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EVALUATION OF PERFORMANCE OF HORIZONTAL AXIS WIND TURBINE BLADES BASED ON OPTIMAL ROTOR THEORY

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Abstract: Wind energy is a very popular renewable energy resource. In order to increase the use of wind energy, it is important to develop wind turbine rotor with high rotations rates and power coefficient. In this paper, a method for the determination of the aerodynamic performance characteristics using NACA airfoils is given for three bladed horizontal axis wind turbine. Blade geometry is obtained from the best approximation of the calculated theoretical optimum chord and twist distribution of the rotating blade. Optimal rotor theory is used, which is simple enough and accurate enough for rotor design. In this work, eight different airfoils are used to investigate the changes in performance of the blade. Rotor diameter taken is 82 m which is the diameter of VESTAS V82-1.65MW. The airfoils taken are same from root to tip in every blade. The design lift coefficient taken is 1.1. A computer program is generated to automate the complete procedure.

Keywords: Chord, Twist, Power coefficient, Aerodynamic design, optimal rotor.

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INTRODUCTION

The power efficiency of wind energy systems has a high impact in the economic analysis of this kind of renewable energies. The efficiency in these systems depends on many subsystems: blades, gearbox, electric generator and control. Some factors involved in blade efficiency are the wind features, like its probabilistic distribution, the mechanical interaction of blade with the electric generator, and the strategies dealing with pitch and rotational speed control. It is a complex problem involving many factors, relations and constraints. The increasing awareness of the general public to climate change and global warming has provided opportunities for wind turbine applications in the UK. The UK claims 40% of the wind energy resources of Europe. Europe leads the world with 70.3% (23GW peak) of the total operational wind power capacity worldwide (Ackermann & Soder, 2002). As well as large wind turbines operating in open areas on- and off-shore, more small-scale wind turbines are being installed and operated by homeowners and small enterprises.

One of the differences between large- and small-scale wind turbines is that small-scale wind turbines are generally located where the power is required, often within a built environment, rather than where the wind is most favorable. In such location, the wind is normally weak, turbulent and unstable in terms of direction and speed, because of the presence of buildings and other adjacent obstructions. To yield a reasonable power output from a small-scale wind turbine located in this turbulent environment, and to justify such an installation economically, the turbines have to improve their energy capture, particularly at low wind speeds and be responsive to changes in wind direction. This means that small-scale turbines need to be specifically designed to work effectively in low and turbulent wind resource areas. Currently, manufacturers of small wind turbines in the UK are not seen to make an attempt to accelerate low-speed airflow prior to reaching the turbines rotating blades.

The aerodynamic and structural design of rotors for horizontal axis wind turbines (HAWTs) is a multi-disciplinary task, involving conflicting requirements on, for example, maximum performance, minimum loads and minimum noise. The wind turbine operates in very different conditions from normal variation in wind speed to extreme wind occurrences. Optimum efficiency is not obtainable in the entire wind speed range, since power regulation is needed to prevent generator burnout at high wind speeds. Optimum efficiency is limited to a single-design wind speed for stall regulated HAWTs with fixed speed of rotation. The development of suitable optimization methods for geometric shape design of HAWT rotors is therefore a complex task that involves off-design performance and multiple considerations on concept, generator size, regulation and loads. In Maalawi & Badawy (2001) based on Glauert (1958) solution of an ideal wind mill along with an exact trigonometric function method, analytical closed form equations are derived and given for preliminary determination of the chord and twist distribution and variation of angle of attack of the wind along the blade is then obtained directly from a unique equation for a known rotor size and refined blade geometry.

Glauert (1958; 1959) originated the basic aerodynamic analysis concept of airscrew propellers and windmills. He applied first the momentum and energy relationships for simple axial flow and then considered the effects of flow rotation after passing through the rotor as well as the secondary flows near the tip and hub regions. Wilson & Lissaman (1974) and Wilson *et al.* (1976) extended the Glauert's work and presented a step by step procedure for calculating performance characteristics of wind turbines. Analysis was based on a two-dimensional blade element strip theory and iterative solutions were obtained for the axial and rotational induction factors. Later on, Wilson (1980) outlined a brief review of the aerodynamics of horizontal axis wind turbines. The performance limits were presented, and short note was given on the applicability of using the vortex-flow model.

In Kishinami *et al.* (2002), aerodynamic performance characteristics of the HAWT were studied theoretically by combination analysis involving momentum, energy and blade element theory by means of the strip element method, and experimentally by the use of a subscale model. Three types of blade were examined under conditions in which the free stream velocity is varied from 0.8 to 4.5 m/s for the open type wind tunnel with an outlet duct diameter of 0.88m. The blade consists of a combination of tapered rectangular NACA44 series. The aeronautical characteristics of the HAWT employing the three different types of blades are discussed by reference of power and torque coefficients, C_p and C_q and to the tip speed ratio, derived from the experimental and analytical results. The output regulation by variable pitch control/fixed pitch stall control method is also discussed in this context.

This paper focuses on the method for performance analysis. The suggested approaches can be divided into two stages, namely; (a) Calculation of the blade geometry, i.e. the chord and twist distributions, and (b) Calculation of elemental power coefficient on the basis of optimal rotor theory. In the first stage, the equations given in Martin & Hansen (2008) are used for getting the chord distribution and, in the second stage, computer programme is developed for getting the other parameter of blade based on Egglestone & Stoddard (1987). This work is completely based on computer program. For validation, the results are compared with power coefficient of VESTAS V82-1.65MW (VESTAS, 2011) which is experimental.

Optimal Rotors

The Glauert annulus momentum equation provides the essential relationships for analyzing each blade segment using two dimensional airfoil data. What is needed in design however is guidance in this infinite parameter space toward that rotor aerodynamics that will be best for one particular purpose. Thus, optimal rotor theory. Glauert presented a blade element analysis to find the "ideal windmill" based on neglecting the airfoil drag but including wake rotation or swirl (Glauert, 1935). This approach was extended by Stewart (1976) to include the effect of drag so that the Glauert analysis becomes a sub case of the Stewart analysis for the case of infinite lift/drag ratio. The Glauert analysis is simple enough and yet accurate enough to be very useful in preliminary rotor design. It can be used directly as a synthesis procedure, whereas almost all other approaches are really analysis procedures.

The results of the Glauert or Stewart ideal wind mill theory is a preferred value of the product, cc_l at each blade segment as a function of local speed ratio x. This yet leaves the choice of c or of c_l, the remaining parameter then fallowing the ideal value of the product cc_l. Although this single-point optimal is extremely useful for first-cut design, it provides no information as to the sensitivity of the performance to off-design operation. That is, it is quite possible to come up with a design that has a very high C_p at one design tip speed ratio X but falls off quite rapidly to either side and so fails to garner the most power over a whole wind spectrum of velocities.

In addition to modifying the Glauert optimal blade element theory to account for drag, Stewart (1976) pointed out that the design freedom in satisfying the optimal product cc_l can be used to find a chord c that will be optimal at two different speed ratios. This approach yields a design that is insensitive to performance degradation for off design conditions. The equations used in Glauert ideal wind mill theory are same as the two dimensional airfoil theory except that the drag coefficient c_d is assumed zero. The effect of wake rotation is included. In a well designed rotor with a carefully shaped airfoil, the lift/drag ratio is very high, on the order of 100 or so, so that the neglect of drag during first cut optimization is justified. Subsequent use of Stewart theory usually results in small corrections to these results.

From Glauert momentum vortex theory

$$\left(\frac{w}{w+\Omega r}\right)=\left(\frac{Bc}{8\pi r}\right)\left(\frac{c_l \sin\phi - c_d \cos\phi}{\sin\phi\cos\phi}\right) \tag{1}$$

$$\frac{a}{(1-a)}=\left(\frac{\sigma R}{8r}\right)\left(\frac{c_l \cos\phi + c_d \sin\phi}{\sin^2\phi}\right) \tag{2}$$

$$\frac{a'}{(1+a')}=\left(\frac{\sigma R}{8r}\right)\left(\frac{c_l \sin\phi - c_d \cos\phi}{\sin\phi\cos\phi}\right) \tag{3}$$

$$\tan\phi=\frac{u}{\Omega r+w}=\frac{V_0(1-a)}{\Omega r(1+a')}=\frac{(1-a)}{x(1+a')} \tag{4}$$

$$\frac{a'}{(1+a')}=\frac{\lambda(\tan\phi-\varepsilon)}{\sin\phi} \tag{5}$$

If c_d is assumed to be zero, dividing **Eq. (3)** by **Eq. (2)** gives:

$$\frac{a'(1-a)}{a(1+a')}=\tan^2\phi \tag{6}$$

Combining this with **Eq. (4)** gives:

$$\frac{a'(1+a')}{a(1-a)}=\frac{V_0^2}{\Omega^2 r^2}=\frac{1}{x^2} \tag{7}$$

At each radius right side of **Eq. (7)** is constant, and therefore the left side must remain constant also, while the power is maximized if the quantity $a'(1-a)$ from **Eq. (5)** is maximized. Performing these operations using Lagrange multipliers yields:

$$a'=\frac{(1-3a)}{(4a-1)} \tag{8}$$

In the windmill state, a' must be positive for positive output torque. Thus for small x, a approaches 1/4 and a' becomes large, whereas for large x, a approaches 1/3 and a' approaches zero.

Substituting a' from **Eq. (8)** into **Eq. (7)** yields the required relationship between a and the local speed ratio x as follows:

$$x=(4a-1)\sqrt{\frac{(1-a)}{(1-3a)}} \tag{9}$$

Miller *et al.* (1978) give a power series in $1/x^2$ as an approximation for the inverse relationship:

$$a\cong\left(\frac{1}{3}\right)-\left(\frac{2}{81x^2}\right)+\left(\frac{10}{729x^4}\right)-\left(\frac{418}{59049x^6}\right)+........ \tag{10}$$

The corresponding relative wind angle ϕ can be found from **Eq. (6)** as given by

$$\tan\phi = \left(\frac{1}{a}\right)\sqrt{(1-a)(1-3a)} \qquad (11)$$

The optimum blade layout in terms of the product of chord c and lift coefficient c_l can be found from **Eq. (1)** with $c_d = 0$.

$$\frac{\sigma c_l X}{8} = \frac{\sigma_l c_l x}{4} = \left(\frac{B\Omega}{8\pi V_0}\right)(cc_l) = \left[\left(\frac{4a-1}{1-2a}\right)\right]\sqrt{(1-a)(1-3a)} \qquad (12)$$

As mentioned previously, there is still some design freedom in that both c and c_l may be varied while their product satisfies **Eq. (12)**. Thus, if chord c is held constant, c_l (and hence α and θ) will follow **Eq. (12)**. Likewise, if c_l (and hence α) is held constant, chord c must vary according to **Eq. (12)**, and the twist angle θ has been specified as a function of r. Alternatively, Stewart (1976) has shown that the freedom in satisfying the product relation, cc_l, can be used to define a blade that is optimal over a wider range, i.e., that is insensitive to off-design conditions. Miller *et al.* (1978) also integrate the C_p relation along the blade to give closed form and series solutions for C_p. Thus, from **Eqs (5)**, **(8)** and **(9)**:

$$dC_p = \left(\frac{12}{X^2}\right)\left[\frac{(4a-1)(1-a)(1-2a)}{(1-3a)}\right]^2 da \qquad (13)$$

which must be integrated from $a = 1/4$ at $r = 0$ to the value of a at the tip, a_T, given by:

$$X = (4a_T - 1)\sqrt{\left(\frac{(1-a_T)}{(1-3a_T)}\right)} \qquad (14)$$

The integration can be done in closed form, resulting in:

$$C_p = \left(\frac{4}{729X^2}\right)\left[\begin{array}{l}\left(\frac{4}{z}\right) - 12\ln\left(\frac{1}{4z}\right) - \left(\frac{4363}{160}\right) + 63z \\ -38z^2 - 124z^3 - 72z^4 - \left(\frac{65}{5}\right)z^5\end{array}\right] \qquad (15)$$

where $z = 1 - 3a_T$. A series expansion valid for large tip-speed ratio X is:

$$C_p = \left(\frac{8}{27}\right)\left[1 - \left(\frac{2}{9X^2}\right)\ln\left[\frac{(27X^2+15)}{8}\right] + \frac{0.053}{X^2} + \ldots\right] \qquad (16)$$

For any tip speed ratio, X, the local speed ratio is $x = Xr/R$ .The parameter a is determined from **Eq. (10)**, and hence ϕ from **Eq. (11)**, the product cc_l from **Eq. (12)**, and C_p from one of the **Eqs (13)** through **(16)**.

To obtain a single point optimum including the effects of drag, Stewart begins by driving local power coefficient:

$$C_{p'} = \frac{\Delta P}{\left(\frac{1}{2}\rho V_0^3 \Delta A\right)} = \frac{\frac{1}{2}\rho W^2 r\Omega B_c(c_l\sin\phi - c_d\cos\phi)\Delta r}{\left[\frac{1}{2}\rho V_0^3(2\pi\Delta r)\right]} \qquad (17)$$

which, by using $W^2 = V_0^2(1-a) + r^2\Omega^2(1+a')^2$ and **Eq. (4)** to eliminate x^2, reduced to $C_{p'} = (1-a)^2(1+\cot^2\phi)ax\lambda(\sin\phi - \varepsilon\cos\phi)$. Then, eliminating λ and expanding $1/(\cot\phi + \varepsilon))$ in Taylor's series of two terms, there results:

$$C_{p'} = 4xa(1-a)(\tan\phi - \varepsilon)(1 + \varepsilon\tan\phi) \qquad (18)$$

The last term $1 + \varepsilon\ \tan\phi$ is quite small, improving the accuracy less than 1 percent for $\varepsilon = 0.01$ and $\phi = 40$ degree. Thus, in most cases, it may be neglected, leaving:

$$C_{p'} = 4xa(1-a)(\tan\phi - \varepsilon) \qquad (19)$$

Since the optimum value of a is found to be quite insensitive to changes in ε, this implies that $C_{p'}$ decreases monotonically as ε increases. By defining local Froude efficiency, $\eta_{F'} = (27/16)\, C_{p'}$ we can relate the performance of each blade element to the ideal value of unity.

After eliminating a':

$$x\tan\phi = 1 - a\left[1 + \left(\frac{\tan\phi - \varepsilon}{\cos\phi + \varepsilon}\right)\right] \qquad (20)$$

For small ε this is closely given by:

$$x\tan\phi = 1 - a\sec^2\phi \qquad (21)$$

If the function, $F = a(1-a)(\tan\phi - \varepsilon)$, is then maximized using $dF/d\phi = 0$, with the relation between a

and ϕ given implicitly by **Eq. (20)**, there results a quadratic equation, as follows:

$$\left[\frac{-a\tan^2\phi}{(1-a)}\right]+\left[\frac{(1-a)}{a}\right]=2+\left[\frac{\varepsilon\sec^2\phi}{(\tan\phi-\varepsilon)}\right] \quad (22)$$

Solving this quadratic, the optimal value of a including the effect of drag is given by

$$\left(\frac{1}{a}\right)=2+\sec\phi[G+\sqrt{(G+H^2)}] \quad (23)$$

where $G=\dfrac{\varepsilon\sec\phi}{2(\tan\phi-\varepsilon)}$ and $H=\dfrac{\tan\phi}{(\tan\phi-\varepsilon)}$

Thus a single point optimization of a rotor- blade element, given ϕ and ε, proceeds as follows:

1. Find a from **Eq. (23)**
2. Calculate ϕ from $\lambda=\dfrac{a\sin\phi}{(1-a)(\cot\phi+\varepsilon)}$
3. Find a' from
$$a'=\frac{a(\tan\phi-\varepsilon)}{[(1-a)(\cot\phi+\varepsilon)-a(\tan\phi-\varepsilon)]}$$
4. Then, $x=\dfrac{(1-a)}{[(1+a')\tan\phi]}$
5. $C_{p'}=4xa(1-a)(\tan\phi-\varepsilon)(1-\varepsilon\tan\phi)$

Aerodynamic Design

The aerodynamic design of optimum rotor blades from a known airfoil type means determining the geometric parameters such as chord length distribution and twist distribution along the blade length for a certain tip-speed ratio at which the power coefficient of the rotor is maximum. For this reason, firstly the change of the power coefficient of the rotor with respect to tip speed ratio should be figured out in order to determine the design tip speed ratio, X corresponding to which the rotor has maximum power coefficient. The blade design parameters will then be according to this design tip speed ratio.

Examining the plots between relative wind angle and local tip speed ratio for a wide range of glide ratios gives us a unique relationship when the maximum elemental power coefficient is considered, and this relationship can be found to be nearly independent of glide ratio and tip loss factor. Therefore, the general relationship can be obtained between optimum relative wind angle and local tip speed ratio which will be applicable for any airfoil type.

$$\frac{\delta}{\delta\phi}\{\sin^2\phi(\cos\phi-x\sin\phi)(\sin\phi+x\cos\phi)\}=0 \quad (24)$$

Equation (24) reveals after some algebra (Hau, 2006):

$$\phi=\left(\frac{2}{3}\right)\tan^{-1}\left(\frac{1}{x}\right) \quad (27)$$

Twist distribution can be easily determined from **Eq. (25)** and, for getting the chord, the relation given in Chap. 8 was used (Martin & Hansen, 2008).

$$C_n=C_l\cos\phi+C_d\sin\phi \quad (26)$$

$$\frac{c}{R}=\frac{8\pi ax\sin^2\phi}{(1-a)BC_nX} \quad (27)$$

Airfoil properties

In this work, eight airfoils are taken, four of them are four digit airfoils and other four are five digit airfoils. Reference (Abott & Vonoenhoff, 1958) is used for taking the properties of airfoils. The selection of airfoil is very important point in designing an efficient rotor (Griffiths, 1977; Hassanein *et al.*, 2000). Griffiths (1977) showed that the output power is greatly affected by the airfoil lift-to-drag ratio, while Hassanein *et al.* (2000) recommended that the airfoil be selected according to its location along the blade to ensure its highest contribution to the overall performance. The numbering system for NACA wing sections of the four-digit series is based on the section geometry.

The first integer indicates the maximum value of the mean-line ordinate y_c in per cent of the chord. The second integer indicates the distance from the leading edge to the location of the maximum camber in tenths of the chord. The last two integers indicate the section thickness in per cent of the chord. Thus the NACA 2412 wing section has 2 per cent camber at 0.4 of the chord from the leading edge and is 12 per cent thick. The numbering system for wing sections of the NACA five-digit series is based on a combination of theoretical aerodynamic characteristics and geometric characteristics.

The first integer indicates the amount of camber in terms of the relative magnitude of the design lift coefficient; the design lift coefficient in tenths is thus three-halves of the first integer. The second and third integers together indicate the distance from the leading edge to the location of the maximum camber; this distance in per cent of the chord is one-half the number represented by these integers. The last two integers indicate the section thickness in per cent of the chord. The NACA 23012 wing section thus has a design lift coefficient of 0.3, has its maximum camber at 15 per cent of the chord, and has a thickness ratio of 12 per cent.

RESULT AND DISCUSSION

Chord Distribution and Twist Distribution

The aerodynamically optimum distribution of chord and twist of the rotor blades depends on the selection of a particular lift and drag coefficient. The lift and drag coefficient are taken from Abott & Vonoenhoff (1958). The rotor taken in this work has the same airfoil profile at inboard, mid-span and outboard stations of the blade. The chord and twist distribution for airfoils taken in this work are given by **Figs 1–5,** as we can see from **Figs 2** and **3**, there is very slight difference in chord distribution in both four digit and five digit airfoils. The blades are twisted to optimize the angle of attack at every element of the blade, the design lift coefficient taken in this work is 1.1 and the tip speed ratio is 7.

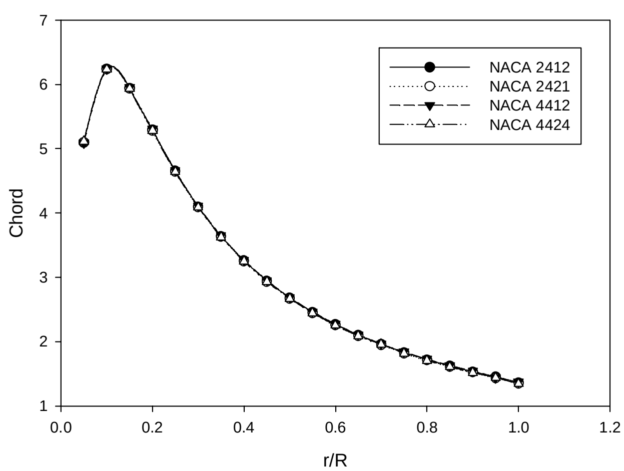


Fig. 1 Chord Distribution for Four Digit Airfoils.

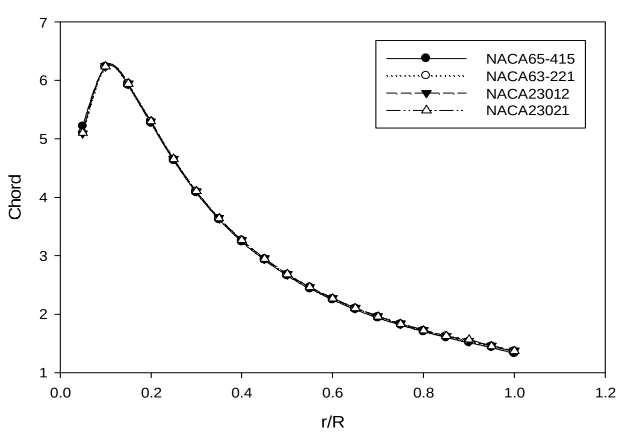


Fig. 2 Chord Distribution for Five Digit Airfoils.

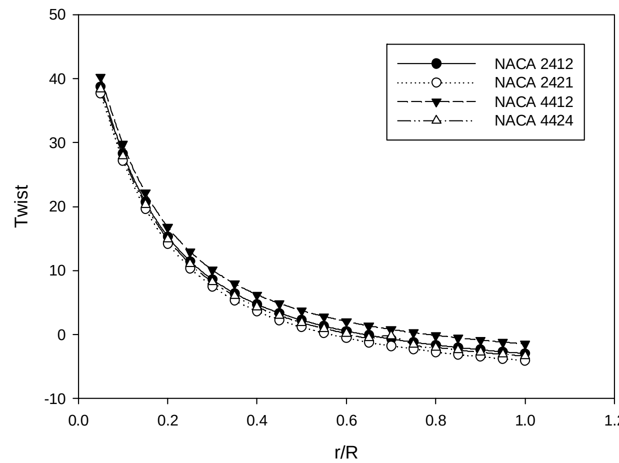


Fig. 3 Twist Distribution for Four Digit Airfoils.

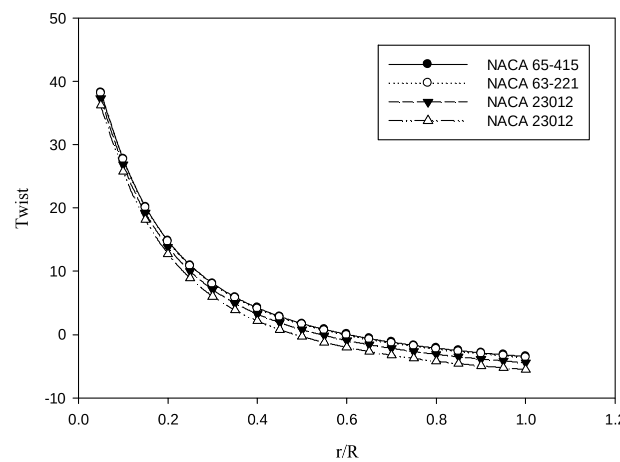


Fig. 4 Twist Distribution for Five Digit Airfoils.

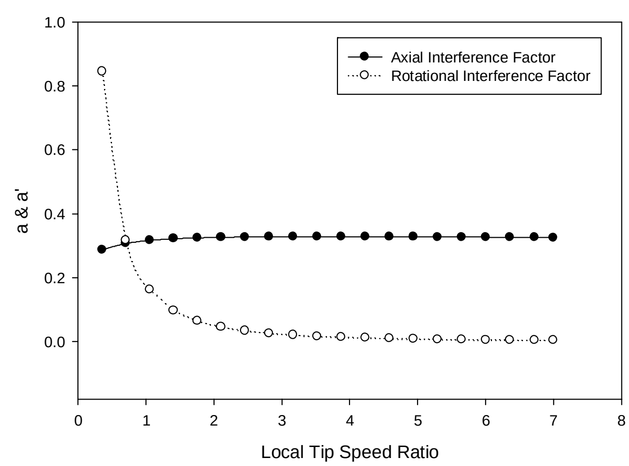


Fig. 5 NACA 2412.

Local Tip Speed Ratio

The Tip Speed Ratio is of very important in the design of wind turbine blade. If the rotor of the wind turbine rotates very slowly, maximum wind will pass as it is through the gap between the rotor blades. Alternatively, if the rotor rotates quickly, the blurring blades will appear like a solid wall to the wind. Therefore, wind turbines are designed with optimal tip speed ratios to extract as much power out of the wind as possible. When a rotor blade passes through the air it leaves turbulence in its wake. If the next blade on the spinning rotor arrives at this point while the air is still turbulent, it will not be able to extract power efficiently from the wind. However, if the rotor span a little more slowly the air hitting each turbine blade would no longer be turbulent. Therefore, the tip speed ratio is chosen so that the blades do not pass through turbulent air. For small local tip speed ratio, axial interference factor approaches 1/4 and rotational interference factor becomes large, whereas for large local tip speed ratio, axial interference factor approaches 1/3 and rotational interference factor approaches zero. **Figures 5–12** gives the difference between the values of *a* and *a*' with respect to local tip speed ratio for both four and five digit airfoils.

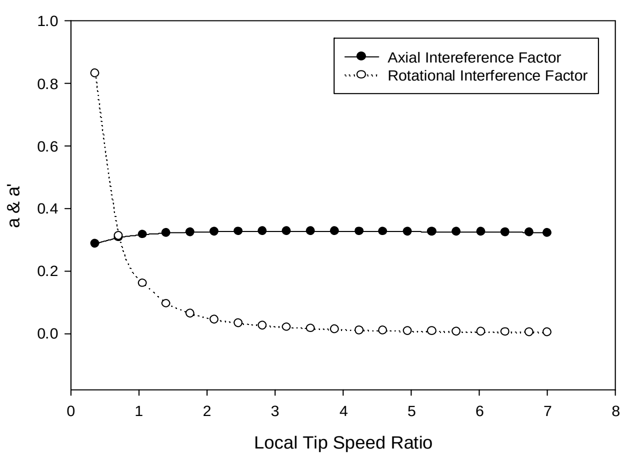


Fig. 6 NACA2421.

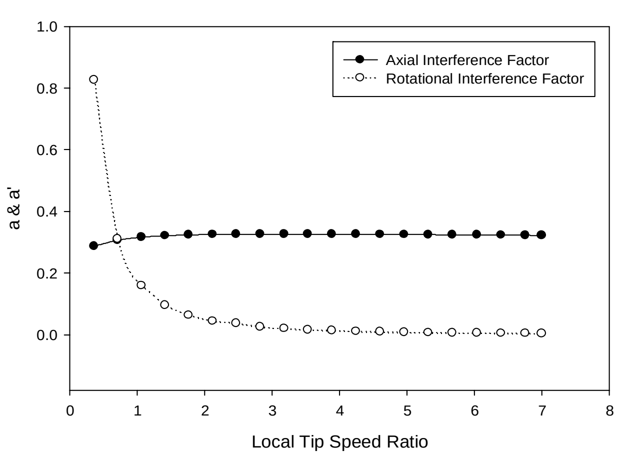


Fig. 7 NACA4424.

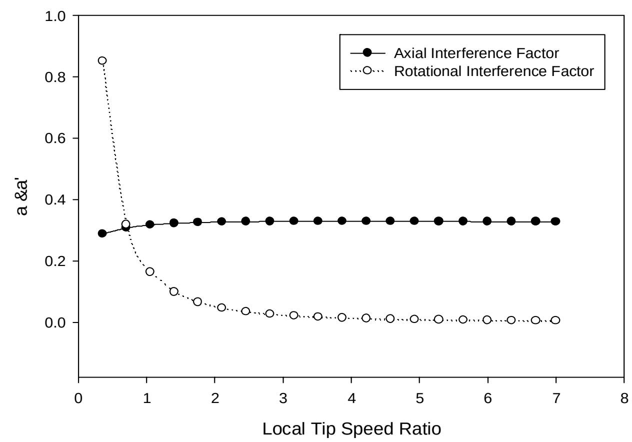


Fig. 8 NACA4412.

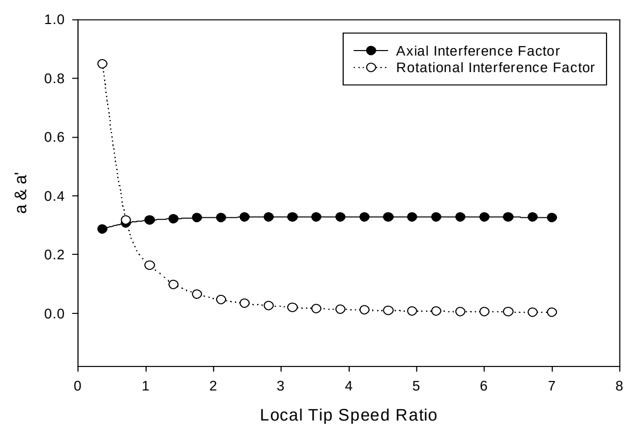


Fig. 9 NACA 23012.

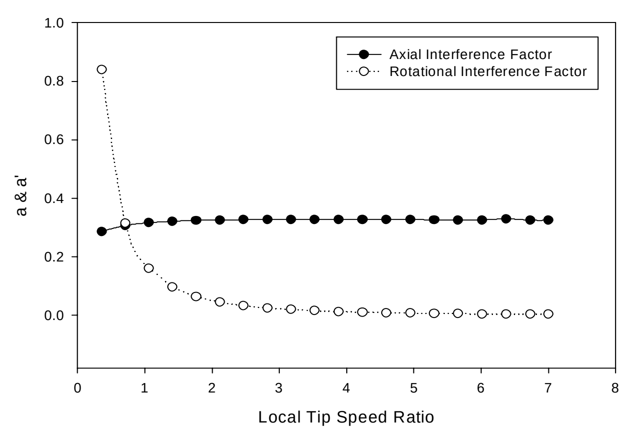


Fig. 10 NACA 3021.

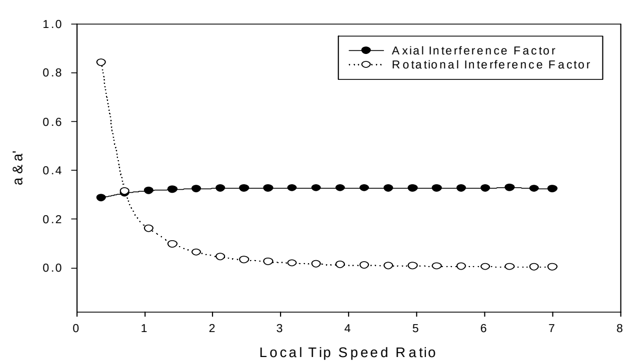


Fig. 11 NACA 6322.

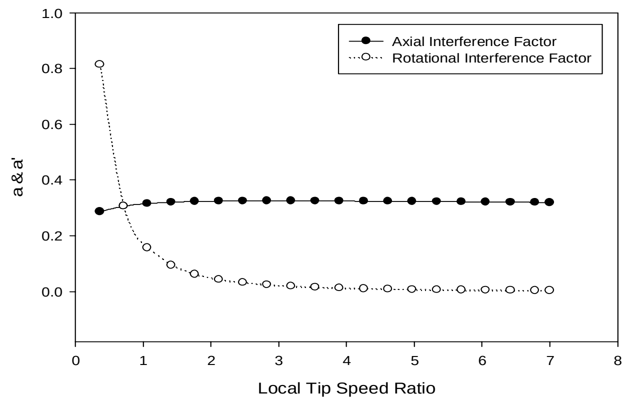


Fig.12 NACA 65415.

Power Coefficient

Figures 13–14 gives the elemental power coefficient of wind turbine blade. For calculating the elemental power coefficient, the blade was divided into 20 elements. As it can be seen from the figures, the performance of NACA 4424 and NACA 23012 are better then other airfoils. **Figure 15** is showing the power coefficient of VESTAS V82-1.65MW.

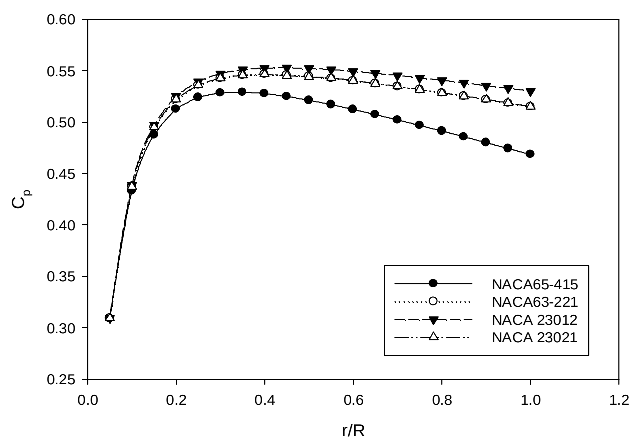
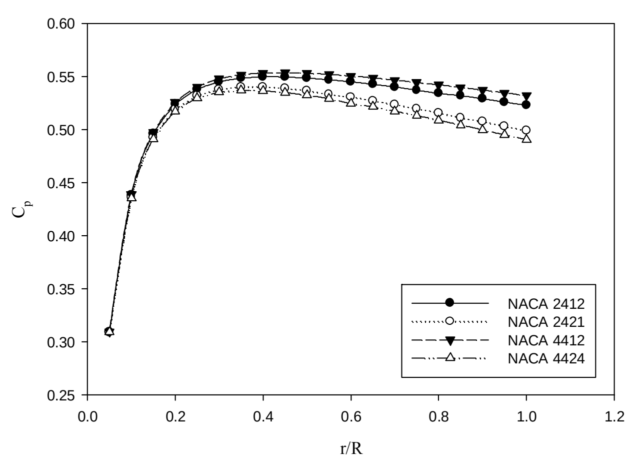


Fig. 13 Power Coefficient for Five Digit Airfoils.



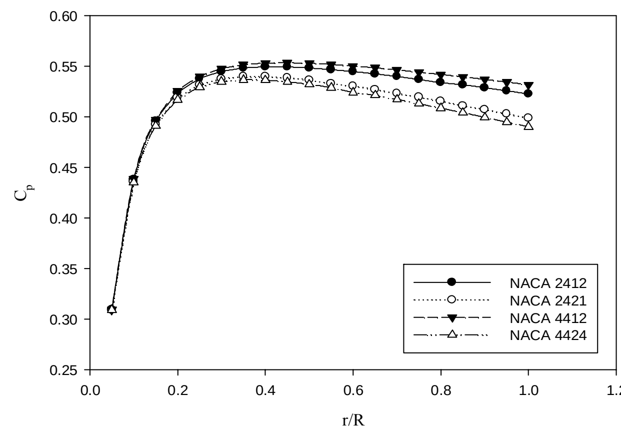


Fig.15 Power Coefficient of VESTAS V82-1.65MW (VESTAS, 2011).

CONCLUSION

A methodology has been presented in this work for designing the wind turbine blade. The approach uses the formulation base on Glauert annulus momentum equation. The selected design parameters include the chord and twist distribution, type of airfoil, tip speed ratio. A computer programme is developed for automating the calculations. Two families of NACA airfoils are taken in this work, i.e. four-digit series and five-digit series. For the selected airfoils and the blade, number the chord and twist decreases as we go from root to tip. The variation of the twist and chord is almost same for all airfoils used in this work. The values of axial interference and rotational interference factor are almost same for all airfoils taken in this work with respect to the local tip speed ratio. The elemental power coefficient calculated in this work depends on local tip speed ratio, axial interference factor, rotational interference factor, angle of relative wind form rotor plane, drag to lift ratio. As it can be seen from **Figs 13–14**, the elemental power coefficient of airfoils NACA 4412 and NACA 23012 are higher than other airfoils. With the help of comparison between the **Figs 13–15**, it is easy to understand that the power coefficient in the present work is more.

Nomenclature

a =Axial interference factor
a' = Rotational interference factor
B = Number of blades
c = Chord
C_l = Section lift coefficient
C_d = Section drag coefficient
r = Local blade radius
A = Rotor swept area
F = Prandtl loss factor

u = Wind speed through rotor plane
x = Local speed ratio
X = Tip speed ratio
V_0 = Free-stream wind velocity
C_p = Power coefficient
ε = Drag/lift ratio
α = Blade segment angle of attack
θ = Angle of blade chord with rotor plane
ϕ = Angle of relative wind from rotor plane
Ω = Rotor angular velocity

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