Tutorial Article

A Tutorial for Modeling Real Options Lattices from Project Cash Flows

Tutorial para a Modelagem de Malha Binomial de Opções Reais a partir do Fluxo de Caixa de Projetos

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ABSTRACT

Context: several methods for evaluating real options have been extensively studied and published. But recombining binomial trees, known as lattices, are perhaps one of the most practical and intuitive approaches to model uncertainty and price project managerial flexibilities for real options applications. Although the Cox, Ross, and Rubinstein (1979) lattice model is simple to implement for financial options, modeling real options lattices requires a different approach such as the one proposed by Copeland and Antikarov (2001), which considers project cash flows as dividends in the lattice model. **Objective:** in this tutorial, we propose a code in an open-source software with intuitive guidelines to help researchers and practitioners model real options lattices from project cash flows. **Method:** our code considers the correct project's volatility estimation, dividend yield modeling, and lattice building. Results: the results show how real options can affect the value of projects. **Conclusions:** as a contribution, this tutorial provides a simple mechanism for analyzing investment opportunities in projects that have uncertainty and flexibility.

Keywords: tutorial; real options; lattice model; project volatility; cash flow.

RESUMO

Contexto: diversos métodos para avaliação de opções reais foram extensivamente estudados e publicados. Mas as árvores binomiais recombinantes, conhecidas como treliças, talvez sejam uma das abordagens mais práticas e intuitivas para modelar incertezas e precificar flexibilidades gerenciais de projetos. Apesar de o modelo de árvore binomial de Cox, Ross e Rubinstein (1979) ser de simples implementação para opções financeiras, essa modelagem para opções reais requer uma abordagem diferente, como a proposta por Copeland e Antikarov (2001), que considera os fluxos de caixa do projeto como dividendos. **Objetivo:** neste tutorial, propomos um código em software aberto com diretrizes intuitivas para ajudar pesquisadores e profissionais a modelar malhas binomiais de opções reais a partir de fluxos de caixa de projetos. **Método:** nosso código considera a estimativa correta da volatilidade do projeto, a modelagem do rendimento de dividendos e a construção da treliça. Resultados: os resultados mostram como as opções reais podem afetar o valor de projetos. **Conclusões:** como contribuição, este tutorial fornece um mecanismo simples para analisar oportunidades de investimento em projetos que possuem incerteza e flexibilidade.

Palavras-chave: tutorial; opções reais; árvore binomial; volatilidade de projetos; fluxo de caixa.

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INTRODUCTION

Real options approach (ROA) was developed to overcome the limitations of the discounted cash flow method by using option-pricing methods to capture the value of any managerial flexibility that may be embedded in a project subject to future uncertainty.

It is a well-established principle in Finance and Economics that the correct measure of an asset are the future cash flows it generates, discounted to the present time at an appropriate risk-adjusted rate. Nevertheless, in the case of real assets, this principle does not account for the uncertainty over the future behavior of the cash flows nor for the value generated by the flexibility some projects have to react to future events. In order to value project cash flows which can flexibly change their trajectory, as future uncertainties are resolved, it is necessary to use a more adequate approach. Given that the flexibility to adapt to changes in expected future cash flows has option-like characteristics, they can only be valued with option-pricing methods.

The real options approach fulfills these conditions and has been widely discussed in the academic and practitioner literature. Although several option-pricing methods are well known and widely used, probably the most intuitive and flexible method is the binomial lattice option-pricing model originally developed by Cox, Ross and Rubinstein (1979) for financial options (CRR model) and then extended by Copeland and Antikarov (2001) for the pricing of real options.

However, even though the CRR binomial lattice model is widely used in the real options literature (Ashuri, Kashani, Molenaar, Lee, & Lu, 2012; Bastian-Pinto, Brandão, & Hahn, 2009; Brandão & Dyer, 2005; Jang, Lee, & Oh, 2013; Kim, Park, & Kim, 2017; Lee & Shih, 2010; Lin & Wesseh, 2013), there are many obstacles and difficulties in its implementation for real options valuation, as its principles are sometimes not very well understood by practitioners. In this tutorial, we guide the user through the sequence of steps necessary for the correct implementation of a real options model based on project cash flow estimation.

We begin by explaining the simplest CRR binomial model and its parameters, showing a direct application with the corresponding R code for a simple application of option valuation. Then we explain the issues and characteristics of a project cash flow model and show how to implement the CRR approach in such a model. We do that by demonstrating the procedure required to correctly estimate the volatility of the project value (V) from the project uncertainties. Next, we show how to incorporate the project cash flows into the CRR lattice using the cash flow dividend rate, as suggested by Copeland and Antikarov (2001), to create the value lattice of the project. Following that, we show how to model the project flexibilities, or real options, as decision nodes in the project value lattice, and determine the expanded value of the project with a backward maximization framework. Finally, we develop an additional R language code specifically to run this model and provide a detailed step-by-step tutorial on parameter determination and use of the model, showing how the code can be customized for particular applications.

VALUING DERIVATIVES WITH THE CRR LATTICE MODEL

Financial options are classified as derivative securities since their value derives in part from the price of another marketed financial asset, known as the underlying asset.

The Cox, Ross and Rubinstein (1979) binomial tree, or lattice model, emulates the Black and Sholes (1973) (B&S) option valuation approach. One advantage of this model is that it allows for the pricing of American type options, which can be exercised at any time before expiration, which is not possible under the B&S model. Although it is a discrete approach, it is accurate enough for most real asset valuation applications.

To implement the CRR model on a derivative of an asset whose current price is S_0, and has a volatility of σ, at each time step the asset value (S) is multiplied by a random variable that can take two values, u or d. For this representation to emulate a lognormal distribution, the values for u and d and the risk neutral probability p must be as shown in equations (1) where σ is the asset volatility and r is the risk-free discount rate.

$$u = e^{\sigma\sqrt{\Delta t}}, d = \frac{1}{u}, and \; p = \frac{(1+r)^{\Delta t} - d}{u - d} \qquad (1)$$

where u and d are respectively the up and down multipliers of the lattice nodes, and p is the risk-neutral probability, which will be used in discounting the lattice nodes.

With these parameters, we can implement the lattice shown in Figure 1, which is a discretization of a geometric Brownian motion (GBM) diffusion process for S. In the last period n, which is the time of expiration of the option or last period of the lattice, the options, which are a maximization process, are exercised on the values of Sn at each node. At the nodes where the options are exercised, the values of Sn change to S'_n. These can be call options or put options. After this step, we move to the previous period ($n - 1$) and perform the same maximization process at each node, but now also considering the value of continuation, which takes in account the present value of the expected future nodes discounted at the risk-free rate and weighted by the probabilities p and $(1 - p)$.

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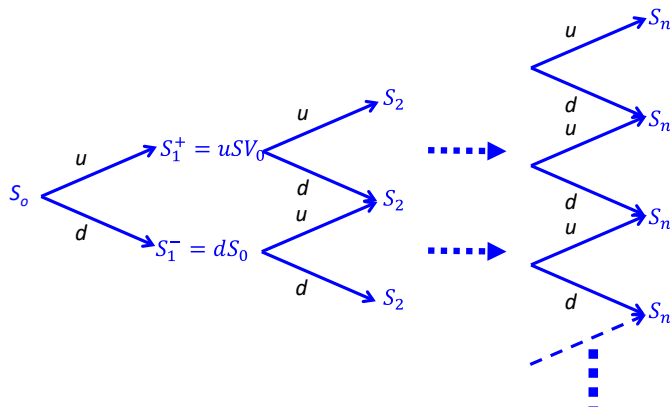
Figure 1. The CRR binomial lattice model.

Source: Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics, 7*(3), 229-263. https://doi.org/10.1016/0304-405X(79)90015-1.

This is a discretization of a geometric Brownian motion (GBM) diffusion process for price S using the volatility σ of this price through the multipliers u and d from equation (1). The recombining feature of the lattice is important since at the final step n, it will have $n + 1$ nodes, instead of 2^n in the case of a non-recombining tree.

This process of backward maximization is summarized by equation (2).

$$maximum\left[S'_{t-1}; \left(S'^+_t p + S'^-_t(1 - p)\right)/(1 + r)\right] \quad (2)$$

where S_t is the asset value at time t, before the exercise of any option, and of S'_t after the exercise of an option.

The whole process is shown in Figure 2, where at the initial step, the bold red arrow indicates the value of the options exercised along the whole lattice.
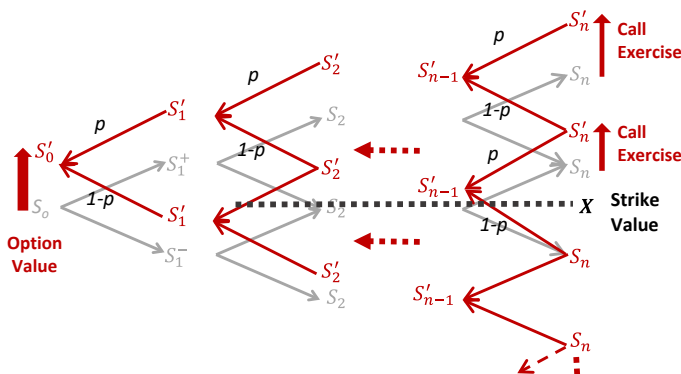
Figure 2. Diagram showing backward discounting of the CRR lattice for a call option.

This figure shows that the call option is exercised above the strike value X. Starting from the last period n after exercising the options, we move backward at each node, weighting the future values by the risk neutral probabilities p and $(1 - p)$ and discounting with the risk-free rate r using equation (2), up to the initial step.

In order to help researchers and practitioners understand how this CRR binomial model works for the

calculation of financial options, we provide a simple code with the open software R (www.r-project.org, retrieved in March 20, 2020), which uses the available fOptions package developed by Wuertz, Setz and Chalabi (2017). This code, available online, evaluates call and put options considering the following parameters: n = 1.5 years (18 months); S_0 = $100; σ = 30% per year; r = 6% per year; call strike price of $120; and put strike price of $90. After running this code, considering dt = 2 months, we find that the call value is $11.00 and the put value is $6.71.

Note that the fOptions package is very well suited for this type of calculation and for plotting the binomial lattices. However, it does not allow us to determine the value of the real options, whose main characteristics are having investment projects as underlying assets and more complex exercise rules. Given this, we develop in this article a specific R code for modeling real options applications based on project cash flows. This code is described in section "R language model". Other packages such as the DerivaGem software that accompanies Hull (1997) publication and that can be found online free of charge are also a good solution for calculating options with CRR lattices.

FROM CASH FLOWS TO A REAL OPTIONS LATTICE

While the same principles of financial options apply to real options, in this case the underlying asset is an investment project, which as a real asset is not traded in the market and thus does not have its market price determined. Additionally, exercise rules for real options are typically significantly more complex than those for financial options, as they may involve multiple exercise opportunities, combinations of simultaneous distinct option types, and multiple uncertainties.

Market asset disclaimer — MAD

Given that the underlying real asset, such as an investment project, is not traded in the market, it is impossible to determine its true value and risk-return characteristics. A simple solution to this problem is to assume that the underlying asset is the project itself and that the true market value of the project is the present value (V_0) of the project. This assumption implies that the traditional present value of the project's cash flows without flexibility is the best non-biased estimator of the market value of the project if it were a traded asset. Copeland and Antikarov (2001) refer to this hypothesis as the market asset disclaimer (MAD).

The first step in the model is to determine the cash flow structure F_t of the project, as shown in equation (3):

$$F_t = [R_t(1 - \gamma) - \lambda_t - \Gamma](1 - \pi) + \lambda_t \qquad (3)$$

where R_t is the total revenue in year t; γ represents variable costs; π is the income tax; λ_t is the depreciation in year t; and Γ represents fixed costs. These are projected for a number of years (n), after which we consider a continuation value (CV). The project value at time $t = 0$, V_0, can be determined with equation (4).

$$V_0 = \sum_{t=1}^{t=n} \frac{F_t}{(1 + \mu)^t} + \frac{CV_n}{(1 + \mu)^n} \qquad (4)$$

where:

$$CV_n = \frac{F_n(1 + g)}{(\mu - g)} \qquad (5)$$

μ is the risk-adjusted discount rate of the project and g is the cash flow perpetuity growth rate.

The purpose is to model the project's value V, with the CRR binomial approach, which allows the real options to be exercised by retroactive induction, or backward maximization, maximizing V along the binomial nodes. When the starting point of the lattice is reached, we will have a V increased by the optimal exercise of real options (RO), which we call expanded present value ($V0^*$), that is: $V_0^* = V_0 + RO$.

Estimating the project's volatility from the income variables

To model CRR's lattice for a given project, we must estimate the volatility σ_V of the project value V. We assume that the revenue R_t is the product of a price and a quantity, where Q is deterministic but the price $\tilde{P}$ is stochastic (indicated by the ~ sign above the variable) with a growth rate of a and volatility σ_p. The project revenues are described by equation (6).

$$\tilde{R}_t = Q\tilde{P}_t \qquad (6)$$

The price $\tilde{P}$ is assumed to follow a geometric Brownian motion (GBM) type of stochastic diffusion process, which can be represented by the differential equation: $dP = \alpha P dt + \sigma_P P dz$, where $dz = \varepsilon\sqrt{dt}, \varepsilon \sim N(0,1)$ is the standard Weiner process.

To estimate the two parameters necessary to model this GBM (a and σ_p), given a historical time series of

prices P_t, with n events, these can be calibrated using the following procedure: first, calculate the log return series of the ($n - 1$) events of the price series with $Ln(P_t/P_{t-1})$. The growth rate, or drift, a can be estimated by calculating the mean of this log return series, and the volatility parameter σ_p by the standard deviation of the same series. These must be in the same time increment, which for project cash flows is in years. If the time series of P_t is provided in a different time interval such as monthly, then the values of a and σ_p must be converted to yearly values. This is done by multiplying the drift parameter by 12 (12 months per year) and the volatility parameter by $\sqrt{12}$. The values thus obtained can now be used in modeling the GBM for the prices in the process described in equations (8), (9), and (10).

For such a stochastic process, the expected value equation is represented by (7).

$$P_t = P_o e^{\alpha(t - t_0)} \qquad (7)$$

The simulation equation is shown in (8):

$$\tilde{P}_t = P_{t-1} e^{[(\alpha - \sigma_P^2/2)\Delta t + \sigma_P N(0;1)]} \qquad (8)$$

where $N(0;1)$ is the normal distribution with mean 0 and standard deviation 1.

Using the proof of Samuelson (1965) according to whom the return rate of a financial asset will follow a random walk, independently of its future cash flows as long as investors have access to all the asset's information, we assume that V will also follow a GBM process. Therefore, future cash flows dependent on multiple uncertainties, even with autoregressive processes, can be combined into a single multiplicative binomial lattice.

To estimate the volatility of V, we use the approach suggested by Copeland and Antikarov (2001), but with the correction made by Brandão, Dyer and Hahn (2012). After estimating the stream of n cash flows with equation (3), and the deterministic initial project value $\bar{V}_0$, we calculate the project value in $t = 1$, with equation (9).

$$\tilde{V}_1 = \sum_{t=1}^{t=n} \frac{\tilde{F}_t}{(1 + \mu)^{t-1}} + \frac{CV_n}{(1 + \mu)^n} \qquad (9)$$

As $\tilde{P}$ follows a GBM stochastic process, $\tilde{F}$ and $\tilde{V}$ will also be the result of this GBM diffusion process. We define the variable $\tilde{Z}$ with equation (10):

$$\tilde{Z} = Ln\left(\frac{\tilde{V}_1}{\bar{V}_0}\right) \qquad (10)$$

Running a Monte Carlo simulation (MCS), we use the standard deviation of the variable $\tilde{Z}$ as the volatility of the stochastic project (σ_V) value $\tilde{V}$. Note that $\bar{V}_0$ is a static value, while $\tilde{V}_1$ is stochastic and will have a new value with every simulated trajectory of $\tilde{F}$. The modification of Brandão et al. (2012) to this procedure points that for the MCS on equation (9), it is necessary to attribute a stochastic value from equation (8), only in the first time period $t = 1$ of the simulation, with the subsequent increments calculated by the expected value of P from equation (7). Otherwise, the yearly volatility estimate will be overestimated and will monotonically increase with the number of cash flow periods of the project.

## CCR lattice applied to real options from cash flow projection

The lattice structure shown in section "Valuing Derivatives with the CCR Lattice Model" prices options on assets that do not pay dividends or cash flows. In the case of assets such as stocks, ongoing projects, and firms that generate a continuous stream of cash flows to the shareholders, some adjustments must be made.

After estimating the cash flows stream of the project in question, and using equation (9), it is straightforward to calculate the stream of V_t values for $t = 1$ to n, before the subtraction of cash flow at this period t, and we call it $V_{\text{ex ante}}$. Consequently, after subtraction of the cash flows F_t we will have the stream of $V_{\text{ex post}}$, or the values of the project ex-post cash flows, or dividends. From these, we estimate the stream or vector of dividend yield δ_t from $t = 1$ to n, as defined in equation (11).

$$\delta_t = F_t / V_{t\,\text{ex ante}} \tag{11}$$

Using this vector of dividend yield, Copeland and Antikarov (2001) proposed a scheme that builds a project value lattice that incorporates the dividends, or cash flows paid out at each step of the process. At each time step t, the values at each node ex ante subtracting of cash flows (V_a) are multiplied by $(1 - \delta_i)$, yielding V_p, or the ex-post project values: $V_p = V_a(1 - \delta_t)$. This model is shown in Figure 3, where the red arrows represent the cash flows subtracted from the project value V_a at each time step. The value of the dividends (D) is calculated by equation (12) at every node of the ex-ante values lattice V_a.

$$D_t = V_{at}\delta_t \tag{12}$$
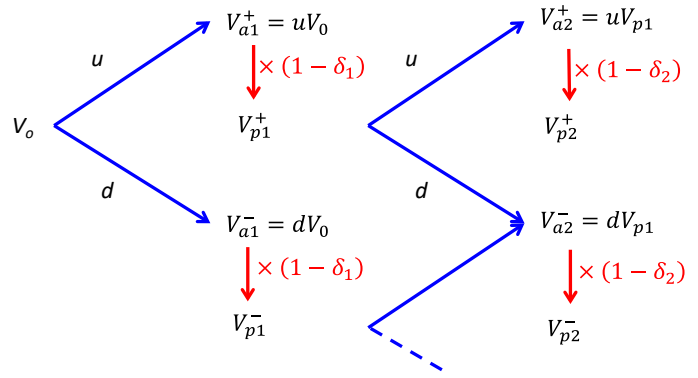
Figure 3. CCR lattice with dividend payment at each node.
This figure presents the values of V ex ante (V_a) and ex post (V_p). The lattice thus 'penalized' by the dividend yields δ_t still is recombining as the one of Figure 1.

As this scheme will produce, at every node, two values (ex ante and ex post), this will yield two lattices (one for values of V_a and another for V_p), which are interdependent. As this can encumber the lattice model, we will simplify the above scheme with the model displayed in Figure 4, where the V_a lattice is bypassed and only the V_p lattice remains, using at each time step the dividend yield δ_t stream, or vector, of the cash flow model to do this.
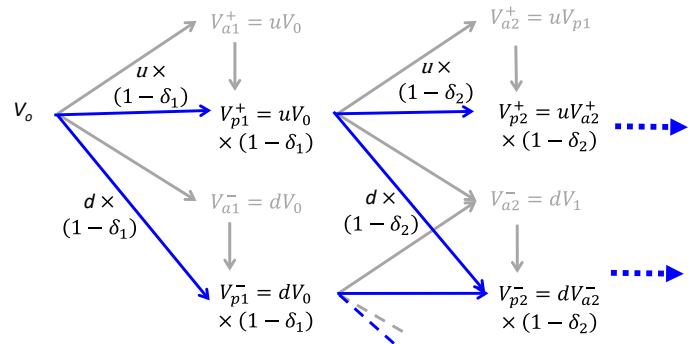
Figure 4. CRR lattice for ex-post values of V_p.
This lattice, simpler and more straightforward than that of Figure 3, provides all the necessary values for the real options estimation (V_p). Note that this lattice remains recombining.

In Figure 4, the V_a values of Figure 3 appear only in light grey and the values of V_p are apparent. As stated, our tutorial model uses this approach as it has fewer steps to implement. Yet it brings another complication when discounting the lattice backward from the last step of the model.

At this last step, the real options are exercised on the values of Vpn, at each node (maximization process) on the

ex-post values, that is, after the cash flow (dividends) have been paid. At the nodes where the options are exercised, the values of Vpn change to V'_{pn}. As with the normal CCR lattice (without dividends), we move to the previous time period $(n-1)$, and perform the same maximization process at each node, also considering the value of continuation, as also done with the simple CRR lattice in chapter 2. But this value, previously calculated with equation (10), now uses the ex-ante values V'_{an} of the last step n, which is equal to the dividends D_n added to V'_{pn}. And as these dividends are the ones already paid by the project, they are the values of V'_{an} multiplied by δ_n. But as the model of Figure 4 has bypassed the estimation of V_a lattice, we must use another approach to calculate Dt. We do this using the following algebra: $D_t = V_{at} - V_{pt} = V_{pt} \times \frac{\delta_t}{(1-\delta_t)}$. The vectors of values are then calculated with equation (13).

$$\delta'_t = \frac{\delta_t}{(1 - \delta_t)} \tag{13}$$

which multiplied to V_{pt} yields the dividend values needed to discount the values lattice. Instead of equation (10), the maximization process is the one shown in equation (14).

$$maximum\left[V'_{p(t-1)}; \left(\left[V'^{+}_{pt} + V^{+}_{pt} \times \delta'_t\right]p + \left[V'^{-}_{pt} + V^{-}_{pt} \times \delta'_t\right](1-p)\right)/(1+r)\right] \tag{14}$$

This discounted process is displayed in Figure 5.
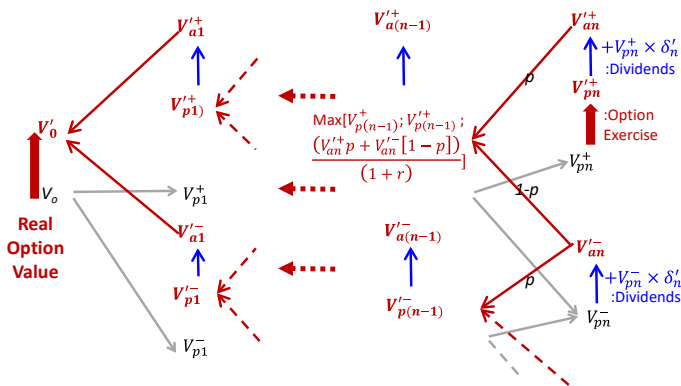
Figure 5. Backward discounting of the lattice for ex-post values of V_p.

Now, as we don't have the values of the ex-ante lattice, we must estimate these by calculating the vector δ'_t. As shown in Figure 2, starting from the last period n after exercising the options, we move backward at each node adding the value of dividends estimated by δ'_n, weighting the future values by the risk-neutral probabilities p and $(1-p)$ and discounting with the risk free rate r, with equation (14) up to the initial step.

The full understanding of the particulars of the cash flow lattice model shown in this section is relevant for real options researchers and practitioners, as this model is widely used in many real options applications. Several authors have applied this model to infrastructure projects (Garvin & Cheah, 2004; Iyer & Sagheer, 2011; Marques, Brandão, & Gomes, 2019; Oliveira, Couto, & Pimentel, 2020; Rakić & Rađenović, 2014), renewable energy (Dalbem, Brandão, & Gomes, 2014; Santos, Soares, Mendes, & Ferreira, 2014; Wesseh & Lin, 2015; Zhang, Zhou, & Zhou, 2014), mining (Miranda, Brandão, & Lazo, 2017), and other fields of research.

R LANGUAGE MODEL

Guidelines for the model routine and numerical example

In order to assist researchers and practitioners model project cash flows under the real options approach, considering the correct volatility estimation and lattice implementation, we propose an intuitive R code. In this code, we exemplify the model described in section "From Cash Flows to a Real Options Lattice" through a numerical example.

In this sense, we assume a hypothetical project requiring an initial capital expenditure of $1.5 million, which will depreciate in 10 years, which also is the projection horizon of the project (n). In addition, it will have a fixed output (Q) of 10,000 units that will sell at a price (P) assumed today of 100 $/unit, but which is expected to grow at a rate a per year: from $t = 1$ to $t = 10$.

As we assume that price (P) is the main source of uncertainty in this project, we will treat it as a stochastic variable that follows a GBM, as mentioned in section "From Cash Flows to a Real Options Lattice". Thus, the code first inputs the parameters shown in Table 1 to simulate the price using the Monte Carlo simulation technique. Note that users can change any of these parameters to adapt the code to their project.

Table 1. Price simulation parameters.

Inputs		
Depreciation duration time (n)	n	10 years
Number of time intervals	i	10
Price growth rate (a)	a	3% (per year)
Price volatility (σ_p)	vol	15% (per year)
Price at $t = 0$ (P_0)	P_0	$100
Number of simulations	nt	10,000

Note. This table presents all the necessary parameters for price simulation.

This simulation results in a matrix (X) of dimension nt x 11, where the first column represents the prices at $t = 0$ ($P_0 = 100$) and the other columns the prices simulated at each time point until the tenth year of the project.

After simulating price, the code estimates the project's revenue using equation (6) and assuming a production (Q) of 10,000 units. Then, through equation (3), the code starts the process of calculating the project's cash flows (F_i), considering fixed costs (Γ) of $300,000 per year, variable costs (γ) of 55% of the project's income (R), an income tax rate (π) of 34% of EBIT (earnings before interest and taxes), and an annual investment to maintain the project (EI) of $50,000. Table 2 summarizes all of these parameters that need to be entered in the code to calculate the cash flows (F_t) and the project value (V_0). As well as the parameters mentioned in Table 1, these can also be changed in the code so that the user can adapt it to the characteristics of their project.

Table 2. Cash flow and project value parameters.

Inputs		
Risk-free rate (r)	r	6% (per year)
Perpetuity growth rate (g)	g	3% (per year)
Discount rate (μ)	k	12% (per year)
Production (Q)	prod	10,000 units
Variable costs (γ)	VC	55% of revenues
Fixed costs (Γ)	FC	$300,000
Investment	I	$1,500,000
Extra investments	EI	$50,000
Income tax (π)	IT	34% (per year)

Note. This table shows all the parameters required to estimate the cash flows and the project value.

As the simulated prices are arranged in a matrix, we need to calculate the cash flow for each year and each price trajectory. In this step, we exclude the first column of the price matrix, because at $t = 0$ the project does not generate cash flow. In this way, we will have a cash flow matrix (FCF) of dimensions nt x 10. In addition, since we consider that this project has a continuation value (CV), we have included a column in this matrix that represents the perpetual cash flows of that project. To calculate perpetuity, the code uses equation (5).

Considering a risk-adjusted discount rate of k = 12% per year, a perpetuity growth rate of g = 3% per year, equation (4), and the NPV equation of the package developed by Signorell, Aho, Alfons, Anderegg, Aragon and Arppe (2016), we find that the project value

is V_0 = $1,661, yielding a net present value (NPV) of $161,549. From this, we can also estimate the dividend yield and, consequently, the present values ex ante and ex post, which allows us to find the variable $\tilde{Z}$, as well as the project volatility (σ_V).

Thus, using Monte Carlo simulation as described in section "Estimating the project's volatility from the income variables", the project volatility is estimated as σ_V = 33% per year. This value can vary slightly as it is the result of a MCS. Given the volatility of the project, we can determine the parameters of the binomial cash flow tree (u = 1.39, d = 0.72, and p = 0.51). With this, the code calculates both the value lattices, using the approach of section "CCR lattice applied to real options from cash flow projection". The value lattices are displayed in Appendix I with both the ex-ante and ex-post values (Figures A1 and A2).

After this, we model two real options on the project lattice. First, an expansion option that is modeled as a call option on the value lattice. It considers that at any time for the next 10 years (t = 1 to 10) the project value can be augmented by 80% (multiplied by 1.8) at a cost of $1,200,000. Second, an abandonment option, modeled as a put option, that considers the project can be sold at a value of its total depreciated investment, minus a discount of 20%.

To incorporate these managerial flexibilities, we need to consider the inputs listed in Table 3. As with the other inputs, these can also be changed so that the code adapts to the project that each user is analyzing.

Table 3. Abandonment and expansion options parameters.

Inputs		
Abandonment factor	abandf	0.8 (per year)
Expansion factor	expf	1.8 (per year)
Expansion cost	expc	$1,200,000

Note. These are the parameters required to calculate the abandonment and expansion options values.

In addition to the inputs mentioned in Table 3, to determine the project value considering the options, we need to find the residual value of the project in case of abandonment. For this, first, we calculate the depreciated asset value each year until year 10 by discounting the depreciation amount from the investments. Then, we multiply these values by the abandonment factor.

Given these parameters, the code calculates the backward discounted lattices for both managerial flexibilities. In summary, following the model of Figure 5,

the code evaluates backward the maximum value between maintaining, abandoning, and expanding the project each year until year 10. Appendix I also presents these lattices (Figures A3 and A4). At the starting step of this lattice, the project value is now $2,109,671, compared to $1,661,448 for the project without options. This yields an incremental value of $448,223 derived from both expansion and abandonment options.

Finally, the code plots the ex-post dividends lattices with and without options so that we can have a better understanding of the results, as shown in Figure 6.

In addition, we also provide the command to plot the same lattices, but on a log scale. This allows better visualization of the results found, especially the exercise boundaries of the Expansion (Call) and Abandonment (Put) options, as shown in Figure 7.
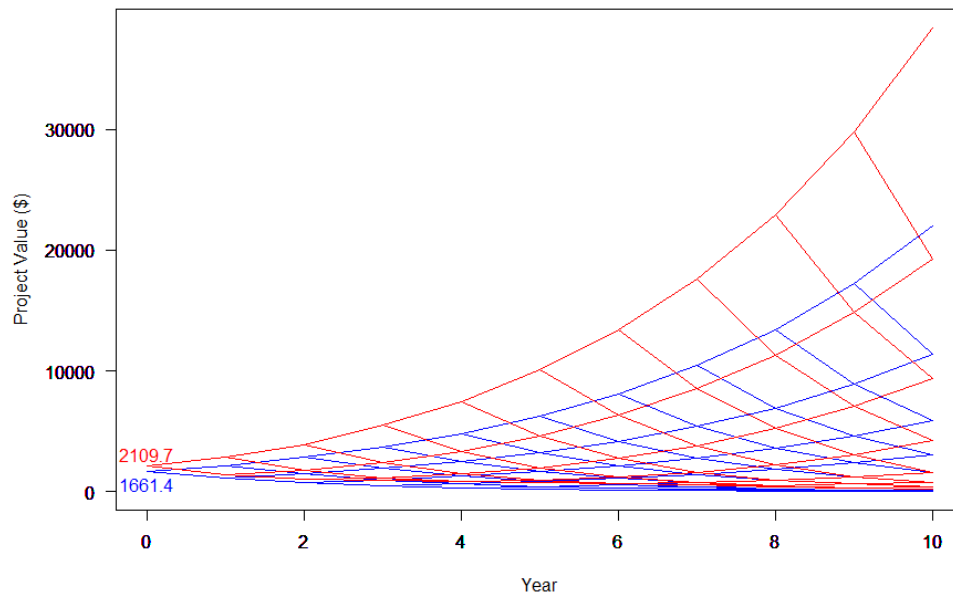
Figure 6. Ex-post lattices with and without abandonment and expansion options.
This figure shows the lattice ex-post dividends considering the abandonment and expansion options (*) and the lattice ex-post dividends without options (*). The values are in $ thousands.
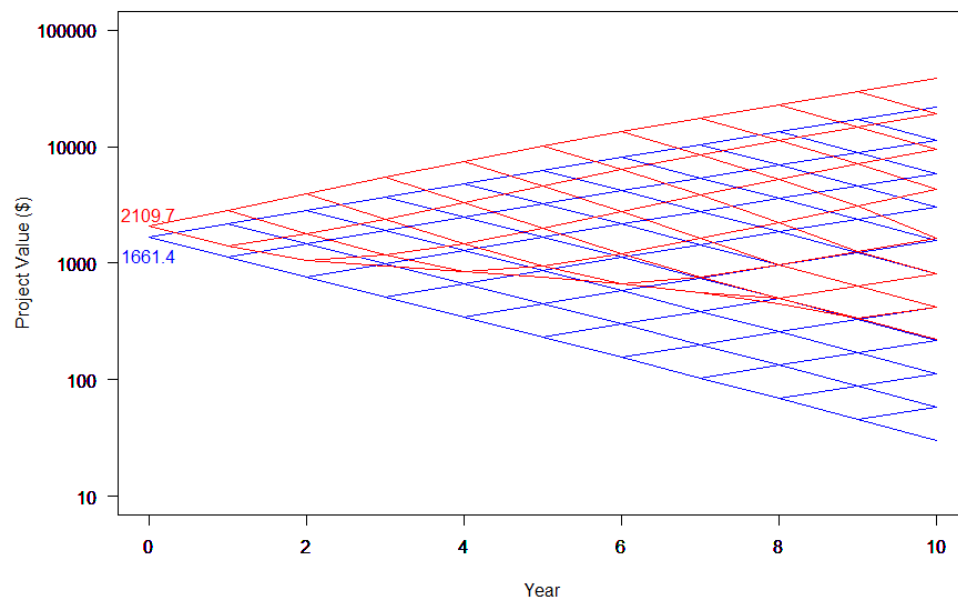
Figure 7. Ex-post lattices with and without abandonment and expansion options in log scale.
This figure shows, in log scale, the lattice ex-post dividends considering the abandonment and expansion options (*) and the lattice ex-post dividends without options (*). The values are in $ thousands.

Discussing the proposed code

The proposed code was built in a very intuitive and simple way so that even beginning researchers can use it in their analysis. However, we believe it is important to provide a general guideline for its correct use. Thus, we draw the reader's attention to a few points. First, the code only models annual cash flows, that is, it only allows the time interval to be equal to 1 ($dt = 1$). Another point is that only the values listed in Tables 1, 2, and 3 can be changed by the users to adjust them to the reality of the projects they are modeling. To stress this point, the following comment — "Parameters: Here, you can change the input values to suit your project" — was included in the appropriate places in the code. Finally, we emphasize that readers should not be concerned if they use the same input values as ours or find output values slightly different from those presented in our article. This variation is common, as the output values are the result of a Monte Carlo simulation.

CONCLUSIONS

This tutorial provides a guide on the sequence of steps required to implement a real options model, based on the estimation of project cash flows. It presents students, researchers, and practitioners the correct real options procedures to calculate the volatility of a project's value; incorporate the project's cash flows into the CRR binomial model using the cash flow dividend rate; and model the managerial flexibilities (real options) of the project. It uses the Copeland and Antikarov (2001) scheme, incorporating the dividends or cash flows paid out, into a CCR lattice modeling, and is adaptable enough to reproduce a great number of managerial flexibilities available to managers.

We believe this tutorial to be relevant for real options students, researchers, and practitioners, as it contributes to the understanding of project cash flow lattice modeling. It provides a simple and practical method for the pricing of real options that can assist decision-makers to analyze investment opportunities in projects where there is uncertainty and flexibility.

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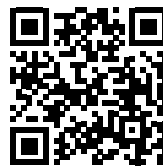
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## APPENDIX I — PROJECT LATTICES (EX ANTE & EX POST)

V Lattice with ex ante values
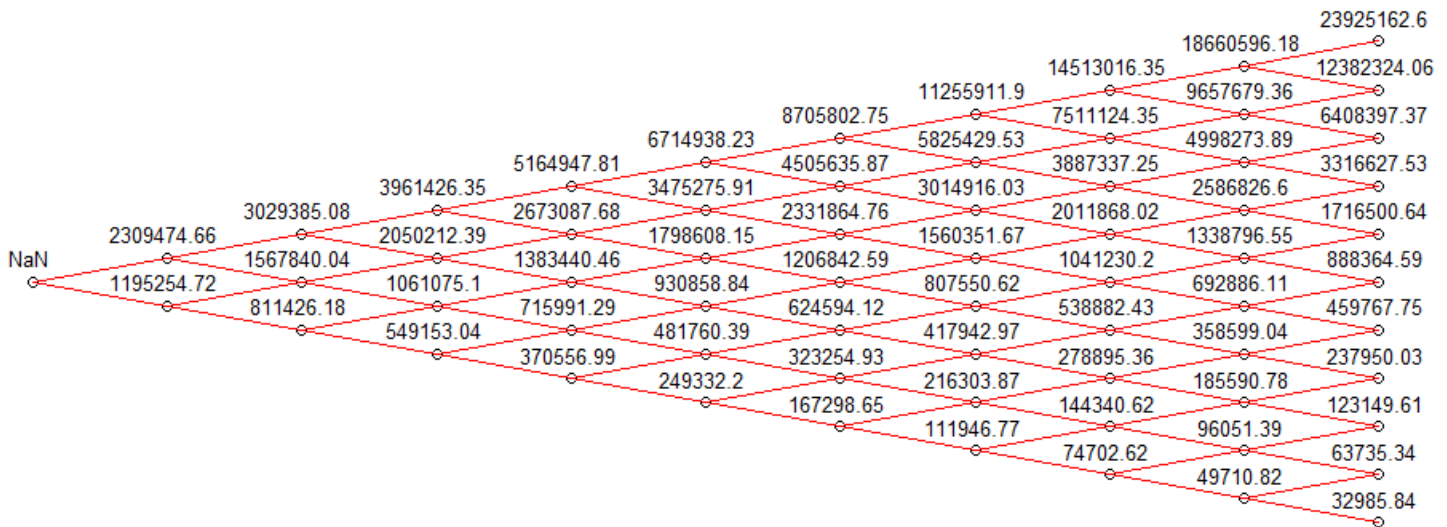
Figure A1. Ex-ante project value lattice.

This is one of the outputs of our R code (Code II) that shows the ex-ante project value lattice. To estimate this, the code uses the approach described in section "CCR lattice applied to real options from cash flow projection". Note that no managerial flexibility was considered in this calculation.

V Lattice with ex post values
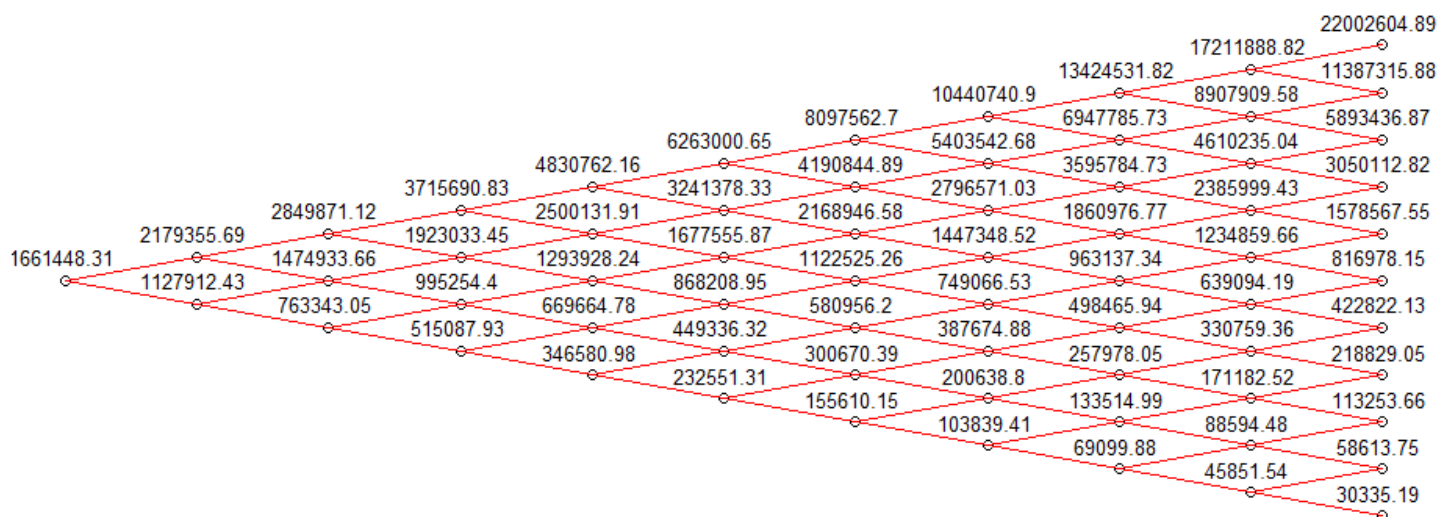
Figure A2. Ex-post project value lattice.

This is one of the outputs of our R code (Code II) that shows the ex-post project value lattice. To estimate this, the code uses the approach described in section "CCR lattice applied to real options from cash flow projection". Note that no managerial flexibility was considered in this calculation.
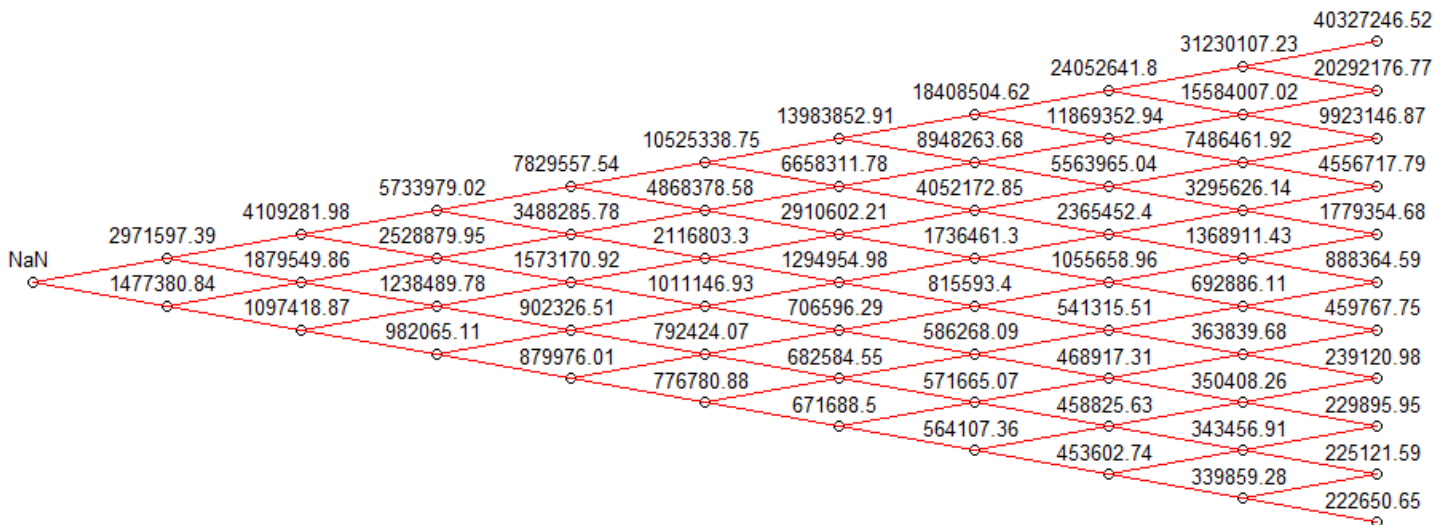
Expansion Option: 80%   expansion , at a cost of: 1,200,000 $

Abandonment option: project can be sold at a value of its depreciated investments, minus: 20%

	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7	t = 8	t = 9	t = 10
Abandonment value:	1,120,000	1,036,000	948,000	856,000	760,000	660,000	556,000	448,000	336,000	220,000

Figure A3. Ex-ante project value lattice with exercise of abandonment and expansion option.

This is one of the outputs of our R code (Code II) that shows the ex-ante project value lattice with exercise of abandonment and expansion options. To estimate this, the code evaluates backward the maximum value between maintaining, abandoning, and expanding the project each year until year 10.

Figure A4. Ex-post project value lattice with exercise of abandonment and expansion options.

This is one of the outputs of our R code (Code II) that shows the ex-post project value lattice with exercise of abandonment and expansion options. To estimate this, the code evaluates backward the maximum value between maintaining, abandoning, and expanding the project each year until year 10.

APPENDIX II — SUMMARY OF VARIABLES USED IN THE R CODE

Table A1. Variables used in the R code.

Description	Variable
Time interval	dt
Price simulation	X
Revenue	R
Operating revenue	RO
Depreciation (λt)	Dep
Earnings before interest and taxes	EBIT
Free cash flow (F_t)	FCF
Perpetuity (CV)	Perp
Present value (V_0)	PV
Net present value	NPV
Ex-ante present value	PVa
Ex-post present value	PVp
Average present value	PVd
Return ($\tilde{Z}$)	lRet
Project volatility (σ_V)	sig
Upside multiplying factor (u)	u
Downside multiplying factor (d)	d
Probability (p)	p
Dividend rate	divr
Dividends	div
Ex-ante lattice	Latta
Ex-post lattice	Lattp
Depreciated asset	Depasset
Residual value	Residual
Ex-ante lattice with options	Lattao
Ex-post lattice with options	Lattpo

Note. This table describes all the variables used in the proposed R code (Code II).