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The Role of an Imitating Firm in a Dynamic Context

Francisco Martínez-Sánchez

–Introduction. –I. The model. –II. The case of monopoly. –III. The case of duopoly. –IV. Comparative static. –Conclusion. –References.

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Resumen: *se analizan los efectos de una empresa imitadora en el comportamiento de una empresa innovadora. La estructura del análisis usada es un modelo de duopolio dinámico de diferenciación de producto vertical, donde tanto el innovador como el imitador compiten simultáneamente en precio y calidad. Se obtiene, que cuando el mercado es pequeño, la presencia de un imitador estimula al innovador a aumentar su proceso de innovación, de modo que la entrada de un imitador no debería ser obstruida; y cuando el mercado es grande, el imitador reduce los incentivos del innovador a invertir sus recursos en I+D y a producir una mayor calidad, de modo que la entrada del imitador no debería ser fomentada.*

Palabras Clave: *líder, imitador, calidad, diferenciación del producto y consumidor.*
Clasificación JEL: L11, L13, L15.

Abstract: *We analyze the effects of an imitating firm on the behavior of the innovating firm. The framework of analysis used is a dynamic duopoly model of vertical product differentiation, where both the innovator and the imitator compete simultaneously in price and quality. We obtain that when the market is small, the presence of an imitator encourages the innovator to increase its innovating process, so the entry of the imitator should not be obstructed; and, when the market is large, the imitator reduces the innovator's incentives to invest its economic resources in R&D and to provide a higher quality, so the entry of the imitator should not be encouraged.*

Keywords: *Leader, Imitator, Quality, Differentiation of Product, Consumer. JEL classification: L11, L13, L15.*

Résumé: *On analyse les effets entrainés par une entreprise imitatrice sur le comportement d'une entreprise innovatrice. Pour ce faire, on utilise un modèle dynamique de duopole de différenciation du produit. Dans ce modelé, autant l'entreprise innovatrice que l'entreprise imitatrice sont en concurrence à la fois sur le prix et la qualité d'un produit. Les résultats montrent que lorsque le marché est petit, l'existence d'une entreprise imitatrice encourage l'entreprise innovatrice à augmenter son processus d'innovation, de telle sorte que l'entrée d'un imitateur ne doit pas être interdite. En revanche, lorsque le marché est grand, l'entreprise imitatrice réduit les motivations de l'entreprise innovatrice à investir ses ressources en Recherche et Développement (R+D) et l'incite à augmenter la qualité de son produit, de telle sorte que l'entrée de l'entreprise imitatrice ne devrait pas être encouragée.*

Mots clef: *Leader, Imitateur, Qualité, Différenciation du Produit, Consommateur. Classification JEL: L11, L13, L15.*

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Introduction

Over the past few years, the topic of imitation has been triggering intensive debate, especially in those areas whose productive processes are closely linked to the development of new technology.

In this paper we analyze the effects of an imitating firm on the behavior of the innovating firm, with the aim of complementing the theses by Schumpeter (1942), and Arrow (1962). In 1942, Schumpeter claimed that the greater the dimension of a firm, the more it invests in R&D, since the larger firms have a easier access to capital markets, face a lower risk, and have economies of scale on R&D. Arrow (1962), on the other hand, made it quite clear that, in many situations, the greater the power of the market, the lower the incentives to invest in research.

The framework of analysis used is a dynamic duopoly model of vertical product differentiation, where both the innovator and the imitator compete simultaneously in price and quality. This framework has been often applied to study the topic of innovation. Bonanno and Haworth (1998) analyze the profit incentive in Cournot and Bertrand duopolies for different degrees of vertical product differentiation. They find that the incentive to introduce

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a cost-reducing innovation is stronger for a Cournot competitor. The same question is faced by Bester and Petrakis (1993), but now under a model of horizontal product differentiation. They show that Cournot competition provides a stronger incentive to innovate that Bertrand competition if the degree of substitutability is low and a weaker incentive if this degree is high. Belleflamme and Vergari (2006), in an oligopoly model of horizontal differentiation, show that different measures of competition (number of firms, degree of product differentiation, kind of competition: Bertrand vs. Cournot) affect incentives to innovate in different ways.

Here we do not consider the possibility of investing in cost-reducing innovation by the firms because we want to focus in product innovation in a dynamic context. In particular, we want to know how change the quality decision of innovating firm when there exist an imitating firm.

The main result is that when the market is small, the presence of an imitator encourages the innovator to provide a higher quality, so that the entry of the imitator should not be obstructed; and, when the market is large, the imitator reduces the innovator's incentives to invest its economic resources in R&D and to provide a higher quality, so the entry of the imitator should not be encouraged.. This is because of the competition is more aggressive when the market is small and there is an imitator, so the innovator finds profitable to invest more in R&D for raising his market power.

The rest of the paper is organized as follows. Section 2 describes the model formally. Section 3 analyzes the case of monopoly. Section 4 analyzes the case of duopoly. Section 5 analyzes the quality decision of innovating firm.

I. The model

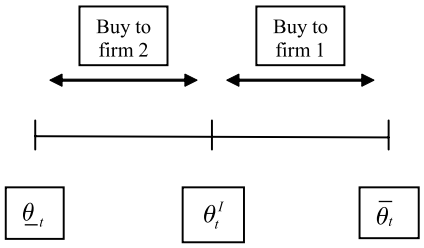
We develop a dynamic model with two periods. We consider two firms: firm 1—the innovator—that improves the quality of its product through investment in R&D, and firm 2—the imitator—that mimics the innovations of firm 1 with one-period lag.

We assume that the quality of the product of each firm will be, at least, as good as it was in the previous period, and the firms can not produce two products in the same period. Moreover, firm 2 cannot surpass the quality obtained by the innovating firm in the previous period.

We assume that consumers only live one period. In each period there is a continuum of consumers indexed by θ_t, $\theta_t \in \left[\underline{\theta}_t, \overline{\theta}_t\right]$, where θ_t, is assumed to follow a uniform distribution, and represents the difference in the consumer's

tastes for the quality of the product. Each consumer is assumed to buy only one unit of the good. For simplicity, we also assumed that all consumers buy a good just once, i.e. the market is covered[1]. The utility of a type θ_t, following Mussa and Rosen (1978), is $q_{jt}\theta_t - p_{jt}$ if he buys from firm j, where p and q represent the price and quality of the goods, and zero otherwise.

Firm's demand functions are obtained as follows. Let θ_t^d be the consumer, in period t, which is indifferent between buying from either firm 1 and 2. From utility function, $\theta_t^d=(p_{1t}-p_{2t})/(q_{1t}-q_{2t})$. A consumer with an index greater than will prefer to buy from firm 1 rather than from firm 2, and consumers with an index lower than θ_t^d will prefer to buy from firm 2 rather than from firm 1. Thus, as the two firms cover the market, the demand for each firm's product is graphically illustrated in Figure 1 and is given by equation (1):[2]



Source: cálculos del autor

Figure 1. *Duopoly*

$$D_{1t}=\overline{\theta}_t-\frac{p_{1t}-p_{2t}}{q_{1t}-q_{2t}},\quad D_{2t}=\frac{p_{1t}-p_{2t}}{q_{1t}-q_{2t}}-\underline{\theta}_t \tag{1}$$

When either firm decides to improve the quality of its goods, it incurs a certain cost. Specifically, if firm j increases the quality of its product in period t up to level $q_{jt}>0$ it incurs a cost, according to the cost function $C_{jt}(q_{jt},q_{j,t-1})=(q_{jt}-q_{j,t-1})^2$. Thus, the profit faced by each firm is defined by $\pi_{jt}(\cdot)=p_{jt}D_{jt}(\cdot)-C_{jt}(\cdot)$. Like Ronnen (1991) and Motta (1993), we consider that the cost incurred by each firm is a quality-improvement fixed cost.

1 Under the assumption that the market is covered, we have that $D_{1t}+D_{2t}=\overline{\theta}_t-\underline{\theta}_t$, where D_{1t} and D_{2t} denote the demand functions of firms 1 and 2 at period t, respectively.

2 Let θ_t^2 be the consumer, in period t, who is indifferent between buying from firm 2 and not buying at all. Since we assume the market is covered in duopoly, a sufficient condition is $\theta_t^2<\underline{\theta}_t$.

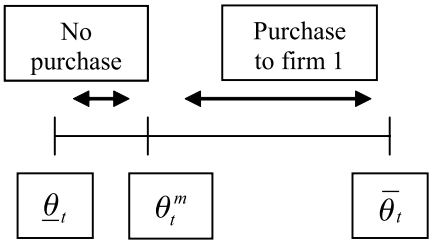
The two-period game is the following. In each period, each firm competes simultaneously in quality and price. Then, consumers decide to buy product 1 or 2 after they have observed firms' prices.

Before analyzing the case of duopoly, we consider a benchmark model with a single firm: the case of monopoly. In the next sections we look for the subgame perfect equilibrium (*SPE*) of the game by backward induction.

II. The case of monopoly

The innovator's demand is obtained as follows. Let θ_t^m be the consumer that is indifferent between buying from firm 1 and not buying at all during period t, that is $\theta_t^m = p_{1t}/q_{1t}$. A consumer with an index higher than θ_t^m will prefer to buy from firm 1 rather than not buy, and consumers with an index lower than θ_t^m will prefer not to buy the innovator's product. We consider the market in monopoly is uncovered, i.e. at least a consumer decides not to buy the good, so a sufficient condition is $\underline{\theta} < \theta_t^m$. Therefore, the demand for the innovating firm's product, which is represented in figure 2, will be:

$$D_{1t}^m = \bar{\theta}_t - \theta_t^m \tag{2}$$



Source: Cálculo del autor

Figure 2. *Monopoly*

The profit function for innovating firm at period 2 is $\pi_{12}(\cdot) = p_{12} D_{12}^m - (q_{12} - q_{11})^2$. From maximizing it we get the following first-order conditions:

$$\begin{aligned} p_{12}^2 &= 2(q_{12} - q_{11}) q_{12}^2 \\ \bar{\theta}_2 &= \tfrac{2p_{12}}{q_{12}} \end{aligned} \tag{3}$$

On solving the system of equations (3) we obtain the interior solutions of period 2:

$$p_{12}=\frac{\overline{\theta}_{12}}{2}\left(q_{11}+\frac{\overline{\theta}_2^2}{8}\right),\ q_{12}=q_{11}+\frac{\overline{\theta}_2^2}{8} \tag{4}$$

The joint profit function is $\pi_1(\cdot)=\pi_{11}(\cdot)+\pi_{12}(\cdot)$. We incorporate solution of period 2 equations (4) in the joint profit. Then we maximize it, and get the following first order conditions:

$$\begin{aligned}&\frac{p_{11}^2}{q_1^2}+\frac{\overline{\theta}_2^2}{4}=2\left(q_{11}-q_{10}\right)\\&\overline{\theta}_1=\frac{2p_{11}}{q_1}\end{aligned} \tag{5}$$

On solving the system of equations (5) we obtain the following interior solutions for period 1:

$$q_{11}^m=q_{10}+\frac{\overline{\theta}_1^2+\overline{\theta}_2^2}{8},\ p_{11}^m=\frac{\overline{\theta}_1}{2}\left(q_{10}+\frac{\overline{\theta}_1^2+\overline{\theta}_2^2}{8}\right) \tag{6}$$

By incorporating the solutions of period 1 equations (6) in the solutions of period 2 equations (4), we obtain the solutions of period 2 in terms of q_{10}, $\overline{\theta}_1$ and $\overline{\theta}_2$.

$$q_{12}^m=q_{10}+\frac{\overline{\theta}_1^2}{8}+\frac{\overline{\theta}_2^2}{4},\ p_{12}^m=\frac{\overline{\theta}_2}{2}\left(q_{10}+\frac{\overline{\theta}_1^2}{8}+\frac{\overline{\theta}_2^2}{4}\right) \tag{7}$$

We can observe how changes in the size of the market affect both quality and price in both periods, independently of the period it occurs. In particular, the appearance of highly sensitive consumers positively affects both variables.

A. Analysis of equilibrium

The values for the demand and the indifferent consumers in equilibrium are the following:

$$\theta_1^m=\frac{\overline{\theta}_1}{2},\ D_{11}^m=\frac{\overline{\theta}_1}{2},\ \theta_2^m=\frac{\overline{\theta}_2}{2},\ D_{12}^m=\frac{\overline{\theta}_2}{2}. \tag{8}$$

Notice that when highly sensitive consumers appear in period t, the demand for the innovating firm's product increases, but only in that period. From this result, we can express the values for the different levels of qualities and prices as follows:

$$\begin{aligned}&p_{11}^m=q_{11}^m D_{11}^m,\quad q_{11}^m=q_{10}+\frac{\left(D_{11}^m\right)^2+\left(D_{12}^m\right)^2}{2},\\&p_2^m=q_{12}^m D_{12}^m,\quad q_{12}^m=q_{10}+\frac{\left(D_{11}^m\right)^2}{2}+\left(D_{12}^m\right)^2.\end{aligned} \tag{9}$$

As we can observe, the demand for the innovator's product positively affects its quality in both periods. Moreover, its price in period t positively depends on its quality and on its demand during that period.

III. The case of duopoly

In this section we look for the *SPE* in the model described in Section 2, i.e. when there is an imitating firm. The profit function of firm 1 at period 2 is $\pi_{12}(\cdot)= p_{12}D_{12}^{d} - (q_{12} - q_{11})^2$. From maximizing it we obtain the first-order conditions:

$$\frac{p_{12}\,(p_{12} - p_{22})}{(q_{12} - q_{22})^2} = 2(q_{12} - q_{11})$$

$$\overline{\theta}_2 = \frac{2p_{12} - p_{22}}{q_{12} - q_{22}} \tag{10}$$

The profit function of firm 2 at period 2 is $\pi_{22}(\cdot)= p_{22}\, D_{22}^{d} - (q_{22} - q_{21})^2$. From maximizing it we obtain the first-order conditions:

$$\frac{p_{22}(p_{12} - p_{22})}{(q_{12} - q_{22})^2} = 2(q_{22} - q_{21})$$

$$\underline{\theta}_2 = \frac{p_{12} - 2p_{22}}{q_{12} - q_{22}} \tag{11}$$

Solving the system of equations consists of the first-order conditions (10) and (11), we obtain the solutions of period 2, which are:

$$q_{12} = q_{11} + \frac{(\overline{\theta}_2 + \underline{\theta}_2)(2\overline{\theta}_2 - \underline{\theta}_2)}{18},\ p_{12} = \frac{2\overline{\theta}_2 - \underline{\theta}_2}{3}\left(q_{11} - q_{21} + \frac{(\overline{\theta}_2 + \underline{\theta}_2)^2}{18}\right),$$

$$q_{22} = q_{21} + \frac{(\overline{\theta}_2 + \underline{\theta}_2)(\overline{\theta}_2 - 2\underline{\theta}_2)}{18},\ p_{22} = \frac{\overline{\theta}_2 - 2\underline{\theta}_2}{3}\left(q_{11} - q_{21} + \frac{(\overline{\theta}_2 + \underline{\theta}_2)^2}{18}\right). \tag{12}$$

In period 1, each firm maximizes its joint profit taking into account the optimal decisions of the second period. The first-order conditions of firm 1 are the following:

$$\frac{p_{11}(p_{11}-p_{21})}{(q_{11}-q_{21})^2}+\left(\frac{2\bar{\theta}_2-\underline{\theta}_2}{3}\right)^2=2(q_{11}-q_{10})$$
$$\bar{\theta}_1=\frac{2p_{11}-p_{21}}{q_{11}-q_{21}} \tag{13}$$

And the first-order conditions of firm 2 are:

$$\frac{p_{21}(p_{11}-p_{21})}{(q_{11}-q_{21})^2}-\left(\frac{\bar{\theta}_2-2\underline{\theta}_2}{3}\right)^2=2q_{21}$$
$$\underline{\theta}_1=\frac{p_{11}-2p_{21}}{q_{11}-q_{21}} \tag{14}$$

Solving the system of equations (13) and (14) we obtain the solutions of period 1:

$$q_{11}^d=q_{10}+\frac{(\bar{\theta}_1+\underline{\theta}_1)(2\bar{\theta}_1-\underline{\theta}_1)}{18}+\frac{(2\bar{\theta}_2-\underline{\theta}_2)^2}{18}$$
$$q_{21}^d=\frac{(\bar{\theta}_1+\underline{\theta}_1)(\bar{\theta}_1-2\underline{\theta}_1)}{18}-\frac{(\bar{\theta}_2-2\underline{\theta}_2)^2}{18}$$
$$p_{11}^d=\frac{2\bar{\theta}_1-\underline{\theta}_1}{3}\left(q_{10}+\frac{(\bar{\theta}_1+\underline{\theta}_1)^2}{18}+\frac{(2\bar{\theta}_2-\underline{\theta}_2)^2+(\bar{\theta}_2-2\underline{\theta}_2)^2}{18}\right) \tag{15}$$
$$p_{21}^d=\frac{\bar{\theta}_1-2\underline{\theta}_1}{3}\left(q_{10}+\frac{(\bar{\theta}_1+\underline{\theta}_1)^2}{18}+\frac{(2\bar{\theta}_2-\underline{\theta}_2)^2+(\bar{\theta}_2-2\underline{\theta}_2)^2}{18}\right)$$

By incorporating the solutions of period 1 equations (15) in the solutions of period 2 equations (12), we obtain the solutions of period 2 in terms of q_{10}, $\bar{\theta}_1$, $\underline{\theta}_1$, $\bar{\theta}_2$ and $\underline{\theta}_2$.

$$q_{12}^d=q_{10}+\frac{(\bar{\theta}_1+\underline{\theta}_1)(2\bar{\theta}_1-\underline{\theta}_1)}{18}+\frac{3\bar{\theta}_2(2\bar{\theta}_2-\underline{\theta}_2)}{18}$$
$$q_{22}^d=\frac{(\bar{\theta}_1+\underline{\theta}_1)(\bar{\theta}_1-2\underline{\theta}_1)}{18}+\frac{3\underline{\theta}_2(\bar{\theta}_2-2\underline{\theta}_2)}{18}$$
$$p_{12}^d=\frac{2\bar{\theta}_2-\underline{\theta}_2}{3}\left(q_{10}+\frac{(\bar{\theta}_1+\underline{\theta}_1)^2}{18}+\frac{\bar{\theta}_2^2+\underline{\theta}_2^2-\bar{\theta}_2\underline{\theta}_2}{3}\right)$$
$$p_{22}^d=\frac{\bar{\theta}_2-2\underline{\theta}_2}{3}\left(q_{10}+\frac{(\bar{\theta}_1+\underline{\theta}_1)^2}{18}+\frac{\bar{\theta}_2^2+\underline{\theta}_2^2-\bar{\theta}_2\underline{\theta}_2}{3}\right)$$

A. Analysis of equilibrium

The values for demands and for the indifferent consumers in equilibrium are the following:

$$\begin{aligned} D_{11}^d &= \tfrac{2\bar{\theta}_1-\underline{\theta}_1}{3}, \quad D_{21}^d = \max\left\{0, \tfrac{\bar{\theta}_1-2\underline{\theta}_1}{3},\right\} \\ D_{12}^d &= \tfrac{2\bar{\theta}_2-\underline{\theta}_2}{3}, \quad D_{22}^d = \max\left\{0, \tfrac{\bar{\theta}_2-2\underline{\theta}_2}{3},\right\} \\ \theta_1^d &= \tfrac{\bar{\theta}_1+\underline{\theta}_1}{3}, \qquad \theta_2^d = \tfrac{\bar{\theta}_2+\underline{\theta}_2}{3}. \end{aligned} \tag{16}$$

To ensure that the imitating firm is always active, we suppose from now on that $\bar{\theta}_t \geq 2\underline{\theta}_t \ \forall t = 1,2$. This inequality could be interpreted as the minimum market size required for an imitating firm to exist. These results can be expressed as follows:

$$\begin{aligned} q_{11}^d &= q_{10} + \tfrac{\theta_1^d D_{11}^d}{2} + \tfrac{(D_{12}^d)^2}{2}, \ q_{21}^d = \tfrac{\theta_1^d D_{21}^d}{2} - \tfrac{(D_{22}^d)^2}{2}, \ p_{11}^d = (q_{11}^d - q_{21}^d) D_{11}^d, \\ q_{12}^d &= q_{10} + \tfrac{\theta_1^d D_{11}^d}{2} + \tfrac{\bar{\theta}_2 D_{12}^d}{2}, \ q_{22}^d = \tfrac{\theta_1^d D_{21}^d}{2} + \tfrac{\underline{\theta}_2 D_{22}^d}{2}, \ p_{12}^d = (q_{12}^d - q_{22}^d) D_{12}^d, \\ p_{21}^d &= (q_{11}^d - q_{21}^d) D_{21}^d, \quad p_{22}^d = (q_{12}^d - q_{22}^d) D_{22}. \end{aligned}$$

As we can show, the demand for either firm's product positively affects the quality of its product in both periods, except in the case of firm 2, in which D_{22} affects negatively q_{21}, due to the fact that a greater demand for its product of low quality in the future implies a curb in the improvement in current quality, in an effort to reduce costs and achieve greater profits. Moreover, the demand of either firm's product only affects its own quality and price but not those of its rival. We also observe that the prices of the firms in each period positively depend on her demand and on the difference of quality of products in each period.

The following table summarizes the evolution of the levels of qualities and prices with respect to the relevant parameters in the model, i.e. the lower and upper bounds of agents' taste, and the market size in both periods. The symbols employed should be interpreted as follows:

(a) The symbol + represents that a variable increases when a parameter increases.

(b) The symbol - represents that a variable decreases when a parameter decreases.

(c) The symbol * means that the sign is true under the assumption $\overline{\theta}_t \geq 2\underline{\theta}_t$.
(d) Finally, ? stands for the case in which the sign can not be determined.

Table 1. *The evolution of the levels of qualities and prices with respect to the relevant parameters in the model*

SIGN ±	$\overline{\theta}_1$	$\overline{\theta}_2$	$\underline{\theta}_1$	$\underline{\theta}_2$	$\overline{\theta}_1 - \underline{\theta}_1$	$\overline{\theta}_2 - \underline{\theta}_2$
q_{11}^d	+	+	$+^*$	−	+	+
q_{21}^d	+	$-^*$	−	$+^*$	+	$-^*$
q_{12}^d	+	+	$+^*$	−	+	+
q_{22}^d	+	+	−	$\begin{cases} + & if\ \overline{\theta}_2 > 4\underline{\theta}_2 \\ - & if\ \overline{\theta}_2 > 4\underline{\theta}_2 \end{cases}$	+	+
p_{11}^d	$+^*$	+	?	$-^*$	+	+
p_{21}^d	$+^*$	$+^*$	$-^*$	$-^*$	+	$+^*$
p_{12}^d	+	+	+	$-^*$	+	−
p_{22}^d	$+^*$	$+^*$	$+^*$	$-^*$	$+^*$	$+^*$

Source: author's estimations.

We observe that the appearance of new consumers who are more sensitive in period 1 implies a quality improvement of both firms' products in both periods, similarly to the monopoly case. However, when highly sensitive consumers increase in the second period, firm 2 reduces its quality in the first period to reduce costs and to differentiate its product from that of firm 1, so it increases its market power.

Notice how both firms react to the disappearance of the less sensitive consumers. If it happens in period 1, they both differentiate their products in an effort to increase their market power in both periods. However, if it happens in period 2, they reduce prices to achieve a competitive advantage in the market.

The increase in market size in period 2 makes the innovator to try to be more competitive in that period, because of he reduces the price and increases the quality of his product in that period. However, firm 2 differentiates its product in $t = 1$ to increase its market power.

IV. Comparative static

In this section, we compare the decisions of the innovating firm in monopoly with its decisions when there exists an imitating firm, under the assumption that the size of the market is high enough for the imitator to decide to enter $\left(\overline{\theta}_t > 2\underline{\theta}_t \forall t = 1,2\right)$.

The difference between the qualities that an innovating firm decides to produce in period 1 in both settings is:

$$q_{11}^d - q_{11}^m = \frac{-\overline{\theta}_1\left(\overline{\theta}_1 - \underline{\theta}_1\right) - 4\underline{\theta}_1^2 + 7\overline{\theta}_2^2 - 4\underline{\theta}_2\left(4\overline{\theta}_2 - \underline{\theta}_2\right)}{72} \tag{17}$$

This difference has a maximum in $\overline{\theta}_1 = 2\underline{\theta}_1$, and his value in this point is

$$q_{11}^d - q_{11}^m\left(\overline{\theta}_1 = 2\underline{\theta}_1\right) = \frac{\overline{\theta}_2\left(7\overline{\theta}_2 - 16\underline{\theta}_2\right) + 4\underline{\theta}_2^2}{72} \tag{18}$$

As we can show, the value of (18) is positive because it is increasing in $\overline{\theta}_2$ and his value is equal to zero when $\overline{\theta}_1 = 2\underline{\theta}_2$. Given the expression (17) is decreasing in $\overline{\theta}_1$ when $\overline{\theta}_1 > 2\underline{\theta}_1$,[3] this difference will reduce until it becomes negative as $\overline{\theta}_1$ increases. Thus, we conclude that when the market in period 1 is small enough, the innovating firm produces a higher level of quality as a duopolist, otherwise, he produces a higher quality as a monopolist.

The difference between the quality that innovating firm decides to produce in period 2 in both settings is:

$$q_{12}^d - q_{12}^m = \frac{-\overline{\theta}_1^2 + 4\underline{\theta}_1\left(\overline{\theta}_1 - \underline{\theta}_1\right) + 6\overline{\theta}_2\left(\overline{\theta}_2 - \underline{\theta}_2\right)}{72} \tag{19}$$

This difference has a maximum in $\overline{\theta}_1 = 2\underline{\theta}_1$. As we can see in (20), the value of (19) in the maximum is positive. Given the expression (19) is decreasing in $\overline{\theta}_1$ when $\overline{\theta}_1 > 2\underline{\theta}_1$,[4] we deduce that: when the market in period 1 is small enough, the innovating firm provides a lower quality as a monopolist; otherwise, he provides a higher quality as a monopolist.

$$q_{12}^d - q_{12}^m\left(\overline{\theta}_1 = 2\underline{\theta}_1\right) = \frac{6\overline{\theta}_2\left(\overline{\theta}_2 - \underline{\theta}_2\right)}{72} > 0 \tag{20}$$

3 $\frac{\partial\left(q_{11}^d - q_{11}^m\right)}{\partial\overline{\theta}_1} = -\frac{\overline{\theta}_1 - 2\underline{\theta}_1}{36}$.

4 $\frac{\partial\left(q_{12}^d - q_{12}^m\right)}{\partial\overline{\theta}_1} = -\frac{\overline{\theta}_1 - 2\underline{\theta}_1}{36}$.

Let $\Delta_t^h = q_{1t}^h - q_{1,t-1}^h$ be the increase of the quality of the product of innovator in period t in the setting $h = \{d, m\}$. In equilibrium, the value of Δ_t^h is as follows:

$$\Delta_1^m = \frac{\bar{\theta}_1^2 + \bar{\theta}_2^2}{8}, \quad \Delta_1^d = \frac{(\bar{\theta}_1 + \underline{\theta}_1)(2\bar{\theta}_1 - \underline{\theta}_1)}{18} + \frac{(2\bar{\theta}_2 - \underline{\theta}_2)^2}{18},$$

$$\Delta_2^m = \frac{\bar{\theta}_2^2}{8}, \qquad \Delta_2^d = \frac{(2\bar{\theta}_2 - \underline{\theta}_2)(\bar{\theta}_2 + \underline{\theta}_2)}{18}.$$

The difference of increase of the quality of innovating firm between monopoly and duopoly in period 1 is showed in the expression (21). This expression has a maximum in $\bar{\theta}_2 = \frac{8}{7}\underline{\theta}_2$, and its maximum value is showed in (22).

$$\Delta_1^m - \Delta_1^d = \frac{\bar{\theta}_1(\bar{\theta}_1 - 4\underline{\theta}_1) + 4\underline{\theta}_1^2 - 7\bar{\theta}_2^2 + 4\underline{\theta}_2(4\bar{\theta}_2 - \underline{\theta}_2)}{72} \tag{21}$$

$$\Delta_1^m - \Delta_1^d\left(\bar{\theta}_2 = \frac{8}{7}\underline{\theta}_2\right) = \frac{\bar{\theta}_1(\bar{\theta}_1 - 4\underline{\theta}_1) + 4\underline{\theta}_1^2 + \frac{36}{7}\underline{\theta}_2}{72} > 0 \tag{22}$$

As we can show, expression (22) is positive because it is increasing in $\bar{\theta}_1$ and expression (23) is positive.

$$\Delta_1^m - \Delta_1^d\left(\bar{\theta}_2 = \frac{8}{7}\underline{\theta}_2, \bar{\theta}_1 = 2\underline{\theta}_1\right) = \frac{\underline{\theta}_2}{14} \tag{23}$$

Given expression (21) is decreasing with respect to $\bar{\theta}_2$ under the assumption that $\bar{\theta}_2 = 2\underline{\theta}_2$,[5] we deduce that: when the market in period 2 is small, the increase of the quality of innovating firm in period 1 is higher when he is a monopolist; otherwise, this increase is higher when he is a duopolist.

The difference of increase of quality of innovating firm between monopoly and duopoly in period 2 is showed in the expression (24). This expression has a minimum in $\bar{\theta}_2 = 2\underline{\theta}_2$, and his minimum value is equal to zero. Given $\Delta_2^m - \Delta_2^d$ is increasing in $\bar{\theta}_2$, we deduce that the increase of quality of innovating firm in period 2 is higher when he is a monopolist.

$$\Delta_2^m - \Delta_2^d = \frac{\bar{\theta}_2(\bar{\theta}_2 - 4\underline{\theta}_2) + 4\underline{\theta}_2^2}{72} > 0 \tag{24}$$

From (25) we conclude that the increase of quality of innovating firm's product is lower in period 2, both monopoly and duopoly.

5 $\frac{\partial(\Delta_1^m - \Delta_1^d)}{\partial\bar{\theta}_2} = -\frac{7\bar{\theta}_2 - 8\underline{\theta}_2}{36}$.

$$\Delta_2^m - \Delta_1^m = -\frac{\bar{\theta}_1^2}{8} \qquad \Delta_2^d - \Delta_1^d = -\frac{(\bar{\theta}_1+\underline{\theta}_1)(2\bar{\theta}_1-\underline{\theta}_1)+(2\bar{\theta}_2-\underline{\theta}_2)(\bar{\theta}_2-2\underline{\theta}_2)}{18} \quad (25)$$

We now compare the decrease of increase of quality of innovating firm between monopoly and duopoly. As can be seen, (26) has a maximum in $\bar{\theta}_1 = 2\underline{\theta}_1$, and his maximum value is showed in (27), which is positive because of we assume $\bar{\theta}_2 > 2\underline{\theta}_2$.

$$\left[\Delta_1^d - \Delta_2^d\right] - \left[\Delta_1^m - \Delta_2^m\right] = \frac{-\bar{\theta}_1^2 + 4\underline{\theta}_1(\bar{\theta}_1 - \underline{\theta}_1) + 4(2\bar{\theta}_2 - \underline{\theta}_2)(\bar{\theta}_2 - 2\underline{\theta}_2)}{72} \quad (26)$$

$$\left[\Delta_1^d - \Delta_2^d\right] - \left[\Delta_1^m - \Delta_2^m\right](\bar{\theta}_2 = 2\underline{\theta}_2) = \frac{4(2\bar{\theta}_2 - \underline{\theta}_2)(\bar{\theta}_2 - 2\underline{\theta}_2)}{72} > 0 \quad (27)$$

Given expression (26) is decreasing with respect to $\bar{\theta}_1$, we deduce that: when the market in period 1 is small enough the decrease of levels of increase of quality is greater when an imitator exist; otherwise, this one is greater when the innovator is a monopolist. The results obtained in this section are summarizes in the following proposition:

Proposition 1 In *equilibrium, we have:*

(a) If the market in period 1 is small enough, the innovating firm provides higher quality as a duopolist. But if the market is big in period 1, he provides lower quality as a duopolist.
(b) If the market in period 1 is small enough, the decrease of increase of innovator's quality is greater when an imitator exists; otherwise, this one is lower when the innovator is a duopolist.
(c) The levels of increase of quality of innovating firm is lower in period 2, both monopoly and duopoly.
(d) If the market in period 2 is small enough, the increase of quality of innovating firm in period 1 is higher when he is a monopolist; otherwise, this increase is higher when he is a duopolist. However, the increase of quality of innovator in period 2 is higher when he is a monopolist independently of the size of the market.

From proposition 1 we conclude that, when the market is small enough in period 1, the innovator invests more intensively in R&D to improve the quality of his product as a duopolist, although the decrease of levels of increase of innovator's quality is greater. However, when the market is high, the opposing result is obtained. This is because of the competition is more aggressive when the market is small and there is an imitator, so the innovator

finds profitable to invest more in R&D for raising his market power. The levels of increase of quality of the innovating firm is lower in period 2 both in the monopoly and duopoly cases, because that period is the last one. Finally, the increase of quality of the innovating firm in period 1 is higher when it is a duopolist and the market in period 2 is big enough because of the innovator tries to differentiate herself from the imitator by raising his market power.

Conclusion

In this paper, we analyze the effects of an imitating firm on the behaviour of an innovating firm. The framework of analysis used is a dynamic duopoly model of vertical product differentiation, where both the innovator and the imitator compete simultaneously in price and quality.

The main result is that when the market is small enough in period 1 the innovating firm provides a higher quality as a duopolist because the innovator invests more intensively in R&D to improve the quality of his product, although the decrease of increase of innovator's quality is greater. However, when the market is high, the opposing result is obtained. This is because of the competition is more aggressive when the market is small and there is an imitator, so the innovator finds profitable to invest more in R&D for raising his market power.

The consequences of our results are: when the market is small, the presence of an imitator encourages the innovator to provide a higher quality, so that the entry of the imitator should not be obstructed; and, when the market is high, the imitator reduces the innovator's incentives to invest its economic resources in R&D and to provide a higher quality, so the entry of the imitator should not be encouraged.

The question that this model addresses could be extended to those situations in which products are subject to externalities in their consumption patterns (Katz and Shapiro, 1985), or those in which a technological improvement by the innovator might increase the cost of the imitator.

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