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The role of relative prices in a generalized new keynesian Phillips curve

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- **Abstract:** Recent literature attributes a very large fraction of the inflation volatility to "cost-push shocks." This paper develops and estimates a microfounded DSGE model that features a first-order role for relative-price dynamics. A measure of relative price disalignments appears in the new Keynesian Phillips curve (NKPC) of the model, resembling a cost-push shock. Impulse response analysis suggests that an alternative interpretation to the effects of exogenous cost-push shocks in the NKPC relies on the endogenous response of relative prices to exogenous shocks that perturb them.

- **Resumen:** Literatura reciente atribuye una fracción importante de la volatilidad de la inflación a los "choques de costos". Este artículo desarrolla y estima un modelo microfundado, dinámico, estocástico y de equilibrio general, el cual establece un papel de primer orden para la dinámica de precios relativos. Una medida de desalineamientos de precios relativos aparece en la nueva curva Keynesiana de Phillips, del modelo, la cual resembla un choque de costos.

    El análisis de impulso respuesta sugiere que una interpretación alternativa a los efectos de los choques de costos exógenos en la curva de Phillips se basa en la respuesta endógena de los precios relativos a choques exógenos que los perturban.

■ *Introduction*

In a dynamic, stochastic and general equilibrium (DSGE) framework, this paper investigates the dynamic effects of stochastic shocks that perturb the distribution of relative prices on output, inflation and the interest rate. As in Carlstrom et al. (2006) or Carvalho (2006), the model assumes that firms are heterogeneous in the average frequency of price adjustments. This heterogeneity gives rise to a first-order role of relative-price dynamics in the model that is made explicit in the new Keynesian Phillips curve (NKPC) of the model. In the standard model with price stickiness as in Calvo (1983), the Phillips curve takes the form:$\pi_t = \beta E_t \pi_{t+1} + s_\psi \psi_t$, where π is the inflation rate, ψ is the marginal cost and $\beta \in (0,1)$ and $s_\psi > 0$ are parameters of the model. Different from the standard model, the model presented here features a generalized Phillips curve of the form: $\pi_t = \beta E_t \pi_{t+1} + s_\psi \psi_t + s_T T_t$, where $s_T > 0$ and the additional term, T, is an endogenous variable that summarizes relative-price dis-alignments in the economy.

The first-order role of relative prices arises in theoretical pricing models very naturally. For example, in state-dependent pricing models in which firms face a lump-sum physical cost of changing prices,[2] the optimal pricing policy calls for a forecast of the whole distribution of relative prices in the economy; of course, without further simplifying assumptions, such model is intractable. Thus, state dependent pricing models such as Sheshinski and Weiss (1977), Caplin and Spulber (1987), Caplin and Leahy (1997), Dotsey et al. (1999) or Gertler and Leahy (2006) make enough assumptions so that the distribution of relative prices remains tractable.[3] On the other hand, time-dependent pricing models—that is, models in which the timing of the price change does not obey to the state of the economy—, such as Taylor (1980) or Calvo (1983), also limit the number of relevant relative prices in the economy. In the case of Taylor (1980) the number of relevant relative prices is imposed exogenously and in the case of Calvo (1983), as mentioned above, the inflation process can be described up to a first-order approximation without explicit role for relative prices.

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2   In such models, firms follow Ss pricing policies. That is, firms allow their relative price to fluctuate within the range (s,S) without adjusting its own nominal price to avoid the lump sum physical cost of changing prices; however, when its relative price hits the upper or lower limits of such range, the firm optimally resets its nominal price.

3   For example, Caplin and Leahy (1997) make assumptions so that the distribution of relative prices remains invariant to shocks. In Dotsey et al. (1999), the number of relevant relative prices is determined endogenously by the calibration of the model; they calibrate the model so that the number or relative prices is small.

Calvo (1983) pricing has become the standard in the literature—see for example Woodford (2003) or Smets and Wouters (2007)—mainly because its tractability since other more "realistic" models do not outperform Calvo pricing regarding its predictions on the dynamics of inflation; however, by the same token, the role of relative prices has been relegated to a second order. From an econometric perspective, it is important to disentangle the endogenous effects of those relative-price dis-alignments from the exogenous cost-push shocks often added to the Phillips curve. For example, the estimated DSGE models of Ireland (2004a), Smets and Wouters (2003) and Smets and Wouters (2007), among many other models, introduce an exogenous shock to firms' markup over marginal cost that appears in the NKPC curve as a cost-push shock that resembles T. Ireland (2004a) finds that cost-push shocks explain about 90% of the U.S. inflation volatility, whereas Smets and Wouters (2003) find that cost-push shocks explain about 33% of inflation volatility in Europe;[4] thus, it seems important to investigate what those shocks might be. The framework presented here explore the possibility that those exogenous cost-push shocks partially capture shifts in relative prices.

Using the Bayesian econometric techniques detailed in An and Schorfheide (2007), I estimate a subset of the deep parameters of the DSGE model that features four behavioral equations: an Euler equation, a generalized Phillips curve, a monetary policy and a equation that governs the dynamics of relative prices.

Additionally, the model contains four exogenous shocks: a preference shock that shifts the Euler equation, a technology shock that perturbs the production costs, a shock to the monetary policy rule and a shock that perturbs relative prices. The estimated parameters of the model are: the parameters that shape all autoregressive exogenous shocks, the parameters of the monetary policy rule, and a subset of parameters that define the Euler equation and the Phillips curve. Moreover, the non-estimated parameters are calibrated in a standard fashion using parameter values widely adopted in the literature.

The estimated means and standard deviations of the parameters are based on a Random-Walk Metropolis-Hastings algorithm. In the estimation I use U.S. quarterly data for the period 1948:1 to 2003:1. The data includes the growth rate of real per-capita GDP, the inflation rate measured with the GDP deflator and the three-month Treasury bill interest rate.

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4   The results for Ireland (2004a) refer to the unconditional variance of inflation for the post-80 sample. The results for Smets and Wouters (2003) correspond to the 100-period ahead inflation forecast error—which in the limit converges to the unconditional variance—due to what they call price-markup shock.

I use the estimated model to analyze the impulse responses of output, inflation and the interest rate to the four exogenous disturbances. In summary, the impulse responses produced by the preference shock, the technology shock and the monetary rule shock are all in line with the vast literature that explores them. However, with the notable exception of de Walque et al. (2006),[5] the shock that perturbs relative prices has not been included in estimated DSGE models— although it has been explored theoretically, for example, in Carlstrom et al. (2006).

The key result of the impulse-response exercise is that a shock that increases the relative prices that are stickier or less flexible causes very strong and very persistent effects on output, inflation and the interest rate. Output falls achieving its lowest point about 8 quarters after the initial impact; inflation increases reaching its peak about 6 quarters after the initial impulse and consequently the interest rate persistently rises. The impulse responses obtained here are consistent with the impulse response to a negative cost-push shock in Ireland (2004a). It Suggests that an alternative interpretation to the effects of exogenous cost-push shocks in the NKPC relies on the endogenous response of relative prices to exogenous shocks that perturb them.

The rest of the paper is organized as follows. Section 2 presents a microfounded DSGE model that motivates the empirical estimation. Section 3 presents a loglinear version of the model extended to facilitate its estimation and a state-space representation of the model and the data available for the estimation. Section 4 presents the estimation of a subset of parameters of the extended model. Section 5 discusses the estimated shocks and the impulse responses of the model. Finally, Section 6 presents some conclusions.

■ *A Model with Heterogeneous Price Stickiness*

The economy is populated by a representative household, a continuum of monopolistic firms of mass one and a monetary authority.

The Household
The household derives satisfaction from the consumption of a basket of a continuum of differentiated goods á la Dixit and Stiglitz (1977) and from leisure. The household's period utility function at t is

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5   De Walque et al. build a substantially larger model than the presented here and estimate it
     using data for the euro area; the results presented here are consistent with their results. See
     Section 5.

$$U(c_t, \ell_t) = \xi_t \left\{ \frac{1}{1-\gamma}(c_t)^{1-\gamma} + k 1 n(1-\ell_t) \right\},$$

where $\gamma > 1$ and $k > 0$ are parameters that shape the consumer's preferences; ξ_t is an exogenous preference shock (detailed below); ℓ_t is the fraction of time allocated to work, with the total endowment of time normalized to one; and

$$c_t \equiv \left[\int_0^1 [c_t(s)]^{(\theta-1)/\theta} ds \right]^{\theta/(\theta-1)},$$

with $\theta > 1$, is the Dixit-Stiglitz aggregator of consumption over varieties of goods $c_t(s)$.

Expenditure minimization yields the demand for the variety $c_t(s)$:

(1)
$$c_t(s) = \left[\frac{p_t(s)}{P_t} \right]^{-\theta} c_t,$$

where

(2)
$$P_t \equiv \left[\int_0^1 [p_t(s)]^{1-\theta} ds \right]^{\frac{1}{1-\theta}}$$

is the utility-based price index.

In period t, the sources of funds come from nominal bonds B_{t-1} purchased at $t-1$ and maturing at t, with an interest rate R_{t-1} income from working a fraction ℓ_t of the endowed time at a nominal wage rate W_t; and from lump-sum transfers of the nominal profits from the monopolistic firms, denoted by Δ_t. The uses of funds are: the consumption of the good $c_t(s)$ purchased at the nominal price $p_t(s)$ for $s \in [0,1]$–note that $\int_0^1 p_t(s)c_t(s)ds = P_t c_t$– and bonds purchased at t. Thus the time-t budget constraint in real terms is:

(3)
$$w_t \ell_t + b_{t-1} \frac{1+R_{t-1}}{1+\pi_t} + \frac{\Delta_t}{P_t} \geq c_t + b_t,$$

where $w_t \equiv W_t / P_t$ is the real wage, b_t are real bonds and $\pi_t \equiv P_t / P_{t-1} - 1$ is the inflation rate.

The household chooses c_t, ℓ_t and b_t or all t to maximize

$$\sum_{i=0}^{\infty} \beta^i E_t U(c_{t+i}, \ell_{t+i})$$

subject to the sequence of budget constraints given by (3) for all t.

Let λ_t denote the Lagrange multiplier associated to the budget constraint, the first-order conditions for c_t, ℓ_t, and b_t, respectively, imply:

(4)
$$\lambda_t = \xi_t c_t^{-\gamma},$$

(5)
$$\lambda_t w_t = \xi_t \frac{k}{1-\ell},$$

and

(6)
$$\lambda_t = \beta E_t \lambda_{t+1} \frac{1+R_t}{1+\pi_{t+1}}.$$

The Firms

As in Calvo (1983), firms do not choose optimally their prices in every period; instead, they only revise prices when they receive a random signal that arrives with exogenous and constant probability. In the periods in which firms do not receive the random signal, they keep the same price of the previous period. However, given their price, firms have to meet demand.

Note that the probability of price changes, i.e. the Calvo probability from now on, pins down the average frequency of price changes. That is, Calvo pricing assumes that the average frequency of price changes is common across all firms. To relax slightly that assumption I extend Calvo pricing by assuming that there are two types of firms with two different average frequency of price changes. This is a first attempt to capture the effects of the large heterogeneity in the frequency of price changes documented, for example, in Bils and Klenow (2004).

Formally, I assume that the continuum of firms can be described by two disjoint sets of firms that are subject to a set-specific Calvo probability to reset prices. The firms in the set of mass μ can reset prices with probability $(1 - \alpha_L)$ the remaining firms can change prices subject to the probability $(1 - \alpha_H)$. Without loss of generality I assume $(1 - \alpha_H) > (1 - \alpha_L)$

Firms have access to the constant returns to scale technology:

(7)
$$y_t(s) = z_t \ell_t(s),$$

where $y_t(s)$ is the output produced by the firm, $\ell_t(s)$ is the amount of labor employed, and z_t is a productivity shock that follows a stationary stochastic process (and detailed below).

Constant returns to scale together with the market clearing condition $c_t(s) = y_t(s)$ and the firm's demand given by (1) yields the profit function in real terms:

$$
(8) \qquad \frac{\Delta_t}{P_t} = \left[\frac{p_t(s)}{p_t} - \psi_t \right] \left(\frac{p_t(s)}{P_t} \right)^{-\theta} c_t.
$$

Where ψ_t is the real marginal cost. Note that the marginal cost is not firm specific because labor is freely mobile and z_t is common across firms.

Optimal New Prices. Any firms choosing an optimal price takes into account the possibility that the price chosen will remain in place in the following periods; thus, the firm maximizes its expected present value adjusted by its Calvo probability. Thus, a firm subject to the Calvo probability $\alpha_j \in \{\alpha_H, \alpha_L\}$ chooses $p_{j,t}(s)$ to maximize

$$
E_t \sum_{i=0}^{\infty} \alpha_j^i \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left[\frac{p_{j,t}(s)}{P_{t+i}} - \psi_{t+i} \right] \left(\frac{p_{j,t}(s)}{P_{t+i}} \right)^{-\theta} c_{t+i},
$$

where $\beta^i \dfrac{\lambda_{t+i}}{\lambda_t}$ is the relevant stochastic discount factor since firms are owned by the consumers. The optimal new price set at t by *any* firm under the Calvo probability $(1 - \alpha_j)$ is

$$
(9) \qquad p_{j,t}^* = \frac{\theta}{\theta - 1} \frac{E_t \sum_{i=0}^{\infty} \left(\beta \alpha_j \right)^i \frac{\lambda_{t+i}}{\lambda_t} \psi_{t+i} (P_{t+i})^{\theta} c_{t+i}}{E_t \sum_{i=0}^{\infty} \left(\beta \alpha_j \right)^i \frac{\lambda_{t+i}}{\lambda_t} (P_{t+i})^{\theta-1} c_{t+i}},
$$

where I dropped the firm-subindex s because the new price $p^*_{j,t}$ is common for all firms subject to the probability $(1 - \alpha_j)$.

The Price Level. It is convenient to rewrite the price index (2), in terms of the price sub-indexes $P_{L,t}$ and $P_{H,t}$ as: $P_t \equiv \left[\mu P_{L,t}^{1-\theta} + (1-\mu) P_{H,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}$,

where

$$
P_{L,t} \equiv \left[\frac{1}{\mu} \int_0^{\mu} \left[p_t(s) \right]^{1-\theta} ds \right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P_{H,t} \equiv \left[\frac{1}{1-\mu} \int_{\mu}^{1} \left[p_t(s) \right]^{1-\theta} ds \right]^{\frac{1}{1-\theta}},
$$

with the proper selection of the index s ∈ [0,1], the integral in the price sub-index $P_{j,t}$ aggregates prices of firms subject to the probability $(1 - \alpha_j)$.

As in the standard Calvo (1983)–Yun (1996) setup, the dynamics of the price sub-indexes can be described using the recursion:

$$(10) \qquad P_{j,t}^{(1-\theta)} = \alpha_j P_{j,t-1}^{(1-\theta)} + \delta_j(1-\alpha_j)(p_{j,t}^*)^{(1-\theta)},$$

with $\delta_j = \mu$ for $j = L$ and $\delta_j = 1 - \mu$ for $j = H$.

Central Bank

To close the model we must specify the monetary policy. I assume that the central bank follows a modified Taylor (1993) rule

$$(11) \qquad \hat{R}_t = \rho_r \hat{R}_{t-1} + \rho_\pi \hat{\pi}_t + \rho_c \hat{c}_t + \hat{E}_t^r,$$

where $\hat{x}_t \equiv \dfrac{dx_t}{\bar{x}}$ denotes log-linear deviations from the steady-state $\bar{x}$ of

the corresponding variable; $\rho_r \geq 0$, $\rho_\pi > 0$ and $\rho_c \geq 0$ are parameters chosen by the central bank; E_t^r is an exogenous monetary shock (detailed below).

■ *The Log-linear Model and A Hybrid Extension*

To solve the model, I log-linearize the first-order conditions of the consumer, the optimal new price of the firm and the price indexes.

Accordingly, let $\hat{x}_t \equiv \dfrac{dx_t}{\bar{x}}$ be the log-linear deviation of the variable x

from its steady-state value $\bar{x}$; for the fractions R and π, I define

$$\hat{x}_t = \frac{d(1 + x_t)}{(1 + \bar{x})}.$$

Using the first-order conditions (4) and (6) I obtain the Euler equation of consumption:

$$(12) \qquad \hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\gamma} E_t \left[\hat{R}_t - \hat{\pi}_{t+1} \right] - E_t \hat{\vartheta}_{t+1}.$$

Note that equation (12) resembles and IS-curve and

$\hat{\vartheta}_{t+1} = \dfrac{1}{\gamma}\left[\hat{\xi}_{t+1} - \hat{\xi}_t\right]$ is a preference shock that shifts consumption.

Appendix A shows that log-linearizing the firm's new price (9), the price index and the price sub-indexes (10) we obtain the generalized new Keynesian Phillips curve (NKPC):

(13)
$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + s_c \hat{c}_t + s_T \hat{T}_t - s_z \hat{z}_t,$$

where the variable
$$\hat{T}_t \equiv \hat{P}_{L,t} - \hat{P}_{H,t}$$

captures the role of relative price deviations as discussed below, and $\quad s_c \equiv s_\psi \dfrac{\gamma k + \bar{c}^{1-\gamma}\bar{w}}{\bar{c}^{-\gamma}\bar{w}(1-\bar{c})},\quad$ with $\quad s_\psi \equiv \left[\mu a_L + (1-\mu)a_H\right],\quad$ where

$a_L \equiv (1-\alpha_L)(1-\beta\alpha_L)/\alpha_L\quad$ and $\quad a_H \equiv (1-\alpha_H)(1-\beta\alpha_H)/\alpha_H$; the

steady-state consumption $\bar{c}$ is given by $\quad \bar{c}^{-\gamma}\left(1-\bar{c}\right) = k\dfrac{\theta}{\theta-1}$ and the

steady-state wage is $\overline{w} = \dfrac{\theta-1}{\theta}$; $s_T \equiv \mu(1-\mu)\left[a_H - a_L\right]$; and $s_z \equiv s_\psi \dfrac{1}{1-\bar{c}}$.

Different from the standard NKPC—see for example Ireland (2004b)—, in the Phillips curve above there is a first-order role for disalignments in relative prices.

The two sets of firms subject to different Calvo probabilities have different price dynamics; the heterogeneity in prices becomes first-order and shows up as an additional term in the Phillips curve.

To build intuition on the role of T, the Appendix A shows that in

equilibrium: $\hat{T}_t = \dfrac{1}{\theta}\left[\hat{C}_{H,t} - \hat{C}_{L,t}\right]$, where $\hat{C}_{j,t}$ is the consumption sub-

basket that aggregates the goods produced by the firm subject to the Calvo probability α_j for $j = L,H$. That is, T summarizes the effects of consumption reallocation given a perturbation that affects asymmetrically to a subset of prices in the economy—such asymmetry is built on the Calvo probabilities. The origin from that perturbation can be any of the stochastic shocks described so far, but additionally, in a more detailed model, a direct perturbation to a subset of prices can arise so

that $\hat{T}_t = \frac{1}{\theta}\left[\hat{C}_{H,t} - \hat{C}_{L,t}\right] + \tau_f \hat{f}_t$. In that case, $\hat{f}_t$ is an exogenous relative-

price shock (and τ_f regulates its variance) that increases prices in the sector with stickier prices and therefore moves resources toward the consumption sub-basket produced by firms with more flexible prices. This shock therefore can be interpreted as a contractionary shock in the sticky-price sector.

The dynamics of $\hat{T}_t$ (see Appendix A) is governed by

$$(14) \qquad \hat{T}_t = \beta\tau_1 E_t\hat{T}_{t+1} + \tau_1\hat{T}_{T-1} - \tau_c\hat{c}_t + \tau_z z_t + \tau_f\hat{f}_t,$$

where $\tau_1 \equiv \left[1 + \beta + (1-\mu)a_L + \mu a_H\right]^{-1}$, $\tau_c \equiv \tau_1(a_H - a_L)\dfrac{\gamma k + \bar{c}^{1-\gamma}\overline{w}}{\bar{c}^{-\gamma}\overline{w}(1-\bar{c})}$,

$$\tau_z \equiv \tau_1(a_H - a_L)\frac{1}{1-\bar{c}},$$

and $\hat{f}_t$ is an ad hoc exogenous shock that directly perturbs relative prices, its variance is governed by the parameter $\tau_f > 0$. The law of motion of the relative-price shock $\hat{f}_t$ is detailed below.

Equations (11), (12), (13) and (14) form a system of four equations in four endogenous variables: $\hat{c}_t$, $\hat{R}_t$, $\hat{\pi}_t$, and $\hat{T}_t$, its solution characterize an equilibrium.

Also the system contains four exogenous stochastic processes: $\hat{\varepsilon}_t$, $\hat{\vartheta}_t$, $\hat{z}_t$ and $\hat{f}_t$, which are detailed below.

It is worth to remark that in order to describe the dynamics of the aggregate variables $\hat{c}_t$, $\hat{R}_t$, $\hat{\pi}_t$ and $\hat{T}_t$ we do not need to make explicit the dynamics of the price sub-indexes $P_{L,t}$ and $P_{H,t}$ or the consumption sub-baskets $C_{H,t}$ and $C_{L,t}$.

Extending The Model
In order to estimate the model, I follow Ireland (2004b) in extending the Euler equation to include lagged consumption and the Phillips curve to include lagged inflation. Thus the system of the hybrid model is:

$$(15) \quad \hat{c}_t = \Phi_c c_{t-1} + (1-\Phi_c)E_t\hat{c}_{t+1} - \frac{1}{\gamma}E_t\left[\hat{R}_t - \hat{\pi}_{t+1}\right] - E_t\hat{\vartheta}_{t+1}$$

$$(16) \qquad \hat{\pi}_t = \Phi_\pi\hat{\pi}_{t-1} + (1-\Phi_\pi)\beta E_t\hat{\pi}_{t+1} + s_c\hat{c}_t + s_T\hat{T}_t - s_z\hat{z}_t$$

$$(17) \qquad \hat{T}_t = \beta\tau_1 E_t\hat{T}_{t+1} + \tau_1\hat{T}_{t-1} - \tau_c\hat{c}_t + \tau_z\hat{z}_t + \tau_f\hat{f}_t$$

$$(18) \qquad \hat{R}_t = \rho_r\hat{R}_{t-1} + \rho_\pi\hat{\pi}_t + \rho_c\hat{c}_t + \hat{E}_t^r$$

where $\Phi_c \in [0,1]$ and $\Phi_\pi \in [0,1]$. Note that when $\Phi_c = \Phi_\pi = \tau_f = 0$ the system boils down to the original microfounded model, thus I let Φ_c and Φ_π to be pinned down by the data.

Following the long tradition started with Kydland and Prescott (1982), I assume that the exogenous shocks are governed by autoregressive processes of order one[6]:

$$(19) \qquad \hat{\xi}_t = \varphi_\xi \hat{\xi}_{t-1} + \varepsilon_{\xi t}$$

$$(20) \qquad \hat{z}_t = \varphi_z \hat{z}_{t-1} + \varepsilon_{zt}$$

$$(21) \qquad \hat{f}_t = \varphi_f \hat{f}_{t-1} + \varepsilon_{ft},$$

and

$$(22) \qquad \hat{E}_t^r = \varphi_r \hat{E}_{t-1}^r + \varepsilon_{rt}$$

where $\varphi_\xi, \varphi_z \varphi_f$ and φ_r are between zero and one, and for empirical purposes I assume $\varepsilon_{xt} \sim N\left(0, \sigma_x^2\right)$ for all innovations ε_{xt}.

Model and Data in a State-Space Representation
The system (15) to (22) can be solved with numerical methods such as the ones presented in Blanchard and Kahn (1980), Uhlig (1999) or Klein (2000). I follow Uhlig (1999) to solve the model; the solution can be written as:

$$(23) \qquad s_{t+1} = As_t + B\varepsilon_{t+1}.$$

where $s_t = \left[\hat{c}_{t-1}\hat{\pi}_{t-1}\hat{T}_{t-1}\hat{R}_{t-1}\hat{\vartheta}_t\hat{z}_t\hat{f}_t\hat{E}_t^r\right]'$ and $\varepsilon_t = \left[\varepsilon_{\xi t} \ \varepsilon_{zt} \ \varepsilon_{ft} \ \varepsilon_{rt}\right]'$. The

matrices A and B are detailed in a technical appendix available from the author upon request.

Moreover assume that we have quarterly data available for output growth, inflation and the interest rate; also assume that the data contains a measurement error. The vector of available data is:

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6   The choice of AR(1) processes for the exogenous shocks allows for some persistence on the shocks without introducing more complex exogenous dynamics to the model such as higher order autoregressive processes.

$$\begin{bmatrix} g_t^d \\ \pi_t^d \\ R_t^d \end{bmatrix} = \begin{bmatrix} 1n(c_t) - 1n(c_{t-1}) - 1n(\overline{g}) \\ 1n(P_t) - 1n(P_{t-1}) - 1n(\overline{\pi}) \\ 1n(R_t) - 1n(\overline{R}) \end{bmatrix} + \begin{bmatrix} v_t^g \\ v_t^\pi \\ v_t^R \end{bmatrix},$$

where v_t^x is the corresponding measurement error.

The model has implications for the data vector $d_t = \begin{bmatrix} g_t^d, \pi_t^d, R_t^d \end{bmatrix}'$. The

relation between the data and the model solution can be described as

(24) $$d_t = Cs_t + v_t,$$

where $v_t = \begin{bmatrix} v_t^g, v_t^\pi, v_t^R \end{bmatrix}'$. $Ev_t v_t'$ is given by the matrix C and the

one-step ahead covariance matrix of the vector s_t —see the technical notes of Ireland (2004a) and Ireland (2004b). The matrix C is straightforward to construct from (23).

Hamilton (1994) discusses how to evaluate the likelihood function

for the data $\{d_t\}_{t=1}^T$ in the system (23)-(24) by using a Kalman filter.

Ireland (2004a) discusses the details for the evaluation of the likelihood function in the context of DSGE models.

■ *Estimation and Calibration*

I estimate some parameters of the model using the Bayesian methods detailed in An and Schorfheide (2007) for the estimation of DSGE models. The nonestimated parameters are calibrated using standard values taken from the literature.

Calibrated Parameters

Table 1 shows the values of the calibrated parameters. γ in the consumer preferences is set to 2, this implies more curvature in the utility than a logarithmic utility in consumption. β is set to 0.99 implying a risk-free real interest rate is about 4%. The value of k is set to 5.25 so that the steady-state fraction of time allocated to labor is 1/3. The parameter θ is set to 8, thus the steady-state firms' markup over marginal cost is about 15%.[7] The

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7   As shown in Rotemberg and Woodford (1995), the choice of the steady-state markup can
     affect quantitatively the impulse responses of the model. However, note that by choos-
     ing κ so that steady-state labor takes a specific value, I ensure that the size of distortion

parameter τ_f in the relative price equation (17) regulates the time-t impact of the relative-price shock, I set it to 1 so that the estimated volatility of innovations to the shock capture that effect.

The parameters α_L, α_H and μ are chosen so that the slope of the Phillips curve, in the space of inflation and marginal cost, s_ψ, is consistent with the estimations of Sbordone (2005). In a purely forward looking Phillips curve, Sbordone estimates a marginal-cost slope of 0.025, I set $\alpha_L = 0.9$, $\alpha_H = 0.7$ and $\mu = 0.9$ so that $s_\psi = 0.024$[8].

Table 1
Calibrated parameters

Parameter	Value	Description
γ	2	Intertemporal elasticity of substitution
β	0.99	Discount factor
k	5.25	Leisure parameter
θ	8	Steady-state markup parameter
τ_f	1	Relative-price-shock parameter
α_L	0.9	Calvo probability
α_H	0.7	Calvo probability
μ	0.9	Mass of firms subject to α_L

Estimated Parameters

The estimation uses quarterly data for the U.S. from 1948:1 to 2003:1, which includes: quarterly changes in seasonally-adjusted real per-capita GDP as measure of output growth; quarterly changes in seasonally-adjusted GDP deflator as measure of inflation; and quarterly averages of the three-month U.S. Treasury bill rate as measure of the interest rate. The vector of estimated parameters contains the backward-looking parameter in the Euler equation (15) Φ_c, the backward-looking parameter in the Phillips curve (16) Φ_π; the parameters of the Taylor rule (18) ρ_r, ρ_π, and ρ_c; the autocorrelation coefficients of the exogenous shocks $\varphi_\xi, \varphi_z, \varphi_f$ and φ_r; and the standard deviations of the innovations to the exogenous shocks $\sigma_\xi, \sigma_z, \sigma_f$ and σ_r.

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of the monopolistic competition, captured by the markup, does not affect steady-state consumption. More importantly, the calibration strategy also ensures that, different from Rotemberg and Woodford (1995), the impulse responses are invariant to the choice of the markup.

8   Note that equation (16) is written in terms of consumption, not marginal cost; however s_c is a function of s_ψ. See (30) in the appendix.

Let Θ be the vector of parameters to estimate and d the data available. To construct the posterior distribution of Θ, I first implement a Kalman filter for the system (23)-(24) to numerically evaluate the likelihood function $L(d|\Theta)$. Then, given a parameter prior-distribution $P(\Theta)$, detailed below, I obtain the posterior distribution (up to a constant): $P(\Theta)L(d|\Theta)$. With the posterior likelihood at hand I use a Random-Walk Metropolis-Hastings algorithm to simulate the posterior distribution of parameters.

Prior Distributions. For the prior distributions I choose the Beta distribution for all parameters constrained between zero and one; the Gamma distribution for parameters constrained to be positive; and the Uniform distribution for all standard deviations. In particular, the backward looking parameters Φ_c and Φ_π the autocorrelation coefficient of the interest rate ρ_r and the autocorrelation coefficients of the exogenous shocks φ_x have a Beta prior distribution with mean 0.5 and standard deviation of 0.1. The other two coefficients in the Taylor rule ρ_π and ρ_c are constrained to be positive and can be larger than one, thus they have a prior Gamma distribution with mean 0.5 and standard deviation 0.1. The prior distributions of the standard deviations of the innovations to the exogenous shocks σ_x are Uniform with range $\left[0,\sqrt{5}\right]$ [9].

Posterior Distributions. To simulate the posterior distribution I use a Random-Walk Metropolis-Hastings algorithmas detailed in Koop (2003). After performing 300 thousand simulations, the first 100 thousand were discarded and the remaining had an acceptance rate of 34%; I use the accepted simulations to calculate the parameter means, standard deviations and posterior quantiles 5 and 95 reported in Table 2. It is worth to mention that I followed the standard practice of checking

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9   Note that the choices of the prior distributions constraint the parameter space so that the estimates are within the parameter space required by the economic model. For example, the exogenous shocks considered here are transitory and not permanent shocks; thus, a Beta prior distribution ensures that the autocorrelation coefficients are between zero and one, otherwise the probability assigned is zero. Similarly, the Gamma distribution ensures that the inflation and consumption coefficients in the monetary rule are positive—and they can be larger than one. However, also note that the priors considered here are "flat priors" in the sense that their standard deviations are large enough so that the posterior mean is largely dictated by the data information contained in the likelihood function. The Uniform distributions chosen for standard deviations of stochastic shocks are also "uninformative" priors—within their range— since any parameter value within their range is equally weighted. Overall, the strategy followed for the choices of priors is to impose some minimum constraints in the parameter space to maintain the economic meaning of parameters, but at the same time I let the data to largely pin down the posterior means.

for the uniqueness of the solution of the model for each parameter draw, so that explosive or sunspot solutions were ruled out by penalizing the likelihood.[10]

Table 2
Posterior Distribution

Parameter	Prior distribution			Posterior distribution			
	Distribution	Mean	Std. Dev.	Mean	Std. Dev.	5%	95%
Φ_c	Beta	0.5	0.1	0.2557	0.0188	0.224	0.2858
Φ_π	Beta	0.5	0.1	0.1686	0.0353	0.1131	0.2292
ρ_r	Beta	0.5	0.1	0.8471	0.0363	0.7847	0.9032
ρ_π	Gamma	0.5	0.1	0.8618	0.1005	0.7059	1.0349
ρ_c	Gamma	0.5	0.1	0.037	0.0069	0.0266	0.049
Exogenous shocks							
φ_ξ	Beta	0.5	0.1	0.71	0.071	0.5852	0.8161
φ_z	Beta	0.5	0.1	0.5007	0.1031	0.331	0.6692
φ_f	Beta	0.5	0.1	0.9579	0.009	0.9423	0.9717
φ_r	Beta	0.5	0.1	0.8159	0.0136	0.7926	0.8374
σ_ξ	Uniform	1.1	0.4	0.0247	0.0023	0.0214	0.0287
σ_z	Uniform	1.1	0.4	0.0001	0.0003	0	0.0006
σ_f	Uniform	1.1	0.4	0.0136	0.001	0.012	0.0155
σ_r	Uniform	1.1	0.4	0.0045	0.0006	0.0037	0.0055

Source: own estimations.

The mean of the backward-looking parameter in the Euler equation (15) Φ_c is estimated in 0.25 with a 90 percent posterior interval of [0.22, 0.28]. Thus, the data prefers a model that departs from the microfounded model presented above. The estimates are consistent with a model in which the consumer smooths the quasigrowth rate of consumption as opposed to the consumption level as assumed by the model above. That is, the consumer shows preferences with habit formation as the ones discussed in Abel (1990) and widely adopted in the literature—see for example Smets and Wouters (2003). The estimated mean of the backwardlooking parameter in the Phillips curve (16) Φ_π is 0.16 with a 90 percent interval of [0.11, 0.22]. An interpretation of Φ_π follows from Gali and Gertler (1999), they propose a hybrid model in which a fraction of firms set prices using

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10   The rationale for this is based on the fact that we do not observe such explosive behavior in the data, thus parameter values that imply such behavior must be very unlikely. To be sure, the algorithm penalizes the likelihood

a backwardlooking rule of thumb and the rest are subject to Calvo pricing; they estimate a fraction of 25% of backward-looking firms, whereas I obtain a 16%. An alternative interpretation of Φ_π is that it captures the effect of price indexation as in Smets and Wouters (2007); Smets and Wouters estimate an effect of indexation of 0.24—surprisingly close to the estimate of Gali and Gertler. These two estimates together with the calibrated parameters identify the Euler equation of consumption and the Phillips curve of the model.

The estimates of the parameters shaping the monetary policy rule are as follows. The estimated mean of the interest rate smoothing parameter ρ_r is 0.84 with a 90% posterior interval of [0.78, 0.90]. This estimate implies that the Central Bank smooths substantially the maneuvering of the interest rate and it is consistent with the estimation of Smets and Wouters (2007) of 0.81. The mean of the coefficient ρ_π is 0.86 with a 90 percent interval [0.70, 1.03]; this is, the Central Bank policy rule has a strong short-term response to movements in inflation that together with the smoothing parameter ensure a non-explosive path of the economy in the sense that ensures determinacy. The parameter ρ_c has a mean of 0.03 with a 90 percent interval [0.02, 0.04]; that is, the Central Bank does not respond strongly to output fluctuations.

In the next section we turn to the estimations of the parameters shaping the exogenous shocks in the context of the impulse responses of the model.

■ *Exogenous Shocks and Impulse Responses*

I use the calibrated and estimated parameters to compute the impulse responses of the model to the exogenous shocks. Each exogenous shock is characterized by two parameters: its autocorrelation coefficient and the standard deviation of its innovations. Table 2 reports the estimated mean of those parameters as well as the standard deviation and the percentiles 5 and 95 of their posterior distribution. However, the dynamic impact of those shocks on output, inflation and the interest rate are analyzed in terms of the impulse responses predicted by the model. Figure 1 shows the impulse responses.

The estimations show that the preference shock ξ or "demand shock" is characterized by an autocorrelation coefficient with mean 0.71 and innovations with standard deviation 0.02. The model predicts that when the economy is hit by a positive innovation to the preference shock of 1 standard deviation, output grows by about 0.8% above its long-run trend. The shift in demand causes inflation to grow by 11 basis points;

Figure 1
Impulse-Responses with Estimated Parameters
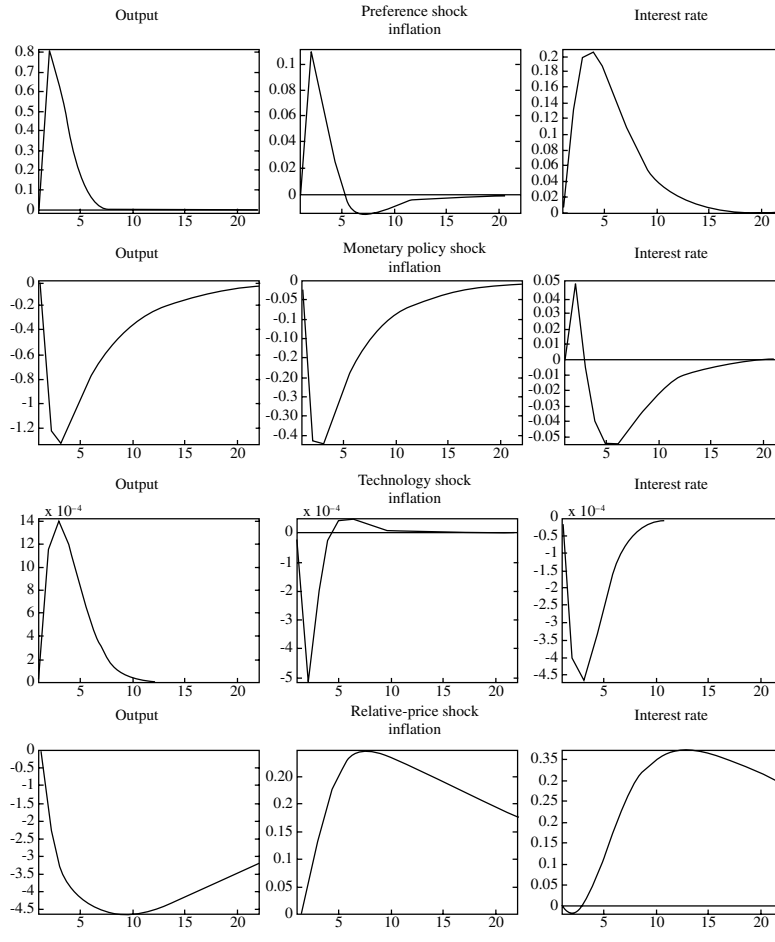
Note: Time in quarters is in the horizontal axis and the vertical axis measures percent deviations from steady-state for output and percent points for inflation and the interest rate.

consequently, the Central Bank increases the interest rate by 25 basis points to bring inflation down.

The monetary policy shock $\in^r$ has an estimated autocorrelation coefficient with mean of 0.81 and innovations with standard deviation of 0.0045. The model predicts that a positive innovation to the monetary policy of 1 standard deviation causes a fall in output of 1.2%; moreover, the fall in demand caused by a higher interest rate—i.e. the monetary shock—drops inflation for about 40 basis points. On impact, the Central Bank responds simultaneously to both effects: the monetary shock by which the interest rate goes up and the fall in output and inflation by which the interest rate goes down according to the monetary policy; the resulting net effect is an increase in the interest rate of about 5 basis points on impact. After the initial impact, the Central Bank endogenously responds to the fall in output and inflation by following the monetary rule and decreases the interest rate to restore the equilibrium.

The technology shock z has an estimated autocorrelation coefficient with mean 0.5 and innovations with standard deviation 0.001. The model predicts that when there is a positive innovation to the technology shock of 1 standard deviation, output mildly increases; the costs of production are reduced (marginal cost drops), thus inflation drops and the Central Bank lowers the interest rate in response. The small responses of output, inflation and the interest rate to the technology shock come from the fact that the estimated standard deviation is surprisingly low; for example, Ireland (2004a) estimates that parameter in 0.0109. One key difference between this estimation and other estimations that find results similar to the ones in Ireland (2004a) is, of course, that I included the relative-price shock that competes with the other shocks in explaining the volatility of output, inflation and interest rates as suggested by the following result.

The shock to the relative price equation (17) has an estimated autocorrelation coefficient with mean 0.95 and innovations with standard deviation of 0.0136. Recall from section 3 that a positive relative-price shock is a contractionary shock to the stickier sector, thus an innovation to the relative-price shock of 1 standard deviation has highly contractionary and persistent effects on output. The model predicts a substantial drop in output that reaches its minimum at 4.5% below its long-term trend about 2 years after the shock; inflation grows slowly to reach an increment of 30 basis points in one year and half; and the Central Bank responds by increasing the interest rate very persistently by about 35 basis points to counterbalance the inflationary pressures. The small magnitude of the impulse responses to a technology shock combined with the large and persistent impulse responses to a relative-price shock suggests that there

is tension between these two shocks to explain the data. However, the estimations obtained here suggest that a larger fraction of the volatility of the data can be explained by the relative-price shock than by the technology shock. Of course, to resolve this apparent trade-off, a much larger model is needed to capture the effects of the technology shock in capital accumulation and investment.

An econometrician estimating the standard Phillips curve extended by an exogenous cost-push shock may indeed be capturing part of the effect of relative price shocks in the cost-push shock. That is, such econometrician would replace T_t in (16) by an exogenous shock. For example, Ireland (2004a) introduces an exogenous cost-push shock in the Phillips curve as a shock to the firm's markup—by making θ an stochastic process. The estimated cost-push shock is highly persistent (with an autocorrelation coefficient of 0.96 and innovations with standard deviation of 0.04) and the estimations yield impulse responses that are similar to the impulse responses to a negative innovation to the relative-price shock. Different from a cost-push shock, the relative price T_t is endogenous.

■ *Summary and Conclusions*

Recent literature has found that cost push shocks explain a substantial percent of the observed volatility of inflation and the interest rates. For example, Ireland (2004a) finds that cost-push shocks explain almost all the volatility of inflation and around 60% of the volatility of interest rates.

The paper presents a microfounded, dynamic stochastic and general equilibrium model that features firms' heterogeneity in the average frequency of price changes. In particular the model extends Calvo (1983) pricing by assuming that there are two sets of firms subject to two different average frequency of price changes. This modification to the standard Calvo pricing yields a generalized Phillips curve of the model in the sense that a measure of relative-price disalignments— that is, deviations in the relative average price of the two sets of firms—appear in the Phillips curve as an additional variable. This additional variable in the Phillips curve resembles a cost-push shock; that is, a shock that perturbs the standard NKPC.

Bayesian estimation of a subset of parameters of the model allows to perform impulse response analysis of the four stochastic shocks featured in the model: preference shocks or demand shocks, technology shocks or supply shocks, monetary policy shocks and shocks to the relative prices. The first three shocks are standard in the literature and the

impulse response of output, inflation and the interest rate obtained here are in line with the widely accepted view in the literature. However, the novel feature of the model allows to explore the effects of relative-price shocks, that is shocks that affect mainly to a sub-set of prices in the economy—but not to all prices symmetrically.

In the model, a shock that increases the relative price of the stickier-price sector has a contractionary effect on that sector and it induces a reallocation of consumption toward the sub-basket of goods produced by firms with more flexible prices. The net effect contracts aggregate consumption. The intuition for the net contraction of consumption follows from the fact that when prices are stickier, the production has to adjust more to accommodate the shock than when prices are more flexible; with more flexible prices, the shock is absorbed in a larger part by prices and thus it has a lesser impact on production—in the limiting case of flexible prices, prices would accommodate the shock and no change in production would occur.

The effects of relative-price shocks are very strong and very persistent on output, inflation and the interest rate, similar to the effects found for cost-push shocks. Thus, the results suggest that an alternative interpretation to the effects of exogenous cost-push shocks in the NKPC relies on the endogenous response of relative prices to exogenous shocks that perturb them.

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■ *A. Appendix:*
  Derivation of the generalized NKPC

Log-linearizing the price index I obtain $\hat{P}_t = \mu\hat{P}_{L,t} + (1-\mu)\hat{P}_{H,t}$; moreover, defining $\pi_{j,t} \equiv \dfrac{P_{j,t}}{P_{j,t-1}}$ for $j = H, L$ we get:

(25) $$\hat{\pi}_t = \mu\hat{\pi}_{L,t} + (1-\mu)\hat{\pi}_{H,t}.$$

The log-linear version of equation (9) can be written as

(26) $$\hat{p}^*_{j,t} = \left(1 - \beta\alpha_j\right)\left(\hat{P}_t + \hat{\psi}_t\right) + E_t\beta\alpha_j\hat{p}^*_{j,t+1}.$$

Using the log-linear versions of equations (10) for $j = H, L$ are

(27) $$\hat{P}_{j,t} = \alpha_L\hat{P}_{j,t+1} + \left(1 - \alpha_j\right)\hat{p}^*_{j,t}$$

Next, let $R_{j,t} \equiv \dfrac{P_{j,t}}{P_t}$ and recall $T_t \equiv \dfrac{P_{L,t}}{P_{H,t}}$. Thus, from the price index we have

(28) $$\hat{R}_{L,t} = (1-\mu)\hat{T}_t \quad \text{and} \quad \hat{R}_{H,t} = -\mu\hat{T}_t.$$

Forwarding (27) and solving for $\hat{p}^*_{j,t+1}$,

I obtain $\left(1 - \alpha_j\right)\hat{p}^*_{j,t+1} = \hat{\pi}_{j,t+1} + \left(1 - \alpha_j\right)\hat{P}_{j,t}.$

Substituting the last equation into the right-hand side of (26), substituting the resulting equation into (27), and rearranging yields

(29) $$\hat{\pi}_{j,t} = \beta E_t\hat{\pi}_{j,t+1} + \frac{\left(1 - \alpha_j\right)\left(1 - \beta\alpha_j\right)}{\alpha_j}\left[\hat{\psi}_t - \hat{R}_{j,t}\right].$$

Next, multiplying (29) for $j = L$ times μ and (29) for $j = H$ times $(1 - μ)$, substituting the resulting equations into (25) and using (28) yields

(30) $$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + s_\psi\hat{\psi}_t + s_T\hat{T}_t$$

where s_ψ and s_T are given in the text.

Next, substituting the first-order condition (4) into (5) and using the market clearing condition $c_t = y_t = z_t \ell_t$ to eliminate ℓ from the resulting

equation, I obtain $\quad \hat{w}_t = \dfrac{\gamma k + \bar{c}^{\,1-\gamma}\,\overline{w}}{\bar{c}^{\,-\gamma}\,\overline{w}\left(1-\bar{c}\right)}\hat{c}_t - \dfrac{\bar{c}^{\,1-\gamma}}{\bar{c}^{\,-\gamma}\left(1-\bar{c}\right)}\hat{z}_t$; moreover, the

marginal cost is given by $\hat{\psi}_t = \hat{w}_t - \hat{z}_t$, thus:

$$\hat{\psi}_t = \frac{\gamma k + \bar{c}^{\,1-\gamma}\,\overline{w}}{\bar{c}^{\,-\gamma}\,\overline{w}\left(1-\bar{c}\right)}\hat{c}_t - \frac{1}{1-\bar{c}}\hat{z}_t,$$

where the steady-state of consumption $\bar{c}$ and the wage rate $\overline{w}$ are given in the text. Substituting the last equation in (30) I obtain the equation (13) in the text.

The second-order difference equation for T_t, (14), is obtained as follows. Rewrite (30) as

(31) $\beta \mathrm{E}_t \hat{R}_{j,t+1} - \left[1+\beta+\left(1-\alpha_j\right)\left(1-\beta\alpha_j\right)/\alpha_j\right]\hat{R}_{j,t} + \hat{R}_{j,t-1} = \hat{\pi}_t - \beta \mathrm{E}_t \hat{\pi}_{t+1}$

$$-\left[\left(1-\alpha_j\right)\left(1-\beta\alpha_j\right)/\alpha_j\right]\hat{\psi}_t,$$

then use (28) to express $\hat{R}_{j,t}$ in terms to T_t, subtract (31) for $j = L$ minus (31) for $j = H$ and collect common terms.

To build intuition on the economic role of the relative prices T that appear in the Phillips curve consider the following. First note that we can rewrite the consumption basket equivalently as:

$$C_t \equiv \left[\mu\left(C_{L,t}\right)^{\frac{\theta-1}{\theta}} + \left(1-\mu\right)\left(C_{H,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where the subbaskets $C_{L,t}$ and $C_{H,t}$ contain the goods produced by the firms subject to the Calvo probabilities α_L and α_H respectively. In particular

$$C_{L,t} \equiv \left[\frac{1}{\mu}\int_0^\mu \left(c_t(s)\right)^{\frac{\theta-1}{\theta}}ds\right]^{\frac{\theta}{\theta-1}} \qquad C_{H,t} \equiv \left[\frac{1}{1-\mu}\int_\mu^1 \left(c_t(s)\right)^{\frac{\theta-1}{\theta}}ds\right]^{\frac{\theta}{\theta-1}},$$

and

where the index s has been chosen appropriately.

This equivalent representation of the consumption basket, together with the price sub-indexes $P_{L,t}$ and $P_{H,t}$ defined in the text, make explicit two subbaskets of consumption. It is straight forward to show $\int_0^1 p_t(z)c_t(z)dz = P_t C_t = \mu P_{L,t} C_{L,t} + (1-\mu)P_{H,t}C_{H,t}$; using this and recalling $T_t \equiv \dfrac{P_{L,t}}{P_{H,t}}$, we can show that the optimal allocation of consumption

implies: $T_t = \left(\dfrac{C_{L,t}}{C_{H,t}}\right)^{-1/\theta}$.

Log-linearizing the equation above, I obtain the condition discussed in the text.