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Kouropoulos, Giorgos P.

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THE EFFECT OF THE COEFFICIENT OF RESTITUTION TO THE PARTICLE COLLECTION EFFICIENCY DURING THE PROCESS OF MAGNETIC FILTRATION – NUMERICAL APPROACH

Giorgos P. Kouropoulos

Department of Energy Technology, Technological Educational Institute of Athens, Greece

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Abstract: In the present study, the effect of the coefficient of restitution to the particle collection efficiency of magnetic filter during magnetic filtration of a fluid will be examined. Initially, are described the available mathematical models that give efficiency of magnetic filtration in relation to various parameters and physical quantities. Also, the role of coefficient of restitution in the magnetic filtration models will be reported. Furthermore, is formulated the basic equation that examines the change of filtration efficiency with regard to the coefficient of restitution. In the last step, a numerical example with given values from a real magnetic filter will be applied and the characteristic curves of magnetic filter will be drawn. The general conclusion that we have is that as the coefficient of restitution increases, the efficiency of magnetic filter decreases. The efficiency of magnetic filter converges in the same value, when there is elastic collision of a filtered particle.

Keywords: magnetic filtration; coefficient of restitution; magnetic filter; magnetic collection efficiency, magnetic filter efficiency

INTRODUCTION

Magnetic filtration is the separation process between solid particles and fluid with magnetic forces. The particles which are included inside the fluid, are retained of the filter collectors have magnetic properties. Magnetic filter is a system that consists of cylindrical magnets. The particles of fluid interact with magnetic field of magnet and are withheld on the collection surface. There are lot of different types of magnetic filters, the most usual are in (**Fig. 1)** and (**Fig. 2)**.

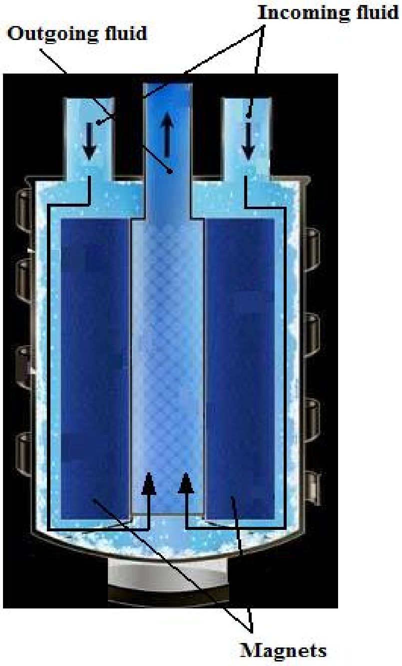

Fig. 1 Typical magnetic filter construction.

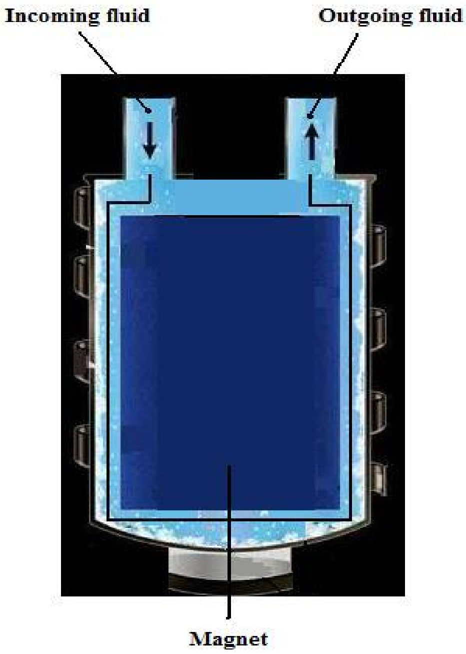

Fig. 2 Typical magnetic filter construction.

There are two orifices for incoming fluid and one orifice for outgoing fluid in the filter in (**Fig. 1**). There is one orifice for incoming and outgoing filtered fluid in the filter in (**Fig. 2**), the magnet have a bigger size than the filter 1.

Environmental technology and magnetic filters.

Magnetic filtration is mainly used in industrial applications for metal separation procedure from a fluid. The magnetic filters have limited breadth of application in environmental technologies. More specifically, magnetic separation is used for algae removal (Yadidia *et al*., 1977), in water cleaning, pollution control and substances removal of water such as phosphorus, orthophosphate, iron oxides and chromium (Ambashta & Sillanpaa, 2010; Gupta *et al*., 2011; Latour, 1973; Lua & Boucher 1984; Morse *et al*., 1998).

The magnetic filtration process for a fluid, have been studied by several researchers and various mathematical models have been referred, more specifically (Briss et *al*., 1980; Gooding, 1980; Luborsky & Drummond, 1975; Polyon *et al*., 2010). There are no research reports for the effect of coefficient of restitution to the collection efficiency in magnetic filtration. The scientific research for the effect of coefficient of restitution in filtration mechanisms of fibrous filters is relatively limited (Chernyakov *et al*., 2011).

THE COEFFICIENT OF RESTITUTION.

The coefficient of restitution C_R or e, was introduced by Isaac Newton in 17th century and is defined as the ratio of a body speed u_A after collision to the body speed u_B before collision (**Fig. 3**).

$$e = \frac{u_A}{u_B} \quad (1)$$

The coefficient values are oscillated between $0 \le e \le 1$, but may be greater than 1 in some specific cases. In a more complicated case which two bodies collide each other, the coefficient of restitution is defined as:

$$e = \frac{U_A - u_A}{u_B - U_B} \quad (2)$$

Where:

u_A: The speed of the first body after collision.
u_B: The speed of the first body before collision.
U_A: The speed of the second body after collision.
U_B: The speed of the second body after collision.

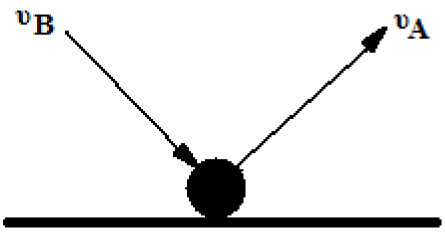

Fig. 3 The collision of a particle in the filter surface.

The calculation of coefficient of restitution requires different difficulty level proportionally to the size of colliding objects. For individual objects with bigger than 1cm diameter, there is the drop test. For objects with micrometrical size, specific measuring devices are required. In case of aerosol that consists of N population of microparticles each one of which have different coefficient of restitution value and are retained in the magnet surface, there is an average coefficient of restitution $\bar{e}$ (Briggs, 1945; Haron & Ismail, 2011).

$$\bar{e} = \frac{1}{N}\sum_{v=1}^{i} e \quad (3)$$

MATHEMATICAL MODELS OF MAGNETIC FILTRATION

Generally, the particle collection efficiency of a fibrous filter as defined as:

$$E = \frac{N_I - N_O}{N_I} \times 100\% \quad (4)$$

Where N_I is the mass of incoming particles towards the filter and N_O is the mass of outgoing particles from the filter. The quantity P = 1 – E, called penetration. The collection efficiency E is correlated with other physical quantities such as speed, density and viscosity of fluid. According to (Polyon *et al*., 2010; Watson, 1975):

$$E = 1 - \exp\left(-\frac{3r_c^2 L F^3}{4\alpha^3}\right) \quad (5)$$

Where:

r_c: The radius of duct that the filtered fluid passes (m).
L: The filter length (m).
F: The packing fraction or packing density of filter (dimensionless).
α: The radius of cylindrical magnet inside the filter (m) **(Fig. 1).**

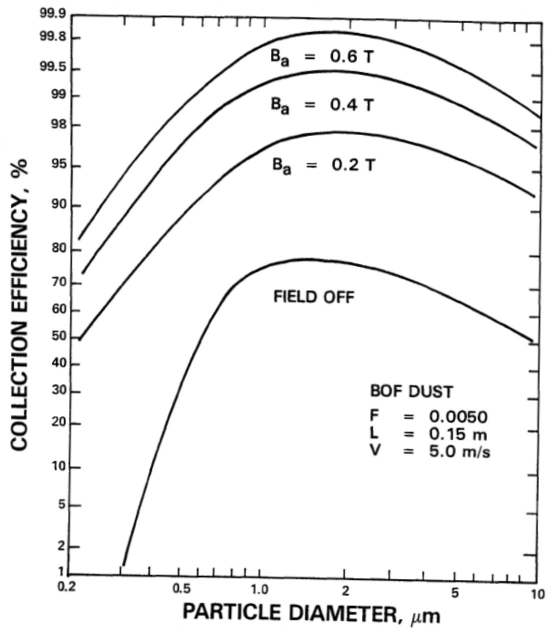

Fig. 4 Typical characteristic curve of penetration degree of a magnetic filter for various magnitude values (Gooding, 1980).

Researchers (Gooding, 1980; Gooding & Felder, 1981) they propose a more complicated and exact mathematical equations for the particle collection efficiency of a magnetic filter, which is equal to:

$$E = 1 - \exp\left[-\frac{F}{(1-F)^2}\frac{4LY_c \sin\theta_c}{\pi^2 r_w}\right] \quad (6)$$

Where:

F: Packing fraction or packing density of the filter. Is the ratio of total wire volume to the total filter volume, (dimensionless).

r_w: The wire radius (m).

L: The filter length (m).

Y_c: Dimensionless parameter Y_c is a function of Stokes number K and magnetic parameter W. It is selected from a diagram (Gooding & Felder, 1981).

θ_c: The critical collision angle (rad).

The angle θ_c is equal to:

$$\theta_c = \frac{1}{2}cos^{-1}\left[\frac{1}{2\mu_0 X H_a^2}\left(e^2\rho_p u_0^2 - \frac{U}{4\pi l r_p^2}\right) - \frac{1}{2}\right] \quad (7)$$

Where:

e: The coefficient of restitution (dimensionless).

μ_0: Magnetic permeability of a vacuum (4π $\times 10^{-7}$ H/m).

ρ_p: The particle density (Kg/m^3).

X: Magnetic susceptibility. Dimensionless parameter that is depended by material of magnet (Schenck, 1996).

H_a: Magnetic field strength (A/m). It is depended on the magnetization B and is selected from magnetization curve of filter magnets.

U: Hamaker constant (J). It is depended on the material of magnet, (Bergstrom, 1997).

u_0: The particle collision speed (m/sec).

r_p: The particle radius ($\times 10^{-6}$ m).

l: Equilibrium distance of magnetic material ($\times 10^{-12}$ m), (Jacobs *et al.*, 1984).

η: Dynamic viscosity of fluid ($Pa \times sec$).

The magnetic parameter W and Stokes number K are equal to:

$$K = \frac{2r_p^2\rho_p u_0}{9r_w\eta} \quad (8)$$

$$W = \frac{\mu_0 X H_a^2}{\rho_p u_0^2} \quad (9)$$

Y_c is a function of the K and W, also K is a function of the particle radius r_p, so the parameter Y_c is a function of the particle radius.

$$\begin{cases} Y_c = f(K,W) \\ K = f(r_p) \end{cases} \Rightarrow Y_c = f(r_p,W)$$

The particle collection efficiency curve of magnetic filter, is govern by a final function of two variables E(r_p,e) that results after from the replacement of **Eqs (7), (8)** and **(9)** to **Eq (6)**.

Table 1. Technical characteristics of magnetic filter

Characteristic	Symbol	Value
Particle density	ρ_p	7870 Kg/m^3
Fluid flow speed	V_0	10 m/sec
Wire radius	r_w	0.002 m
Dynamic viscosity	η	0.001154 Pa×sec
Filter length	L	0.2 m
Coefficient of rest.	$\bar{e}$	0 – 1
Packing fraction	F	0.008
Hamaker const. (FeO)	U	10.1$\times 10^{-20}$ J
Magnetic suscep. (Iron)	X	200000
Magnetic field strength	H_a	100 A/m
Equilibrium separation	l	191$\times 10^{-12}$ m

NUMERICAL EXAMPLE

The average coefficient of restitution of aerosol with microparticles will have various values in the range 0≤**e**≤ **1** for E(r_p) curves. Equilibrium separation distance of a material can be determined by Lenard-Jones equation. It is related by van der Waals forces (Jacobs *et al.*, 1984). Hamaker constant is related on interaction between magnetic material and filtered fluid in the filter, also is determined experimentally (Bergstrom, 1997).

Determination of $Y_c(r_p)$ function.

The parameter Y_c is a function of two variables, W and K. It is selected by a diagram (Gooding & Felder, 1981, pp. 199). The **Table 2** below presents the W, K and Y_c for various values of the particle radius r_p. Next, the change function $Y_c(r_p)$ with computational simulation of least square method will be extracted.

Table 2. Parameter Y_c for various values of r_p.

r_p ($\times 10^{-6}$ m)	Magnetic param. W	Stokes number K	Y_c
0	0.006	0	0
0.05	0.006	0.017	0.002
0.10	0.006	0.07	0.007
0.15	0.006	0.157	0.009
0.20	0.006	0.278	0.08
0.25	0.006	0.435	0.15
0.30	0.006	0.626	0.25
0.35	0.006	0.852	0.35
0.40	0.006	1.113	0.41

0.45	0.006	1.409	0.49
0.50	0.006	1.74	0.55
0.55	0.006	2.105	0.6
0.60	0.006	2.505	0.65
0.65	0.006	2.94	0.7
0.70	0.006	3.41	0.71
0.75	0.006	3.914	0.75
0.80	0.006	4.454	0.76
0.85	0.006	5.028	0.79
0.90	0.006	5.636	0.81
1	0.006	6.959	0.83
2	0.006	27.83	0.96
3	0.006	62.63	0.985
4	0.006	111.3	0.992
5	0.006	174	0.995

From the values of the **Table 2** and least square method applying, results the **Eq (10)** with typical error R = 0.9996 and SSE = 0.001094.

$$Y_c(r_p) = \frac{1.011 r_p^2 - 0.1472 r_p + 0.02611}{r_p^2 - 0.1132 r_p + 0.1398} \quad (10)$$

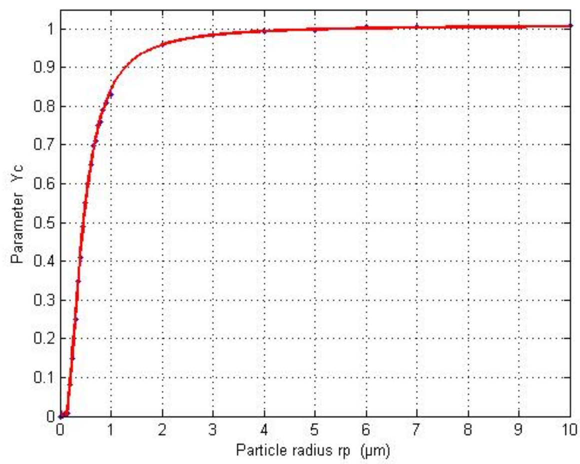

Fig. 5 The change curve of the function (10).

Extracting curves

If we substitute the numerical values of **Table 1** into **Eqs (6), (7)** and **Eq (10)** to **(6)** then will have the following curves $E(r_p, e)$ for various values of coefficient of restitution. The final functions of characteristic curves for magnetic filter are below:

$$\theta_c = \frac{1}{2}\cos^{-1}\left[\left(\frac{1}{\omega}\right)\left(156.65e^2 - \frac{0.0084}{r_p^2} - \frac{1}{2}\right)\right] \quad (11)$$

$$E(r_p, e) = 1 - \exp\left[-0.329 Y_c(r_p) \times \omega \sin\theta_c\right] \quad (12)$$

where ω=57.296 is the conversion factor of angle θ from rad to degrees.

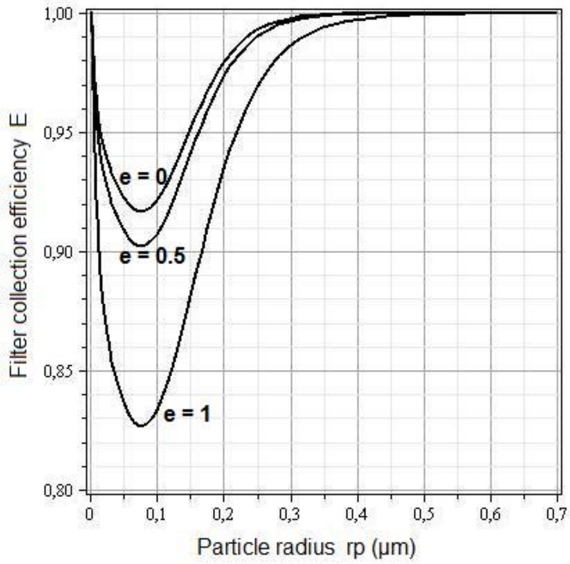

Fig. 6 Characteristic curves of magnetic filter efficiency for various values of coefficient of restitution, e=0, e=0.5 and e=1.

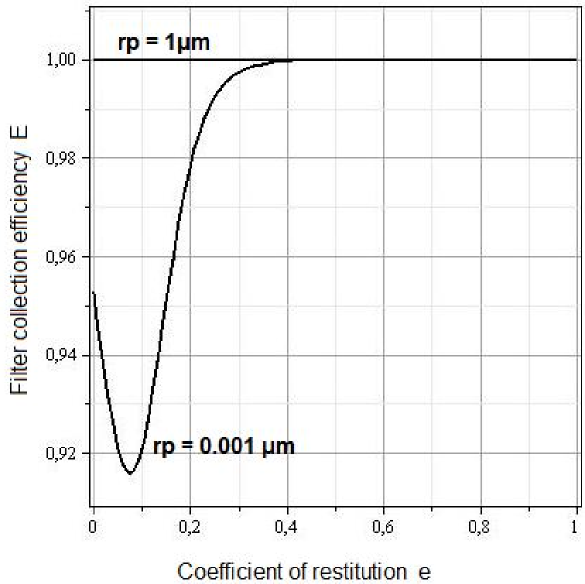

Fig. 7 The change curves of filter collection efficiency with regard to coefficient of restitution for various values of particle radius, r_p =1μm and r_p =0.001μm.

The functions (11) and (12) can be used for extracting of change curves of filter collection efficiency with regard to the coefficient of restitution for various values of particles radius **(Fig. 7)**.

The **Eqs (6)** and **(7)** can be solved by coefficient of restitution e and result a function of two variables e(E, r_p), so the change curves can be extracted.

$$E = 1 - \exp\left[-\frac{F}{(1-F)^2}\frac{4LY_c\omega\sin\theta_c}{\pi^2 r_w}\right] \Leftrightarrow$$

$$\Leftrightarrow \ln(1-E) = -\frac{F}{(1-F)^2}\frac{4LY_c\omega\sin\theta_c}{\pi^2 r_w} \Leftrightarrow$$

$$\Leftrightarrow \sin\theta_c = -\frac{1}{\omega}\frac{\ln(1-E)}{\frac{4LY_cF}{\pi^2 r_w(1-F)^2}} \Leftrightarrow$$

$$\Leftrightarrow e = \sqrt{A[B[1+2\omega\cos[2\sin^{-1}C]]]} \quad (13)$$

Where:

$$A = \frac{1}{\rho_p u_0^2} \quad (14)$$

$$B = \frac{U}{4\pi l r_p^2} + \mu_0 X H_a^2 \quad (15)$$

$$C = -\frac{1}{\omega}\frac{\ln(1-E)}{\frac{4LY_cF}{\pi^2 r_w(1-F)^2}} \quad (16)$$

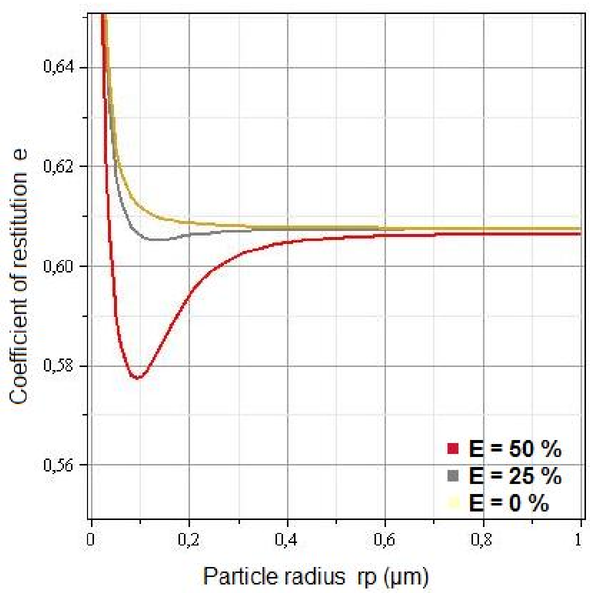

Fig. 8 The change curves of coefficient of restitution with regard to the particle radius for various values of filter collection efficiency, E=50%, E=25% and E=0%.

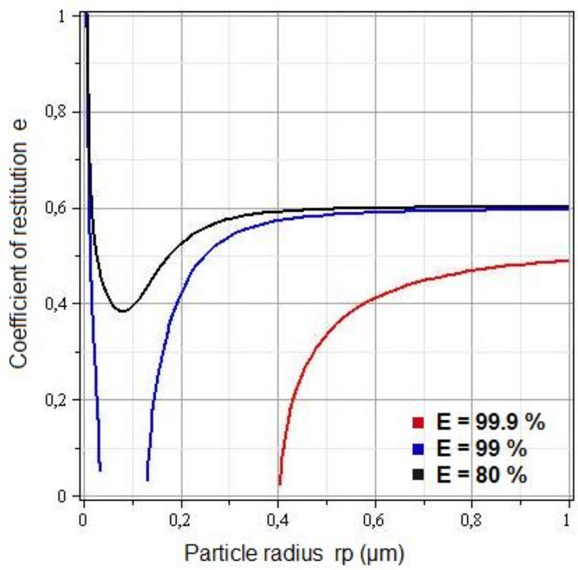

Fig. 9 The change curves of coefficient of restitution with regard to the particle radius for various values of filter collection efficiency, E=99.9%, E=99% and E=80%.

CONCLUSIONS

The characteristic curve of a magnetic filter in **(Fig. 6)**, is similar to the characteristic curve of a fiber filter medium (Kowalski *et al.*, 1999)**.** The collection efficiency of magnetic filtration decreases as the coefficient of restitution increases. The decrease in **(Fig. 6)** is more apparent in the local minimum of the function E(r_p) in the range 0.05 ≤ r_p ≤ 0.075.

In the **(Fig. 7),** the effect of the coefficient of restitution to the collection efficiency is annihilated as the particle radius increases. The curve E(r_p) remains constant when $r_p \geq$ 1μm. In **(Fig. 8)** and **(Fig. 9)**, the coefficient of restitution decreases as the collection efficiency of magnetic filter increases. The coefficient of restitution presents local minimum in the function e(r_p) in the range 0.05 ≤ r_p ≤ 0.075.

The curve of the function e(r_p), presents discontinuities in the range 0.05 ≤ r_p ≤ 0.4 and for collection efficieny values E = 99% and 99.9%, this is due to the specificities of mathematical model, **Eqs (13 - 16)**. Generally, the particle collection efficiency of magnetic filter converges in the same value **(Figs. 6, 7, 8)**, when there is elastic collision of a filtered particle upon the collection surface of magnet. Elastic collision has coefficient of restitution equal to 1.

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