Application of an advanced soil constitutive model to the study of railway vibrations in tunnels through 2D numerical models: a real case in Madrid (Spain)

Aplicación de un modelo constitutivo avanzado del suelo al estudio de vibraciones ferroviarias en túneles mediante modelos numéricos 2D: un caso real en Madrid (España)

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Abstract
In the last few years, different numerical models for the study of railway vibrations in tunnels have been developed. Virtually all of them assume an elastic and linear behaviour of the soil. In this article the influence of soil constitutive model is investigated, comparing an “advanced” model of the soil called Hardening Soil model with small-strain stiffness (HSsmall) and the Mohr-Coulomb (MC) model. Moreover, the influence of soil stiffness has been studied when this is considered in the range of small strains (E50) or in the range of very small strains (E0). These models have been applied to a real case through a 2D finite element model formulated in the time domain, where it is concluded that both soil stiffness and the amplitude of the maximum tangential strain are the most important geotechnical parameters when estimating the deformational parameters and the constitutive models of the soil most adequate for the study of railway vibrations in tunnels.

Keywords: advanced soil constitutive model; soil stiffness; railway vibrations; tunnel; building

Resumen
En los últimos años, diferentes modelos numéricos han sido desarrollados para el estudio de vibraciones ferroviarias en túneles. Todos ellos han considerado el comportamiento del suelo como elástico y lineal. En este artículo es estudiada la influencia del modelo constitutivo del suelo, comparando un modelo avanzado denominado Hardening Soil con rigidez en pequeñas deformaciones (HSsmall) y el modelo Mohr-Coulomb (MC). Además, la influencia de la rigidez del suelo ha sido estudiada cuando ésta es considerada en el rango de pequeñas deformaciones (E50) o en el rango de muy pequeñas deformaciones (E0). Estos modelos han sido aplicados a un caso real mediante un modelo numérico de elementos finitos 2D formulado en el dominio del tiempo, en el que se concluye que tanto la rigidez del suelo como la amplitud de la deformación tangencial alcanzada en el suelo son los parámetros geotécnicos más importantes cuando se estiman los parámetros deformacionales y el modelo constitutivo del suelo más adecuados para el estudio de vibraciones ferroviarias en túneles.

Palabras Claves: modelo constitutivo avanzado del suelo; rigidez del suelo; vibraciones ferroviarias; túnel; edificio

Introduction

The study of railway vibrations in tunnels has become an issue of first importance for researchers due to the important growth of railway transport systems in the 21th century, mainly in populated areas where the disturbance caused to the inhabitants can become very important. Several numerical models have been developed, notably those proposed by Jones and Hunt (2012), Hussein and Hunt (2006; 2007), Clouteau et al. (2005; 2006), Andersen and Jones (2006), Sheng et al. (2006), Forrest and Hunt (2006a; 2006b), Galvín et al. (2010), Rieckh et al. (2012), all of them formulate in the frequency and wavelength domain. In the time domain the models presented by Gardien and Stuit (2003) and Deng et al. (2006) are noteworthy. All of these models have assumed a soil behaviour model that is either linear elastic or linear elastic perfectly plastic in the case of numerical models formulated in the time domain.

The behaviour of the soil is not linear, showing a degradation of its stiffness in the presence of shear stresses. In the attempt to rigorously model soil behaviour, advanced constitutive models have been developed such as the Hyperbolic model (Duncan & Chang, 1970), the Hardening soil model (Schanz et al., 1999) and, recently, the Hardening soil model with small-strain stiffness (Benz, 2006; Plaxis, 2011), where the initial soil stiffness (E0) is considered in the range of the very small strains ($\gamma<10^{-6}$).

Description of the problem

To the authors’ knowledge, there is no analysis or application of advanced models for the simulation of soil behaviour to the study of railway vibrations in tunnels, while they have been found in cases of high-speed trains (Alves Costa et al., 2010) running on ground surface in which a dynamic model known as Equivalent Linear model has been applied. Also, Hall (2003) and Madshus and Kaynia (2000) have recognized the importance of considering the nonlinear behaviour of the soil and its possible influence on trains running on ground surface. The main objective of this article is to apply the advanced constitutive model Hardening soil model with small-strain stiffness (HSsmall) to a real case of railway vibrations in tunnels in the city of Madrid. In addition, the results are also compared with those resulting from the Mohr-Coulomb model when the stiffness is considered as E0 (very small strains) and as E50 (small strains). The response in the building and on the ground surface is compared and some recommendations are made regarding its application for the study of railway vibrations in tunnels, which could be important for previous design of railway tunnels.

2D finite element method to study of railway vibrations in tunnels

Numerical models used in 2D have been developed in Plaxis software. The numerical solution scheme is the finite element method formulated in the time domain. Finite elements have triangular form, where interpolation polynomial functions are used and provide a second-order interpolation of displacements. The time integration scheme is implicit and the β Newmark method is applied. The values of coefficients α and β have been considered as 0.25 and 0.5 respectively because they involve an unconditionally stable analysis (Bathe, 1982). Finite element size in dynamic analysis formulated in the time domain depends mainly on S-wave velocity and expected frequency in every material. Ten elements per wavelength have been applied in this research, as explained by Unterberger and Hochgatter (1997). Generally, this criterion is only restrictive for ground elements although if any structures, such as concrete tunnel invert and slab track, are represented as continuum element (not as beam or plate), an amount of element is required. A minimum requirement is 3-4 elements in tunnel invert, increasing numerical precision with density mesh (Clouteau et al., 2006; Unterberger & Hochgatter, 1997). While the element size is determined by lower S-wave velocity, time step is imposed by higher P-wave velocity and size element (Unterberger & Hochgatter, 1997). In the context of underground railway vibrations, this implies that the stiffness of tunnel invert determines the time step for the whole mesh, although in ground elements the time step would be larger (Unterberger & Hochgatter, 1997). The following equation may be used for a generic element (Plaxis, 2011):

$$\Delta t_{critical} = \frac{I_e}{\alpha \cdot \sqrt{\frac{E(1-\upsilon)}{\rho(1+\upsilon)(1-2\upsilon)}} \cdot \sqrt{1+\frac{B^4}{4S^2}-\frac{B^2}{2S}\left[1+\frac{1-2\upsilon}{4}\frac{2S}{B^2}\right]}} \quad (1)$$

In the above equation, the term B and S respectively denote the largest dimension of the finite element and the surface area of the finite element. The first root term represents the compression wave velocity. The factor α depends on the element type. Ie is the average length of an element. In a finite element model, the critical time step is equal to the minimum value of Δt according to Eq. (1). The boundary conditions most used in dynamic finite element models are viscous boundaries (absorbent boundaries). The theoretical basis is that incremental stress in boundary would be absorbed without reflection (Plaxis, 2011). Viscous boundaries are implemented in Plaxis software, using Lysmer and Kuhlmeyer's formulation (1969) and are based on Impedance Ratio. The normal and shear stresses absorbed by a damper in x-direction is (Plaxis, 2011):

$$\sigma_n = -C_1 \rho V_p; \ \tau = -C_2 \rho V_s \dot{u}_y \quad (2)$$

Here, ρ is the density of the materials. Vp and Vs are the pressure wave velocity and shear wave velocity, respectively. C1 and C2 are relaxation coefficients that have been introduced in order to improve the effect of the absorption. The experience gained until now shows that the use of C1=1 y C2=0.25 results in a reasonable absorption of waves at the boundary (Plaxis, 2011). It is clear that wave propagation in the ground is a completely three-dimensional phenomenon, whereby a priori it cannot be rigorously studied in 2D plane strain models (Metrikine & Vrouwenvelder, 2000). Nevertheless, Fernández (2014) has proposed an approximate methodology in 2D numerical models, applied in this research, which has shown an acceptable fit between numerical results and real measurements in buildings.

The hardening soil model with small-strain stiffness (HSsmall)
The strain range in which soils can be considered truly elastic is very small. The soil stiffness decays with increasing shear strain. The HSsmall model is based on the Hardening soil model (Schanz et al., 1999; Plaxis, 2011), but considering the complete stress-strain curve including the range from very small strains. The Hardening soil model and the Hardening soil model with small-strain stiffness were explained in detail by Schanz et al. (1999) and Benz (2006) respectively. The main features of the HSsmall model are summarized below. The stress-strain relationship can be simply described from the secant shear modulus of elasticity as follows (Plaxis, 2011):

$$\tau = G_s \gamma = \frac{G_0 \gamma}{1+0.385\frac{\gamma}{\gamma_{0.7}}} \quad (3)$$

The above equation is a modification of the relationship given by Hardin and Drnevich (1972) and proposed by Santos and Correia which is included in software Plaxis (2011). Once the direction of loading is reversed, the stiffness regains a maximum recoverable value which is in the order of the initial soil stiffness (Plaxis, 2011). Then, while loading in the reversed direction is continued, the stiffness decreases again. A strain history dependent, multi-axial extension of the Hardin-Drnevich relationship is therefore needed in order to apply it in the Hardening Soil model. Such an extension has been proposed by Benz (2006) in the form of the small-strain overlay model. Benz derives a scalar valued shear strain γhist by the following projection (Plaxis, 2011):

$$\gamma_{hist} = \sqrt{3}\frac{\|H\Delta e\|}{\|\Delta e\|} \quad (4)$$

Where Δe is the actual deviatoric strain increment and H is a symmetric tensor that represents the deviatoric strain history of the material. Whenever a strain reversal is detected the tensor H is partially or fully reset before the actual strain increment Δe is added. The scalar value shear strain γ=γhist calculated in Eq. 4 is applied subsequently used in Eq. 3 (Plaxis, 2011). In this way Masing's rules are applied within this material model. This stiffness reduction curve reaches far into the plastic material domain. In the HSsmall model, stiffness degradation due to plastic straining is simulated with strain hardening. In this model, the small-strain stiffness reduction curve is therefore bounded by a certain lower limit, determined by conventional laboratory tests (Plaxis, 2011):

- The lower cut-off of the tangent shear modulus Gt is introduced at the unloading-reloading stiffness Gur which is defined by the material parameters Eur and vur (Plaxis, 2011):

$G_t \geq G_{ur}$ where $G_{ur} = \frac{E_{ur}}{2(1+\nu_{ur})}$ and $G_t = \frac{E_t}{2(1+\nu_{ur})}$ (5)

- The cut-off shear strain γcut-off can be calculated as (Plaxis, 2011):

$$\gamma_{cut-off} = \frac{1}{0.385}\left(\sqrt{\frac{G_0}{G_{ur}}} - 1\right)\gamma_{0.7} \quad (6)$$

Within the HSsmall model, the quasi-elastic tangent shear modulus is calculated by integrating the secant stiffness modulus reduction curve over the actual shear strain increment. The HSsmall model applied to dynamic problems shows hysteretic damping in cyclic loading. The amount of hysteretic damping depends on the amplitude of the applied load and the corresponding strain amplitudes. This fact means that it is not necessary, with the use of this model, a Rayleigh damping (dependent on frequency), so that the phenomenon of damping is more real (Plaxis, 2011). However, it has been shown that in the case of very small strains this model exhibits small damping due to the fact that the hysteresis loops of cyclic shear strain cover a very small area (Brinkgreve et al., 2007).
Case study: railway line c-7 in Madrid (Spain)

Figure. 1. Train "Serie 446 Renfe". Source: Self-elaboration

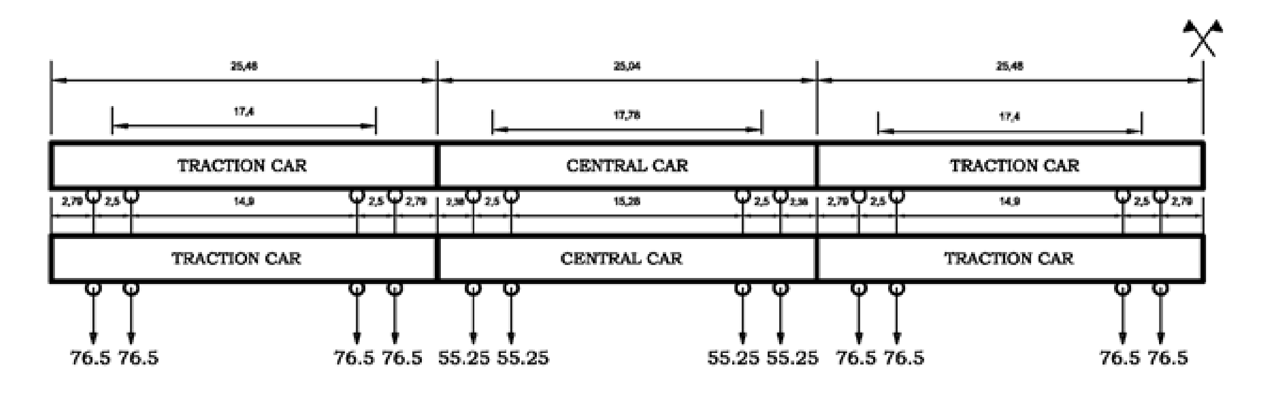


Figure 2. Cross section of the study case (in metres). Source: Self-elaboration

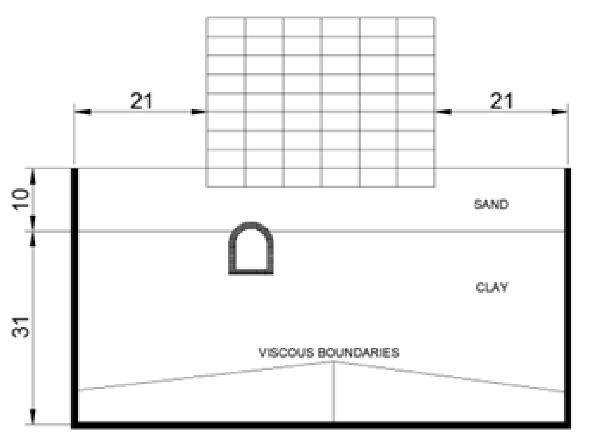
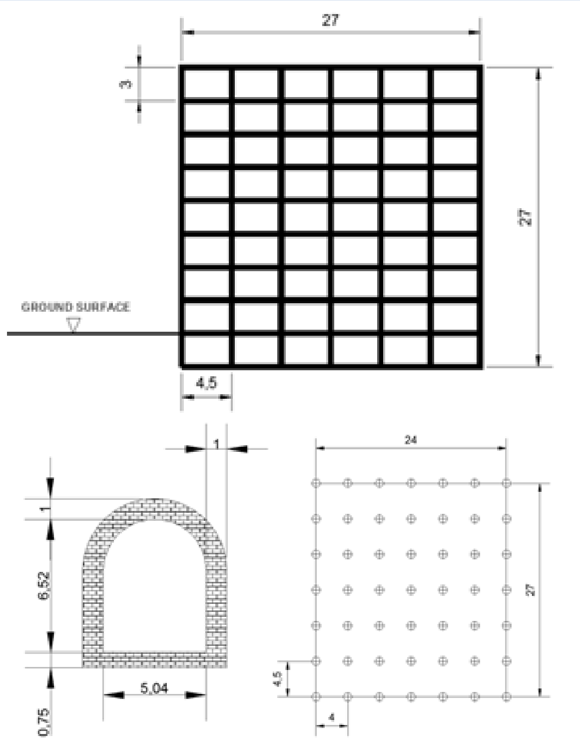


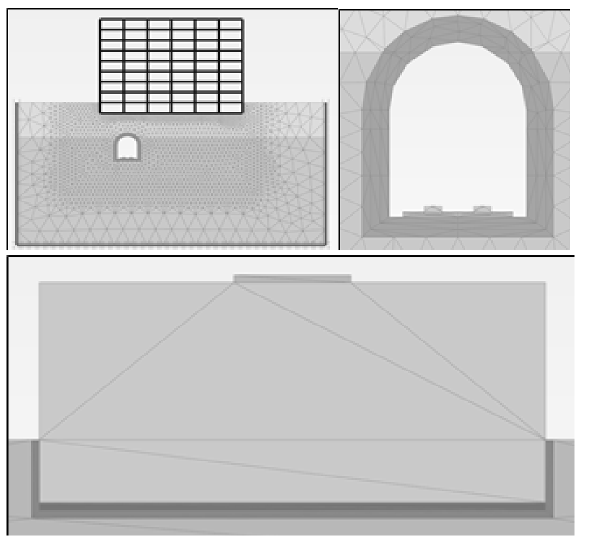
Figure 3. PTunnel section, building floor plan and building cross-section (in metres). Source: Self-elaboration



Description

Two real cases have been studied in the Madrid railway line C-7, due to complaints of the inhabitants of the building located above tunnel. The measurements have been taken on rail, slab, tunnel invert and 5th floor of building. All measurements have been carried out with trains in operation, where that train type is the same in the two cases, varying the speed between them. The train type is "serie 446 Renfe" (Fig. 1). The vibrations induced by several trains have been measured, two of which have been selected for this article: Train 818: speed 52 km/h and Train 812: speed 43 km/h. The scheme of the analyzed section is shown in Fig. 2, where the mesh size used in finite element model is bounded. The railway track is a slab track with Stedef twin block system, with double damping through rail pad and microcellular pad under block. The rail is UIC-54, the separation between blocks is 0.6 m., its geometric section is shown in Fig. 3 and the 2D numerical model is shown in Fig. 4.

Figure 4. Variation of the EI modulus of the pre slab 2. Source: Self-elaboration, 2008.



Since the maximum frequency of the acceleration of vibration in building is around 50 Hz, this has been chosen as characteristic frequency to determine the size of finite elements. Thus, the maximum dimension of the element in the soil would be 1 meter, while in the other materials there is no size restriction since the actual dimension is less than required. The time step has been 2.5x10-5 s., which in this case is determined by the block elements, having a maximum dimension of 0.08 meters.

Table. 1. Characteristic of railway track and tunnel invert in Madrid railway line C-7 (developed by the authors)

PARAMETERS	RAILPAD	BLOCK	MICROCELLULAR PAD	RUBBER BOOT	SLAB	TUNNEL INVERT
Mechanical behaviour γ (kN/m3)	Linear-Elastic 9.5	Linear-Elastic 25	Linear-Elastic 9.5	Linear-Elastic 9.5	Linear-Elastic 25	Linear-Elastic (masonry) 25
E' (kN/m2)	71.43x103 (Dynamic stiffness = 200 kN/mm)	30x106	3.07x103 (Dynamic stiffness = 40 kN/mm)	22.22x103 (Dynamic stiffness = 40 kN/mm)	30x106	5x106
v'	0	0.2	0	0	0.2	0.2

Material properties

The strain-stress parameters of track and tunnel invert are summarized in Table 1. The building has seven floors and its properties are summarized in Table 2. The geotechnical profile is constituted by two soils:

-Sand (locally called "Arena de miga"): from the surface to 10 metres of depth.

-Low to medium plasticity clay (locally called "Tosco"): from 10 metres of depth to indefinite depth.

The stress-strain parameters of the soils are shown in Table 3, for both the Mohr-Coulomb model and the HSsmall model.

Table. 2. Building characteristics. Source: Self-elaboration

PARAMETERS	FLOORS	COLUMNS	UNDERGROUND BUILDING WALLS
Structural element	Plate	Plate	Plate
Mechanical behaviour	Linear-Elastic	Linear-Elastic	Linear-Elastic
γ (kN/m3)	25	25	25
Thickness (m)	0.3	0.35	0.4
Dimensions (m)	-	0.35x0.35	-
E (kN/m2)	30x106	30x106	30x106
v	0.2	0.2	0.2
G (kN/m2)	12.5x106	12.5x106	12.5x106
I (m4)	2.25x10-3	1.25x10-3	5.33x10-3

The soil parameters such as γap, E50, ϕ' and c' have been obtained from Sanhueza and Oteo (2009) and according to Rodriguez Ortiz (2000). To estimate the values of E0, G0, the relationships given by Alpan (1970), by Vucetic and Dobry (1991) and by Ishibashi (1992) have been used, resulting in similar values. The "m" parameter has been set by the authors in order to find a good fit with the experimental values that were shown by Sanhueza and Oteo (2009) and by Rodríguez Ortiz (2000), since these correspond with a large number of real measurements through triaxial and pressuremeter tests. The material damping has been estimated through Rayleigh damping and the considered values are shown in Table 4. Frequency range has been between 0.5-50 Hz in order to take into account the main expected frequencies in the soil and in the building. In concrete (block, slab and building) and masonry elements it has been estimated in line with Newmark and Hall (1982). The soil damping has been estimated according to formulations given by Ishibashi and Zhang (1993), depending on maximum estimated shear strains reached, and in the elastomeric materials (rail pad, microcellular pad and rubber boot) it has been estimated in line with Maes and Guillaume (2006) and Thompson and Verheij (1997). The Rayleigh damping curve for sand and clay is shown in Fig. 5. It is possible check that the damping has been selected in order to obtain a value of 1-3% within range of interest of expected frequencies.

Table 3. Soil parameters. Source: Self-elaboration

PARAMETERS	SAND ("ARENA DE MIGA")	CLAY ("TOSCO")
γ_{ap} (kN/m3)	20	21
E_{50} (kN/m2)	60x103+2000z	150x103+4000x(z-10)
E_0 (kN/m²)	320x103+6000z	550x103+8000x(z-10)
v_{ur}	0.2	0.2
v'	0.3	0.3
φ'	35º	30º
c' (kN/m²)	5	30
K_0	0.4264	0.5
E_{50}^{ref} (kN/m²)	80x103	150x103
E_{oed}^{ref} (kN/m²)	80x103	150x103
E_{ur}^{ref} (kN/m²)	160x103	300x103
m	0.1	0.3
K_0	0.4264	0.5
G_o^{ref} (kN/m²)	158.3x103	229.2x103
$\gamma_{0.7}$	2x10-4	2x10-4
p_{ref} (kN/m²)	85.28	85.28
φ'	35º	30º
c' (kN/m²)	5	30

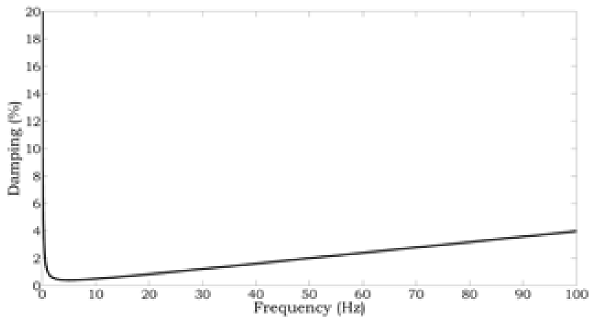
Train loads

In order to compare more accurately the numerical results obtained through the different models of behaviour of the soil, the real train load (not the quasi-static load) has been considered, as the main objective of this paper is to show the influence of the use of advanced soil constitutive models and models parameters to the study of railway vibrations in tunnels. Since real acceleration rail measurements are available and that in Plaxis software it is possible to introduce loads as a dynamic prescribed displacement, this form has been selected to compare the different models. In addition, we have followed the approximate methodology outlined by Fernandez (2014) for the consideration of the magnitude of the applied train load in 2D numerical models formulated in the time domain. In the numerical model the train load has been applied on the rail pad.

Table 4. Rayleigh damping. Source: Self-elaboration

MATERIAL	DAMPING (%)	FREQUENCY RANGE (Hz)	RAYLEIGH COEFFICIENTS α	RAYLEIGH COEFFICIENTS β
Rail pad/ Microcellular pad	10	0.5-50	0.622	0.0006303
Block/Slab	2.5	0.5-50	0.1555	0.0001575
Rubber boot	5	0.5-50	0.311	0.00031515
Tunnel invert	2.5	0.5-50	0.1555	0.0001575
Sand	2	0.5-50	0.1244	0.000126
Clay	2	0.5-50	0.1244	0.000126
Building elements	2.5	0.5-50	0.1555	0.0001575

Figure 5. Rayleigh damping curve for sand and clay. Source: Self-elaboration



Results and discussion

The numerical results are compared with the real measurements on the 5th floor of the building in the fig. 6 and fig. 7. Moreover, the numerical results are also shown on the ground surface although there are not real measurements at that point. The computational cost in the different numerical models has been higher in the HSsmall model than in the Mohr-Coulomb model. In fact, the computing time has been 4.8 hours/1 second and 1.25 hours/1 second for the HSsmall model and the Mohr-Coulomb model respectively. It is relevant to note that the use of advanced soil constitutive models implies, in the cases studied, a computing time four times higher than simple constitutive models such as the Mohr-Coulomb model.

The numerical results in building show a better fit with the real measurements when the soil is modelled through the HSsmall model than when it is considered the Mohr-Coulomb model with a stiffness value of E50, especially in the time domain. When the stiffness is considered as E0 in the Mohr-Coulomb model, the numerical results fit with an acceptable accuracy with the real measurements and they are similar to those obtained through the HSsmall model, although slightly less accurate. The reason for this latter fact is that in this case the soil stiffness remains substantially unchanged by the wave motion and in the range of very small strains. In general it could be said that the soil stiffness degradation by railway traffic in tunnels might be possible in case of soft soils. In the studied cases, the soils are considered medium stiff and stiff. Considering the error in the maximum acceleration peaks, it can be concluded that in the HSsmall model it is around 15-20% whereas in the Mohr-Coulomb model with stiffness E50 it is 35-45%. In the Mohr-Coulomb model with stiffness E0 the error is around 20-25%. However, in these cases there are not very important differences between the numerical results in the building when the two constitutive soil models are used and when the soil stiffness is considered in small or very small strains. The reason of this fact is that the building could act as a filter.

Arguably, the most accurate numerical results are shown in the HSsmall model, whereas the use of stiffness E50 in the Mohr-Coulomb model provides higher values than the real measurements. In the case of the Mohr-Coulomb model with stiffness E0 the numerical results are slightly lower than the real measurements. Thus, it can be argued that the use of an advanced soil constitutive model does not provide great advantages over the Mohr-Coulomb model in the building when the soil stiffness is considered in very small strains. In general, the most accurate fit of the HSsmall model in comparison with the Mohr-Coulomb model with stiffness E0 does not seem to be worthy in terms of the computing time, since this is 4 times higher and it implies a much higher computational effort. In contrast, the influence of the soil stiffness is very important on the ground surface. In the same way that in the building, the use of the HSsmall model over the Mohr-Coulomb model provides similar numerical results if the soil stiffness is regarded in very small strains, whereas there are large differences in numerical results when the soil stiffness is considered in small strains in the Mohr-Coulomb model. In these cases the peak acceleration on the ground surface is 6 times higher if the soil stiffness is regarded in small strains in the Mohr-Coulomb model than if the HSsmall model is used or the soil stiffness is considered in very small strains according to the Mohr-Coulomb model. The existence of such differences on the ground surface may be owing to the fact that the vibration at these points is only controlled by the soil stiffness while the building vibration is controlled by its own stiffness (which does not change with the different numerical models), in addition of acting as a filter. Moreover, the influence of soil-structure interaction seems to be low, at least in these cases. For soft soils, soil-structure interaction would play a more important role. In the cases studied here, it seems that the best way to consider the soil stiffness in the study of railway vibrations in tunnels is the use of an advanced soil constitutive model, such as the HSsmall model in spite of its high computational cost, which could be acceptable in 2D numerical models but which is unacceptable in 3D numerical models nowadays. To minimize the computational cost, the Mohr-Coulomb model could be used with soil stiffness in very small strains if there were not soft soils. In the case of soft soils, the use of advanced constitutive models may reasonably be more accurate because soil stiffness degradation could occur. This fact would be more relevant in soils surrounding the tunnel.

Figure 6. Vertical acceleration train 818 (a, b, c, d, e and f: 5th floor; g and h: ground surface). Source: Self-elaboration

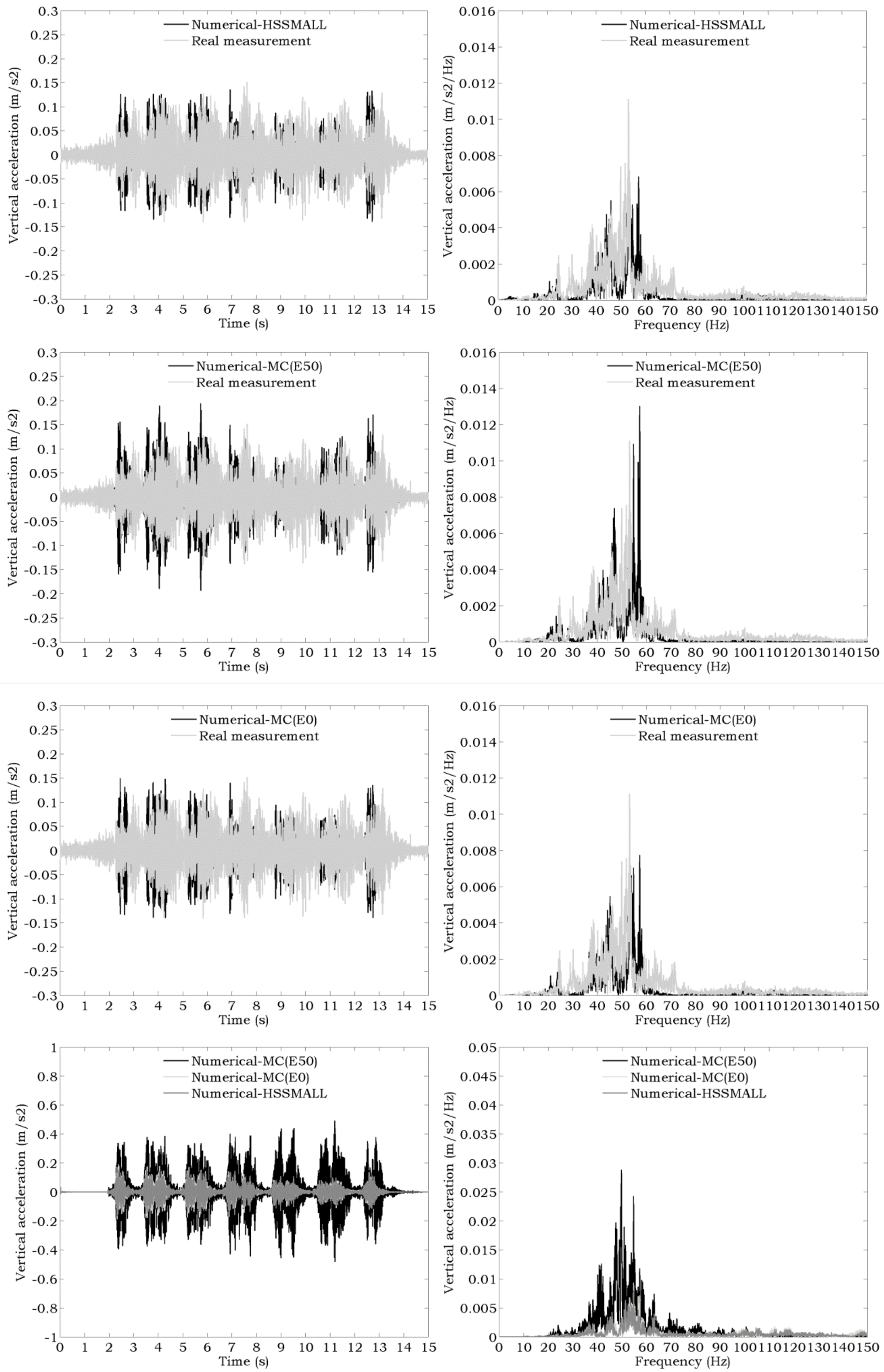
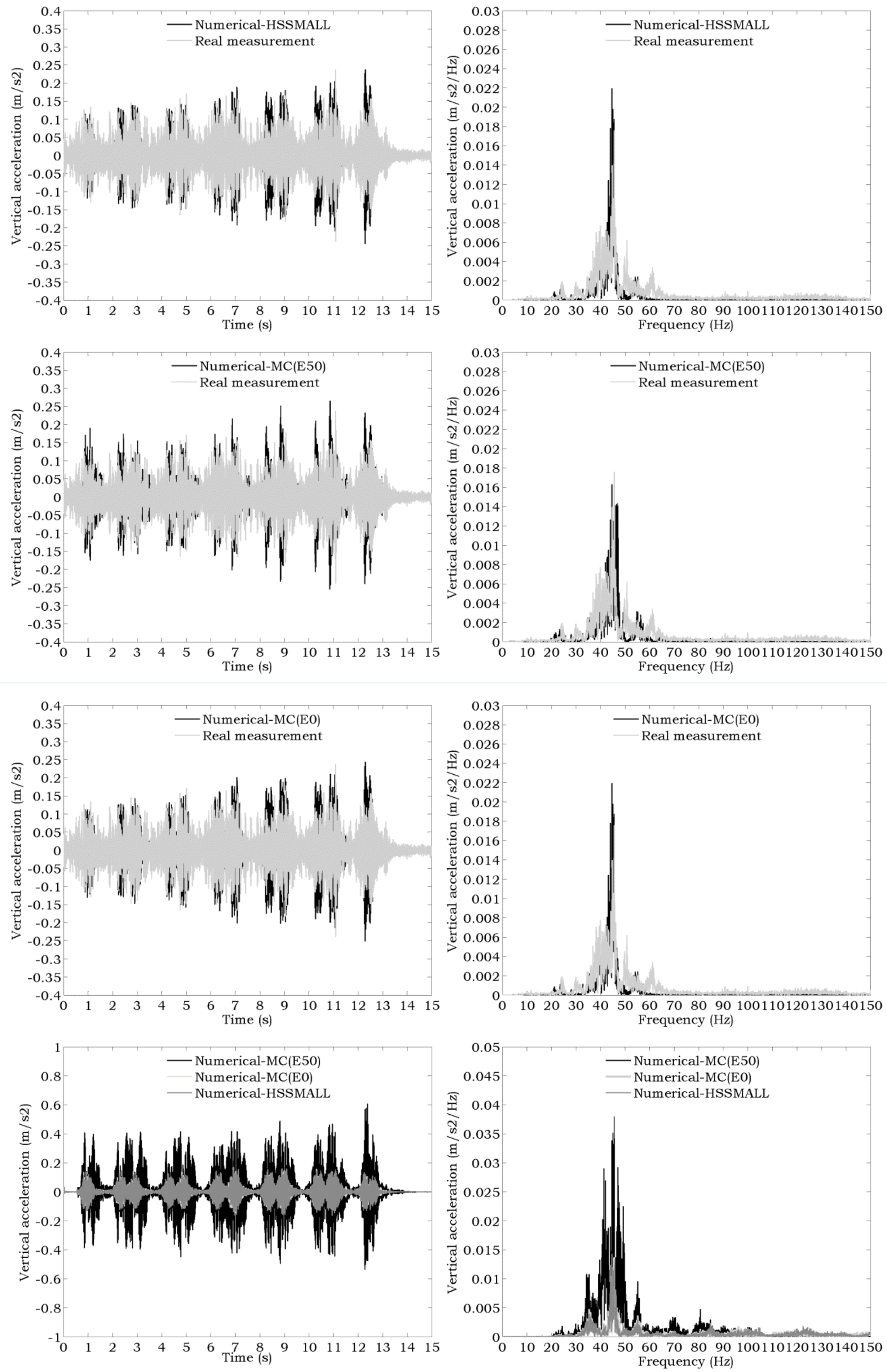


Figure 7. Wood-Concrete slab modeled with beam elements. Source: Self-elaboration, 2008..



Conclusions

This article has shown the influence of an advanced soil constitutive model called HSsmall model in the study of railway vibrations in tunnels, showing a better fit with real data than the results provided by the Mohr-Coulomb model with stiffness E50. However, when the Mohr-Coulomb model is considered with stiffness E0, the use of the HSsmall model does not provide a clear advantage because the results between them are quite similar in the 2 cases studied. Then, it would be possible to conclude that the use of advanced soil constitutive models in the study of railway vibrations in tunnels offers little advantage over the Mohr-Coulomb model, as long as soil stiffness is considered in very small strains and there are not soft soils.

Concerning the consideration of soil stiffness in the range of small strains or very small strains, it is noteworthy that no extremely significant differences have been found in the building but very significant ones on the ground surface. It is likely that there would be more differences in cases of soft soils if soil stiffness were considered in the range of small or very small strains, since in these cases the soil-structure interaction would play a more relevant role. The authors, in the light of the results, think that the use of advanced soil constitutive models could have more influence in the study of railway vibrations in tunnels in soft soils and, in general, with trains running on the surface, where the computational cost could be justified.

Although it seems that the most appropriate soil stiffness to study railway vibrations in tunnels is in the range of very small strains and this has been shown in comparison with real measurements in buildings, the authors believe that the use of stiffness E50 could be adequate due to during the process of tunnelling is produced a relevant degradation of Gs (shear strains γ ≈10-4-10-3) although this topic should be investigated in another research because it has not been taken into account in this article.

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