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MARKET VALUE CALCULATION AND THE SOLUTION OF CIRCULARITY BETWEEN VALUE AND THE WEIGHTED AVERAGE COST OF CAPITAL WACC

CÁLCULO DO VALOR DE MERCADO E A SOLUÇÃO DA CIRCULARIDADE
ENTRE VALOR E CUSTO MÉDIO PONDERADO DE CAPITAL CMEPC

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ABSTRACT

Most finance textbooks present the Weighted Average Cost of Capital (WACC) calculation as: $WACC = K_d \times (1-T) \times D\% + K_e \times E\%$, where K_d is the cost of debt before taxes, T is the tax rate, $D\%$ is the percentage of debt on total value, K_e is the cost of equity and $E\%$ is the percentage of equity on total value. All of them precise (but not with enough emphasis) that the values to calculate $D\%$ y $E\%$ are market values. Although they devote special space and thought to calculate K_d and K_e , little effort is made to the correct calculation of market values. This means that there are several points that are not sufficiently dealt with: Market values, location in time, occurrence of tax payments, WACC changes in time and the circularity in calculating WACC. The purpose of this note is to clear up these ideas, solve the circularity problem and emphasize in some ideas that usually are looked over. Also, some suggestions are presented on how to calculate, or estimate, the equity cost of capital.

KEYWORDS

Weighted average cost of capital; Firm valuation; Capital budgeting; Equity cost of capital; Market value.

RESUMO

A maioria dos livros apresenta o cálculo do custo médio ponderado de capital como: $CMePC = K_d \times (1-T) \times D\% + K_e \times E\%$, em que K_d é o custo de capital de terceiros antes dos tributos; T , alíquota de imposto; $D\%$, o percentual dos empréstimos em relação ao ativo total; K_e , o custo do capital próprio; e $E\%$, o percentual do patrimônio líquido sobre o ativo total. Todos eles necessitam (mas não com suficiente ênfase) que os valores para calcular $D\%$ e $E\%$ sejam valores de mercado. Muito embora se dedique atenção especial para calcular K_d e K_e , pouco esforço é destinado ao cálculo correto de seus valores de mercado. Isso significa que há

inúmeros pontos que não são suficientemente esclarecidos: valores de mercado, localização no tempo, ocorrência do pagamento de tributos, a variação do CMePC ao longo do tempo e circularidade no cálculo do CMePC. O objetivo deste artigo é esclarecer esses conceitos, resolver o problema da circularidade e discutir algumas ideias geralmente ainda obscuras. Igualmente, algumas sugestões são apresentadas para o cálculo ou estimação do custo do capital próprio.

PALAVRAS-CHAVE

Custo médio ponderado de capital; Avaliação da empresa; Orçamento de capital; Custo do capital próprio; Valor de mercado.

1 INTRODUCTION

Most finance textbooks (BENNINGA; SARIG, 1997; BREALEY; MYERS; MARCUS, 2004; COPELAND; KOLLER; MURRIN, 1995; DAMODARAN, 1996; GALLAGHER; ANDREW JR., 2000; VAN HORNE, 1998; WESTON; COPELAND, 1992) present the Weighted Average Cost of Capital (WACC) calculation as:

$$WACC = K_d \times (1-T) \times D\% + K_e \times E\% \quad (1)^1$$

Where K_d is the cost of debt before taxes, T is the tax rate, $D\%$ is the percentage of debt on total value, K_e is the cost of equity and $E\%$ is the percentage of equity on total value. All of them precise (but not with enough emphasis) that the values to calculate $D\%$ y $E\%$ are market values. Although they devote special space and thought to calculate K_d and K_e , little effort is made to the correct calculation of market values. This means that there are several points that are not sufficiently dealt with:

1. Market values are calculated period by period and they are the present value at WACC of the future cash flows.
2. These values to calculate $D\%$ and $E\%$ are located at the beginning of period t , where the WACC belongs. From here on, the right notation will be used.
3. $K_d \times (1-T)$, the after tax cost of debt, implies that the tax payments coincides in time with the tax accrual. (Some firms could present this payment behavior, but it is not the rule. Only those that are subject to tax withheld from their customers, pay taxes as soon as they invoice their goods or services.)

¹ This formula is derived in Appendix A.

4. Because of 1, 2 and the existence of changing macroeconomic environment (say, inflation rates), WACC changes from period to period.
5. That there exists circularity when calculating WACC. In order to know the firm value it is necessary to know the WACC, but to calculate WACC, the firm value and the financing profile are needed.
6. That we obtain full advantage of the tax savings in the same year as taxes are paid. This means that earnings before interest and taxes (Ebit) are greater than or equal to the interest charges.
7. There are no losses carried forward.
8. That (1) implies a definition for K_e , the cost of equity, in most cases they use;

$$K_{e_t} = K_{u_t} + (K_{u_t} - K_d) \times (1-T) \times D\%_{t-1}/E\%_{t-1} \quad (2)$$

This formula is derived in Appendix B. This is the typical formulation of K_e , but it has to be said, it only applies to perpetuities and not to finite periods.

In this expression, K_{e_t} is the levered cost of equity, K_{u_t} is the cost of unlevered equity, K_d is the cost of debt, T is the tax rate, $D\%_{t-1}$ is the proportion of debt on the total market value for the firm, at $t-1$ and $E\%_{t-1}$ is the proportion of equity on the total market value for the firm, at $t-1$. It can be shown that equation 2 results from the assumption that the discount rate for the tax savings. In this case that rate is K_d and expression 2 is valid only for perpetuities. When working with n finite it can be shown that the expression for K_e changes for every period (THAM; VÉLEZ-PAREJA, 2001c). The assumption behind K_d as the discount rate is that the tax savings are a non-risky cash flow.

9. The only source of tax shields is the interest expenses.

The purpose of this work is to clear up these ideas, solve the circularity problem and emphasize in some ideas that usually are looked over.

2 THE MODIGLIANI-MILLER PROPOSAL

The basic idea is that under a scenario of no taxes, the firm value does not depend on how the stakeholders finance it. This is the stockholders (equity) and creditors (liabilities to banks, bondholders, etc.) The reader should examine this idea in an intuitive manner and she will find it is reasonable. Because of this idea, Franco Modigliani and Merton Miller (MM from here on) were awarded the Nobel Prize in Economics. They proposed that with perfect market conditions, (perfect and complete information, no taxes, etc.) the capital structure does not affect the value of the firm because the equity holder can borrow and

lend and thus determine the optimal amount of leverage. The capital structure of the firm is the combination of debt and equity in it.

That is, V^L the value of the levered firm is equal to V^{UL} the value of the unlevered firm.

$$V^L = V^{UL} \quad (3)$$

And in turn, the value of the levered firm is equal to V^{Equity} the value of the equity plus V^{Debt} the value of the debt.

$$V^L = V^{Equity} + V^{Debt} \quad (4)$$

What does it imply regarding the Weighted Average Cost of Capital (WACC)? Simple. If the firm has a given cash flow, the present value of it at WACC (the firm total value) does not change if the capital structure changes. If this is true, it implies that the WACC will remain constant no matter how the capital structure changes. This situation happens when no taxes exist. To maintain the equality of the unlevered and levered firms, the return to the equity holder (levered) must change with the amount of leverage (assuming that the cost of debt is constant)

One of the major market imperfections are taxes. When corporate taxes exist (and no personal taxes), the situation posited by MM is different. They proposed that when taxes exist the total value of the firm does change. This occurs because no matter how well managed is the firm, if it pays taxes, there exists what economists call an externality. When the firm deducts any expense, the government pays a subsidy for the expense. It is reflected in less tax. In particular, this is true for interest payments. The value of the subsidy (the tax saving) is $T \times K_d \times D$, where the variables have been defined above.

Hence the value of the firm is increased by the present value of the tax savings or tax shield.

$$V^L = V^{UL} + V^{TS} = V^D + V^E \quad (5a)$$

Associated to equations (4) and (5a) there exists correlated cash flows, as follows:

$$FCF + TS = CFD + CFE \quad (5b)$$

Where FCF is free cash flow, TS is tax savings, CFD is cash flow to debt and CFE is cash flow to equity.

When a firm has debt there exists some other contingent or hidden costs associated to the fact to the possibility that the firm goes to bankruptcy. Then,

there are some expected costs that could reduce the value of the firm. The existence of these costs deters the firm to take leverage up to 100%. One of the key issues is the appropriate discount rate for the tax shield. In this note, we assert that the correct discount rate for the tax shield is K_u , the return to unlevered equity, and the choice of K_u is appropriate whether the percentage of debt is constant or varying over the life of the project.

In this work the effects of taxes on the WACC will be studied. When calculating WACC two situations can be found: with or without taxes. In the first case, as said above, the WACC is constant, no matter how the firm value be split between creditors and stockholders. (The assumption is that if inflation is kept constant, otherwise, the WACC should change accordingly.) When inflation is not constant, WACC changes, but due to the inflationary component and not due to the capital structure. In this situation, WACC is the cost of the assets, K_A , or the cost of the firm, K_u and at the same time is the cost of equity when unlevered. This means,

$$K_{u_t} = K_d \times D_{t-1}\% + K_e \times E_{t-1}\% \quad (6)$$

This K_u is defined as the return to unlevered equity. The WACC is defined as the weighted average cost of debt and the cost of levered equity. In a MM world K_u is equal to WACC without taxes. When taxes exist, the WACC calculation will change taking into account the tax savings.

If it is true that the cost K_u , is constant, K_e , the cost of equity changes according to the leverage. Here for simplicity we assume that the K_u is constant, but this assumption is not necessary. If the K_u is changing then in each period, the WACC will change as well, not only for the eventual change in the financing profile, but for the change in K_u . In any case, K_e has to change in order to keep K_u constant or in order to be consistent with the changing K_u .

The cost of equity, K_e is:

$$K_{e_t} = (K_{u_t} - K_d \times D\%_{t-1})/E\%_{t-1} = K_{u_t} + (K_{u_t} - K_d) \times D\%_{t-1}/E\%_{t-1} \quad (7)^2$$

This equation is proposed by Harris and Pringle (1985) and is part of their definition of WACC³. A complete derivation for K_e and WACC can be found in Tham and Vélez-Pareja (2002, 2004b). The K_e is derived under different assumptions for the discount rate for the tax savings and for perpetuities and finite periods. Note the absence of the $(1-T)$ factor.

² This formula is derived in Appendix B.

³ This was the original proposal by MM in a seminal paper published in 1958, but corrected in 1963.

As before, it can be shown that equation 7 results from the assumption that the discount rate for the tax savings is K_u and it can be shown that K_e , defined in equation 7, is the same for finite periods and for perpetuities, see Tham and Vélez-Pareja (2004a, 2004b). The assumption behind K_u as the discount rate is that the tax savings are a strictly correlated to the free cash flow.

What is the meaning of equation 7? Since K_u and K_d are constant, we see that the return to levered equity K_e is a linear function of the debt-equity ratio. It should be no surprise that there is a positive relationship between K_e , the return to levered equity and the debt-equity ratio. Since the debt holder has a prior claim on the expected cash flow generated by the firm, relative to the debt holder, the risk to the equity holder is higher and the equity holder demands a higher return to compensate for the higher risk. The higher the amount of debt, given a constant total value, the higher is the risk to the equity holder, who is the residual claimant.

Equation 7 shows the relationship between the K_e , the return to levered equity and the debt-equity ratio. Table 1 shows the relationship between D , the amount of debt, the debt-equity ratio, E , the amount of equity and K_e , the return to levered equity.

TABLE 1

RELATIONSHIP BETWEEN D , THE AMOUNT OF DEBT, THE DEBT-EQUITY RATIO AND K_e , THE RETURN TO LEVERED EQUITY FOR $K_u = 15.1\%$ AND $K_d = 11.2\%$

| Debt, D | Equity, E | D/E Ratio | K_e |
|-----------|-------------|-------------|--------|
| 0 | 1,000 | 0.00 | 15.10% |
| 100 | 900 | 0.11 | 15.53% |
| 200 | 800 | 0.25 | 16.08% |
| 300 | 700 | 0.43 | 16.77% |
| 400 | 600 | 0.67 | 17.70% |
| 500 | 500 | 1.00 | 19.00% |
| 600 | 400 | 1.50 | 20.95% |
| 700 | 300 | 2.33 | 24.20% |
| 800 | 200 | 4.00 | 30.70% |
| 900 | 100 | 9.00 | 50.20% |

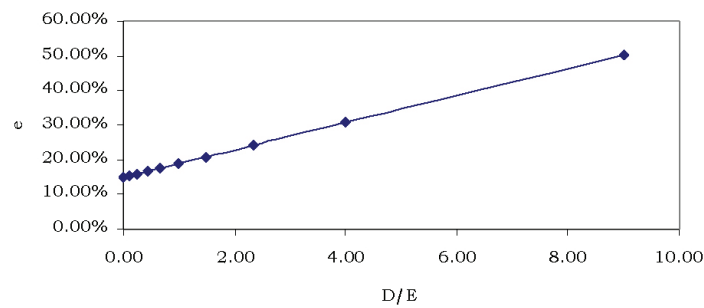
Source: Elaborated by authors.

If the amount of debt is \$ 100, the debt-equity ratio is 0.11 and the return to levered equity is 15.53%. As presented in Figure 1, note that there is a linear relationship between K_e , the return to levered equity and the debt-equity ratio.

FIGURE 1

K_e AS A FUNCTION OF D/KE

K_e as a function of D/E



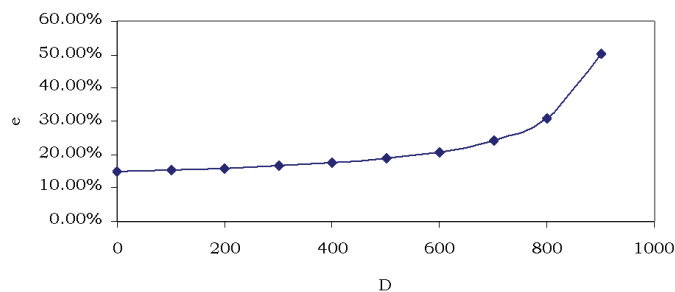
Source: Elaborated by authors.

If the amount of debt increases from 100 to 200, the return to levered equity increases by 0.43 percentage points, from 15.1% to 15.53%. However, the relationship between K_e , the return to levered equity and the amount of debt D is non-linear (remember that $E = \text{Total value} - D$ and $D/(V - D)$). If the amount of debt increases from 500 to 600, the return to levered equity increases by 1.95 percentage points, from 19% to 20.95%.

FIGURE 2

K_e AS A FUNCTION OF D

K_e as a function of D



Source: Elaborated by authors.

As can be seen in Appendix A, WACC after taxes can be calculated as:

$$WACC_t = Kd_t \times (1-T) \times D\%_{t-1} + Ke_t \times E\%_{t-1} \quad (8)$$

The values for D% and E% have to be calculated on the total value of the firm for the beginning of each period. This is the well known expression for the weighted average cost of capital.

It can be shown that under the assumption of the discount rate of tax savings is K_u , the WACC for the FCF can be expressed as (THAM; VÉLEZ-PAREJA, 2002, 2004a):

$$WACC_t = K_u - TS_t/TV_{t-1} \quad (9)$$

Where TS means tax savings and TV is the total levered value of the firm. This means that $Kd \times T \times D\%$ is the same as $Kd \times T \times D/TV$ and in general, we call TS to the tax savings $-Kd \times D \times T$. However, it must be said that the tax savings are equal to $Kd \times D \times T$ only when taxes are paid in the same year as accrued. The implicit assumption in (9) is that we consider the actual tax savings earned and when they occur. This new version of WACC has the property to give the same results as (8) and what is most important, as TS is the actual tax savings earned, it takes into account the losses carried forward (LCF), when they occur. This problem has been studied by Vélez-Pareja and Tham (2001a, 2001b) and Tham and Vélez-Pareja (2004b).

If the Capital Asset Pricing Model (CAPM) is used, it can be demonstrated that there is a relationship between the betas of the components (debt and equity) in such a way that

$$\beta_{t \text{ firm}} = \beta_{t \text{ debt}} D_{t-1}\% + \beta_{t \text{ stock}} Ke_{t-1}\% \quad (10)$$

If $\beta_{t \text{ stock}}$, $\beta_{t \text{ debt}}$, $D_{t-1}\%$ and $E_{t-1}\%$ are known, then K_u can be calculated as

$$K_u = R_f + \beta_{t \text{ firm}} (R_m - R_f) \quad (11)$$

Where R_f is the risk free rate of return and R_m is the market return and $(R_m - R_f)$ is the market or equity risk premium. And this means the K_u can be calculated for any period.

3 CALCULATIONS FOR K_e AND K_u

The secret is to calculate K_e or K_u . If K_e is known for a given period, the initial period, for instance, K_u can be calculated. On the contrary, if K_u is known

Ke can be calculated. For this reason several options to calculate Ke and Ku are presented. In order to calculate Ke, we have several alternatives:

1. With the Capital Asset Pricing Model (CAPM). This is the case of a firm that is traded at the stock exchange, it is traded on a regularly basis and we think the CAPM works well. However, it has to be said that if we know the value of the equity (it is traded at the stock exchange) it is not necessary to discount the cash flows to calculate the value.

2. With the Capital Asset Pricing Model (CAPM) adjusting the betas. This is the case for a firm that is not listed at the stock exchange or if registered, is not frequently traded and we believe the model works well. It is necessary to pick a stock or industry similar to the one we are studying (from the same industrial sector, about the same size and about the same leverage). This is called the proxy firm.

• Example:

The beta adjustment is done with⁴:

$$\beta_{nt} = \beta_{proxy} \frac{\left[1 + \frac{D_{nt}}{E_{nt}}(1-T) \right]}{\left[1 + \frac{D_{proxy}}{E_{proxy}}(1-T) \right]} \quad (12)$$

Where, β_{nt} is the beta for the stock not registered at the stock exchange; D_{nt} is the market value of debt, E_{nt} is the equity for the stock not registered in the exchange; D_{proxy} is the market value of debt for the proxy firm, E_{proxy} is the market value of equity for the proxy firm.

For instance, if you have a stock traded at the stock exchange and the beta is β_{proxy} of 1.3, a debt D_{proxy} of 80, E_{proxy} worth 100, and we desire to estimate the beta for a stock not listed in the stock exchange. This non-traded stock has a debt D_{nt} of 70 and equity of E_{nt} of 145 and a tax rate of 35%, and then beta for the non-traded stock can be adjusted as

$$\beta_{nt} = \beta_{proxy} \frac{\left[1 + \frac{D_{nt}}{E_{nt}}(1-T) \right]}{\left[1 + \frac{D_{proxy}}{E_{proxy}}(1-T) \right]} = 1.3 \frac{\left[1 + \frac{70}{145}(1-35\%) \right]}{\left[1 + \frac{80}{100}(1-35\%) \right]} = 1.12$$

⁴ Based on Hamada (1969). This assumes Kd as the discount rate for the TS and perpetuities.

This is easier said than done. Although we have illustrated the use of the formula, we have to recall that the market value of equity for the non traded firm is not known. That value is what we are looking for. Hence, there will be a circularity when using this approach.

3. Subjectively and assisted by a methodology such as the Analytical Hierarchy Process developed by Tom Saaty and presented by Cotner and Fletcher (2000) applied to the owner of the firm. With this approach the owner given a leverage level estimates the perceived risk. This risk premium is added to the risk free rate and the result would be an estimate for K_e .

4. Subjectively as 3, but direct. This is, asking the owner, for a given value level of debt and a given cost of debt, what is the required return to equity?

5. An estimate based on book value (given that these values are adjusted either by inflation adjustments or asset revaluation, so the book value is a good proxy to the market value).

An example: assume a privately held firm. Tax rate is 35%.

TABLE 2

FINANCIAL INFORMATION OF HYPOTHETICAL FIRM

| YEAR | ADJUSTED BOOK VALUE FOR EQUITY E | DIVIDENDS PAID D | RETURN $R_t = ((E_t + D_t)/E_{t-1}) - 1$ |
|------|-------------------------------------|------------------|---|
| 1990 | \$ 1,159 | \$ 63 | |
| 1991 | \$ 1,341 | \$ 72 | 21.92% |
| 1992 | \$ 2,095 | \$ 79 | 62.12% |
| 1993 | \$ 1,979 | \$ 91 | -1.19% |
| 1994 | \$ 3,481 | \$ 104 | 81.15% |
| 1995 | \$ 4,046 | \$ 126 | 19.85% |
| 1996 | \$ 3,456 | \$ 176 | -10.23% |
| 1997 | \$ 3,732 | \$ 201 | 13.80% |
| 1998 | \$ 4,712 | \$ 232 | 32.48% |
| 1999 | \$ 4,144 | \$ 264 | -6.45% |
| 2000 | \$ 5,950 | \$ 270 | 50.10% |

Source: Elaborated by authors.

TABLE 3

ADDITIONAL MACROECONOMIC INFORMATION

| YEAR | NOMINAL RISK FREE RATE OF INTEREST ⁵ R_f | INFLATION RATE | REAL INTEREST RATE $i_r = (1+R_f)/(1+i_f)-1$ | RETURN TO EQUITY $K_{et} = ((D_t + E_t)/E_{t-1}) - 1$ | RISK PREMIUM $i_0 = K_{et} - R_{ft} \times (1-T)$ |
|------|---|-------------------|--|--|--|
| 1990 | 36.3% | | | | |
| 1991 | 30.6% | 26.8% | 3.0% | 21.92% | 2.0% |
| 1992 | 28.9% | 25.1% | 3.0% | 62.12% | 43.3% |
| 1993 | 26.3% | 22.6% | 3.0% | -1.19% | -18.3% |
| 1994 | 26.3% | 22.6% | 3.0% | 81.15% | 64.1% |
| 1995 | 15.8% | 19.5% | -3.1% | 19.85% | 9.6% |
| 1996 | 16.3% | 21.6% | -4.4% | -10.23% | -20.8% |
| 1997 | 21.2% | 17.7% | 3.0% | 13.80% | 0.0% |
| 1998 | 51.7% | 16.7% | 30.0% | 32.48% | -1.1% |
| 1999 | 16.4% | 13.0% | 3.0% | -6.45% | -17.1% |
| 2000 | 12.9% | 9.6% | 3.0% | 50.10% | 41.7% |
| 2001 | | Expected 10% | Average 4.4% | | Average 10.3% |

Source: Elaborated by authors.

Estimated risk free rate for 2001:

$$R_{f_{2001}} = ((1 + i_{f_{est}})(1 + i_{r_{avg}}) - 1) \times (1 - T) = ((1 + 10\%)(1 + 4.4\%) - 1) \times (1 - 0.35) = 9.61\%$$

$$\text{Cost of equity } K_e = R_{f_{2001}} + i_{0 \text{ average}} = 9.61\% + 10.30\% = 20.0\%$$

6. Calculate the market risk premium as the average of $R_m - R_f$, where R_m is the return of the market based upon the stock exchange index and R_f is the risk free rate (say, the return of treasury bills or similar). Then, subjectively, the owner could estimate if he prefers, in terms of risk, to stay in the actual business or to buy the stock exchange index basket. If the actual business is preferred, then one could say that the beta of the actual business is lower than 1, the market beta, and

⁵ This information is based on actual data for nominal risk free rates in the Colombian bond market.

the risk perceived is lower than the market risk premium, $R_m - R_f$. This is an upper limit for the risk premium of the owner. This upper limit could be compared with zero risk premium, the risk free rate risk premium which is the lower limit for the risk perceived by the equity owner.

If the owner prefers to buy the stock exchange index basket, we could say that the actual business is riskier than the market. Then, the beta should be greater than 1 and the perceived risk for the actual business should be greater than $R_m - R_f$.

In the first case, the owner could be confronted with different combinations – from 0% to 100% – of the stock exchange index basket and the risk free investment and the actual business. After several trials, the owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk, or simply, the market risk premium ($R_m - R_f$) times the proportion of the stock exchange index basket accepted. In fact what has been found is the beta for the equity holders in the actual business.

In the second case one must choose the highest beta found in the stock exchange index basket. This beta should be used to multiply the market risk premium $R_m - R_f$, and the result would be an estimate of the risk premium for the riskiest stock in the index. This might be an upper limit for the risk perceived by the owner. In case this risk is lower than the perceived risk by the owner, it might be considered as the lower limit. In case that the riskier stock is considered riskier than the actual business, then the lower limit is the market risk premium, $R_m - R_f$. In this second case, the owner could be confronted with different combinations – from 0% to 100% – of the stock exchange index basket and the riskiest stock and the actual business. After several trials, the owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk. That is, the market risk premium ($R_m - R_f$) times the proportion of the stock exchange index basket accepted plus the risk premium for the riskiest stock in the index (its beta times the market risk premium, $R_m - R_f$) times the proportion accepted for that stock.

In both cases the result might be an estimation of the risk premium for the actual business. This risk premium could be added to the risk free rate and this might be a rough estimate of K_e .

If K_e , $D\%$ and $E\%$ are known, then K_u is calculated with (6). As it is necessary to know the market values that are the result of discounting the future cash flows at WACC, then circularity is found, but it is possible to solve it with a spreadsheet.

Another option is to calculate K_u directly. One of the following alternatives could be used:

1. Using the CAPM and unlevering the beta and using equation (13), which is derived from (12).

$$\beta_{nt} = \frac{\beta_{proxy}}{1 + \frac{D_{proxy}}{E_{proxy}}(1 - T)} \quad (13)$$

With this beta we apply CAPM to obtain K_u .

2. According to MM, the WACC before taxes (K_u) is constant and independent from the capital structure of the firm. Then we could ask the owner for an estimate on how much she is willing to earn assuming no debt. A hint for this value of K_e could be found looking how much she could earn in a risk free security when bought in the “secondary” market. On top of this, a risk premium, subjectively calculated must be included.

3. Another way to estimate K_u is assessing subjectively *the risk for the firm* and this risk could be used to calculate K_u using CAPM with the risk free rate. Cotner and Fletcher (2000) present a methodology to calculate the risk of a firm not publicly held⁶. This methodology might be applied to the managers and other executives of the firm. This would give the risk premium for the firm. As this risk component would be added to the risk free rate, the result is K_u calculated in a subjective manner. A hint that could help in the process is to establish minimum or maximum levels for this K_u the minimum could be the cost of debt before taxes. The maximum could be the opportunity cost of owners, if it is perceptible this is, if it has been “told” by them or if, by observation, it is known observing were they are investing (other investments made by them).

This K_u is in accordance to the actual level of debt. It has to be remembered that K_u is, according to MM, constant and independent from the capital structure of the firm. This K_u is named in other texts as K_A cost of the assets or the firm, for instance, Ruback (2002), or K_u cost of unlevered equity, for instance, Fernández (1999a, 1999b).

If K_u is estimated directly and we wish to estimate the WACC (or the K_e), then circularities will be present. However, as will be shown below, the total value of the firm can be calculated with K_u using the Capital Cash Flow, CCF, and no circularities will be present and there is no need to calculate the leverage ratio for every period.

⁶ In fact, in the article the authors say that the methodology is to calculate the risk of the cost of capital, although at the end they say it is to define the risk for the equity cost. The way the methodology is presented allows thinking that it is the firm risk that is dealt with and this risk is added to the risk free rate. With this, the cost of capital before taxes for the firm is found. This would be K_u .

4 AN EXAMPLE FOR CALCULATING WACC AND THE FIRM VALUE

For a better understanding of these ideas, an example is presented. This example is done assuming that the discount rate for TS is K_u . In this example it is assumed that K_u is the correct discount rate for tax savings.

Assume a firm with the following information:

- The cost of the unlevered equity K_u 15.1%
- Cost of Debt, K_d 11.2%
- Tax rate 35.0%

The information about the initial investment, free cash flows, debt balances and initial equity is presented in Table 4.

TABLE 4

FREE CASH FLOW AND INITIAL INVESTMENT

| YEAR | 0 | 1 | 2 | 3 | 4 |
|---------------------------------|------------|------------|------------|------------|------------|
| Free cash flow FCF ⁷ | | 170,625.00 | 195,750.00 | 220,875.00 | 253,399.45 |
| Debt at end of period, D | 375,000.00 | 243,750.00 | 75,000.00 | 37,500.00 | |
| Initial equity investment | 125,000.00 | | | | |
| Total initial investment | 500,000.00 | | | | |

Source: Elaborated by authors.

The WACC calculations are made estimating the debt and equity participation in the total value of the firm for each period and calculating the contribution of each to the WACC after taxes. As a first step, we will not add up these components to find the value of WACC and we will calculate the total firm value with the WACC set at 0. We will construct each table, step by step, assuming that WACC is zero. Remember that $D_{t-1}\% = D_{t-1}/V_{t-1}$, where D is market value of debt, and V is the total firm value.

⁷ In the FCF at year 4 we assume there is a terminal value, that takes into account the value added by the firm from year 5 to infinity. This is a very important issue in firm valuation because experience shows that more than 50% of the firm value might be provided by terminal value. The subject is not addressed in detail because it is beyond the scope of this paper. It is a complex issue and the purpose of this text is to illustrate how to involve market values in the calculation of WACC. The interested reader can read several papers on this at http://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=145648.

As said, the first step is to calculate the value with an arbitrary value for WACC, for instance, zero. See this in the next table. Our table for WACC and Total Value will appear as presented in Table 5.

TABLE 5

WACC CALCULATIONS

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|------------|------------|------------|------------|---|
| WACC after taxes (Debt + equity contributions) | | | | | |
| Total value TV, at t-1 and WACC = 0 | 840,649.45 | 670,024.45 | 474,274.45 | 253,399.45 | |

Source: Elaborated by authors.

We use a well known formulation in finance:

$$V_t = \frac{CF_{t+1} + V_{t+1}}{1 + WACC_{t+1}} \quad (14)$$

Where CF is cash flow, V is market value and WACC is the weighted average cost of capital.

- Example: firm value at end of year 3 is $(253,399.45 + 0) / (1 + 0\%) = 253,399.45$.

For year 2 it will be $(253,399.45 + 220,875.00) / (1 + 0\%) = 474,274.45$ and so on for the other years.

We do this to avoid a division by zero. Done this we can calculate temporary values for D%, E% and Ke. The contribution of debt to WACC (temporary results) is presented in Table 6.

TABLE 6

WACC CALCULATION – CONTRIBUTION OF DEBT TO WACC (TEMPORARY RESULTS)

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|--------|--------|--------|--------|-------|
| Debt | | | | | |
| Relative weight of debt D% (Debt balance at t-1)/Total value of firm at t-1) | 44.61% | 36.38% | 15.81% | 14.80% | |
| Cost of debt after taxes $K_d \times (1 - T)$ | | 7.28% | 7.28% | 7.28% | 7.28% |
| Contribution of debt to WACC, $K_d \times (1 - T) \times D_{t-1} \%$ | | 3.25% | 2.65% | 1.15% | 1.08% |

Source: Elaborated by authors.

The same procedure is used to estimate the contribution of equity to WACC, as presented in Table 7.

TABLE 7

**WACC CALCULATION – CONTRIBUTION OF
EQUITY TO WACC (TEMPORARY RESULTS)**

| YEAR | 0 | 1 | 2 | 3 | 4 |
|---|--------|--------|--------|--------|--------|
| Equity | | | | | |
| Relative weight of equity $E\% = (1-D\%)$ | 55.39% | 63.62% | 84.19% | 85.20% | |
| Cost of equity $Ke = (Ku_t - Kd) \times D\%_{t-1} / E\%_{t-1}$ | | 18.24% | 17.33% | 15.83% | 15.78% |
| Contribution of equity to WACC = $E\%_{t-1} \times Ke$ | | 10.10% | 11.03% | 13.33% | 13.44% |

Source: Elaborated by authors.

It is recommended that the last arithmetic operation be the WACC calculation as the sum of the debt and equity contribution to the cost of capital.

At this point we recommend to set the spreadsheet to handle circularities following these instructions:

1. Select the Office Button at the top left and select Excel Options (down to the right) in Excel (2007).
2. Select Formula.
3. Enable Iterations.
4. Click Ok.

This procedure can be done before starting the work in the spreadsheet or when Excel declares the presence of circularity. After these instructions are done, then, the WACC can be calculated as the sum of the debt and equity contribution to the cost of capital.

Now we can proceed to formulate the WACC as the sum of the two components: debt contribution and equity contribution. When the WACC is calculated, previous tables will be shown as in Table 8.

TABLE 8

**WACC CALCULATION – CONTRIBUTION
OF DEBT TO WACC (FINAL)**

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|--------|--------|--------|--------|-------|
| Debt | | | | | |
| Relative weight of debt D% (Debt balance _{t-1} /Total value _{t-1}) | 61.68% | 47.38% | 19.39% | 16.94% | |
| Cost of debt after taxes $K_d \times (1-T)$ | | 7.28% | 7.28% | 7.28% | 7.28% |
| Contribution of debt to WACC $K_d \times (1-T) \times D_{t-1} \%$ | | 4.49% | 3.45% | 1.41% | 1.23% |

Source: Elaborated by authors.

The same procedure is used to estimate the contribution of equity to WACC, as shown in Table 9.

TABLE 9

**WACC CALCULATION – CONTRIBUTION
OF EQUITY TO WACC (FINAL)**

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|--------|--------|--------|--------|--------|
| Equity | | | | | |
| Relative weight of equity $E\% = (1-D\%)$ | 38.32% | 52.62% | 80.61% | 83.06% | |
| Cost of equity $K_{e_t} = K_{u_t} + (K_{u_t} - K_d) \times D\%_{t-1}/E\%_{t-1}$ | | 21.38% | 18.61% | 16.04% | 15.90% |
| Contribution of equity to WACC $E\% \times K_e$ | | 8.19% | 9.79% | 12.93% | 13.20% |

Source: Elaborated by authors.

Note that the cost of equity – K_e – is larger than K_u as expected, because K_u is the cost of the stockholder, as if the firm were unlevered⁸. When there is debt

⁸ As MM say that K_u is constant and independent from the capital structure, it will be equal to K_u when debt is zero. This K_u is WACC before taxes. And this is the condition for the validity of the first proposition of MM.

– Ke calculation – necessarily Ke ends up being greater than Ku, because of leverage. With these values it is possible to calculate the firm value for each period.

If Ke_1 is known, as it was said above, Ku is found with (6). Excel solves the circularity that is found and the same values result.

Now we have our final table with WACC and value obtained simultaneously as follows in Table 10.

TABLE 10

WACC CALCULATIONS (FINAL)

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|------------|------------|------------|------------|-------|
| WACC after taxes (Debt + equity contributions) | | 12.7% | 13.2% | 14.3% | 14.4% |
| Firm value a end of t | 607,978.04 | 514,457.73 | 386,835.85 | 221,433.06 | |

Source: Elaborated by authors.

Notice that WACC results in a lower value than Ku . WACC is after taxes.

Using (14) and from tables 14 and 10, we have that the firm value at end of year 3 is $(253,399.45+0)/(1+14.4\%) = 221,433.06$.

For year 2 it will be $(221,433.06 + 220,875.00)/(1+14.3\%) = 386,835.85$ and so on for the other years.

The reader has to realize that the values 14.4% and 14.3%, etc. are not calculated from the beginning because they depend on the firm value that is going to be calculated with the WACC. In this case circularity is generated. This is solved allowing the spreadsheet to make enough iteration until it finds the final numbers.

With the WACC values for each period the present value of future cash flows and the NPV are calculated (Table 11).

TABLE 11

NPV CALCULATIONS

| YEAR | 0 |
|-----------------------------|------------|
| Present value of cash flows | 607,978.04 |
| Initial total investment | 500,000.00 |
| NPV | 107,978.04 |

Source: Elaborated by authors.

If the initial investment is 500,000, then, NPV is 107,978.04.

The same result can be reached calculating the present value for the free cash flow assuming no debt and discount it at K_u , or what is the same, at WACC before taxes and add up the present value of tax savings at the same rate of discount, K_u . Myers proposed this in 1974 and it is known as Adjusted Present Value APV. Myers and all the finance textbooks teach that the discount rate for the TS should be the cost of debt. However, the tax savings depend on the firm profits. Hence, the risk associated to the tax savings is the same as the risk of the cash flows of the firm rather than the value of the debt. Hence, the discount rate should be K_u . For this reason the tax savings are also discounted at K_u . This way, the present value for the free cash flows discounted at WACC after taxes coincides with the present value of the free cash flow assuming no debt discounted at K_u and added to the present value of the tax savings discounted at the same K_u .

The use of K_u to discount the tax savings has been proposed by Tham (1999, 2000) and Ruback (2000). Tham proposes to add to the unlevered value of the firm (the present value of the FCF at K_u), the present value of the tax savings discounted at K_u . Ruback presents the Capital Cash Flow and discount it at K_u . The CCF is simply the FCF plus the tax savings so:

$$CCF = FCF + \text{Tax savings} \quad (15)$$

$$\begin{aligned} PV(\text{FCF at WACC after taxes}) &= PV(\text{FCF without debt at } K_u) + \\ PV(\text{Tax savings at } K_u) &= PV(\text{CCF at } K_u) \end{aligned} \quad (16)$$

TABLE 12

CALCULATION OF VALUE AND APV WITH K_u

| YEAR | 0 | 1 | 2 | 3 | 4 |
|---|---|------------|------------|------------|------------|
| Interest payments | | 42,000.00 | 27,300.00 | 8,400.00 | 4,200.00 |
| Tax savings $TS = T \times I$ ($T=35\%$) | | 14,700.00 | 9,555.00 | 2,940.00 | 1,470.00 |
| Free cash flow FCF | | 170,625.00 | 195,750.00 | 220,875.00 | 253,399.45 |
| Capital Cash Flow (CCF) = FCF + Tax savings | | 185,325.00 | 205,305.00 | 223,815.00 | 254,869.45 |
| K_u | | 15.10% | 15.10% | 15.10% | 15.10% |
| PV (CCF) at K_u | | 607,978.04 | | | |

(continue)

TABLE 12 (CONTINUATION)

CALCULATION OF VALUE AND APV WITH K_u

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--------------------------------------|---|------------|---|---|---|
| Adjusted NPV (APV) | | | | | |
| PV(FCF at K_u) | | 585,228.51 | | | |
| PV(TS at K_u) | | 22,749.53 | | | |
| PV(FCF at K_u) + PV(TS at K_u) | | 607,978.04 | | | |
| NPV | | 107,978.04 | | | |

Source: Elaborated by authors.

Notice that the same result is reached with the three methods. At this time, the reader can test that equation (9) matches with WACC from Table 10. For instance, for year 1, $15.1\% - 14,700/607,978.04 = 15.1\% - 2.42\% = 12.68\%$.

From the point of view of equity valuation, the value is calculated with the present value of the free cash flow discounted at WACC minus the debt at 0. This value also can be reached with the equity cash flow (CFE) and it is equal to

$$\text{CFE} = \text{FCF} + \text{TS} - \text{Cash flow to debt before taxes CFD} \quad (17)$$

TABLE 13

CALCULATING THE VALUE OF EQUITY WITH CFE

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|------------|------------|------------|------------|------------|
| Free cash flow FCF | | 170,625.00 | 95,750.00 | 20,875.00 | 53,399.45 |
| Tax savings TS = $T \times I$ (T = 35%) | | 14,700.00 | 9,555.00 | 2,940.00 | 1,470.00 |
| CFD = Interest + principal payment | | 173,250.00 | 196,050.00 | 45,900.00 | 41,700.00 |
| CFE | | 12,075.00 | 9,255.00 | 177,915.00 | 213,169.45 |
| K_e | | 21.38% | 18.61% | 16.04% | 15.90% |
| PV(CFE at K_e) | 232,978.04 | 9,948.31 | 6,428.52 | 106,499.41 | 110,101.80 |

Source: Elaborated by authors.

FCF is taken from Table 4, TS comes from Table 12 and CFD comes from tables 4 and 12.

When the present value of CFE at K_e , is calculated the same result is obtained. This is, $607,978.04 - 375,000 = 232,978.04$. This means that the right discount rate to discount the CFE is K_e , and its discounted value is consistent with the value calculated with the FCF.

In table 13 we calculated the market value of equity using the market value calculated before. However, this is not an independent method when we use the values from other method. In order to calculate the market value of equity in an independent way we will use the same procedure utilized for the calculation with WACC. The difference is that we will calculate again the value of K_e . The first table with K_e equal to zero is Table 14.

TABLE 14

INITIAL TABLE TO CALCULATE THE
MARKET EQUITY VALUE (TEMPORARY)

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|------------|------------|------------|------------|------------|
| Free Cash Flow FCF | | 170,625.00 | 195,750.00 | 220,875.00 | 253,399.45 |
| Interest charges | | 42,000.00 | 27,300.00 | 8,400.00 | 4,200.00 |
| Debt payment | | 131,250.00 | 168,750.00 | 37,500.00 | 37,500.00 |
| CFD | | 173,250.00 | 196,050.00 | 45,900.00 | 41,700.00 |
| Tax savings TS | | 14,700.00 | 9,555.00 | 2,940.00 | 1,470.00 |
| CFE = FCF – CFD + TS | | 12,075.00 | 9,255.00 | 177,915.00 | 213,169.45 |
| Relative weigh to debt D% | 47.6% | 37.8% | 16.1% | 15.0% | |
| Relative weigh to equity E% | 52.4% | 62.2% | 83.9% | 85.0% | |
| $K_{e_t} = K_{u_t} + (K_{u_t} - K_d) \times D\%_{t-1}/E\%_{t-1}$ | | | | | |
| Debt at end of period | 375,000.00 | 243,750.00 | 75,000.00 | 37,500.00 | - |
| Market value of equity | 412,414.45 | 400,339.45 | 391,084.45 | 213,169.45 | |
| Total value | 787,414.45 | 644,089.45 | 466,084.45 | 250,669.45 | - |

Source: Elaborated by authors.

The final table for this calculation is as follows,

TABLE 15

**INDEPENDENT CALCULATION OF
MARKET EQUITY VALUE (FINAL)**

| YEAR | 0 | 1 | 2 | 3 | 4 |
|--|------------|------------|------------|------------|------------|
| Free Cash Flow FCF | | 170,625.00 | 195,750.00 | 220,875.00 | 253,399.45 |
| Interest charges | | 42,000.00 | 27,300.00 | 8,400.00 | 4,200.00 |
| Debt payment | | 131,250.00 | 168,750.00 | 37,500.00 | 37,500.00 |
| CFD | | 173,250.00 | 196,050.00 | 45,900.00 | 41,700.00 |
| Tax savings TS | | 14,700.00 | 9,555.00 | 2,940.00 | 1,470.00 |
| CFE = FCF – CFD + TS | | 12,075.00 | 9,255.00 | 177,915.00 | 213,169.45 |
| Relative weight of debt D% | 61.7% | 47.4% | 19.4% | 16.9% | |
| Relative weight of equity E% | 38.3% | 52.6% | 80.6% | 83.1% | |
| $Ke_t = Ku_t + (Ku_t - Kd) \times D\%_{t-1}/E\%_{t-1}$ | | 21.4% | 18.6% | 16.0% | 15.9% |
| Debt at end of period | 375,000.00 | 243,750.00 | 75,000.00 | 37,500.00 | - |
| Market value of equity | 232,978.04 | 270,707.73 | 311,835.85 | 183,933.06 | |
| Total value | 607,978.04 | 514,457.73 | 386,835.85 | 221,433.06 | - |

Source: Elaborated by authors.

Observe that working independently we reach the same values for equity, total value and Ke . Observe as well that the NPV for the equity holder is the same as the NPV for the project (firm).

TABLE 16

NPV CALCULATIONS FROM EQUITY INVESTMENT

| YEAR | 0 |
|-----------------------------|------------|
| Present value of cash flows | 232,978.04 |
| Initial equity investment | 125,000.00 |
| NPV | 107,978.04 |

Source: Elaborated by authors.

The investment from the equity holders was 125,000 and hence NPV for them is 107,978.04. There is no surprise that both NPVs are identical. The very definition of NPV says that NPV for the firm (project) is the same as the NPV for the equity holder.

Summarizing, the different methodologies presented to calculate the total value of the firm are⁹:

Total Value for the firm $V = PV(\text{FCF at WACC})$

Total Value for the firm $V = PV(\text{FCF at } K_u) + PV(\text{TS at } K_u)$

Total Value for the firm $V = PV(\text{CCF at } K_u)$.

Market value of equity $E_{mv} = TV - D$

Market value of equity $E_{mv} = PV(\text{CFE at } K_e)$.

All these calculations give identical results.

In this example, it is shown in Table 17.

TABLE 17

A COMPARISON OF VALUES BY DIFFERENT APPROACHES

| METHOD | VALUE | EQUITY VALUE = VALUE - DEBT | NPV EQUITY = NPV FIRM |
|--------------------------------------|------------|--------------------------------|-----------------------|
| PV(FCF at $WACC_t$) Table 10 | 607,978.04 | 232,978.04 | 107,978.04 |
| PV(FCF at K_u) + PV(TS at K_u) | 607,978.04 | 232,978.04 | 107,978.04 |
| PV(FCF+TS at K_u) Table 12 | 607,978.04 | 232,978.04 | 107,978.04 |
| PV(CFE at K_e) Table 15 | | 232,978.04 | 107,978.04 |

Source: Elaborated by authors.

The value of equity is the price that the owners would sell their participation in the firm and this is higher than the initial equity contribution of 125,000.

When using K_d as the discount rate for the TS, we find a higher value and full consistency as we did with the assumption the discount rate for the TS is K_u (in this example). In short, ALL methods if properly calculated yield the same firm and equity value and identical NPV for the firm and for the equity holder. See Tham and Vélez-Pareja (2004b) and Vélez-Pareja and Burbano-Pérez (2008).

5 CONCLUSIONS

The misuse of WACC might be due to several reasons. Traditionally there have not been computing tools to solve the circularity problem in WACC calcula-

⁹ There exist other methodologies, but they do not coincide among them. See Taggart Jr. (1991).

tions. Now it is possible and easy with the existence of spreadsheets. Not having these computing resources in the previous years it was necessary to use simplifications such as calculating just one single discount rate or in the best of cases to use the book values in order to calculate the WACC.

Here a detailed (but known) methodology to calculate the WACC has been presented taken into account the market values in order to weigh the cost of debt and the cost of equity. By the same token a methodology based on the WACC before taxes K_u , constant (assuming stable macroeconomic variables, such as inflation) that does not depend on the capital structure of the firm has been presented.

The most difficult task is the estimation of K_u , or alternatively, the estimation of K_e . Here, a methodology to estimate those parameters is suggested. If it is possible to estimate K_u from the beginning, it will be possible to calculate the total and equity value independently from the capital structure of the firm, using the CCF approach or the Adjusted Present Value approach and discounting the tax savings at K_u .

In summary, the different methodologies presented to calculate the total value of the firm are consistent and yield identical values, as presented in Table 18.

TABLE 18

SUMMARY

| METHOD | TOTAL VALUE | EQUITY VALUE |
|---|-------------|--------------|
| PV(FCF at $WACC_t$) | 607,978.04 | 232,978.04 |
| PV(FCF at K_u) + PV(Tax savings at K_u) | 607,978.04 | 232,978.04 |
| PV(FCF + TS at K_u) | 607,978.04 | 232,978.04 |
| PV(CFE at K_e) | | 232,978.04 |

Source: Elaborated by authors.

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APPENDIX A

TRADITIONAL WACC FOR A FINITE STREAM OF FREE CASH FLOW (FCF)

In this appendix, we derive the traditional WACC for a finite stream of free cash flow. Consider a finite stream of cash flows where FCF_i is the free cash flow in year i . Similarly, CFE_i is the cash flow to equity in year i , CFD_i is the cash flow to debt in year i , and TS_i is the tax shield in year i , based on the value of the debt at the end of the previous year $i-1$.

In any year i , the capital cash flow (CCF) is equal to the sum of the free cash flow and the tax shield.

$$CCF_i = FCF_i + TS_i \quad (A1)$$

Also, in any year i , the capital cash flow is equal to the sum of the cash flow to equity and the cash flow to debt.

$$CCF_i = CFE_i + CFD_i \quad (A2)$$

Combining equation A1 and equation A2, we obtain,

$$FCF_i + TS_i = CFE_i + CFD_i \quad (A3)$$

Returns and taxes

The return to unlevered equity in year i is Ku_i , the return to levered equity in year i is Ke_i , the cost of debt in year i is Kd_i and the discount rate for the tax shield in year i is ψ_i . We assume only corporate tax τ . Furthermore, the corporate tax rate is constant. If the debt is risk-free, then the cost of debt is equal to the risk-free rate r_f .

M & M world

The unlevered value in year i is V_i^{un} , the levered value in year i is V_i^L , the (levered) equity value in year i is E_i^L , the value of debt in year i is D_i and the value of the tax shield in year i is V_i^{TS} .

With perfect capital markets in an M & M world, we make the following assumptions. In any year i , the levered value is equal to the sum of the unlevered value and the value of the tax shield.

$$V_i^L = V_i^{Un} + V_i^{TS} \quad (A4)$$

Also, in any year i , the levered value is equal to the sum of the value of (levered) equity and value of debt.

$$V_i^L = E_i^L + D_i \quad (A5)$$

Combining equation A4 and equation A5, we obtain,

$$V_i^{Un} + V_i^{TS} = E_i^L + D_i \quad (A6)$$

The expressions for the unlevered value, the (levered) equity value, the value of debt and the value of the tax shield are shown below. In any year $i-1$, the value is equal to the cash flow discounted by the appropriate discount rate.

$$V_{i-1}^{Un} = \frac{FCF_i}{1+Ku_i} \quad (A7.1)$$

$$E_{i-1}^L = \frac{CFE_i}{1+Ke_i} \quad (A7.2)$$

$$D_{i-1} = \frac{CFD_i}{1+Kd_i} \quad (A7.3)$$

$$V_{i-1}^{TS} = \frac{TS_i}{1+\psi} \quad (A7.4)$$

Substituting equation A7.1 to equation A7.4 in equation A3, we obtain:

$$(1 + Ku_i) \times V_{i-1}^{Un} + (1 + \psi_i) \times V_{i-1}^{TS} = (1 + Ke_i) \times E_{i-1}^L + (1 + Kd_i) \times D_{i-1} \quad (A8.1)$$

Substituting equation A6 into equation A8.1 and simplifying, we obtain,

$$Ku_i \times V_{i-1}^{Un} + \psi_i \times V_{i-1}^{TS} = Ke_i \times E_{i-1}^L + Kd_i \times D_{i-1} \quad (A8.2)$$

The weighted average cost of capital with the FCF

Let W_i be the WACC in year i based on the FCF_i . Then in year $i-1$, the levered value is equal to the FCF in year i discounted by W_i .

$$V_{i-1}^L = \frac{FCF_i}{1+W_i} \quad (A9.1)$$

Rewriting equation A9.1, we obtain that

$$FCF_i = (1 + W_i) \times V_{i-1}^L \quad (A9.2)$$

From equation A3, we know that

$$FCF_i = CFE_i + CFD_i - TS_i \quad (A10)$$

Substituting equation A9.2, and equation A7.2 to equation A7.4 into equation A10, we obtain:

$$(1 + W_i) \times V_{i-1}^L = (1 + Ke_i) \times E_{i-1}^L + (1 + Kd_i) \times D_{i-1} - (1 + \psi_i) \times V_{i-1}^{TS} \quad (A11)$$

Simplifying equation A11.1 we obtain:

$$V_{i-1}^L + W_i \times V_{i-1}^L = Ke_i \times E_{i-1}^L + Kd_i \times D_{i-1} - (1 + \psi_i) \times V_{i-1}^{TS} + E_{i-1}^L + D_{i-1} \quad (A12.1)$$

$$W_i \times V_{i-1}^L = Ke_i \times E_{i-1}^L + Kd_i \times D_{i-1} - (1 + \psi_i) \times V_{i-1}^{TS} \quad (A12.2)$$

We know that the tax shield in year i is equal to the tax rate τ times the cost of debt times the value of debt at the end of the previous year i-1.

$$TS_i = \tau \times Kd_i \times D_{i-1} \quad (A13)$$

Substituting equation A7.4 and equation A13 into equation A12.2, we obtain the traditional formulation of the WACC.

$$W_i \times V_{i-1}^L = Ke_i \times E_{i-1}^L + Kd_i \times D_{i-1} - \tau \times d_i \times D_{i-1} \quad (A14.1)$$

$$W_i = \frac{E_i}{V_{i-1}^L} \times Ke_i + \frac{D_{i-1}}{V_{i-1}^L} + Kd_i \times (1 - \tau) \quad (A14.2)$$

The WACC is a weighted average of the cost of equity and the cost of debt, where the cost of debt is adjusted by the coefficient $(1 - \tau)$ and the weights are the market value of equity and market value of debt, as percentages of the levered market value. Equation B14.2 is equation 1 in the text.

APPENDIX B

DERIVING KE FOR A PERPETUITY

List of symbols

| | | |
|---|---|-------|
| K_u | The cost of the unlevered equity | |
| K_d | The cost of debt (assumed constant) | |
| D | Market value of debt | |
| K_{e_n} | Levered cost of equity at year n | |
| E^L | Market value of levered equity | |
| ψ_n | Appropriate discount rate for tax savings at year n | |
| V^{TS} | Value of TS | |
| V^{ND} | Value of unlevered firm | |
| V^L | Value of levered firm | |
| $V^{TS} = \tau \times K_d \times D / \psi$ | | (B1a) |
| $\psi \times V^{TS} = \tau \times K_d \times D$ | | (B1b) |
| $V^{ND} = FCF / K_u$ | | (B2a) |
| $V^{ND} \times K_u = FCF$ | | (B2b) |
| $E^L = Z / K_e$ | | (B3a) |
| $E^L \times K_e = Z = FCF - K_d \times D + \tau \times K_d \times D$ | | (B3b) |
| $E^L \times K_e = V^{ND} \times K_u - K_d \times D + \psi \times V^{TS}$ | | (B4a) |
| $E^L \times K_e = [V^L - V^{TS}] K_u - K_d \times D + \psi \times V^{TS}$ | | (B4b) |
| $K_e \times E^L = K_u \times E^L + (K_u - K_d) \times D - (K_u - \psi) \times V^{TS}$ | | (B4c) |
| $K_e = K_u + (K_u - K_d) \times D / E^L - (K_u - \psi) \times V^{TS} / E^L$ | | (B4d) |

This is the most general formulation for K_e , the cost of levered equity.

- Case I

Assume $\psi = K_d$ and perpetuities

$$K_e = K_u + (K_u - K_d) \times D / E^L - (K_u - K_d) \times \tau \times D / E^L \quad (B4e)$$

Reorganizing

$$K_e = K_u + (K_u - K_d) \times (1 - \tau) \times D / E^L \quad (B4f)$$

This equation C4f is equation (2) in the text. For a finite horizon we have to use B4d.

- Case 2

Assume $\psi = K_u$

$$K_e = K_u + (K_u - K_d) \times D/E^D \quad (B4g)$$

This is equation (7) in the text. This formula is valid for finite horizons and perpetuities.