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Theoretical and computacional analysis of the fixing of ossicular chain

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México

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Resumen

Presentamos un modelo matemático para describir la dinámica de la membrana timpánica ante una fijación de la cadena osicular. El modelo se ha derivado de la ecuación de onda libre, considerando primero semi-membranas, las cuales resultan de líneas nodales del centro al eje de la membrana. Posteriermente se considera el caso donde sólo hay una línea nodal, la cual modela la región donde está conectado el martillo a la membrana. Se ha usado MAPLE para realizar las simulaciones del modelo, se han observado las vibraciones de la membrana timpánica con la cadena osicular fija. Se muestra que la región donde el martillo se conecta a la membrana permanece fija.

***Palabras Clave*:** modelo matemático, membrana timpánica, cadena osicular, simulación

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Abstract

We present a mathematical model to describe the dynamic of the tympanic membrane in the fixing of the ossicular chain. The model has been derived from the equation of free wave, considering first semi-membranes which resulting in nodal lines of center at the edge of membrane. Then we consider the case where only there is a nodal line, it which model the region where the malleus is connected to the membrane. Has been used MAPLE to perform simulations of model, have been observed the simulations of the tympanic membrane with the fixing of the ossicular chain. It shows the fixed region where the malleus is connected to the tympanic membrane.

Keywords: mathematical model, tympanic membrane, ossicular chain, simulation

1. Introduction

Diseases of the ear are very common and it is important to identify their causes in order to solve the problem or prevent them. The tympanosclerosis is the term used to describe a sclerotic or hyaline change of the submucosal tissue of the middle ear. It appears to be an end-product of re-current acute or chronic ear infection. Calcification and ossification may occur. Its presence may affect the ultimate prognosis for hearing improvement. If there is progressive hearing impair-ment, there may be ossicular fixation by tympanosclerosis. A second operation for stapedectomy is required all times. The myringosclerosis is the term applied to describe deposits of hyaline masses within the fibrous layer of the tympanic membrane, but it is also generally referred to as tympanosclerosis of the tympanic membrane. It is often necessary to remove these hyaline masses for a successful myringoplasty. This lack of vibration leads to hearing loss that continues to deteriorate over time [1]. The otosclerosis it is a disease where there is fixing of the ossicular chain at the level of incudoestapedial articulation, is the most common cause of hearing loss in the middle ear for young adults and usually affects both ears [2].

In particular, the study of the dynamics of the human tympanic membrane (TM) is essential for understanding the hearing mechanism of the middle ear (ME) (see fig. 1). Computerized theoretical modeling of the human ME have been extensively carried out. Several pathological conditions such as stiffness, fixation of the ossicular chain, chain disarticulation, etc., have been also analyzed [3], [4], [5]. In recent years, finite-element models (FEM) of the ME have become of general use, due in part to the modern computational power and the feasibility of this technique of modeling very complex systems such as the ME. Three-dimensional FEM of the ME including the TM, external auditory meatus, ossicular chain, ME cavity and ME ligaments and muscles, as also morphologic data and boundary conditions have been developed. Several FEM have particularly emphasized the role of both the geometric and mechanical properties of the TM and the coupling of the manubrium on the eardrum. Bornitz et al [6] used a FEM of the human ME for parameter estimation of the TM, by comparison of the natural frequencies and mode shapes of the TM between the model and the specimens. Drescher et al [7] studied the geometric properties of a cadaver specimen human TM and its coupling with the malleus by using a finite shell model. The mechanical coupling between the TM and the manubrium was also investigated by Funnell [8]. He demonstrated the critical role of curvature in the behavior of the eardrum. Mechanical properties of the manubrium were examined by Funnell et al [9, 10] by using a FEM of cat eardrum. They found that a significant degree of manubrial bending occurs in the model. Lesser and Williams [11] applied FEM in a two-dimensional cross-sectional model of the TM and the malleus. The shape of the displaced TM was found to be sensitive to several factors such as the elastic modulus of the membrane and the presence and position of the rotation of the malleus. Also, Decraemer and Khanna [12] found that the description of the motion of the manubrium of cat requires of a rotational and a translational component, instead of a pure rotation, as classically assumed.

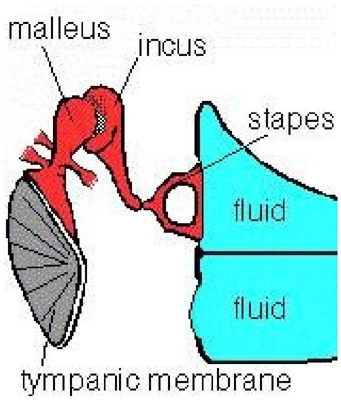

Figure 1. A diagram of the middle ear anatomy, showing the tympanic membrane and the malleus.

In this paper we present a mathematical model to describe the dynamics of the tympanic membrane in the fixing of the ossicular chain. Also, a computer animation using the maple software package is provided for a circular membrane. In section 2 we present the mathematical model derived from the equation of free wave, considering semi-membranes which resulting in nodal lines of center at the edge and then we consider the case where only there is a nodal line it which model the region where the malleus is connected to the membrane. In section 3 we show numerical simulations using the MAPLE software package. Finally, section 4 is concerned to the discussion and conclusion.

2. Mathematical model

The mathematical model that we propose is derived from the equation of free wave, which is in polar coordinates and to describe the region where is connected the membrane and the malleus is introduced through a semi-membrane. This model has been used by Alvarado-Anell et al [13] together with the case forced [14] to describe the behavior of a tympanic membrane heals, but not previously thought to apply it in a fixing of the ossicular chain. Now we use to describe the dynamic of the tympanic membrane in this problem.

By cutting a sector of angle α, with $0 \le \alpha \le 2\pi$, a semi-membrane is obtained. It occupies the domain $D_\alpha = \{(r,\theta): 0 \le r \le b, \alpha \le \theta \le 2\pi\}$, where b is the radius of the membrane. Now, we consider

$$c^2 \nabla^2 z = \frac{\partial^2 z}{\partial t^2}, \quad (r,\theta) \in D_\alpha \times [0,\infty), \tag{1}$$

subject to the initial conditions $z(r,\theta,0) = F(r,\theta)$, $\frac{\partial z}{\partial t}(r,\theta,0) = 0$ where $(r,\theta) \in D_\alpha$ and is fixed following the boundary condition $z(r,\theta,t) = 0$ where $(r,\theta,t) \in \partial D_\alpha \times [0,\infty)$, and ∂D_α is the boundary of the domain D_α. The eq. (1) is solved using the method of separation of variables proposing the solution $z(r,\theta,t) = R(r)\Theta(\theta)T(t)$. We observe that in D_α the function $\Theta(\theta)$ is no longer 2π-periodic, from (1) we get

$$\Theta(\theta) = \cos\left(n\pi \frac{\theta-\alpha}{2\pi-\alpha}\right) + \sin\left(n\pi \frac{\theta-\alpha}{2\pi-\alpha}\right). \tag{2}$$

Therefore, after applying the initial conditions and boundary the modes of vibration for a semi-membrane, determined for the angle $2\pi - \alpha$, are

$$z(r,\theta,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[A_{nm}\cos\left(n\pi\frac{\theta-\alpha}{2\pi-\alpha}\right) + B_{nm}\sin\left(n\pi\frac{\theta-\alpha}{2\pi-\alpha}\right)\right] J_{k(n)}\left(\frac{\mu_{m,k(n)}}{b} r\right)\cos\left(\frac{c}{b}\mu_{m,k(n)} t\right), \tag{3}$$

where

$$A_{nm} = \frac{4}{b^2(2\pi-\alpha)J^2_{k(n)+1}(\mu_{m,k(n)})}\int_0^b\int_\alpha^{2\pi} rF(r,\theta)J_{k(n)}\left(\frac{\mu_{m,k(n)}}{b} r\right)\cos\left(n\pi\frac{\theta-\alpha}{2\pi-\alpha}\right) dr d\theta \tag{4}$$

and

$$B_{nm} = \frac{4}{b^2(2\pi-\alpha)J^2_{k(n)+1}(\mu_{m,k(n)})}\int_0^b\int_\alpha^{2\pi} rF(r,\theta)J_{k(n)}\left(\frac{\mu_{m,k(n)}}{b}r\right)\sin\left(n\pi\frac{\theta-\alpha}{2\pi-\alpha}\right)drd\theta\ . \qquad (5)$$

The subscript $k(n)$ is given by

$$k(n) = \frac{n\pi}{2\pi-\alpha}, \qquad (6)$$

and the parameter $\mu_{m,k(n)}$ represents the zeros of Bessel function of order $k(n)$. This solution describes the behavior of semi-membranes formed by an angle $2\pi-\alpha$ and have a radius b. Currently we are interested in the case where $\alpha=0$ in (3), that is where there is a circular membrane with n nodal lines. After only we consider the case for $n=1$ because just there is one nodal line which represents the effect of the manubrium in the TM. The solution for $\alpha=0$, $n=1$ and considering an initial deformation $F(r)$ is reduced to

$$z(r,\theta,t) = \sum_{m=1}^{\infty} B_m J_{1/2}\left(\frac{\mu_m}{b}r\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{c}{b}\mu_m t\right), \qquad (7)$$

where

$$B_m = \frac{8}{\pi b^2 J^2_{3/2}(\mu_m)}\int_0^b F(r)J_{1/2}\left(\frac{\mu_m}{b}r\right)rdr\ . \qquad (8)$$

The solution (7) can represent the case of the fixing of the ossicular chain, this mean what no there is transmission of the sound to inter ear. But with this solution we can analyzed how the rest of the membrane vibrate when the malleus is fixed and to think in an alternative to solve deafness, for example, a magnet placed on the side of the membrane that vibrates and even detect changes in the field with a sensor and translated as sound.

3. Simulation

The simulation is achieved by introducing into MAPLE the solutions obtained with the proper parameters for the TM. The figure 2 show a sequence of the simulation for an oscillation with a duration of $0.0686s$ obtained considering the frequency of vibration of the membrane ($t = 2\pi/\omega$). We can also observe clearly how the region, which represents the hammer connection with the membrane, does not vibrate. We only show the first mode of vibration for being the most representative, but we can get each of them and the overlap of all (see figs 3 and 4). The initial deformation depends of r and multiplied by a parameter $a = 0.01$ that determines the initial deformation according to the size of the eardrum, this is $F(r) = a\sqrt{b^2 - r^2}$ where b is the radius of TM $0.005m$. The density that was considered is $\rho = 1200Kg/m^3$, the tension was obtained considering a pressure $(P = 0.002Pa)$ generated by the sound of a musical instrument played softly around $60dB$, this is, $T = PA$, where A is the area of the membrane and the speed of propagation in the membrane is given by $c = \sqrt{T/\rho}$. The figure 5 show the amplitude of the TM, which is about $50\mu m$ and is consistent with experimental results [15], [16]. The simulations were made using the software MAPLE version 11, with a processor AMD Turion 64, 2.21 GHz and 960 MB RAM.

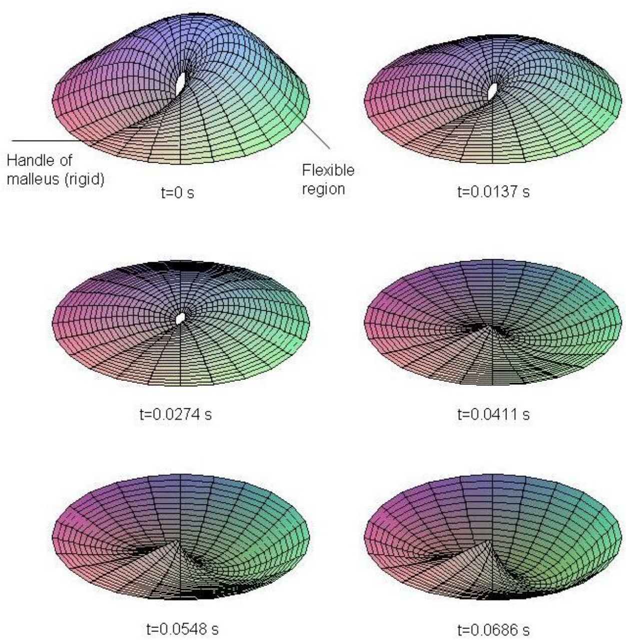

Figure 2. First mode of vibration of the tympanic membrane obtained in MAPLE. The amplitude of the membrane is larger in the flexible region than in the part of the hundle of malleus (rigid).

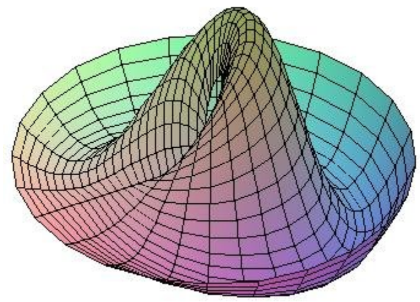

Figure 3. Second mode of vibration of the tympanic membrane

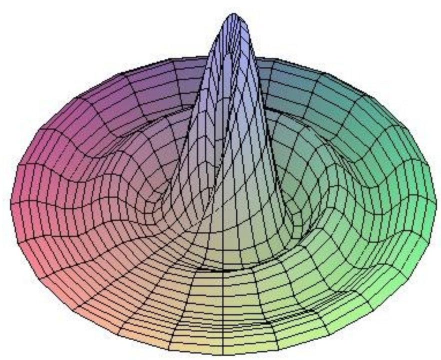

Figure 4. Sum of the firts five modes of vibration of the tympanic membrane.

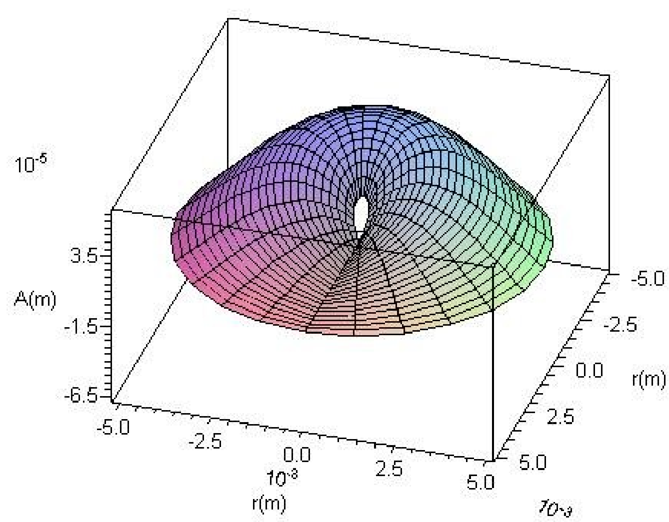

Figure 5. Maximun vibration amplitude of the tympanic membrane ($\approx 50\mu m$).

4. Discussions and Conclusions

The TM is a relatively complex membrane. Have such a density that is a function of the position, is composed of different layers, having a number of them that varies from one place to another and the tension also varies with the position. All these factors make its vibration relatively complex, being significantly different from the vibrations of a single membrane, like a drum.

The objective of this study is that physicians can use it as a method of diagnostic or research, for example for creating prosthesis or detection of deafness. Once we have seen and analyzed simulations of our model, we would like to be able to determine whether it is possible to use the model achieved so far for the diagnosis of diseases of the auditory system that manifest themselves as changes in kinematics of the eardrum. Another hand, we have proposed a model to simulate the vibrations of the TM which is relatively simple when we compare it with other models proposed in the literature, based primarily on finite element simulations. Likewise, the proposed model has the potential that the input parameters can be adjusted to simulate various pathological conditions, which in principle is an advantage, since it could be used to predict or determine the conditions which change, under a given condition, vibrations of the membrane. More specifically it is important to emphasize that have been simulated modes of vibration of the TM, introducing real physiological parameters and modeling the effect that the handle of the malleus is on the membrane, this through to propose a solution to the vibration consisting initially of a semi-membrane. This proposal is a relatively simple and yet novel, to describe the actual situation. The output parameters of the simulation, such as the amplitude of vibration and level curves, reproduced reasonably well reported experimentally $(50\mu m)$. Moreover, it should be noted that the problem of determining the shape and vibration of the TM, has enormous interest and relevance, both from the medical point of view, because it is a viable mechanism for diagnosis, as well as from the point of view mathematical and computational physicist. Thus, the relevance of the work is reflected in the design of increasingly modern devices for the measurement of the amplitude of tympanum at different frequencies. As regards to the research presented in this paper, the proposed model is based on a package mathematically simple, computationally easy to implement in commercial software standards, physically quite understandable based on the simplicity of the description of the phenomena involved and medically viable and interesting because reproduced consistent with the results reported in the literature, both amplitude and vibration modes obtained other models with more sophisticated or experimentally measured by different techniques. Future rechearch with this model, we allow identify, in detail, the place of fixing of the ossicular chain.

Acknowledgments

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