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Using metaheuristics for Optimizing Satellite Image Registration

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Abstract. This paper deals with the optimization techniques used for Satellite Images Registration (SIR). Our approach besides requiring less execution time improves the quality of the results. The proposed approach is based on a newly concept called "points of interest of an image". Once extracted such points will serve as inputs for the similarity estimator whose type is the Sum Square Differences (SSD). This procedure is used to evaluate digital images registration. Implementations of Simulated Annealing and a Genetic Algorithm are used to solve the SIR problems. Numerical experiments are conducted to assess the performances of the SA and GA.

Keywords: Image registration, Simulated Annealing, Genetic Algorithms, Interesting Points, Satellite images, Optimization.

1 INTRODUCTION

This paper addresses the problem of optimization techniques when dealing with images registration. Such a problem has received great attention from researchers in different fields. This treatment has been used in a variety of applications such as images processing, medical imaging, territory surveillance, environmental protection, among others.

Images registration is the process of mapping a reference image(RI) with an image in which changes have occurred, called Image to Readjust (or Transformed Image : TI), by applying a series of transformations. Image registration is characterized by four criteria (Gradeux, 2008), (Barrilot, 1999), (Brown, 1992). The first criterion consists of features derived from images to be registrated. Such features make an easy use of the registration process. The second criterion compares images by computing the similarity between images parameters (e.g. brightness, intensity, etc). The model of deformation defines the mode of transformation between the reference image and the transformed image (affine, rigid, elastic…). The last criterion deals with the optimization strategy whose objective is to find the best transformation mode abiding to a given image similarity criterion as well as to a search of the space defined by deformations.

The images registration problem is investigated by several authors. For instance, Maintz and Viergever in (Maintz, & Viergever, 1998) present an interesting survey of medical image registration. A state of the art of images registration is presented in Vincent Noblet's thesis (Noblet, 2006). More recently, Gardeux in (Gardeux, 2008), presents a critical view of image registration and puts forward related works dealing with images registration optimization methods.

The present work addresses the rigid satellite image registration in the monomodal case. Here, the reference image and the transformed image are assumed to have identical resolution representing the same target. Distortions are given according to translations and/or rotation, while the images similarity is defined by SSD. This latter is evaluated on the basis of an extracted set, of specific points, which we refer to as *points of interest*. The main objective of our work is to propose an approach to reduce the execution time and to improve the quality of the registration. To this end, a mathematical model for images registration is built on the basis of the four criteria defined previously. This mathematical model takes into account all specifications of satellite images by exploiting the notion of points of interest. As the obtained mathematical model is difficult to solve optimally, metaheuristics namely SA and GA are proposed. The efficiency of our approach will be demonstrated empirically obtained by numerical experiments.

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The rest of this paper is organized as follows. In the next section some definitions and basic concepts are given. The image registration mathematical model is proposed in Section 3, and the solution methods are given in section 4. Numerical experiments are presented in Section 5, while the conclusion and some suggestions are briefly highlighted in section 6.

2 DEFINITIONS AND BASIC CONCEPTS

In this section, definitions and concepts used in the rest of the present paper are given.

- *Remote sensing* is the analysis and interpretation of the information in the images, taken on a target on the ground, by a satellite monitoring.
- *Remote sensing satellites* are earth observation satellites. There are two kinds of such satellites, those using the radiation from the sun to gather data on a target (passive satellites), and those that produce their own radiation (active satellites). The latter are useful when there is an insufficient amount of sunlight to illuminate the target.
- *Satellite image* is a digital image completely defined by a matrix of the values of intensities from gray. These values range between 0 and 255.
- *Point of interest (characteristic point) is distinctive, discernible and distinguishable in an image by properties of intensity and brightness compared to its neighbors (corners, lines, curves...). It is used to identify the object.*

In the digital image processing, the problem is to allow the computer to make comparisons between images. This problem is commonly known as “Computer Vision” (Horaud & Monga, 1995). Our proposal consists in extracting a set of points of interest from the images which can be used to establish correspondences between the images.

3 MATHEMATICAL MODELING

The Satellite Images Registration problem can be summarized by the following key points:

- Determination of distortions between images *RI* and *TI*, representing the same target and obtained by the same source.
- Determination of all the possible combinations of translations and/or rotation applicable under the rigid images registration.

Basic hypotheses

The main hypotheses on which our model is based include:

- The translations can be done according to the x-axis and/or the y-axis. Δx is associated with a translation along X and Δy along the Y. These Δx and Δy represent the translation of the lines and the columns on the matrix corresponding to the image respectively.

- Δx and Δy take whole values only.

$$-W_R \leq \Delta x \leq W_R, \text{ and } -L_R \leq \Delta y \leq L_R. \quad (1)$$

- In practice, the angle of rotation, between the two images, (RI and TI) does not reach extreme values. Without loss of generality, we take $\Delta\theta$ to rotate between discrete values $<-\theta°, +\theta°>$. The rotation will be compared to the origin (0, 0), located at the northwest corner of the image. The interval is centered to achieve the positive and negative values of the angles.

A solution, in our case, is defined by a triplet (Δx, Δy, Δθ), where Δθ is the angle of rotation, Δx is a translation along x and Δy is a translation along y.

To measure the quality of registration, we compare the similarities between pixels of the two images at point (u_1, v_1) in the image I_1, to the point (u_2, v_2) in the image I_2, in the square mask (2n +1) × (2n +1). Equation (2) shows the similarity measure between these pixels, which minimizes the sum of the differences in intensity on the correlation window:

$$\mathrm{SSD}(u_1,v_1)=\sum_{i=-n}^{n}\sum_{j=-n}^{n}\left[(\mathrm{I}_1(u_1+i,v_1+j)-\mathrm{I}_2(u_2+i,v_2+j))^2\right] \tag{2}$$

Where I_1 and I_2 are intensity functions in each image.

This SSD criterion is very sensitive to variations in illumination between two images. Its measure is inversely proportional to the similarity. If the similarity is perfect, the calculation of the SSD between the two images will lead to zero (Blanc, 1998).

The calculation of the sum of the SSD on all the points of an image is a classical procedure to measure the similarity between the images. It has the advantage of being simple to use but its major drawback is that it does not take into account the characteristics of the digital images (answers to the filters), which make the algorithms easy to use but not very robust. To refine this approach, we propose to associate the notion of points of interest with the calculation procedure. In other words, research will focus on one restricted set of characteristic points only. This will enhance the robustness and yield a substantial reduction in the execution time. The sum of SSD between points of interest in the image *RI* and their corresponding points in the image *TI* will give the measure of similarity.

$$Z=\sum_{k=1}^{m}\sum_{i=-n}^{n}\sum_{j=-n}^{n}\left[(\mathrm{I}_{\mathrm{RI}}(x_k+i,y_k+j)-\mathrm{I}_{\mathrm{TI}}(x_k+i,y_k+j))^2\right] \tag{3}$$

Where *m* is the number of points of interest, I_{RI} and I_{TI} are input functions of the data matrix.
After the transformation $T_{\Delta x,\Delta y,\Delta\theta}$, for evaluating the similarity of two images, we calculate the sum of SSD between each point of interest in the RI and, if it exists, its corresponding coordinates in the TI. The corresponding coordinates in the TI, can be obtained by the inverse function ($T^{-1}_{\Delta x,\Delta y,\Delta\theta}$) associated with the transformation used.

The objective function is given by (4) and consists of minimizing (3) on the set of points of interest.

$$MinZ=\sum_{k=1}^{m}\sum_{i=-n}^{n}\sum_{j=-n}^{n}\left[(\mathrm{I}_{\mathrm{RI}}(x_k+i,y_k+j)-\mathrm{I}_{\mathrm{TI}}(T^{-1}_{\Delta x,\Delta y,\Delta\theta}(x_k+i,y_k+j)))^2\right] \tag{4}$$

The selection of points of interest in the image is carried out using the well-known procedure based on the concept of Harris's score which associates a score with each point (pixel. The largest value refers to the special point in the image, see (Horaud, &Monga, 1995).

The Mathematical Model of the Rigid Registration Problem
The mathematical model (5) associated with the rigid satellite images registration is as follows:

$$MinZ=\sum_{k=1}^{m}\sum_{i=-n}^{n}\sum_{j=-n}^{n}\left[(\mathrm{I}_{RI}(x_k+i,y_k+j)-\mathrm{I}_{\mathrm{TI}}(T^{-1}_{\Delta x,\Delta y,\Delta\theta}(x_k+i,y_k+j)))^2\right]$$

$$\begin{aligned} &-L_R \le \Delta x \le L_R \\ &-W_R \le \Delta y \le W_R \\ &-\theta_0 \le \Delta\theta \le \theta_0 \\ &\Delta x, \Delta y, \Delta\theta \in Z \end{aligned} \tag{5}$$

I_{RI} et I_{TI} are defined above as a function on a matrix (image) reading data. The model associated with the rigid satellite images registration is an integer mathematical program with bounded variables.

4 RESOLUTION METHODS

The unusual nature of the objective function (using input functions I (x, y) for each point of the image) and the impossibility of enumerating all combinations of Δx, Δy and $\Delta\theta$ are the main reasons behind the use of approximate methods. For the registration of satellite images, we use optimization methods based on metaheuristics framework, namely Simulated Annealing (SA) and Genetic Algorithms (GA). These methods are adapted and implemented in dedicated software (NUMERICAL).

Simulated Annealing

Simulated annealing (SA), introduced for the first time into the field of optimization, by Kirkpatrick and al. (1983), consists in a generic probabilistic metaheuristic for solving optimization problems. SA is a Monte Carlo based method for numerical optimization that implies the principles of thermodynamics and is motivated by an analogy to annealing in solids (for more information see also Dreo & al. 2003). It performs optimization without prior knowledge of the problem structure or of any particular solution strategy. Simulated annealing is a local search method, in which candidate solutions are randomly chosen from a neighborhood ***N(s)*** of a current solution ***s*** and accepted with probability 1 if the energy function ***Z(s')*** associated with the new solution ***s'*** is better than ***Z(s)*** or with probability $e^{-(\Delta Z/T)}$, where $\Delta Z = Z(s') - Z(s)$ and ***T*** is control parameter, called Temperature. The pseudo-code of Simulated Annealing is given in Fig. 1.

```
Procedure Simulated annealing
(0)     Initialization step
        s^0 ∈ S, s=s^0; Z(s) = Z (s^0);
        T:  initial temperature;
        α : Factor for temperature decreasing;
        Best solution: s*= s; Z(s*)=Z(s);
(1)     Iterative improvement step
 Repeat
 |      Repeat
 |      |   Generate a neighborhood for s: N(s); // s current solution
 |      |  Selection randomly s′ ∈ N(s);
 |      |    ΔZ = Z (s′) − Z(s);
 |      |  if (ΔZ < 0)
 |      |     {s = s′ ; Z(s′)= Z(s);
 |      |       if (Z(s′) < Z(s*)) { s= s* ; Z(*)=Z(s’); // best so far solution
 |      |      } else{ random (p) according an uniform distribution U[0, 1];
 |      |              accept move (s=s’); with probability p<= e − (ΔZ /T);
 |      |            }
 |      Until ¬ end of stage
 |      T= α*T ;
 Until  ¬ stop criterion
Return (s*)
```

Fig. 1. Pseudo-code of the basic Simulated Annealing

Adaptation of Simulated Annealing to SIR

In order to use the Simulated Annealing for solving the SIR problem, we need to adapt all operational parameters of the method to this specific problem.

- *An initial solution* given to Simulated Annealing can be defined by any combination between the variables in their respective fields. In the case of satellite image registration defined in our study, the changes undergone by TI are centered (balanced with respect to the origin), and values of the solution sought rarely reach the limit values. The configuration (dx,dy,dθ)=(0,0,0) is used an initial solution.

- *A move strategy* which defines the neighborhood structure is given by a procedure of changes along X, Y (translations) and to θ (rotation);

- *An evaluation function* consists in the objective function defined above in Section 3.

- *Initial temperature:* We adopted after simulations, the most appropriate value as defined by (Aarts, & Laarhoven, 1985), $T_{init} = r*MAX\ (\Delta Z)$; where r>> 1 (typically r = 10) and *MAX (ΔZ)* is the maximum gap between two energies of the system. In our case $Max\ (\Delta Z) = 25 * 255 = 6375$; T_{init} = 70 000.

- We define *the number of iterations* for each level of temperature as ***k * MAX = 100***.

- *The decrease of temperature*: two ways are followed for the definition of the value of the parameter α. The first way consists in setting α to a given value (lower and close to 1), the second approach, proposed in (Yagouni, & Ait Haddadene 2005), is called variable cooling. Here the value of α is a function calculated by integrating changing values of parameters of the method throughout the optimization process and the data of the instance to be solved.

- *The stopping criterion* used in our application is the minimum temperature, we set the threshold $T_{final} = 0.1$

Parallelization of Simulated Annealing applied to image registration satellites

We implemented the massive parallel version of simulated annealing (MPSA) (Fig.1), where the set of feasible solutions S is partitioned into k subsets S_i generated by the combination of conditions related to the definition of variables: dx> 0; dx <0, dy > 0, dy <0, dt> 0, dt <0.

The initial solution taken for each process is s° = (dx,dy,dθ) = (0,0,0).

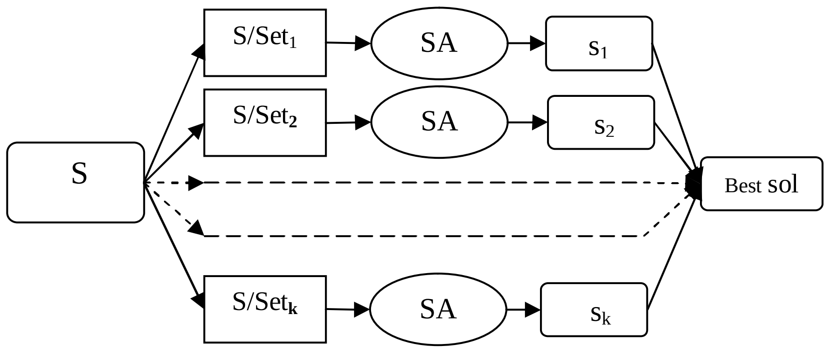


Fig. 1 Massive Parallel Simulated Annealing

The advantages of this method compared to conventional simulated annealing are mainly in a best exploration of the space solutions, which ensures a better quality result, and a substantial reduction of the execution time.

Genetic Algorithms

Genetic Algorithms (GAs) have been developed by Holland in the 1970s and applied for optimization problems in the 1980s. They are a very popular class of Evolutionary Algorithms. Their main idea consists in making a population of solutions evolve by applying judiciously operators known as genetic operators. There are *Selection, Crossover* and *Mutation* in simple general procedures which can be adapted to many different search and optimization problems for which the GAs are able to find good solutions. In order to use a genetic algorithm to solving the SIR problem, we need to apply a set of transformations to this problem, define the genetic representation of the individuals and the genetic operators.

Adaptation of Genetic Algorithm (GA) for satellite image registration

For an adaptation of a GAs to SIR, we had to set and adjust all the parameters of this method, such as:

- Coding of a solution;
- Population size, number of generations;
- Operators of selection, crossover and mutation;
- Fitness function (evaluation);
- Termination condition.

Coding of a solution

The representation of a problem in the genetic algorithms (GAs) is a key issue. It has a direct influence on the performance and on the choice of genetic operators. For an illustration, the encoding used in our case is given in Table 1:

- The X translation is coded in 10-bit binary: chx[i];
- The Y translation is coded in 10-bit binary: chy[i];
- The rotation θ is encoded in binary on 8 bits: chθ[i].

*Table 1. Shape of a chromosome **i***

chx[i]	0	0	0	0	0	1	1	0	0	0
chy[i]	1	1	0	0	0	0	0	0	1	0
chθ[i]	1	0	0	0	0	0	0	0		

Significant bit (sign) Low bit

The most significant/initial bit of every line, with High=1 (negative value) and Low=0 (positive value).
In general we have:

$$
\begin{aligned}
chx[i] &= \sum_{k=0}^{8} chx[i] \times 2^k \\
chy[i] &= \sum_{k=0}^{8} chy[i] \times 2^k \\
ch\theta[i] &= \sum_{k=0}^{6} ch\theta[i] \times 2^k
\end{aligned}
\tag{6}
$$

Taking the example of the chromosome *i* in Table 1, we get:

$$
\begin{aligned}
chx[i] &= +24 \\
chy[i] &= -258 \\
ch\theta[i] &= 0
\end{aligned}
$$

Then $(\Delta x, \Delta y, \Delta\theta)$ =(chx[i], chy[i], chθ[i])= (24,-258,0).

Size of the population and the number of generations

Taking as reference the maximum size that a satellite image can reach and the high number of combinations of translations and rotations, we fixed the size of the population to 900 individuals. After our initial tests and simulations we have chosen a compromise between the population size and the number of iterations, and the value for the number of generations is taken Nbgen = 40.

Operators of selection, crossover and mutation

This operator represents the fact that fitter individuals have a greater chance to survive and to produce offspring. We use an elitist procedure to preserve a number of the best individuals of the t^{th} generation in the $(t+1)^{th}$ generation and to be operated (crossover) we use a tournament selection. For the crossover we used 3 cutoff points (x,y,θ) as shown in Fig. 2. Like with the crossover operator, the mutation will be performed on the three lines of the selected chromosome. Indeed, if a solution is mutated, the gene that undergoes the change will be designated at random and independently in each line $(\Delta x, \Delta y, \Delta\theta)$. A mutation is the change in allele by its complement. The crossover will be used with a probability $P_c \in [0.1, 0.9]$ and the mutation with $P_m \in [0.001, 0.1]$.

Fitness function (evaluation)

We use the SSD function (section 3) as fitness function in our algorithm.

Termination condition

We terminate the GA after ***Nbgen*** generations are used.

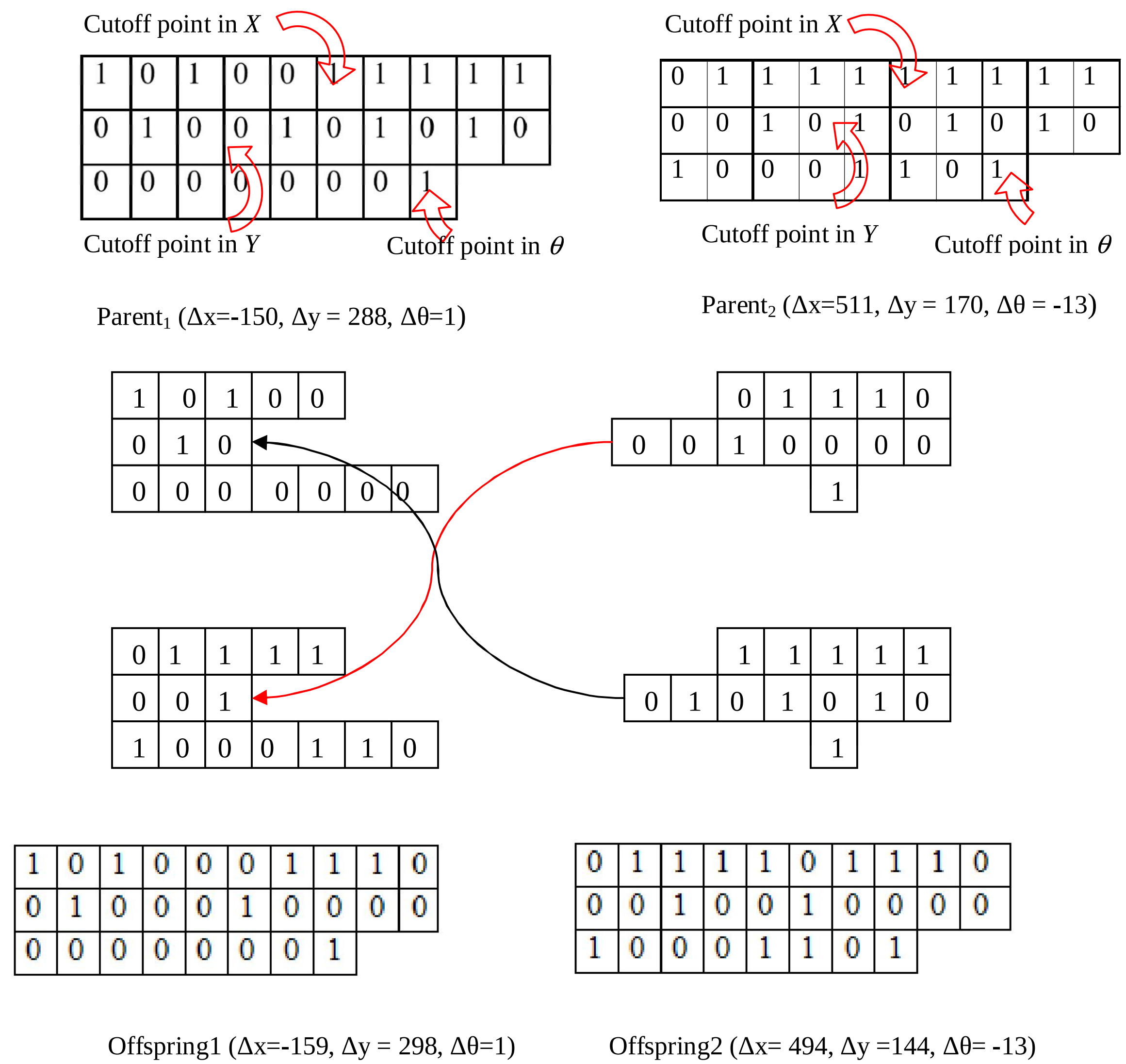


Fig.2. Crossover operator

Parallelization of GA applied to Satellite Images registration

Concerning parallelization, we opted for a migration model in the implementation of the GA (Dréo, & al.,2003) adapted to the image registration: given a parallel architecture composed of ***p*** populations, $\boldsymbol{P_i}$, $\boldsymbol{i \in \{1..p\}}$, that supports the migration of the best individuals between units.

A flow chart of the GA is given in Fig.3:

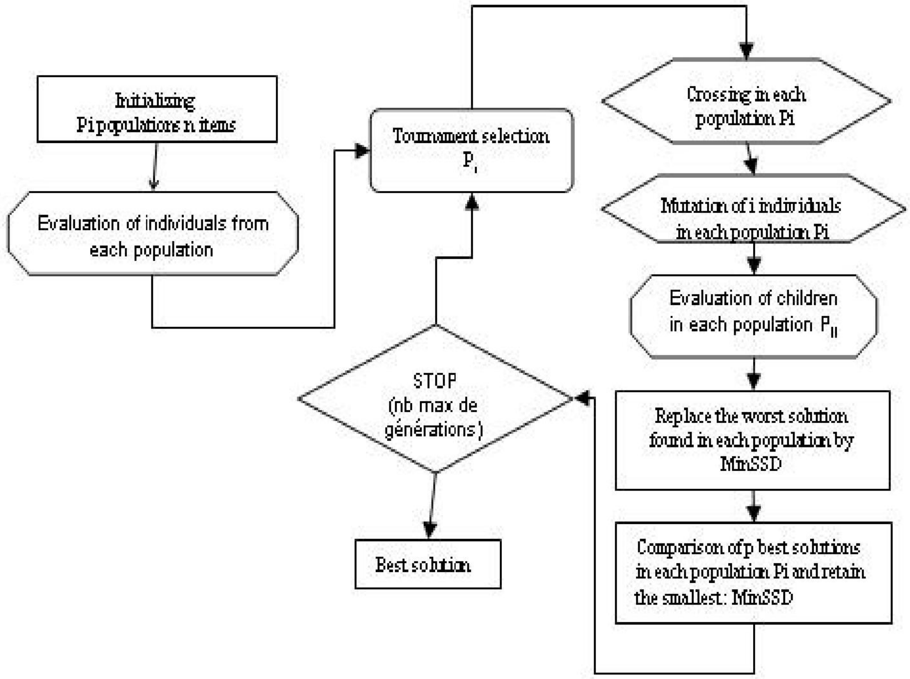


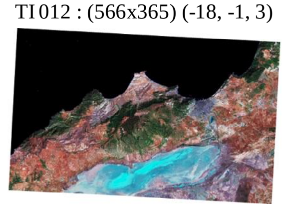
Fig. 3. Parallel Genetic Algorithm

5 Computational results

Sample of tested Images

RI 01 : (549x336) TI 011 : (490x284) (59, 52, 0) TI 012 : (566x365) (-18, -1, 3)





RI 02 : (761x429) TI 021 : (742x485) (-28, 40, 7) TI 022 : (745x478) (31, -50, -7)



Application in Radiology

RI 05 : (383x506) TI051 : (349x455) (0,51,-2) TI 052 : (401x522) (0,-16,-2)



Some results

Methods implemented in sequential and parallel versions have been tested over twenty images, Tables 2,3,4,5. 100 runs have been executed and the results are summarized in Table 6:

Table. 2 Results of SA.

	Exac	*Appro.*	*Div.*
I. 01	88	2	10
I. 02	100	0	0
I. 03	0	0	100
I. 04	100	0	0
I. 05	38	6	56
I. 06	6	0	94
I. 07	100	0	0
I. 08	84	0	16
I. 09	56	3	41
I. 10	15	0	85
I. 11	69	16	15
I. 12	4	19	77
I. 13	72	6	22
I. 14	76	2	22
I. 15	97	3	0
I. 16	89	0	11
I. 17	100	0	0
I. 18	48	5	47
I. 19	33	6	61
I. 20	100	0	0
Average	63,7	3,4	32,8

Table. 3 Results of PSA

	Exact	*Appr.*	**Div**
I. 01	98	0	2
I. 02	100	0	0
I. 03	4	0	96
I. 04	100	0	0
I. 05	20	0	80
I. 06	38	0	62
I. 07	100	0	0
I. 08	92	0	8
I. 09	36	0	64
I. 10	10	0	90
I. 11	65	0	35
I. 12	0	12	88
I. 13	100	0	0
I. 14	100	0	0
I. 15	98	0	2
I. 16	100	0	0
I. 17	100	0	0
I. 18	44	4	52
I. 19	76	0	24
I. 20	100	0	0
Average	69.05	0.8	30.15

Table.4 Results of GA.

	Exact	*Appro.*	*Div.*
I. 01	60	16	24
I. 02	18	0	82
I. 03	34	26	40
I. 04	54	30	16
I. 05	0	0	100
I. 06	0	0	100
I. 07	74	12	14
I. 08	0	0	100
I. 09	2	8	90
I. 10	50	8	42
I. 11	21	18	61
I. 12	0	8	92
I. 13	26	0	74
I. 14	86	0	14
I. 15	72	15	13
I. 16	72	16	12
I. 17	89	4	7
I. 18	55	4	41
I. 19	70	14	16
I. 20	28	8	64
Average	40.55	9.35	50.1

Table.5 Results of PGA

	Exact	*Appro.*	**Div.**
I. 01	80	6	14
I. 02	26	0	74
I. 03	78	6	16
I. 04	20	0	80
I. 05	0	0	100
I. 06	0	1	99
I. 07	58	6	36
I. 08	96	0	4
I. 09	0	2	98
I. 10	26	4	70
I. 11	0	0	100
I. 12	0	0	100
I. 13	96	0	4
I. 14	88	1	11
I. 15	90	4	6
I. 16	70	6	24
I. 17	36	22	42
I. 18	60	8	32
I. 19	32	18	50
I. 20	78	1	21
Average	46,7	4,25	49,0

Table.6 Comparative results between SA, PSA, GA and PGA

	Opt. Solution	App. Solution	Div.
SA	63,75	3,40	32,85
PSA	69.05	0.80	30.15

	Exact Sol.	App. Sol.	Div.
GA	40.55	9.35	50.10
PGA	46,70	4,25	49,05

SA: Simulated Annealing.
PSA: Parallel Simulated Annealing.
GA: Genetic Algorithm.
PGA: Parallel Genetic Algorithm.

Comments on the computational results

- In the case of the use of Simulated Annealing, the results (Table 6) show that, both sequential and parallel version are particularly suited to the problem, the rate of convergence to the global solution is over 63% whereas the genetic algorithms recorded a convergence rate not exceeding 50%.
- All the methods applied give the exact solution when they converge; this explains the relatively small rate of approximate solutions obtained (3rd Column).
- The fourth column shows that the average rates of divergence are around 50% for genetic algorithms. However, these rates are relatively low for RS.
- The implemented methods in their parallel versions have improved the rate of convergence to the exact solution (6%).

6 CONCLUSIONS

In this study we adapted and implemented two metaheuristics to optimize the rigid registration of satellite images. The idea of reducing an image by its points of interest has significantly increased the quality of solutions produced and the speed of obtaining them (reducing the execution time of treatment).

We studied and tested the two algorithms, implemented their improvements in their sequential and parallel versions. The low execution time of sequential algorithms, obtained due to the introduction of points of interest, leads us to suggest the use of two algorithms in their sequential version, because the improvement due to parallelization is in fact not significant.

Our contribution, beyond the application of appropriate optimization techniques, is the introduction of points of interest. Once these points are extracted, they are associated with an appraiser to assess the quality of treatment.

Future works and the prospect potential for this study will focus on the extension to multimodal images with practical applications in many fields such as, medical imaging, remote monitoring.

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