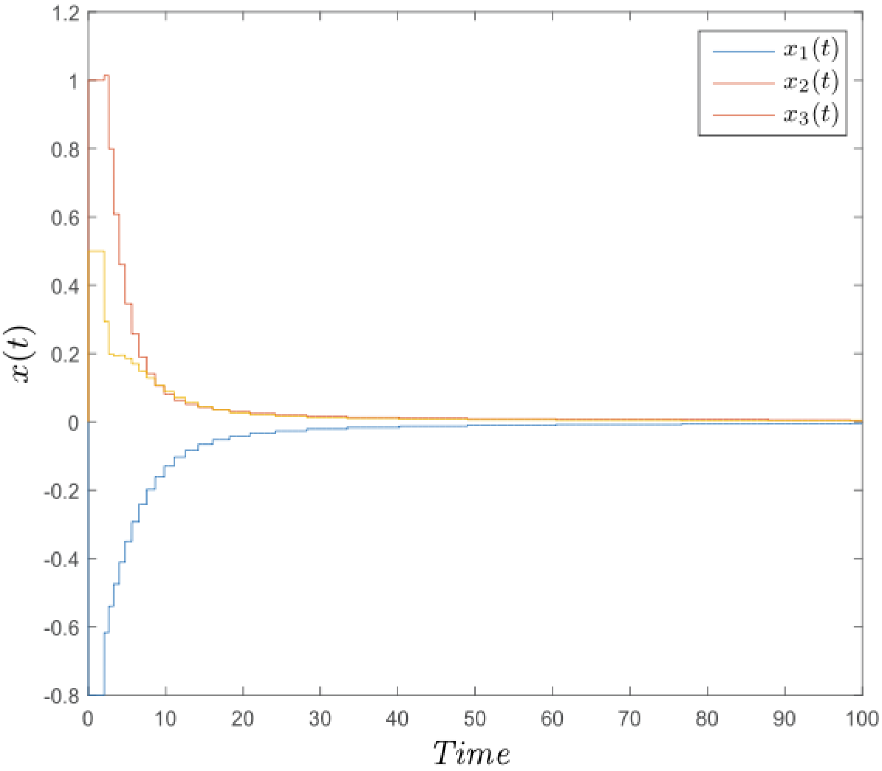

Figure 2
States response with EIC.

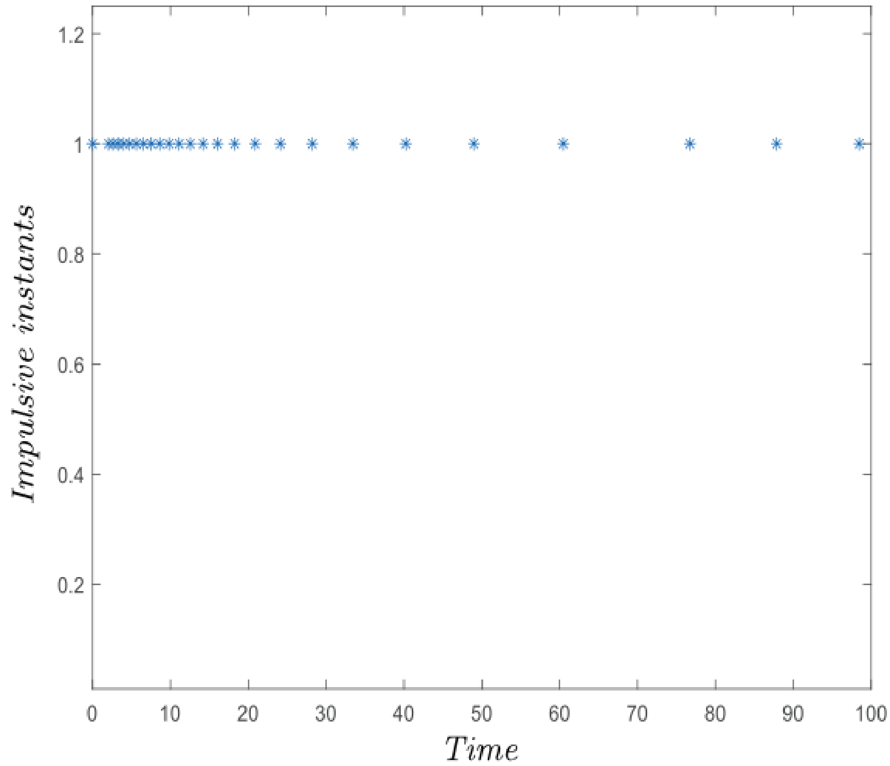

Figure 3.
Impulsive instants.

The simulation results with the same initial condition $x(0) = [-0.8, 1, 0.5]^T$. as no impulsive control is presented in Fig. 2, which shows the fractional-order system (11) is stable with the event-triggered impulsive control (EIC).

The impulsive instants are depicted in Fig. 3, which implies the Zeno-behavior is excluded.

Example 2. Synchronization of fractional-order jerk chaotic systems.

$$D^{\alpha}x(t) = Ax(t) + f\left(x(t)\right), \tag{13}$$

where $\alpha = 0.99$, $f(x) = [0, 0, |x_1| + 1]^T$, and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -0.95 & -0.6 \end{bmatrix}.$$

Construct the slave system as follows:

$$D^{\alpha}y(t) = Ay(t) + f\left(y(t)\right) - u(t), \tag{14}$$

where $u(t)$ is the control input and will be designed later.

Denote $\delta(t) = (x_1(t) - y_1(t), x_2(t) - y_2(t), x_3(t) - y_3(t))^{\mathrm{T}}$ as the synchronization error. Then by master system (13) and slave system (14) we have

$$D^{\alpha}\delta(t) = A\delta(t) + f\big(x(t)\big) - f\big(y(t)\big) + u(t). \tag{15}$$

In order to avoid continuous update of the controller, the following event-triggered impulsive controller will be used:

$$\begin{aligned} u(t) &= \delta(t_k) = x(t_k) - y(t_k) && \textbf{for } t \in [t_k, t_{k+1}), \\ \delta(t) &= C\delta(t^-) && \textbf{for } t = t_k, \end{aligned}$$

where the impulsive instants tk are defined by (3). $x(t_k)$ and $e(t)$ in (4) are replaced by $\delta(t_k)$ and $\delta(t_k) - \delta(t)$, respectively. The control gain matrix C is given as follows:

$$C = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.35 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}.$$

Then one can choose that $l_1 = 2.5042, l_2 = 1, l_3 = 0.35$. Let $\beta = 0.2, T = 0.07$. By simple computation we have $\theta = 0.0354$. Thus,

$$\frac{\ln l_3}{T} + \frac{\ln E_{\alpha}[(l_1 + l_1 l_2 + \frac{\beta l_1 l_2}{1-\beta})T^{\alpha}]}{\theta} = -3.4885 < 0.$$

Therefore, by Theorem 2 the zero solution of system (15) is exponential stability, which implies that the exponential synchronization between the master system (13) and slave system (14) can be achieved.

Let initial conditions be $x_0 = [1.3, -2, 1.5]^{\mathrm{T}}$ and $y_0 = [-0.8, 0, 0.4]^{\mathrm{T}}$. The simulation results are depicted in Figs. 4–6, which show that not only the synchronization between the master and slave system with the event-triggered impulsive controller can be achieved, but also the Zeno-behavior of the impulsive instants is excluded.

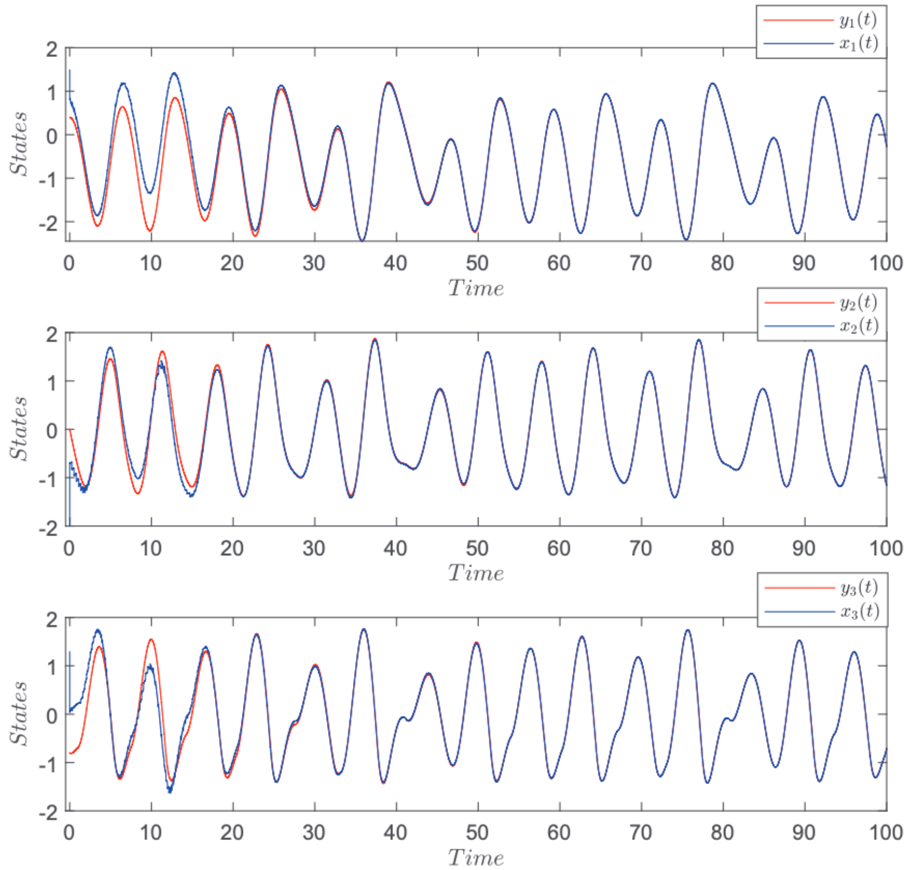

Figure 4.
States of master and slave systems.

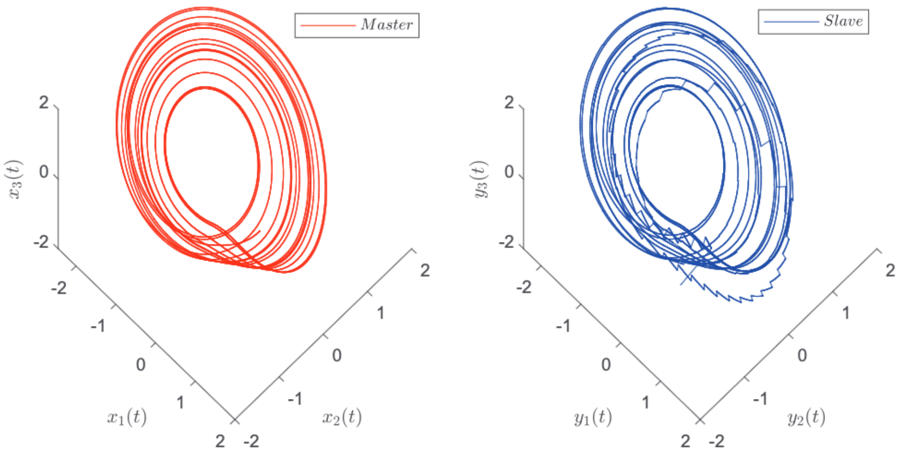

Figure 5.
Chaotic phenomenon of master system and slave system.

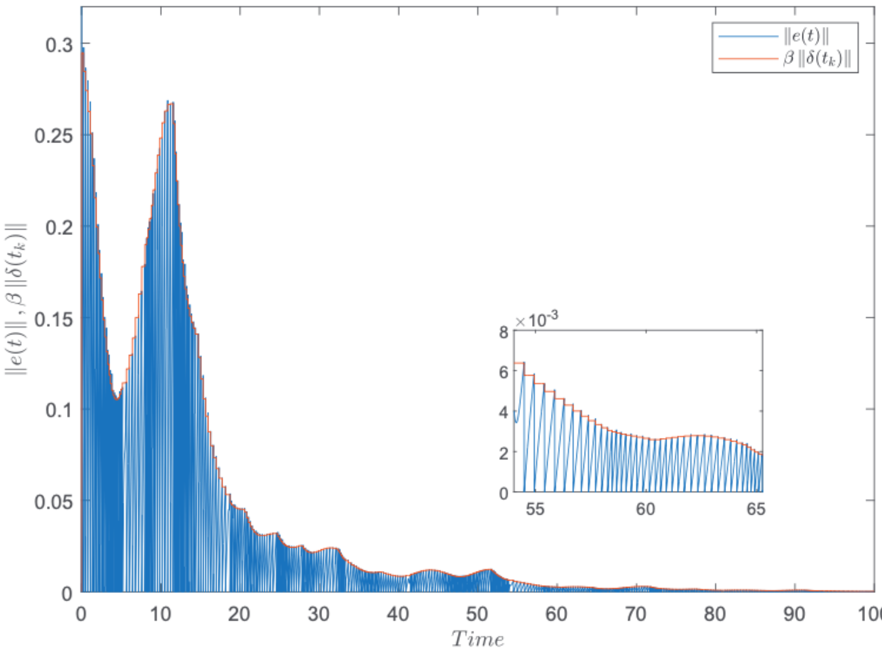

Figure 6.
ke(t)k and βkδ(tk)k.

5 Conclusions

Combining the advantages of impulsive control and event-triggered control, event-triggered impulsive stabilization of fractional-order continuous-time dynamic system is discussed. The impulsive instants depend on the states of the system and are not given in advance. Based on the stability theory and inequality technique, some sufficient conditions on exponential stabilization of nonlinear and linear fractional-order dynamic systems are presented, respectively. It proves that there is no Zeno-behavior for the impulsive instants determined by some designed events. As an application of the obtained theoretical results, the synchronization of fractional-order jerk chaotic systems is also presented in the simulation example. More general fractional-order dynamic systems and eventtriggered impulsive control problem of multi-agent systems will be considered in future study.

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