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A CASE-BASED ROADMAP FOR LATERAL TRANSSHIPMENT IN SUPPLY CHAIN INVENTORY MANAGEMENT

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ABSTRACT

Manufacturers and wholesalers are increasingly cost conscious in response to today’s hyper-competitive environment. Lateral transshipment (LT) has been proposed as a viable solution to drive total inventory costs down whilst increasing customer service level. Our study proposes five LT decision rules with a case-based roadmap to guide professional inventory management. Results of this large fast moving consumer goods case study company demonstrate superior inventory management performance with implementing a combined reactive and proactive LT strategy to determine whether to transship emergency stock from other warehouse or to backorder from suppliers, size of transshipment, favorite wholesaler, preferred supplier, and extra quantity for preventive LT, which are the key LT decision points among the professional supply chain management practitioners.

Keywords: Lateral transshipment; inventory management; decision rules; roadmap

1. INTRODUCTION

Due to increasing market competition, manufacturers and wholesalers are becoming more cost conscious and responsive to the changing market needs. One supply chain strategy is to conserve a low inventory level just enough for instantaneous availability for use or sales purpose. However, the resulting surge in the risk of stock outages and substandard customer service level make the cost saving in lower inventory level problematic to justify. To manage these adversities, manufacturers and
wholesalers usually increase the flexibility in their inventory systems by adopting a mix of emergency lateral transshipment (LT) from other warehouses at a higher cost, while at the same time backordering from their usual suppliers to match the stochastic demand. Decision rules on LT that support this decision making process have practical value for inventory management practitioners.

Assume that the manufacturer with its regional distribution centres or all wholesalers adopt the periodic review policy to replenish inventory from their external suppliers. Under the periodic review policy, the unsatisfied demands in the previous period, the inventory position and the expected demand in the current period are analyzed for planning orders at the beginning of the next period. Unsatisfied demands in this period will be treated as initial demands, or surplus order will be added to the inventory position of the next period. If the wholesaler does not apply a (R,Q) review policy to order quantity Q at reorder point R, its inventory position could drop below R or even close to zero. Though holding costs can decrease as a result, there is a possibility that its inventory position may occasionally become negative. This potentially increases the back-order costs. One possible solution is to apply LT from other wholesalers with surplus stock to replenish wholesaler with stock deficiency within a short period of time.

Within the context of LT, a supply chain structure consists of multiple retailers, wholesalers and suppliers, as shown in the Fig.1.

![Supply chain structure diagram](image)

**Fig.1. Lateral transshipment in a supply chain structure**

All the warehouses belong to the same company. The black solid lines represent possible transshipments among various warehouses within the company. For modeling simplicity, assume only LT between a pair of wholesalers (sender and receiver) $W_i$ and $W_j$, $i$ not equal $j$, where $i$ and $j = 1, 2, \ldots$, total number of warehouses. Similarly, $W_i$ places orders pairwise with a single supplier, instead of multiple suppliers.

Supply chain management (SCM) models that streamline the flow of goods to
optimise total inventory costs, customer service level (Banerjee, Burton, & Banerjee, 2003), total number of stockouts (Jonsson & Silver, 1987), or other performance criteria have been the research concentration for the last three decades. However, previous research into manufacturer and wholesaler inventory management has captured complex criteria drawn from diverse theoretical frameworks that are problematic in real world practice. For example, one earlier study conducted by Axsäter (2003) to derive a decision rule for determining quantity to be transshipped, depending on the complete state of the systems with various input parameters. This decision rule is optimal and can be repeatedly applied as a heuristic for SCM practice. However, the highly mathematical probability analysis may not be easily comprehensible to ordinary managers, and thus not validated in real world practice. Therefore, it remains a need for simpler and more readily applicable decision rules for the LT decision making process.

Further model proposed by Olsson (2009) investigated an optimal ordering policy under complete pooling, but the optimal solution is restricted to systems with only two locations due to problem complexities. Our study extends to a multi-location setting to develop decision rules for reactive LT to fulfill existing inventory shortage due to urgent demand that cannot be satisfied from the stock on hand. The decision rules are derived for determining whether it is more cost effective to transship urgent orders or to backorder all outstanding orders from suppliers, the size of transshipment, the favorite wholesaler to transship and the favorite supplier to order. Further extension also covers the preventive extra LT, which occurs before an inventory shortage emerges.

Apart from less complexity in calculations, the data requirement of our proposed approach can be sourced from previous corporate transaction records, and thus enables adoption of this model by SCM practitioners. However, this new approach does not undermine previous scholarly work, but builds upon it by proposing a more pragmatic decision model for the SCM environment. This model can be applied to a real context with multiple warehouses of manufacturers/wholesalers and multiple suppliers with variable lead times. This proposed approach is validated through the illustration of a practical application of the model in a large fast moving consumer goods (FMCG) case study company.

The next sections reviews the relevant literature, the decision rules derived from the proposed mathematical model with application to the transshipped decision making process, numerical illustration and roadmap through a case study that has practical values and implications for management. The last section concludes with a discussion of the effectiveness and limitations of the decision model, with suggestions for further research.

2. LITERATURE REVIEW

System behaviors of lateral transshipment (LT) models are largely derived from analytical method. The analytical LT models have progressively been upgraded in the literature (Archibald, 2007; Wong, Cattrysse, & Oudheusden, 2005; Wong, Van Houtum, Cattrysse, & Oudheusden, 2006). However, this advancement in the LT research field also added structural complexities that can be too cumbersome for deriving the optimal solutions, especially when elements of the demand and supply are modeled as stochastic processes. Since exact analysis is often mathematically intractable, search procedures are designed to determine the optimal solutions. To
overcome the weakness of analytical method, simulation can generate numerical systems representation of the complex LT models for assessing interactions of key elements and their causal relationships in a large number of simulation runs, for evaluation of the optimal performance of various competing models. Publication of simulation studies in the LT literature are fewer (Banerjee et al., 2003; Burton & Banerjee, 2005; Glazebrook, Paterson, Rauscher, & Archibald, 2014). However, there is an overwhelming consensual call among logistics visionaries to endorse simulation to supplement the traditional analytical method (Davis-Sramek & Fugate, 2007). Therefore, results of the analytical models of our study are validated by Monte-Carlo simulation.

In recent comprehensive overviews of LT (Lee, Jung, & Jeon, 2007; Paterson, Kiesmüller, Teunter, & Glazebrook, 2011; Seidtscher & Minner, 2013; Wong et al., 2006), two major types of transshipments are identified on the basis of timing of the transshipments. They are proactive and reactive transshipments. Proactive LT is also known as preventive LT, which is initiated at a predetermined point in time before an inventory shortage appears for the purpose of preventing future stockout. Inventory levels of different locations at the same echelon are rebalanced to prevent future stockout or reduce the risk of future stockout (Banerjee et al., 2003; Bertrand & Bookbinder, 1998; Diks & De Kok, 1996; Gross, 1963; Jonsson & Silver, 1987; Lee et al., 2007; Tagaras & Vlachos, 2002; Tiacci & Saetta, 2011). On the other hand, reactive LT, also known as emergency LT, which is triggered in response to an existing inventory shortage as demand has been realized. When a retailer experiences with stock deficiency, inventory is transferred from a warehouse or retailer with surplus stock on hand (Krishnan & Rao, 1965; Olsson, 2010; Robinson, 1990). Shipments are assumed to be fast enough to fill the shortage in stock (Tiacci & Saetta, 2011). There are subtle differences between the models within the previous reactive LT studies (Archibald, 2007; Archibald, Black, & Glazebrook, 2009; Hu, Watson, & Schneider, 2005; Huo & Li, 2007; Lau & Nakandala, 2012; Lee, 1987). However, matching the source to the receiving destination optimally remains a research challenge.

In the comprehensive literature review by Paterson et al. (2011), the reactive LT research is categorized under periodic review and continuous review. For periodic-review LT studies, the focus in the literature is frequently on single echelon system. Hu et al. (2005) investigate a multiple retailer distribution system where emergency transshipments are permitted to determine the effect of transshipments on ordering policies. They compared the (s, S) ordering policy, i.e. order quantity up to S at reorder point s, with a simplified policy that assumes free and instantaneous transshipments. Their findings suggested that for a small number of stores and small transshipment costs relative to the holding and stock-out costs, inventory policies may be obtained from a simplified model using zero transshipment costs but using transshipments as a means to solve emergency situations. Otherwise, a model without transshipments can be used.

Several heuristic decision rules have been derived from our analytical model. Our model is further validated through simulation experiments to compare among different scenarios, and test under various conditions, to select a particular transshipment that minimizes the total inventory costs. This approach is similar to the previous research studies of Archibald (2007) using decomposition method to develop three structured heuristic transshipment policies and Archibald et al. (2009) using approximate solution method to determine the optimal transshipment.

On the other hand, an equal amount of research studies on reactive LT also use the continuous review policy to transship whenever there is a stock out or potential
stock out. Results are not significantly different from the periodic review policy research. Based on the number of echelons, reactive LT studies under continuous order review can also be categorized into single-echelon or two-echelon systems. For the studies in reactive LT with single-echelon system, representative research studies include Kukreja et al. (2001), Wong et al. (2006), Wong et al. (2007), and Huo and Li (2007).

Most of the literature assume two-echelon, which is common even in E-tailers. In recent years, the advance of e-Commerce information technology also draws attention to researchers to examine LT in this virtual environment. Customers can benefit from fast access to wider product information and purchasing choices in the online channel, with price comparison to discover the best deal, and minimal probability of stockout, though longer delivery lead time. For example, Amazon.com has no distribution centers, and books are sent from publishers to customers in direct shipments. Nevertheless, its distribution structure has recently been reconfigured to a small number of distribution centers to accommodate both direct from publishers and from own warehouses to customers. Likewise, traditional “bricks and mortar” in other industries also face similar challenge of designing a combination of distribution channels.

Cai (2010) investigated LT under the supply chain structure of one-echelon dual-channel model, i.e. manufacturer constructs virtual shop as an online sales channel in additional to existing physical retail outlets. Examples include Apple, IBM, HP, Lenovo, Acer, Cisco, Nike, Adidas, Estee Lauder, and many other retailers have implemented virtual shops. Apart from benefiting customers, the dual channel (Chen, Kaya, & Ozer, 2008; Huang, Yang, & Zhang, 2012) also provides the normal benefits through retail channel, e.g. touch and feel experience of the products and services, reduce the chance of sales return in failed product and poor delivery quality. Furthermore, the order fulfilment in the dual-channel SCM is usually supported by drop shipping with the advantages of risk pooling to reduce the costs of inventory, transportation and stockout. On the other hand, He et al. (2014) apply unidirectional transshipment under the two-echelon dual channel model and find that the transshipment price mechanism always coordinates the supply chain.

A new online-to-offline (OTO) business model that combines the retail and direct channels has been designed to enhance the traditional dual-channel supply chain and drop-shipping model (Zhao, Wu, Liang, & Dolgui, 2015). The OTO supply chain takes advantage of the concept of LT to fulfill customer order from inventories stored at the nearest retailer’s warehouse, thus not only to reduce the stockout risk but with service level improvement, minimal transshipment costs, and lower inventory costs (Belgasmi, Said, & Ghédira, 2008). The combined proactive and reactive transshipment approaches of our study can also be implemented in the dual channel and OTO business model to achieve the desired outcomes.

This study considers a combination of proactive and reactive LT, and investigates the possibility of using LT to fulfill not only the outstanding urgent orders but also the extra quantities for fulfilling the expected demand at the beginning of the scheduling period in order to avoid high backorder and penalty costs. It builds on previous research conducted by Lau and Nakandala (2012), which developed decision rules for the selection of LT to meet the outstanding demand that cannot be fulfilled by the stock on hand, by including the possible scenario of sourcing extra LT to meet the demand during the scheduling period and then determining the relevant decision rules.
Banerjee et al. (2003) and Burton and Banerjee (2005) compare the performance of a proactive redistribution policy (Transshipment Inventory Equalisation, TIE) to a simple reactive transshipment method (Transshipment Based on Availability, TBA). However, unlike our study, their studies analyzed these two methods separately, rather than combining the proactive and reactive LT into an integrated model.

3. RESEARCH METHOD

Our research approach utilizes an analytical LT model to derive the five LT decision rules, as detailed in the Appendix, to assist supply chain practitioners to implement LT to improve supply chain systems performance by minimizing total inventory costs where stockout is one key cost component. Relevant input indicators have been identified when designing the roadmap for implementing LT. Data collection from the case study company reveals the baseline and the problems associated with the no LT scenario as benchmark for comparison against the LT solution. Tapping into the literature, expert opinions, and management experiences, our study has investigated various interventions and evaluation criteria in a comparative study to determine why one intervention is most suitable to improve inventory management. Relevant results from our study will be summarized to facilitate implementation of LT as part of the inventory management systems. This practical solution can easily be generalized to other companies to gain similar operational benefits.

3.1 Generic Decision rules for LT

Our study and other earlier literature (Alfredsson & Verrijdt, 1999; Archibald et al., 2009; Axsäter, 2006, 2007; Burton & Banerjee, 2005; Dada, 1992; Evers, 2001; Lee et al., 2007; Minner, 2003; Minner & Silver, 2005; Minner, Silver, & Robb, 2003; Wee & Dada, 2005) of analytical and simulation experiments have identified several generic decision concepts and rules that can straightforwardly be comprehended by supply chain practitioners, including:

Rule a: Purchasing, backordering, and holding costs have a crucial impact on LT decision.

Rule b: Backordering is a good approximation for lost sales, provided service level is sufficiently high.

Rule c: LT should not be applied if transshipment lead time is longer than supplier replenishment lead time.

Rule d: Providers with the most stock surplus transship to those with the most shortage.

Rule e: Providers should consider future demand and only transship extra stock surplus.

Rule f: Preventive LT policies are particularly suitable when holding costs are dominant.

Rule g: Reactive LT policies perform better where transshipment costs are relatively lower.

However, these simple decision concepts and rules do not address the fundamental LT decisions explored in the literature, e.g. whether to apply LT or not, selection of the preferred wholesaler, optimal size of transshipment, selection of the preferred supplier, timing of extra transshipment, etc. Furthermore, for large chains with thousands of retail stores, a large number of policies is prohibitive in practice. While the complex systems and policies may mathematically generate optimal solutions, the
practical advantages of simplifying and categorizing into parsimonious policies that embody easily implementable decision rules, irrespective of store differences across geographies and products, can be substantial. Our research aims to develop the LT decision rules that can easily be comprehended and applied by ordinary managers in their LT decision making process to minimize total inventory costs.

### 3.2 Proposed decision rules for LT

As derived from the Appendix, our research study identifies five heuristic decision rules for LT decision support, i.e. whether to apply LT or not, selection of the preferred wholesaler, optimal size of transshipment, selection of the preferred supplier, timing of extra transshipment. There are five strategies examined by our model using the analytical method and simulation experiments. This study has applied the identified decision rules in a comparative study to examine the following set of transshipment strategies:

1. the proposed two-step decision rule
2. no LT
3. LT for only initial outstanding demand
4. LT to satisfy half of the expected demand during the supplier lead time
5. LT to satisfy the total expected demand during the supplier lead time

The input data were compiled from historical corporate database of this case study company, and computed jointly by the operations management and accounting departments. The three key cost components of the total inventory costs were measured as defined above, and examined jointly with various combination of supplier lead time and transshipment costs. To verify feasibility of the proposed LT model, 1,000 simulations of different scenarios have been run. The simulation results confirm the superiority of our proposed two-step LT model documented in the Appendix. Our proposed two-step decision flow chart, as summarized in Fig.2 below, generates the lowest total inventory costs among these five different transshipment strategies.
This flow chart captures the sequential decision steps of the five LT decision rules, as derived from the Appendix. The first step applies the first decision rule. If the condition $-p_{ij} - E(L_{ij})b_i + q_{ik} < 0$ is satisfied, then implement LT, otherwise, place the order with the preferred supplier that minimizes the total inventory costs (the fourth decision rule). Once LT is selected as the preferred option, the inventory manager applies the second decision rule to select the favorite warehouse that charges the lowest unit LT cost. Also, the third decision rule is applied for calculating the optimal size of transshipment.

Based on the Appendix, the analytical derivation of the five decision rules to serve as heuristic guide for LT decision support can be summarized as follow.

1. First decision rule: whether to apply LTs or not

The total cost function, as in equation 3 of the Appendix, is linear with respect to the quantity of transshipment $x$. When the tangent of this linear function $-p_{ij} - E(L_{ij})b_i + q_{ik}$ is negative, the total inventory costs decrease as the quantity transshipped increase. Hence the decision rule for determining whether LT should be implemented is,

$$-p_{ij} - E(L_{ij})b_i + q_{ik} < 0$$

or

$$q_{ik} < p_{ij} + E(L_{ij})b_i$$  \(1\)

If the condition of this decision rule, as defined by equation (1), is satisfied, the higher the quantity transshipped, the lower the total inventory costs for the wholesaler $W_i$. Otherwise, the wholesaler $W_i$ should decide to fulfill the demand by ordering only from the supplier $S_j$ if the above decision rule is not satisfied.
2. Second decision rule: selection of the preferred wholesaler

As a corollary of the above decision rule that the higher the quantity transshipped, the lower the total inventory cost to the wholesaler $W_i$. This suggests the favorite preference should be given to wholesaler $W_k$ that could transship at the lowest LT cost $q_{ik}$

3. Third decision rule: optimal size of transshipment

This manufacturer or wholesaler only orders the initial outstanding demand net of existing inventory at $t = 0$ which is $d_i(0) - l_i(0)$ from another wholesaler due to the higher unit cost of transshipment from another wholesaler, as compared with the unit purchasing cost from suppliers. For simplicity of the model, we assume that the preferred sending wholesaler has sufficient stock to deliver the transshipment to the receiving wholesaler. Hence the optimal size of transshipment, $\mu_k$ is defined as,

$$\mu_k = d_i(0) - l_i(0)$$

Therefore, the size of the transshipment can be either 0 or $\mu_k$ in this model. When the decision rule in equation (1) is not satisfied, there will not be any transshipment, and the maximum of $\mu_k$ is transshipped when the condition is satisfied.

4. Fourth decision rule: selection of the preferred supplier

The wholesaler $W_i$ can source from any one of its suppliers. The selection decision is derived by global minimization of the total inventory cost function in equation 3 of the Appendix, with a fixed $x (x \neq 0)$ and a known $W_k$. Hence, the decision rule is given by the condition that satisfies

$$C_i = \min (C_{i1}, C_{i2}, ..., C_{ij}, ..., C_{iN_i})$$

where $N_i$ is the number of suppliers of the wholesaler $W_i$.

5. Fifth decision rule: determine the extra quantity of transshipment

The time $K$ corresponding to the point of intersection between the two cost functions $c_i^t(t)$ and $c_i^s(t)$, as shown in Fig.1 of the Appendix, determines the extra quantity to be transshipped. $K$ is the maximum integer less than $\frac{p_{ij} - a_{ik} + b_i E(L_{ij})}{h_i + b_i}$. Therefore, $K$ must be non-negative and within the range of $1 < K \leq \frac{p_{ij} - a_{ik} + b_i E(L_{ij})}{h_i + b_i}$. And the extra quantity transshipped is determined by $\delta x = \delta_i(K)$.

4. CASE STUDY

This case study company is a major establishment in the FMCG sector, with one national and five regional distribution centers in each of the five major cities in Australia to serve a diverse range of retail stores across wide geographies. Since most POS systems in its retail stores have real time access to sales and inventory data, continuous review policy seems to be feasible and preferred. However, certain limitations are violating the other conditions for continuous review policy, making periodic review policy a necessity, including pre-determined schedule, fixed contracts
confirmed with customers and shipping companies, simultaneous delivery of a variety of goods, batch update in ERP inventory databases, and inventory decisions are made as per predefined cycles. Therefore, it is more appropriate for this case study company to adopt a periodic review policy.

The objective of our research is to measure current inventory management performance and improve its performance by implementing the LT decision rules so as to minimize total inventory costs. The key decision is the optimal division of inventory between central warehouse and among retail stores. Higher customer service level can be achieved when more inventories are positioned at retail stores, but with the associated increase in holding and transportation costs, thus, an optimal balance is vital to achieving the cost objective. The advantage of positioning more inventories in the national and regional distribution centers is risk pooling that reduce the total systems inventory costs. However, this is not an efficient configuration to restore subsequent inventory imbalances across the regional distribution centers and retail stores and cause shipment delay that may adversely impact on customer service level if lateral shipment is not part of the normal replenishment process.

Recently, advances in information technology enhances the operations of LT. Cachon and Fisher (2000) quantified the potential value of information sharing in a single warehouse, multi-retailer setting, with identical retailers, batch ordering, fixed shipment lead time, periodic review inventory policy. By comparing the total supply chain costs in both with and without information sharing scenarios, however, the value of information sharing is only 2.2%, which is much less than the benefits from the just-in-time (JIT) configuration of shorter lead times and smaller batch sizes, approximately 20% each. Therefore, sophisticated communication systems, though beneficial for information sharing within the supply chain, can be over-engineered with inadequate return on investment. Simple and fast communication of inventory and demand status among the regional distribution centers and retail stores should suffice the LT infrastructure with high potential of performance gains.

4.1 Numerical illustration of the proposed decision rules for LT

To determine the optimal quantity and timing of LT, the three key cost components of the total inventory costs should be minimized. Input parameters were jointly identified by the operations management and accounting departments. The input data were compiled from historical corporate database of this case study company. Computation of the total inventory costs comprises the following three key cost components:

1. **Purchasing costs** include all the labor, equipment, and related resources engaged in planning order, requisition, and monitoring and controlling the progress of order activities, transportation and shipping, receiving, inspection, handling and storage, accounting and auditing costs.

2. **Backordering costs** incurred when stock on hand is not available to meet customer demand which include lost sales, estimated loss of future sales and goodwill due to customer dissatisfaction, and contractual penalties of non-or late deliveries. However, it is largely resorted to judgment and thus generally ignored in inventory costing due to its estimation uncertainty.

3. **Holding costs** include interest on loans to finance inventory or opportunity costs of inventory investment; storage related costs (rent, provision
of facilities, heating, cooling, lighting, security, refrigeration, administrative, handling and storage, transportation); product depreciation, deterioration, spoilage, damages, and obsolesce; insurance and taxes. It amounts to approximately 15% to 30% of the total inventory costs, but is difficult to calculate with high degree of accuracy, and is often underestimated.

Based on the derived five decision rules, our proposed approach is illustrated numerically below with examples from this FMCG case study company.

1. First decision rule: whether to apply LTs or not

Based on corporate databases, each of the three key cost components, i.e. the purchasing, backordering, and holding costs, as defined in section 3.2, are computed by the joint collaboration of the operations management and accounting departments. An example from this case study company:

\[ q_{ik} = \text{unit LT cost from wholesaler } W_k \text{ to wholesaler } W_i = $5.0 \]

\[ E(L_{ij}) = \text{expected lead time of supplier } S_{ij} = 4 \text{ days} \]

\[ b_i = \text{unit backordering cost of } W_i \text{ per unit time} = $2.0 \]

\[ h = \text{unit holding cost for wholesaler } W_i \text{ per unit time} = $2.2 \]

\[ p_{ij} = \text{unit selling price charged by supplier } S_{ij} \text{ to wholesaler } W_i = $2.2 \]

Applying the equation \[ -p_{ij} - E(L_{ij})b_i + q_{ik} = -$2.2 - 4 \times $2.0 + $5.0 = -$5.2, \] which is negative. Therefore, LT should be implemented in this situation, in accordance with the first decision rule.

2. Second decision rule: selection of the preferred wholesaler

To select the preferred wholesaler, so long as \[ -p_{ij} - E(L_{ij})b_i + q_{ik} \] is negative, the wholesaler will be included in the preferred list of wholesalers for consideration, and the favorite preference is given to wholesaler \( W_k \) that could transship at the lowest LT cost \( q_{ik} \). For example, based on the following inputs in Table 1, if each case from 1 to 3 represents an individual wholesaler, then the wholesaler in case 1 should be the favorite preference since its LT cost \( q_{ik} \) is the lowest.

Table 1. Inputs for wholesaler selection through the application of the LT decision rules

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tr>
<td>( q )</td>
<td>5</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>( E(L) )</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>( b )</td>
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<td>2</td>
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<tr>
<td>( h )</td>
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<td>2</td>
<td>2</td>
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<tr>
<td>( p )</td>
<td>2.2</td>
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</tr>
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3. Third decision rule: optimal size of transshipment

The optimal size of the LT is designed to fulfill the initial outstanding demand net of existing inventory at \( t = 0 \) when the condition for LT is satisfied, i.e. \( d_i(0) - l_i(0) \). If \( d_i(0) \) is 6,000 units and \( l_i(0) \) is zero, then the optimal size of the LT is 6,000 units.
4. Fourth decision rule: selection of the preferred supplier

To select the preferred supplier, the total inventory costs will be computed for all the suppliers, and the order will be placed onto the supplier that generates the minimal total inventory costs.

5. Fifth decision rule: determine the extra quantity of transshipment

To determine the optimal timing for preventive extra transshipment, the value $K$ is the maximum integer less than \( \frac{p_{ij} - q_{ik} + b_i E(L_{ij})}{h_i + b_i} \). Therefore, an example of this company is:

- \( q_{ik} \): the unit LT cost from the wholesaler \( W_k \) to wholesaler \( W_i \) = $5.0
- \( E(L_{ij}) \): expected lead time of the supplier \( S_j \) = 4 days
- \( b_i \): unit backordering cost at \( W_i \) per unit time = $2.0
- \( h_i \): unit holding cost for the wholesaler \( W_i \) per unit time = $2.0
- \( p_{ij} \): the unit selling price by the supplier \( S_j \) to the wholesaler \( W_i \) = $2.2

Applying the equation \( \frac{p_{ij} - q_{ik} + b_i E(L_{ij})}{h_i + b_i} = (\frac{2.2 - 5.0 + 4 \times 2.0}{2.0 + 2.0}) = 1.3 \). The maximum integer less than 1.3 is 1, therefore \( K = 1 \). The size of LT is \( \hat{d}_1(K) = \hat{d}_1(1) \), i.e. the expected demand for period 1.

5. DISCUSSION AND CONCLUSION

Based on the case-based roadmap as a feasible solution, we recommend our proposed two-step LT decision rules to the professional inventory management practitioners on the basis of the evidence of achieving superior inventory management performance and return, as compared with the other four strategies. By following these five decision rules for LT decision support, inventory management practitioners are in a better informed position to optimize their inventory management systems to determine whether it is more cost effective to transship emergency orders or to backorder all outstanding orders from suppliers, the size of transshipment, the favorite wholesaler, and the preferred supplier. Further coverage of extra quantity for preventive LT, which occurs before an inventory shortage emerges, can also be examined.

Our study investigates the possibility of LT for fulfilling not only urgent demand at the beginning of the scheduling period, but also the expected demand during supplier lead time. Based on the case study results, we recommend a combined reactive and proactive approach to LT in a manufacturer/wholesaler environment where LT are more expensive and instantaneous. We are unaware of any other study which has closely resembled a similar scope, and this makes our contribution to the LT knowledge base remarkably novel.

The main advantage of these five decision rules is their ease of applicability to inventory management and implementation by professional inventory management practitioners. The data requirements for the application of these proposed decision rules are not complex and cumbersome to collect. These LT decision rules require only the unit purchasing cost from suppliers, unit transshipment cost from other wholesaler, own unit backordering cost, and the expected lead time from its suppliers; while the decision
rule on extra LT quantity requires only their own unit holding cost, in addition to the previous set of data. These data requirements can be sourced from historical corporate transactions and cost records of the manufacturer/wholesaler.

Our study performs the calculations with Matlab to verify the feasibility of the proposed model and assess the effectiveness of the proposed decision rules. However, real-world computation for the LT decision support could well be implemented via commonly available spreadsheet software. On the limitation side, applicability of the proposed decision rules is appropriate for less dynamic business environments. Since this case-based roadmap has not been validated by large-sampled statistical modeling, further studies may apply these five generic LT decision rules to other industries to create the generalization possibility, and augment the model-based design to accommodate cross industry and company differences. Furthermore, future research can extend to accommodate the effects of business dynamics, and validate the proposed decision rules in highly dynamic business environments.

REFERENCES


Appendix

A1. Minimization of the total inventory costs for the decision of lateral transshipment

The total inventory costs of a wholesaler consist of three components: purchasing costs $P_{ijk}$ of both the supplier orders and LT from the other wholesalers; backordering costs $B_{ijk}$ for unfulfilled retailer demands; and holding costs $H_{ijk}$ for carrying inventory to meet potential demand.

$$C_{ijk}(x) = P_{ijk}(x) + B_{ijk}(x) + H_{ijk}(x)$$

Substituting the functions of purchasing, backordering, and holding costs in equation 5, the total inventory costs during the scheduling period is specified as,

$$C_{ijk}(x) = p_{ij}(d_i(0) - l_i(0) + \tilde{B}_j - x) + q_{ik}(x) + (d(0) - l(0) - x)E L_{ij} b_i + b_i \int_{t=0}^{E_{L_{ij}}} \tilde{d}(t) - \tilde{d}(t-1)\{E(L_{ij}) - t\}dt + \int_{t=E(L_{ij})+1}^{E_{L_{ij}}+1} \{\tilde{d}(t) - \tilde{d}(t-1)\}\{t - E(L_{ij})\}h_i dt$$

After collecting terms, the equation can be rewritten as,
\[ C_{ijk}(x) = \left\{ -p_{ij} - E(L_{ij})b_i + a_{isa} \right\}x + p_{ij} \{ d_i(0) - l_i(0) + \bar{D}_{ij} \} + \{ \hat{d}_i(0) - l_i(0) \} E(L_{ij})b_i + \\
\sum_{t=1}^{n_{ij}} \left[ \hat{d}(t) - \hat{d}(t-1) \right] E(L_{ij}) - t \right\} dt + \sum_{t=E(L_{ij})+1}^{n_{ij}} \left[ \hat{d}(t) - \hat{d}(t-1) \right] \left( t - E(L_{ij}) \right) \hat{h}_i \ dt \]

where 

\( W_i \) = the \( i \)th wholesaler,

\( N_i \) = the total number of suppliers to the wholesaler \( W_i \),

\( S_{ij} \) = the \( j \)th supplier of the wholesaler \( W_i \),

\( p_{ij} \) = the unit selling price by \( S_{ij} \) to \( W_i \),

\( q_{ia} \) = the unit intra-shipment cost for \( W_i \) to intraship from \( W_k \),

\( b_i \) = unit back-order cost at \( W_i \) per unit time,

\( h_i \) = unit holding cost for \( W_i \) per unit time,

\( t_a \) = start of the scheduling period, \( t = 0 \),

\( g_{ij}(t) \) = delivery lead time probability mass function of \( S_{ij} \),

\( L_{ij} \) = lead time of \( S_{ij} \) with duration equal to \( L_{ij} \) times unit time interval,

\( L_{ij}^{max} \) = the maximal lead time of \( S_{ij} \),

\( d_i(0) \) = the initial retailer demand at \( t = 0 \) appearing at \( W_i \),

\( \lambda_i(t) \) = the retailer arrival intensity during the \( t \)th time interval at \( W_i \),

\( f_{nm} \) = the probability of \( n \) retailers arriving at \( W_m \) with a total demand of \( m \),

\( \hat{d}_i(t) \) = the expected retailer demand at wholesaler \( W_i \) in the \( r \)th time interval,

\( \hat{D}_i \) = the expected retailer demand at wholesaler \( W_i \) over \( L_{ij}^{max} \)

**A2. Decision rule for extra lateral transshipment**

When both extra transshipment and backordering are considered as viable potential solutions, identifying the choice between these two sourcing options and order characteristics for cost minimization is required. When the demand at time \( t = 0 \) is \( \hat{d}_i(0) \) and \( \hat{d}_i(0) \), \( \hat{d}_i(1) \), \( \hat{d}_i(2) - \hat{d}_i(1) \), \( \ldots \), \( \hat{d}_i[E(L_{ij})-1] - \hat{d}_i[E(L_{ij})-2] \) for the case of \( t = 2, \ldots, E(L_{ij})-1 \). If \( a \) is the quantity transshipped, then the size of backorder is \( \hat{d}_i(t) - \hat{d}_i(t-1) - a \). The inventory costs of current demand \( c_i(t) \) can be expressed as the purchasing costs of LT and regular supply, the holding costs of the delivered quantity, and the backordering costs of the unfulfilled demand. Hence,

\[ c_i(t) = q_i a + p_i [\hat{d}_i(t) - \hat{d}_i(t-1) - a] + t h_i a + b_i [E(L_{ij}) - t] [\hat{d}_i(t) - \hat{d}_i(t-1) - a] \]
Rewriting (4), we derive (5).
\[ c_i(t) = p_i[\hat{d}_i(t) - \hat{d}_i(t-1)] + b_i[E(L_{ij}) - t][\hat{d}_i(t) - \hat{d}_i(t-1)] + \{q_i - p_i + th_i - b_i[E(L_{ij}) - t]\}a \]  

Considering the linear relationship between the size of the LT \( a \) and the total inventory costs \( c_i(t) \) at time \( t \), as shown in the equation (5), the following two conditions can be derived for cost minimization.

1) If \( q_i - p_i + th_i - b_i[E(L_{ij}) - t] > 0 \), then the total inventory costs \( c_i(t) \) increase with the size of the LT \( a \). Hence, \( a \) is set to zero, i.e. LT should not be opted for as a viable solution.

2) If \( q_i - p_i + th_i - b_i[E(L_{ij}) - t] < 0 \), then the total inventory costs \( c_i(t) \) decrease with the size of the LT \( a \). The LT becomes a preferred option and \( a = \hat{d}_i(t) - \hat{d}_i(t-1) \). When \( q_i - p_i + th_i - b_i[E(L_{ij}) - t] = 0 \), \( a \) can be any integer between 0 and \( \hat{d}_i(t) - \hat{d}_i(t-1) \). Hence the decision should be either complete demand fulfillment by LT or complete backordering.

When the conditions for LT are satisfied at time \( t \), the total inventory costs at \( t \) can be specified as
\[ c_i^T(t) = q_i[\hat{d}_i(t) - \hat{d}_i(t-1)] + \int_{-\infty}^{\infty} h_i[\hat{d}_i(t) - \hat{d}_i(t-1)]dt = q_i[\hat{d}_i(t) - \hat{d}_i(t-1)] + th_i[\hat{d}_i(t) - \hat{d}_i(t-1)] \]  

Otherwise, the total demand during time \( t \) is satisfied through regular supplies, and the total inventory costs can be specified as
\[ c_i^S(t) = p_i[\hat{d}_i(t) - \hat{d}_i(t-1)] + \int_{t}^{E(L_{ij})} b_i[\hat{d}_i(t) - \hat{d}_i(t-1)]dt = p_i[\hat{d}_i(t) - \hat{d}_i(t-1)] + b_i[E(L_{ij}) - t][\hat{d}_i(t) - \hat{d}_i(t-1)] \]  

Comparing these two equations of 6 and 7, it is observed that \( c_i^T(t) \) increases with \( t \) while \( c_i^S(t) \) decreases with \( t \). The behaviors the total inventory costs in these two conditions are shown in Fig.1.

![Cost behaviors at time t](image-url)

Fig.1. Cost behaviors at time \( t \) when the demand is fulfilled either by lateral transshipment or regular supplies
Let $K$ be the corresponding time at the intersection point between the two cost functions, as shown in Fig. 1, satisfying the following condition $d_i(K) \leq d_i(K + 1)$, for $K = 0, 1, \ldots, L_{ij} - 1$. For $0 < t \leq K$ under the LT scenario, the extra LT produces a lower total inventory costs than backordering with supplier. For $E(L_{ij}) > t > K$, this condition reverses, with LT becoming more costly than fulfilling the demand via regular supply, as shown in Fig. 1.

These conditions for $0 < t \leq K$ and $E(L_{ij}) > t > K$ can be written as

$$q_{ik}[d_i(K) - d_i(K - 1)] + h_i[K[d_i(K) - d_i(K - 1)] + b_i[E(L_{ij}) - K][d_i(K) - d_i(K - 1)]$$

$$q_{ik}[d_i(K + 1) - d_i(K)] + h_i[K + 1][d_i(K + 1) - d_i(K)] > p_{ij}[d_i(K + 1) - d_i(K)] + b_i[E(L_{ij}) - K - 1][d_i(K + 1) - d_i(K)]$$

Hence $K$ should satisfy the following condition.

$$\frac{p_{ij} - q_{ik} + b_iE(L_{ij})}{h_i + b_i} - 1 < K \leq \frac{p_{ij} - q_{ik} + b_iE(L_{ij})}{h_i + b_i}$$

(10)

Where $p_{ij}$ and $q_{ik}$ are unit purchasing cost and LT cost respectively, $b_i$ is unit backordering cost, $E(L_{ij})$ is the expected lead time, and $h_i$ is the unit holding cost. The fraction $\frac{p_{ij} - q_{ik} + b_iE(L_{ij})}{h_i + b_i}$ can be used for identifying $K$. Equation (10) is applicable for any wholesaler $i$ when replenishing from supplier $j$ and receiving LT from wholesaler $k$.

A staged approach for the decision is proposed below.

**Step 1: Decision on lateral transshipment as a reactive transshipment**

The following condition needs to be tested for the cost effectiveness of LT to fulfill the urgent demand outstanding at the start of the scheduling period. If the condition is satisfied, then LT should be selected. Otherwise, only backordering should be selected.

$$q_{ik} - p_{ij} - b_iE(L_{ij}) < 0$$

(11)

**Step 2: Decision on extra lateral transshipment as a preventive transshipment**

This step should be considered only when the condition (11) is satisfied. The integer value $K$ that fulfils the condition (11) determines the size of the extra transshipment. If $K = 0$, then the wholesaler $W_i$ should use LT to fulfill only the outstanding demand at time $t = 0$. If $K > 0$, then the wholesaler $W_i$ should order extra LT with the size of the expected demand at time $t = K$, i.e. $d_i(K)$.

According to the transshipment condition given in (11), $q_{ik} - p_{ij} - b_iE(L_{ij}) < 0$ and consequently, $\frac{p_{ij} - q_{ik} + b_iE(L_{ij})}{h_i + b_i} > 0$. Since $K$ is the maximum integer less than $\frac{p_{ij} - q_{ik} + b_iE(L_{ij})}{h_i + b_i}$, therefore, $K$ must be non-negative and within the range of $\left[\frac{p_{ij} - q_{ik} + b_iE(L_{ij}) - h_i - b_i}{h_i + b_i}, \frac{p_{ij} - q_{ik} + b_iE(L_{ij})}{h_i + b_i}\right]$.