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SELF BALANCING SYSTEM FOR ROTATING MECHANISMS

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ABSTRACT

A self balancing system analysis is presented which utilizes freely moving balancing bodies (balls) rotating in unison with a rotor to be balanced. Using Lagrange´s Equation, we derive the non-linear equations of motion for an autonomous system with respect to the polar coordinate system. From the equations of motion for the autonomous system, the equilibrium positions and the linear variational equations are obtained by the perturbation method. Because of resistance to motion, eccentricity of race over which the balancing bodies are moving and the influence of external vibrations, it is impossible to attain a complete balance. Based on the variational equations, the dynamic stability of the system in the neighborhood of the equilibrium positions is investigated. The results of the stability analysis provide the design requirements for the self balancing system.

Keywords: Self balancing system, variational, Rayleigh disipation function.

RESUMEN

Se presenta el análisis de un sistema de autobalance el cual utiliza bolas libres de movimiento rotando con el rotor que será balanceado. Se usa la ecuación de Lagrange para derivar un sistema de ecuaciones no lineales para un sistema autónomo con respecto a un sistema de coordenadas polares. De las ecuaciones de movimiento, se obtienen ecuaciones linealizadas variacionalmente y posiciones de equilibrio por el método de perturbación. A causa de la resistencia al movimiento, la excentricidad y el movimiento de los cuerpos libres que son provocados por la influencia de vibraciones externas, hace imposible obtener un balanceo completo. Basado en el método variacional, se investiga el comportamiento dinámico del sistema en la frontera de la posición de equilibrio. Los resultados del análisis de estabilidad proveen los requerimientos de diseño para el sistema de autobalance.

Palabras clave: Sistema de autobalance, método variacional, función de disipación de Rayleigh.

INTRODUCTION

The rotation of unbalanced rotor produces vibration and introduces additional dynamic loads. Particular angular speeds encountered in presently built modern rotating machinery, impose rigorous requirements concerning the unbalance of rotating mechanisms. In the system, however, where the distribution of masses around the geometric axis of rotation varies during the operation of a machine or each time the machine is being restarted, the conventional balancing method becomes impracticable. Therefore, self balancing methods are practiced in such systems where the role of fixed balancing bodies is performed, either by a body of liquid or by a special arrangement of movable balancing bodies (balls or rollers) which are suitably guided for free movement in predetermined directions. In the case when a body of liquid is self balancing the attainable degree of balance does not exceed 50% of initial unbalance of rotating parts [1]. In fact, however, there are a lot of reasons rendering the attainment of such a high degree of balance practically impossible. Self balancing systems are used to reduce the imbalance in washing machines, machining tools and optical disk drives such as CD-ROM and DVD drives. In self balancing systems, the basic research was initiated by Thearle [2], [3], Alexander [4] and Cade [5]. Analysis for various self balancing systems can be encountered in references [6-9]. Equations obtained are for non-autonomous systems, these equations have limitations on complete stability analysis.

Chung and Ro [9] studied the stability and dynamic behavior of an ABB for the Jeffcott rotor. They derived the equations of motion for an autonomous system by using the polar coordinates instead of the rectangular

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coordinates. Hwang and Chung [10] applied this approach to the analysis of an ABB with double races. Chung and Hang [11] studied ABB for a rotor with a flexible shaft. In that case they adopted Stodola-Green rotor instead of the Jeffcott model. In this study, authors got a similar analysis for a flexible shaft and two self balancing systems on the ends. Describing the rotor centre with polar coordinates, the non-linear equations of motion for an autonomous system are derived from Lagrange´s equation. After a balanced equilibrium position and linearized equations in the neighborhood of the equilibrium position are obtained by the perturbation method and theoretically it shows that after critical speed rotor can be balanced. The system has a small lubrication on its balls and they are collocated themselves by inertial motion upper first natural frequency.

EQUATIONS OF MOTION

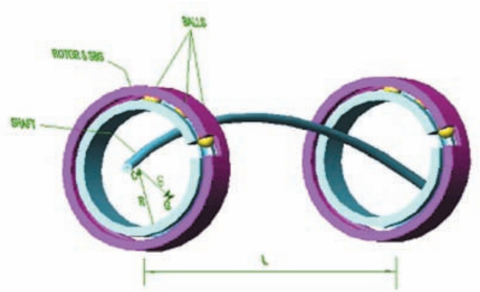
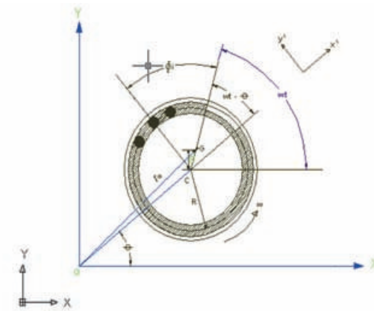


Fig. 1 Self balancing system on the ends of rotor.

The rotor with double self balancing system is shown in Fig. 1, where the shaft is supporting two self balancing systems on the ends. It is assumed that the shaft mass is negligible compared to the rotor mass. The XYZ coordinate system is a space-fixed inertia reference frame end the points C and G of both rotors are centroid and mass centre respectively.

Point O may be regarded as projection of the centroid C onto the axis O´Z. The ball balancer consists of a circular rotor with a groove containing balls and a damping fluid. The balls move freely in the groove and the rotor spins with angular velocity ω. It is assumed that deflection of the shaft is small so may be assumed that he center C moves in the XY-plane. As shown in Fig. 2, the centroid C may be defined by the polar coordinates r and θ. The mass centre can be defined by eccentricity ε and angle ωt, for the given position of the centroid and the angular position of the ball B_i is given by the pitch radius R and the angle ϕ_i.

a)



b)

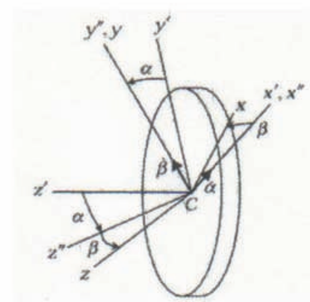


Fig. 2 Schematic representation of self balancing system, a) Front view, b) Euler angles.

Describing the rigid body rotations of the rotor with respect to the X and Y-axis, Euler angles are used, which give the orientation to the rotor-fixed xyz-coordinate system relative to the space-fixed XYZ-coordinate system. In this case, the Euler angles of ωt, α and β are used as shown in Fig. 2. A rotation through an angle ωt about the Z-axis results in the primed system. Similarly rotation α about x´-axis and a rotation β about y´´-axis results double primed and xyz-coordinate systems respectively. In matrix form:

$$x = \Gamma_\beta x'' \qquad x'' = \Gamma_\alpha x' \qquad x' = \Gamma_\omega X \tag{1}$$

and rotation matrices:

$$\Gamma_\beta = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$
$$\Gamma_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \tag{2}$$
$$\Gamma_\omega = \begin{bmatrix} \cos\omega t & \sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} X &= X\hat{I} + Y\hat{J} + Z\hat{K} \\ x' &= x'i' + y'j' + z'k' \\ x'' &= x''i'' + y''j'' + z''k'' \\ x &= xi + yj + zk \end{aligned} \tag{3}$$

in which all components are unit vectors along associated directions respectively.

First step is considering the kinetic energy of the rotor with the self balancing system. The position vector of the mass centre G can be expressed using the rotation matrices:

$$\Gamma_\beta = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$
$$\Gamma_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \tag{4}$$
$$\Gamma_\omega = \begin{bmatrix} \cos\omega t & \sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$r_G = \Gamma_\beta \Gamma_\alpha \Gamma_\omega r_{OC/XYZ} + r_{CG} \tag{5}$$

where

$$r_{OC/XYZ} = r(\cos\theta \hat{I} + \sin\theta \hat{J}) \qquad r_{CG} = \varepsilon i' \tag{6}$$

Using a common generalized coordinate ψ defined by

$$\psi = \omega t - \theta \tag{7}$$

After matrix product the position vector of the mass centre, r_G:

$$r_G = [r\cos\beta\cos\psi - r\sin\alpha\sin\beta\sin\psi]i - r\cos\alpha\sin\psi\, j + [r\sin\beta\cos\psi + r\sin\alpha\cos\beta\sin\psi]k \tag{8}$$

And the position vector of the Ball:

$$r_{Bi} = [r\cos\beta\cos\psi - r\sin\alpha\sin\beta\sin\psi + R\cos\phi_i]i - [r\cos\alpha\sin\psi + R\sin\phi_i]j + [r\sin\beta\cos\psi + r\sin\alpha\cos\beta\sin\psi]k \tag{9}$$

We are supposing that two balls at beginning of this study, the kinetic energy T is given by:

$$T = \frac{1}{2}M\left[\frac{dr_G}{dt}\right]^2 + \frac{1}{2}m\sum_{i=1}^{2}\frac{dr_{Bi}}{dt} * \frac{dr_{Bi}}{dt} + \frac{1}{2}\omega^T J\omega \tag{10}$$

where J is the inertia Matrix and ω is the angular velocity vector of the rotor; m is the mass of ball and M is the mass of rotor:

$$J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \tag{11}$$

$$\omega = (-\omega\cos\alpha\sin\beta + \alpha\cos\beta)i + (\omega\sin\alpha + \beta)j + (\omega\cos\alpha\cos\beta + \alpha\sin\beta)k \tag{12}$$

in which J is the mass moment of inertia about x,y,z-axis. Neglecting gravity and the torsional and longitudinal deflections of the shaft, the potential energy, or the strain energy, results form the bending deflections of the shaft. As shown in Fig. 1, the shaft can be regarded as a beam with loads on ends, which is fixed at Z=L/4 from ends. The shaft deflections in the X and Y directions:

$$\begin{aligned} D_X &= r\cos\theta \\ D_Y &= r\sin\theta \end{aligned} \tag{13}$$

For the given rotation angles α and β, the rotation angles about the X- and Y-axis:

$$\begin{aligned} \Phi_X &= \alpha\cos\omega t - \beta\cos\alpha\sin\omega t \\ \Phi_Y &= \alpha\sin\omega t + \beta\cos\alpha\cos\omega t \end{aligned} \tag{14}$$

Since the deflection and slope at Z=L/4, in the ZX-plane are D_X and Φ_Y while those in the ZY-plane are D_Y and Φ_X, the deflection curves of the shaft in the ZX- and ZY- planes:

$$\begin{aligned} \delta_X &= \frac{3D_X - L\Phi_Y}{L^2}Z^2 - \frac{2D_X - L\Phi_Y}{L^3}Z^3 \\ \delta_Y &= \frac{3D_Y + L\Phi_X}{L^2}Z^2 - \frac{2D_Y - L\Phi_X}{L^3}Z^3 \end{aligned} \tag{15}$$

The strain energy V due to the shaft bending:

$$V = \frac{1}{2} EI \int_0^L \left[\left(\frac{\partial^2 \delta_X}{\partial Z^2} \right)^2 + \left(\frac{\partial^2 \delta_Y}{\partial Z^2} \right)^2 \right] dZ \tag{16}$$

where E is Young's modulus and I is the area moment of inertia of the shaft cross-section.

By the way, Rayleigh's dissipation function F for two discs can be represented by:

$$F = c_t \left(r^2 + r^2\theta^2 \right) + c_r \left(\alpha^2 + \beta^2 \right) + D \sum_{i=1}^{2} \phi_i^2 \tag{17}$$

where c_t and c_r is the equivalent damping coefficient for translation and rotation respectively and D is the viscous drag coefficient of the balls in the damping fluid.

The equations of motion are derived from Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} + \frac{\partial F}{\partial \dot{q}_k} = 0 \tag{18}$$

In this formulation q_k are the generalized coordinates. For the given system, the generalized coordinates are r, ψ, α,β and $\phi_{1,2}$; therefore, the dynamic behavior of the self balancing system is governed by 2+4 independent equations of motion. Under the assumption that r, ψ, α,β are small and its products too, the equations of motion are simplified and linearized in the neighborhood by perturbation method :

$$\begin{gathered} r = \tilde{r} + \Delta r \qquad \psi = \tilde{\psi} + \Delta\psi \qquad \alpha = \tilde{\alpha} + \Delta\alpha \\ \beta = \tilde{\beta} + \Delta\beta \qquad \phi_i = \phi_i + \Delta\phi_i \end{gathered} \tag{19}$$

In this case each above equation has two components; the coordinates for equilibrium positions and their small perturbations. It is considered $\tilde{r}$ =0 in equilibrium position. And the linearized equations of motion:

$$\begin{gathered} \left(M+2m\right)r + 2c_t r + \left[\frac{12EI}{L^3} - \left(M+2m\right)\omega^2 \right] r - \\ \frac{6EI}{L^2}\alpha \sin\tilde{\psi} - \frac{6EI}{L^2}\beta\cos\tilde{\psi} - mR\sum_{i=1}^{2}\left[\phi_i \sin\left(\tilde{\phi}_i + \tilde{\psi}\right)\right] + \\ 2\omega\phi_i \cos\left(\tilde{\phi}_i + \tilde{\psi}\right) - \omega^2\phi_i \sin\left(\tilde{\phi}_i + \tilde{\psi}\right) = 0 \end{gathered} \tag{20}$$

$$\begin{gathered} -2\left(M+2m\right)\omega r - 2c_t\omega r - \frac{6EI}{L^2}\alpha\cos\tilde{\psi} + \\ \frac{6EI}{L^2}\beta\sin\tilde{\psi} - mR\sum_{i=1}^{2}\left[\phi_i \cos\left(\tilde{\phi}_i + \tilde{\psi}\right)\right] - \\ 2\omega\phi_i \sin\left(\tilde{\phi}_i + \tilde{\psi}\right) - \omega^2\phi_i\cos\left(\tilde{\phi}_i + \tilde{\psi}\right) = 0 \end{gathered} \tag{21}$$

$$\begin{gathered} \left(J + mR^2 \sum_{i=1}^{2} \sin^2\tilde{\phi}_i \right)\alpha - mR^2\beta\sum_{i=1}^{2}\cos\tilde{\phi}_i \sin\tilde{\phi}_i + 2c_r\alpha + \\ \left(J_z - 2J_Y\right)\omega\beta - \frac{6EI}{L^2} r\sin\tilde{\psi} - mR^2\omega^2\beta\sum_{i=1}^{2}\cos\tilde{\phi}_i\sin\tilde{\phi}_i + \\ \left[\frac{4EI}{L} + \left(J_Z - J_Y\right)\omega^2 + mR^2\omega^2\sum_{i=1}^{2}\sin^2\tilde{\phi}_i \right]\alpha = 0 \end{gathered} \tag{22}$$

$$\begin{gathered} -mR^2\alpha\sum_{i=1}^{2}\cos\tilde{\phi}_i\sin\tilde{\phi}_i + \left(J + mR^2\sum_{i=1}^{2}\cos^2\tilde{\phi}_i \right)\beta - \\ \left(J_Z - 2J_X\right)\omega\alpha + 2c_r\beta - \frac{6EI}{L^2} r\cos\tilde{\psi} \\ -mR^2\omega^2\alpha J\sum_{i=1}^{2}\cos\tilde{\phi}_i\sin\tilde{\phi}_i + \\ \left[\frac{4EI}{L} + \left(J_Z - J_Y\right)\omega^2 + mR^2\omega^2\sum_{i=1}^{2}\cos^2\tilde{\phi}_i \right]\beta = 0 \end{gathered} \tag{23}$$

$$\begin{gathered} mR^2\phi + D\phi_i - mR\left[r\sin\left(\tilde{\phi}_i + \tilde{\psi}\right) - 2\omega r\cos\left(\tilde{\phi}_i + \tilde{\psi}\right)\right] + \\ mR\left[-\omega^2 r\sin\left(\tilde{\phi}_i + \tilde{\psi}\right)\right] = 0, i = 1,2 \end{gathered} \tag{24}$$

It is assumed in the above 4 equations:

$$\begin{gathered} \tilde{r} = \tilde{\alpha} = \tilde{\beta} = 0 \\ \sum_{i=1}^{2}\cos\tilde{\phi}_i = -\frac{M\varepsilon}{mR}, \sum_{i=1}^{2}\sin\tilde{\phi}_i = 0 \end{gathered} \tag{25}$$

SIMULATION

The mass moments of inertia, J_X=J_Y and J_Z are given by:

$$J_X = J_Y = \frac{1}{4}MR^2 \qquad J_Z = \frac{1}{2}MR^2 \tag{26}$$

The balanced equilibrium position can be represented:

$$\tilde{r} = \tilde{\alpha} = \tilde{\beta} = 0$$
$$\tilde{\phi}_1 = -\tilde{\phi}_2 = -\tan^{-1}\sqrt{\left(\frac{2mR}{M\varepsilon}\right)^2 - 1} \qquad (27)$$

Small perturbations of the generalized coordinates from the balanced position can be written as:

$$\begin{aligned} r &= X_r e^{it} \\ \alpha &= X_\alpha e^{it} \\ \beta &= X_\beta e^{it} \\ \phi_1 &= X_{\phi 1} e^{it} \phi_2 = X_{\phi 2} e^{it} \end{aligned} \qquad (28)$$

and λ is an eigenvalue. Substituting equations (27) and (28) into equations (20)-(23) and using the Pitagoras identity equation, the condition that equations (28) have non-trivial solutions can be expressed as the characteristic equation given as

$$\sum_{k=0}^{12} c_k \lambda^k = 0 \qquad (29)$$

where the coefficients c_k(k=0,1,….12) are functions of ω, M, m, R, L, ε, E, I, D, c_t and c_r. The explicit expressions of c_k are omitted of this paper. The Routh-Hurwitz criteria provide a sufficient condition for the real parts of all roots to be negative. The following geometry parameters are considered:

$$\begin{aligned} \omega_o &= \sqrt{\frac{48EI}{ML^3}} \\ \zeta_t &= \frac{c_t}{4}\sqrt{\frac{L^3}{3MEI}} \\ \zeta_r &= \frac{c_r}{4}\sqrt{\frac{L}{JEI}} \end{aligned} \qquad (30)$$

And ω_o is the reference frequency; ζ_t and ζ_r are dimensionless damping factors for translation and rotation. In this paper the stability of the balancer are studied for the variations of the non-dimensional system parameters such as ω/ω_o versus ε/R. There are some parameters to be considered:
L/R=2, and $m/M = \varepsilon/R = D/mR^2\omega_o = \zeta_t = \zeta_r = 0.01$.

Next figure shows two different areas of equilibrium and non-equilibrium balls position taking ε/R and ω/ω_o as reference. You can see that equilibrium area is very little.

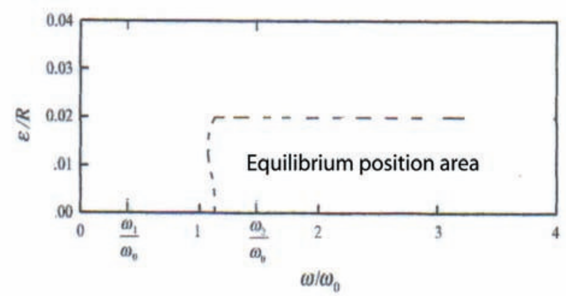


Fig. 3 Possible equilibrium position for variations of rotating speed.

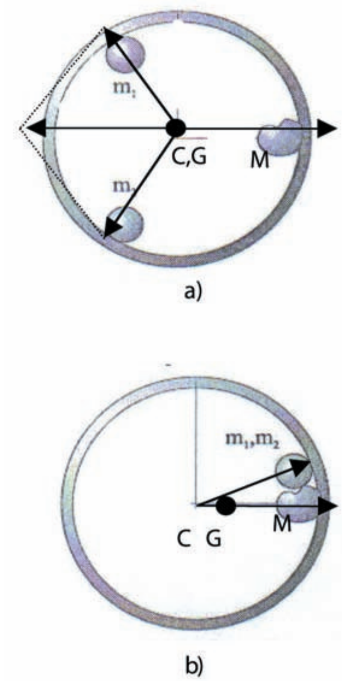


Fig. 4 Schematic representation of two positions of rotor. a) Equilibrium position, b) Non-equilibrium position.

CONCLUSIONS

The balancing bodies of self balancers do not assume positions which ensure complete balancing a rotor. Effective positions of a balancing body differ by $\Delta\phi_t$ from equilibrium position. Other reasons may also appear such as the rubbing of balancing bodies against the sides of drums within they are disposed, irregularities of shape or axially asymmetrical weight distribution of rolling balancing bodies. The positions errors are relative large ones and the larger they are the higher is the coefficient of resistance to rolling motion and the higher is the ratio ω/ω_o (when is greater than 1). In order to reduce these errors it would be necessary to change the method of guiding the balancing bodies, for example air cushion, bodies suspended by magnetic or electrostatic forces.

To obtain the balancing, ω greater than the first natural frequency. The fluid damping D and the dissipation for translation c_t are essential to obtain balancing, but dissipation for rotation c_r is not. The stability of the system have been analyzed with the linear variational equations and the Routh-Hurwitz criteria.

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