



Journal of Urban and Environmental  
Engineering

E-ISSN: 1982-3932

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Journal of Urban and Environmental Engineering, vol. 9, núm. 1, 2015, pp. 66-72  
Universidade Federal da Paraíba  
Paraíba, Brasil

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# **DYNAMIC MATHEMATICAL MODEL OF URBAN SPATIAL PATTERN (RESIDENTIAL CHOICE OF LOCATION): MOBILITY VS EXTERNALITY**

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Received 6 March 2014; received in revised form 16 September 2015; accepted 19 September 2015

## **Abstract:**

Household's residential choice of location determines urban spatial pattern (e.g sprawl). The static model which assumes that the choice has been affected by distance to the CBD and location specific externality, fails to capture the evolution of the pattern over time. Therefore this study proposes a dynamic version of the model. It analyses the effects of externalities on the optimal solution of development decision as function of time. It also derives the effect of mobility and externality on the rate of change of development pattern through time. When the increasing rate of utility is not as significant as the increasing rate of income, the externalities will delay the change of urban spatial pattern over time. If the mobility costs increase by large amount relative to the increase of income and inflation rate, then the mobility effect dominates the effects of externalities in delaying the urban expansion.

**Keywords:** dynamic model; externality; mobility; urban spatial pattern

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## INTRODUCTION

Mobility in term of distance from urban centre affects directly household's decision of residential location. The decision in aggregate will shape the urban spatial pattern. The most observed urban spatial pattern experienced by many urban centres is sprawl, i.e low density development that expands in an unlimited and non-contiguous (leapfrog) way outward from the solidly built up core of a metropolitan area. The ease of mobility has been blamed as one factor that determines sprawl (Sierra Club 1998; Brueckner 2000; Nechyba and Walsh 2004). Sprawl is generally regarded as emerging from market forces subject to various market failures, suggesting that it does not produce an efficient pattern of urban development (Brueckner 2000; Ewing 2008), with higher costs than benefits. Moreover, the economic activities in a sprawled urban area are prone to any change in the ease of mobility, such as the increase of gasoline price.

More recent studies (Irwin and Bockstael 2004; Caruso, Peeters et al. 2007; Fitriani 2011) show that in addition to mobility, location specific externalities created by the surrounding land uses, also contributed to the emergence of sprawl. Those studies generally agree that development decision of a household is a trade off between mobility and externalities. The magnitude of each effect will be valuable information to plan a better transportation infrastructure.

Fitriani and Harris (2011) analyse the effect of mobility and externalities on the urban spatial pattern based on a static model of residential choice of location with externalities. The model however fails to explain the evolution of the pattern through time. It motivates this study to develop a dynamic version of the model, which accommodates both mobility and spatial externalities. The model will be useful to analyse the effect of each factor on the rate of change of the urban spatial pattern as function of time. This study also analyses the situation in which one effect dominates the other.

## THEORETICAL MODEL

The theoretical model is based on the Residential Choice of Location with Externalities from Fitriani (2011) with additional time dimension. It extends the monocentric open city model to accommodate several externalities. It follows the formulation of the crowding externalities model of Fujita (1989), which assumes that the neighbourhood land offers a 'green' type of externality as a decreasing function of neighbourhood density. The neighbouring density in fact might create another type of externality namely a 'social' externality, thus following Caruso et al. (2007), this study also accommodates this type of externality. Both types of

externality are defined as functions of neighbourhood density.

## Assumptions and Optimal Solution

A monocentric city is assumed, in which all job opportunities are located in the CBD and accessible from any location. One of the following agents, a householder or farmer, occupies the space. Households are all identical and composed of a single worker/consumer, who trades off accessibility, space and environmental amenities when choosing residential location. They commute to the CBD for work, rent a residential space and consume composite goods. They enjoy environmental amenities in the form of their neighbourhood land use externalities.

Both farmer and household have von Thunen's type of bid rent for their location. The bid rent for agricultural use by the farmer depends on the productivity and the distance to the CBD, the market of their product, whereas the bid rent of a household depends on commuting costs and on the combination of the two types of neighbourhood externality.

The landowner is absentee and he sells the land to the highest bidder within a competitive land market. Furthermore, with an open city assumption, households from the 'rest of the world' may migrate into the city as long as they can obtain a utility surplus. The migration thus leads to the growth of the city around the CBD, which is assumed to provide enough employment. Finally, for simplicity, it is assumed here that the city is linear: the CBD is the initial point and the specific location in the city is  $r$  units of distance away from it. However, the properties of the model can still be generalized into a more realistic circular city assumption.

To choose the residential location, all households have identical utility functions, which depend on a non spatial composite good  $z$ , a residential lot space  $s$  and location specific neighbourhood 'green' externalities ( $G(.)$ ) and 'social' externalities ( $A(.)$ ).  $s$  is the average space occupied by one household which is defined as the inverse of density. It is also assumed that  $s$  includes the amount of greenspace enjoyed by one household. Location specific externalities are the product of the neighbouring land use. Both types of externality are functions of location specific neighbourhood density. Distance to the CBD  $r$  is used in this case to specify the location specific neighbourhood density and neighbourhood externalities, such that the households' utility is defined as follows:

$$U(z, s, G[s(r)^{-1}], A[s(r)^{-1}]) = z^\alpha s^\beta G[s(r)^{-1}]^k A[s(r)^{-1}]^\gamma \quad (1)$$

It is further assumed that:

$\frac{\partial U}{\partial z} > 0, \frac{\partial U}{\partial s} > 0, \frac{\partial U}{\partial G[s(r)^{-1}]} > 0, \frac{\partial U}{\partial A[s(r)^{-1}]} > 0$ , and  $\alpha + \beta = 1, \alpha, \beta > 0$ . The taste of households for the 'green' and 'social' externalities is represented by  $\lambda > 0$  and  $\gamma > 0$  respectively.

The 'green' externalities  $G(\cdot)$  is defined as a decreasing function of neighbourhood density:

$$G[s(r)^{-1}] = [s(r)^{-1}]^\theta = s(r)^\theta, \frac{dG[s(r)^{-1}]}{d[s(r)^{-1}]} < 0, \quad (2)$$

and the 'social' externalities  $A(\cdot)$  is an increasing function of neighbourhood density:

$$A[s(r)^{-1}] = [s(r)^{-1}]^\phi = s(r)^{-\phi}, \frac{dA[s(r)^{-1}]}{d[s(r)^{-1}]} > 0. \quad (3)$$

In the equilibrium it is assumed that all households at the same distance  $r$  consume on average the same amount of land or space such that  $s(r) = s$ . Using this definition for each function defining each externalities in **Eq. (2)** and **Eq. (3)**, the utility function in **Eq. (1)** can be simplified as:

$$U(z, s) = z^\alpha s^{\beta+c}, \quad (4)$$

with  $c = \lambda\theta - \gamma\phi$  and additional assumption  $\theta > \phi$ . The last assumption guarantees that the utility is an increasing function of externalities.

The utility function in **Eq. (4)** is used to define the composite good  $z$  as a function of certain utility level  $u$  and lot size  $s$ , as follows:

$$z(s, u) = u^{1/\alpha} s^{-(\beta+c)/\alpha} = U s^{-(\beta+c)/\alpha} \quad (5)$$

with  $U = u^{1/\alpha}$ .

The utility function in **Eq. (1)** or **Eq. (4)** will be maximized to decide the household's bid rent for an optimal combination of distance to the CBD and the lot size of the residential choice, subject to the budget. Then, time dimension is added into the following maximization problem such that the decision will be functions of time  $\tau (0 \leq \tau < \infty)$ :

$$\max_{r, z, s} U(z, s), s.t. \ z + R(r, s, \tau) + D(r, \tau) = Y(\tau), \quad (6)$$

where

$R(r, s, \tau)$  is the location rent or land value at size  $s$ , location  $r$ , and time  $\tau$ ,

$D(r, \tau)$  is the cost of transportation or mobility as function of distance/location  $r$ , and time  $\tau$ .

$Y(\tau)$  is the household income as function of time  $\tau$ .

The transportation cost function in **Eq. (6)** will be in the form of:

$$D(r, \tau) = dr. \quad (7)$$

Parameter  $d$  in **Eq. (7)** reflects the ease of mobility (e.g. the gasoline price). The higher the value of  $d$  the more costly the mobility will be. The optimal solution will be affected by any change of this parameter. On the other hand, the same analysis can be applied to evaluate the effect of externalities on the optimal solution. Since the presence of externalities has been summarized in the form of coefficient  $c$  of the utility function in **Eq. (4)**, the effect of externalities on the optimal solution is analysed based on the change of the value of  $c$ .

The household's decision of residential location is then combined with developer's decision of development strategy. The developer's problem is to choose an optimal development time  $t$  and the lot size  $s$ , by maximizing the present value of the future net revenue of developing land of size  $s$  at time  $t$ , as follows:

$$\max_{s, t} p(r, s, t) = \int_0^t e^{-\gamma\tau} R^A(\tau) d\tau + \frac{e^{-\gamma t}}{s} \int_t^\infty e^{-\gamma(\tau-t)} R(r, s, \tau) d\tau - B(s) \quad (8)$$

$s.t. \ t, s \geq 0$

where

$t$  is the time when the location at distance  $r$  to the CBD is converted for residential at size  $s$

$B(s)$  is the development cost for developing land at size  $s$ . It is assumed that the cost is given externally and constant through time.

$\gamma$  is the discount rate for the future revenue and other related costs.

$R(r, s, \tau)$  is the land rent for residential at distance  $r$  to the CBD, at size  $s$  at time  $\tau$ .

$R^A(\tau)$  is the land rent for agricultural use, up to time  $\tau$ . It is zero by assumption.

The optimal solution for the developer's problem in is the land rent offered by the developer  $p(r, s, t)$  for an  $(s, t)$  development strategy at location  $r$ . It is obtained by deriving the following first order conditions of the optimization problem in **Eq. (8)**:

$$\frac{\partial p(r, s, \tau)}{\partial t} = 0 \quad (9)$$

$$R(s, r, t) - \gamma B(s) = s R^A(t)$$

and

$$\frac{\partial p(r, s, \tau)}{\partial s} = 0 \quad (10)$$

$$\bar{R}(s, r, t) - \gamma B(s) = s [\bar{R}_s(s, r, t) - \gamma B_s(s)]$$

The following relations are hold by combining the above conditions (Eq. (9) and Eq. (10)) with the household's decision dynamically:

$$Y(t) - D(r(t), t) - Z(s(t), u(t)) = s(t)R^A(t) \quad (11)$$

and

$$\bar{Y}(t) - \bar{D}(r(t), t) - \bar{Z}(s(t), t) = -s(t)\bar{Z}_s(s(t), t), \quad (12)$$

where  $R^A(t)$  is the land rent when it is in agricultural use, and every notation with bar represents the present value of the future value of the corresponding variable. The last term in the left hand side of Eq. (11) is defined using the composite good function in Eq. (5) as:

$$\begin{aligned} Z(s(t), u(t)) &= z(s(t), u(t)) + \gamma B(s) \\ &= U(t)s(t)^{-\frac{\beta+c}{\alpha}} + \gamma B(s) \end{aligned} \quad (13)$$

where  $U(t) = u(t)^{\frac{1}{\alpha}}$ . The present value of the average future value of  $Z$ ,  $Y$  and  $D$  in Eq. (12) are defined respectively as:

$$\begin{aligned} \bar{Z}(s(t), t) &= \gamma \int_t^\infty e^{-\gamma(\tau-t)} Z(s(t), u(\tau)) d\tau \\ &= \bar{U}(t)s(t)^{-\frac{\beta+c}{\alpha}} + \gamma B(s), \end{aligned} \quad (14)$$

$$\bar{Y}(t) = \gamma \int_t^\infty e^{-\gamma(\tau-t)} Y(\tau) d\tau, \quad (15)$$

and

$$\begin{aligned} \bar{D}(r, t) &= \gamma \int_t^\infty e^{-\gamma(\tau-t)} D(r, \tau) d\tau \\ &= \gamma \int_t^\infty e^{-\gamma(\tau-t)} dr d\tau = dr. \end{aligned} \quad (16)$$

Also, the partial derivative of Eq. (14) with respect to  $s$  for the right hand side of Eq. (12) is defined as:

$$\bar{Z}_s(s(t), t) = -\frac{\beta+c}{\alpha} s(t)^{-\frac{\beta+c}{\alpha}-1} \bar{U}(t). \quad (17)$$

The solution of the household's bid rent as an optimal combination of distance to the CBD  $r$  and the lot size of the residential choice  $s$  as functions of time are obtained by substituting Eq. (13) until Eq. (17) into the first order conditions in Eq. (11) and Eq. (12). They are:

$$s(t) = \left[ \frac{\left(1 + \frac{\beta+c}{\alpha}\right) \bar{U}(t) - U(t)}{\bar{Y}(t) - Y(t)} \right]^{\frac{\alpha}{\beta+c}} \quad (18)$$

and

$$r(t) = \frac{1}{d} \left[ \frac{\left(1 + \frac{\beta+c}{\alpha}\right) Y(t) \bar{U}(t) - \bar{Y}(t) U(t)}{\left(1 + \frac{\beta+c}{\alpha}\right) \bar{U}(t) - U(t)} - \gamma \beta \right] \quad (19)$$

### Rate of Change of the Optimal Solution

The optimal distance to the CBD  $r$  and the optimal lot size of the residential choice  $s$  and their rate of change are functions of time. In order to analyse the rate of change of the optimal solution, the following assumption are needed:

$$U(\tau) = \begin{cases} U_0 e^{a\tau}, & \text{for } 0 \leq \tau \leq T \\ U_0 e^{aT}, & \text{for } \tau > T \end{cases} \quad (20)$$

and

$$Y(\tau) = \begin{cases} Y_0 e^{b\tau}, & \text{for } 0 \leq \tau \leq T \\ Y_0 e^{bT}, & \text{for } \tau > T \end{cases} \quad (21)$$

where  $U_0 > 0, Y_0 > 0, 0 \leq a < b < \gamma$ . Those assumptions ensure that income and utility are increasing functions of time, with the rate of increase below the discount rate  $\gamma$ , and the increasing rate of income is greater than the increasing rate of utility. The following relations are defined:

$$\bar{U}(t) = \gamma \int_t^\infty e^{-\gamma(\tau-t)} u(\tau) d\tau = \frac{U_0 \gamma e^{at}}{\gamma - a} \quad (22)$$

as the present value of average future utility, using the definition in Eq. (20) and

$$\bar{Y}(t) = \gamma \int_t^\infty e^{-\gamma(\tau-t)} Y(\tau) d\tau = \frac{Y_0 \gamma e^{bt}}{\gamma - b} \quad (23)$$

as the present value of average future income, using the definition in Eq. (21). These two last definitions are substituted into the optimal lot size of residential  $s$  in Eq. (18) and the optimal distance to the CBD  $r$  in Eq. (19) such that the following holds:

$$\begin{aligned} s(t) &= \left[ \frac{\gamma - b}{\gamma - a} \left( \frac{a + \gamma(\beta+c)}{b} \right) \frac{U_0}{Y_0} \right]^{\frac{\alpha}{\beta+c}} e^{\frac{(a-b)\alpha}{\beta+c} t} \\ &= K_1 e^{\frac{(a-b)\alpha}{\beta+c} t}, \end{aligned} \quad (24)$$

where  $K_1 = \left[ \frac{\gamma - b}{\gamma - a} \left( \frac{a + \gamma(\beta+c)}{b} \right) \frac{U_0}{Y_0} \right]^{\frac{\alpha}{\beta+c}} > 0$ , and

$$r(t) = \frac{1}{d} \left\{ Y_0 e^{bt} \frac{1}{\gamma - b} \left[ 1 + \frac{(\gamma - a)(1 - \gamma)}{\left( 1 + \frac{\beta + c}{\alpha} \gamma \right) - (\gamma - a)} \right] - \gamma B \right\}$$

$$= \frac{1}{d} (e^{bt} K_2 - \gamma B), \quad (25)$$

$$\text{where } K_2 = Y_0 \frac{1}{\gamma - b} \left[ 1 + \frac{(\gamma - a)(1 - \gamma)}{\left( 1 + \frac{\beta + c}{\alpha} \gamma \right) - (\gamma - a)} \right] > 0.$$

The positive value of  $K_1$  leads to the following property of the first partial derivative of **Eq. (24)** with respect to time  $t$ :

$$\frac{\partial s(t)}{\partial t} = K_1 \frac{(a - b)\alpha}{\beta + c} e^{\frac{(a-b)\alpha}{\beta+c}t} < 0, \text{ for } a < b \quad (26)$$

Whereas the fact that  $K_2 > 0$  ensures the non-negativity of the partial derivative of **Eq. (25)** with respect to time  $t$ :

$$\frac{\partial r(t)}{\partial t} = \frac{1}{d} K_2 b e^{bt} > 0, \text{ for } b > 0 \quad (27)$$

Those two results show that along time the average lot size of residential will be smaller, and that the residential location will be further away from the CBD.

## THE EFFECT OF THE CHANGE OF THE VALUE OF PARAMETERS ON THE DEVELOPMENT PATTERN

The parameters involved in this model are parameters which define externalities and mobility. This section focuses on the analysis of how the change of each parameter value will affect the optimal solutions (**Eq. (24)** and **Eq. (25)**) and their rate of change (**Eq. (26)** and **Eq. (27)**).

### The Change of Parameter Value for Externalities ( $c$ )

At the same point of time, the effects of the change of  $c$  on the optimal lot size  $s$  and the distance to the CBD  $r$  are analyzed based on the first partial derivative of **Eq. (24)** and **Eq. (25)** respectively with respect to  $c$ . Whereas the first partial derivative of **Eq. (26)** and **Eq. (27)** with respect to  $c$  are used to analysed the effects of externalities on the rate of change of the solutions.

The optimal lot size in as function of time in **Eq. (24)** can be redefined as:

$$s(t) = \Delta_1 (\Delta_2 + \gamma c)^{\frac{\alpha}{\beta+c}} e^{\frac{\Delta_3 \alpha}{\beta+c}t} = K_1 Z, \quad (28)$$

where

$$\Delta_1 = \frac{\gamma - b}{\gamma - a} \frac{U_0}{b Y_0} > 0, \Delta_2 = a + \gamma b > 0, \Delta_3 = a - b < 0$$

$$K_1 = \Delta_1 (\Delta_2 + \gamma c)^{\frac{\alpha}{\beta+c}} > 0, Z = e^{\frac{\Delta_3 \alpha}{\beta+c}t} > 0$$

such that the partial derivative of **Eq. (28)** with respect to  $c$  is obtained based on the partial derivative of its components ( $K_1$  and  $Z$ ) with respect to  $c$ . Following relations are derived consequently:

$$\frac{\partial K_1}{\partial c} = K_1 \frac{\alpha}{\beta + c} \left( \frac{\ln(\Delta_2 + \gamma c)}{\beta + c} + \frac{\gamma}{\Delta_2 + \gamma c} \right) > 0 \quad (29)$$

and

$$\frac{\partial Z}{\partial c} = Z \left( -\Delta_3 \frac{\alpha}{(\beta + c)^2} t \right) > 0 \quad (30)$$

Those derivatives are useful to predict the following direction of the change of the optimal lot size  $s$  at the fixed point of time with respect to  $c$ :

$$\frac{\partial s}{\partial c} = s' = K_1' Z + K_1 Z' > 0 \quad (31)$$

It implies that at a fixed point of time, the externalities leads to a larger residential lot size.

In a similar manner, the following partial derivative of the rate of change of the optimal lot size **Eq. (26)**, with respect to  $c$  is obtained:

$$\begin{aligned} \frac{\partial^2 s}{\partial t \partial c} &= K_1' \Delta_3 \frac{\alpha}{(\beta + c)} e^{\frac{\Delta_3 \alpha}{(\beta+c)}t} - \\ &K_1 \Delta_3 \frac{\alpha}{(\beta + c)^2} e^{\frac{\Delta_3 \alpha}{(\beta+c)}t} \left( 1 + \Delta_3 \frac{\alpha}{(\beta + c)} t \right) \\ &= K_1 \Delta_3 \frac{\alpha}{(\beta + c)} e^{\frac{\Delta_3 \alpha}{(\beta+c)}t} \times \\ &\left( \frac{\alpha}{\beta + c} \left( \frac{\ln(\Delta_2 + \gamma c)}{\beta + c} + \frac{\gamma}{\Delta_2 + \gamma c} \right) - \frac{1}{(\beta + c)} - \Delta_3 \frac{\alpha}{(\beta + c)^2} t \right) \end{aligned} \quad (32)$$

In  $\Delta_3$  it is assumed that

Since in  $\Delta_3$   $a \ll b$  (the utility increases at a higher rate than income), then **Eq. (32)** is negative. It implies

that over time the average lot size will be smaller, but the relation in **Eq. (32)** predicts that when the increasing rate of utility (due to residential's amenity) is not compatible with the increasing rate of income, the externalities will reduce the decreasing rate of residential lot size.

A similar analysis is applied for the optimal residential distance to the CBD. It uses the modified version of **Eq. (25)** as follows:

$$r(t) = \frac{1}{d} (e^{bt} K_2 - \gamma B) \quad (33)$$

where

$$\begin{aligned} K_2 &= Y_0 \frac{1}{\gamma - b} \left( 1 + \frac{(\gamma - a)(1 - \gamma)}{1 + \frac{\beta}{\alpha} + \frac{\gamma}{\alpha} c - (\gamma - a)} \right) \\ &= Y_0 \frac{1}{\gamma - b} \left( 1 + \frac{\Delta_5}{\Delta_6 + \frac{\gamma}{\alpha} c} \right) > 0 \end{aligned}$$

and

$$\frac{\partial K_2}{\partial c} = K_2 \left( \frac{-\Delta_5 \frac{\gamma}{\alpha}}{\left( \Delta_6 + \frac{\gamma}{\alpha} c \right)^2} \right) \left( 1 + \frac{\Delta_5}{\Delta_6 + \frac{\gamma}{\alpha} c} \right)^{-1} < 0 \quad (34)$$

Those relations are used to predict the effect of externalities on the optimal residential distance to the CBD at the same point in time, such as:

$$\frac{\partial r}{\partial c} = \frac{1}{d} \frac{\partial K_2}{\partial c} < 0 \quad (35)$$

**Eq. (35)** implies that at a certain point in time, the presence of externalities shorten the residential distance to the CBD.

The externalities also affect the rate of change of distance to the CBD over time. It can be shown that the derivative of **Eq. (27)** with respect to  $c$  is negative:

$$\frac{\partial^2 r(t)}{\partial t \partial c} = \frac{\partial K_2}{\partial c} \frac{1}{d} b e^{bt} < 0 \quad (36)$$

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implying that due to the externalities, the residential distance to the CBD over time is increasing at a decreasing rate. In other words, the externalities might delay the urban expansion over time.

### The Change of Mobility Parameter

In this study the ease of mobility, represented by parameter  $d$ , is assumed due to the low price of the

gasoline. But in general, the availability and the quality of road infrastructure also affect this parameter. The higher the value of  $d$  (e.g. the increase of gasoline market price), the more difficult or the more expensive the mobility is. Therefore it is necessary to analyse how the change of  $d$  affecting the rate of change of residential lot size over time **Eq. (26)** and the rate of change of residential distance to the CBD over time **Eq. (27)**. But, it can be shown that the parameter  $d$  takes part only in **Eq. (27)**. Thus, the ease of mobility only affects the latter rate of change.

Since the partial derivative of **Eq. (27)** with respect to  $d$  is:

$$\frac{\partial^2 r(t)}{\partial t \partial d} = -\frac{1}{d^2} K_2 b e^{bt} < 0, \quad (37)$$

it can be said that the high mobility costs delays the urban expansion.

### Comparing the Effects

Generally, the results in **Eq. (36)** and **Eq. (37)** indicate that externalities and mobility can be used to control urban expansion. But in what conditions one factor dominates the other. By comparing the results, the following condition holds if the mobility effect is expected to dominate the externalities:

$$\frac{\partial^2 r(t)}{\partial t \partial c} > \frac{\partial^2 r(t)}{\partial t \partial d} \Leftrightarrow d > -K_2 \frac{1}{\frac{\partial K_2}{\partial c} / \frac{\partial c}{\partial c}} \quad (38)$$

The relation in **Eq. (33)** indicates that  $K_2$  represents the information about income, discount rate, the increasing rate of income and the increasing rate of utility. Using the fact that the partial derivative of  $K_2$  with respect to  $c$  is negative, **Eq. (38)** implies that when the mobility costs is higher than the combined factors in the right hand side, then the mobility will be more dominant in delaying the urban expansion.

### CONCLUSIONS AND IMPLICATIONS

The analysis in this study leads to the several results. The change of urban spatial pattern over time is reflected through the decrease of the average residential lot size and the increase of the residential distance to the CBD over time. The latter indicates the urban expansion over time. But there are two factors that can be utilized to alter or delay the change, namely externalities and mobility.

The externalities affect the rate of change of both the residential lot size and the residential distance to the CBD over time. When the increasing rate of utility is not as significant as the increasing rate of income, the externalities will delay the change of urban spatial

pattern over time. Thus, due to the externalities, the residential lot size decreases at a decreasing rate over time, and the residential distance to the CBD increases also at a decreasing rate over time.

Urban expansion in term of the residential distance to the CBD might be deterred by the high mobility costs. If the costs increase by large amount relative to the increase of income and inflation rate, then mobility will have an influential part in delaying the urban expansion.

Those results should be incorporated to formulate policies to reshape the future urban spatial pattern. Both externalities and mobility costs serve that purpose. By observing the market characteristics and understanding the direction of the externalities as well as the pattern of mobility, policy makers must allocate public facility or road infrastructure accordingly such that the planned urban spatial pattern will be realized.

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