Investigaciones Regionales
ISSN: 1695-7253
investig.regionales@uah.es
Asociación Española de Ciencia Regional
España

Zhang, Wei-Bin
Economic Globalization and Interregional Agglomeration in a Multi-Country and Multi-Regional Neoclassical Growth Model
Investigaciones Regionales, núm. 34, 2016, pp. 95-121
Asociación Española de Ciencia Regional
Madrid, España

© *Investigaciones Regionales – Journal of Regional Research, 34 (2016) – Páginas 95 a 121*
Sección ARTÍCULOS

Economic Globalization and Interregional Agglomeration in a Multi-Country and Multi-Regional Neoclassical Growth Model

Wei-Bin Zhang *

ABSTRACT: The purpose of this paper is to extend the well-known Uzawa two-sector model for a national economy to a global economy with any number of countries and any number of regions within each country. It studies international and interregional economic development with interactions among wealth accumulation, amenity change and economic structure under assumptions of profit maximization, utility maximization, and perfect competition. The model shows endogenous interregional and international trade patterns on the basis of microeconomic foundation. We deal with the complicated issues by applying Zhang's alternative approach to households' behavior. We simulate the motion of the multi-country and multi-region global economy, identify the existence of an equilibrium point, and confirm the stability of the equilibrium point. We carry out comparative dynamic analysis with regard to the total factor productivity of region's industrial sectors, total factor productivity of region's service sectors, propensities to save, regional amenity parameters, propensities to consume housing, and the national population effect on amenity.

JEL Classification: R11; O18.

Keywords: global economy; spatial agglomeration; regional, national and global growth; amenity.

Globalización económica y procesos de aglomeración interregionales en un modelo neoclásico de crecimiento multi-país y multi-región

RESUMEN: El objetivo de este artículo es generalizar el conocido modelo de Uzawa de dos sectores para una economía nacional para su aplicación en contextos globales con muchos países y muchas regiones dentro de cada país. Se estudia

Acknowledgements: The author is grateful to the constructive comments of the two anonymous referees. The author is also grateful for the financial support from the Grants-in-Aid for Scientific Research (C), Project No. 25380246, Japan Society for the Promotion of Science.

* Ritsumeikan Asia Pacific University. Jumonjibaru, Beppu-Shi, Oita-ken, 874-8577 Japan. Home Tel.: 0977-73-9787; Office Tel.: 0977-78-1020; Fax: 0977-78-1123; E-mail: *wbz1@apu.ac.jp*.

Received: 19 september 2015 / Accepted: 10 march 2016.

el desarrollo económico internacional e interregional a través de un modelo de equilibrio general donde se considera la acumulación de riqueza, el intercambio de recursos y la estructura económica bajo los supuestos de maximización de beneficios y utilidad y competencia perfecta. Para ello se aplica el modelo alternativo de Zhang sobre el comportamiento de los hogares simulándolo para un contexto multi-país y multi-región en una economía global, identificando la existencia del equilibrio y confirmando la estabilidad del mismo. Adicionalmente, se realiza un análisis comparativo dinámico para la productividad total de los factores, tanto del sector industrial como del terciario en cada región, así como de la propensión del ahorro, la asignación regional de recursos, la propensión al consumo de los hogares y los efectos de la población.

Clasificación JEL: R11; O18.

Palabras clave: recursos; crecimiento; aglomeración espacial; globalización; modelos neoclásicos.

1. Introduction

An important phenomenon of modern economic activity is rapid spatial agglomeration within nations and among global economies. Rapid national spatial agglomeration and globalization are taking place simultaneously in a well-connected world economy. Nevertheless, few theoretical economic models address these issues within a single comprehensive framework. The purpose of this study deals with dynamics of global and local economic interactions within a analytical framework proposed by Zhang. We develop a neoclassical economic growth model with multiple (any number of) countries in the world economy and multiple (any number of) regions in each country to explain economic mechanisms and dynamics of economic globalization with spatial agglomeration within a compact analytical framework. This is a challenging task as it is not even easy to model a multi-country growth model with endogenous wealth accumulation or to model a multi-regional growth model of a single economy with endogenous wealth accumulation. In fact, there are few multi-regional or multi-national growth models in economic theory. We build the model on the basis of some well-known models in economic growth theory and regional economics with an alternative approach proposed more than two decades ago by Zhang (1993b).

With regard to capital mobility and international and interregional trade, this study is based on the neoclassical growth trade model. Findlay (1984) shows that almost all the trade models developed before the 1960s are static in the sense that the supplies of factors of production are given and do not vary over time. The classical Ricardian theory of comparative advantage and the Heckscher-Ohlin theory are not dynamic since labor and capital stocks (or land) are exogenous. Early trade models with capital movements are proposed by MacDougall (1960) and Kemp (1961). Moreover, most of trade models with endogenous capital in the contemporary literature are either limited to two-country or small open economies (for instance, Wong, 1995; Jensen and Wong, 1998; Obstfeld and Rogoff, 1998). The model in this study overcomes this shortcoming as it suits for explaining trade patterns between any number of national and interregional economies. There are some neoclassical growth models with international trade. For instance, Oniki and Uzawa (1965) and Bardhan (1965) study trade patterns between two economies in the Heckscher-Ohlin modeling framework with fixed savings rates. Deardorff and Hanson (1978) propose a two country trade mode with different saving rates across countries. There are some other growth models with international trade (*e. g.*, Brecher *et al.*, 2002; Nishimura and Shimomura, 2002; Bond *et al.,* 2003; Ono and Shibata, 2005). This study differs from these studies in modelling behavior of households. Rather than assuming a constant saving rate like in the Solow model or a constant discounting factor in the utility function like in the Ramsey model, this study uses the utility function proposed by Zhang in modelling household behavior.

A few metropolitan areas such as Tokyo and Shanghai in the contemporary global economy are increasingly attracting more people. Bairoch (1993) shows that rapid agglomeration is a modern phenomenon. Kuznets (1966) attributes rapid disparities between regional economic developments to industrialization. In fact, the early development theories by Myrdal (1957) and Hirschman (1958) argue for dynamic interactions between industrial growth and the geographical concentration of industry: industrialization attracts resources to a given location and the resulting agglomeration stimulates growth. Nevertheless, the traditional works could not illustrate motion of interregional dynamics. They could only generally discuss dynamic mechanisms without providing any concrete illustration of motion of dynamic systems. The reason is that to deal with these complicated issues one needs tool for analyzing high-dimensional nonlinear dynamics. However it is only in recent years that scientists can properly deal with highly dimensional dynamics with the help of computer. This study demonstrates the motion of global economies with multiple countries and multiple regions with the help of computer. It should be remarked that in the contemporary literature spatial economists propose interregional dynamic models based on different factors of nonlinear dynamics (*e. g.*, Fujita *et al.,* 1999; Forslid and Ottaviano, 2003). Nevertheless, most of the formal models in economic geography have not succeeded in including capital/wealth accumulation as endogenous processes of industrialization and agglomeration. A reason for the lacking of wealth accumulation in interregional economic dynamics is that there is no proper microeconomic foundation of household behavior for including the growth factor. Our approach differs from the so-called new economic geography (*e. g.*, Krugman, 1991, 1993; Ottaviano *et al.*, 2002; Pflüger, 2004; Charlot, 2006; Picard and Tabuchi, 2010) mainly in that we treat capital/wealth accumulation as an endogenous process of spatial dynamics. In almost all the dynamic models of the new economic geography, physical capital is completely neglected and regional amenities do not play a significant role in determining land rent and population mobility. Tabuchi (2014: 50) observes, «The scopes of most of the theoretical studies published thus far have been limited to two regions in order for researchers to reach meaningful analytical results. The two-region NEG models tend to demonstrate that spatial distribution is dispersed in the early period (high trade costs or low manufacturing share) and agglomerated in one of the two regions in the late period (low trade costs or high manufacturing share). However, it is no doubt that the two-region NEG models are too simple to describe the spatial distribution of economic activities in real-world economies. Since there are only two regions, their geographical locations are necessarily symmetric, and thus diverse spatial distributions cannot occur.» Not only do we include wealth accumulation as endogenous processes of the global growth, but also we take account of environmental dynamics.

This study takes account of environmental dynamics by treating regional amenity as an endogenous variable. Amenity plays an important role in explaining spatial agglomeration. Spatial amenities have increasingly caused attention from spatial scientists (Glaeser *et al.*, 2001; Partridge *et al.*, 2008a, 2008b; Chen *et al.*, 2013). There is a large body of literature on amenities and spatial economics, for instance, equilibrium ideas by Graves (1979) and Roback (1982), turnaround migration theory by Brown *et al.* (1997), life cycle studies by Clark and Hunter (1992), research on rural development by Deller *et al.* (2001). Zhang (1993a) first introduced spatial amenity into utility in a general equilibrium framework. Public services, local transportation systems, accessibilities, pollution, and human relations such as discrimination all involve externalities and affect amenities. Chen *et al.* (2013) point out that «Empirical evidence demonstrates a growing link between the presence of high-valued natural amenities —including pleasant climate and proximity to lakes, oceans, forests, and mountains— and higher rates of population and income growth in the U.S.». They develop a two-region model in which labor distribution, production externalities and natural resources are endogenous. They also show that strong preference for natural amenities tend to lead population dispersion. Chen (2013: 256) review: «Number of empirical studies have demonstrated the positive association between rural growth and natural amenities (*e. g.*, McGranahan, 1999; Shumway and Otterstron, 2001; and Kim *et al.*, 2005), none have examined the pattern of relative population distribution across amenity-based areas nor sought to develop a theoretical model of amenity-led migration that explains this distribution». This study explains endogenous amenity-led migration as a determinant of population distribution. This paper is an extension of Zhang's two-country growth model (Zhang, 1996). This study generalizes Zhang's study mainly by extending the two countries to any number of countries and simulates motion of the multi-country economy rather than only examining the steady state. This study differs from Zhang (1996, 2016) in that the previous studies by Zhang deal with a national economy with multiple regions. This paper integrates the multi-regional growth model and multi-country growth model in a compact analytical framework. The paper is organized as follows. Section 2 defines the multi-country and multi-regional model with capital accumulation and economic structure. Section 3 identifies the differential equations which are applied to simulate the model of the global economy plots the motion of the economic dynamics, demonstrates the existence of a unique equilibrium point, and proves the stability of the equilibrium point. Section 4 carries out comparative dynamic analysis with regard to the total factor productivity of region's industrial sectors, total factor productivity of region's service sectors, propensities to save, regional amenity parameters, propensities to consume housing, and the national population effect on amenity. Section 5 concludes the study. The main analytical results of Section 3 are proved in the appendix.

2. The multi-country and multi-regional growth model

This study is based on Zhang's dynamic two-region (Zhang, 1996, 2016) and two-sector trade model (Zhang, 1995). These models are influenced by the neoclassical trade theory with capital accumulation (Uzawa, 1961; Oniki and Uzawa, 1965; Brecher *et al.*, 2002; Sorger, 2003). The global economy consists of multiple national economies and each national economy consists of multiple regions. Each region produces one capital good and one consumer good (services). There is only one capital good in the global economy. The capital good can be used for capital accumulation and for consumption. Households own assets of the economy and distribute their incomes to consume and save. The production sectors use capital and labor as inputs. Exchanges take place in perfectly competitive markets. The production sectors sell their products to households or to other sectors and households supply their labor and assets to the production sectors. The factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. All the savings volunteered by households are absorbed by firms. We require saving and investment to be equal at any point in time.

The global economy consists of J countries, indexed by $j = 1, ..., J$. Country j consists of Q_j regions, indexed by $q = 1, ..., J$. We assume perfect competition in all the markets both internationally and nationally. The capital good is traded without any barriers. We neglect transport costs. The omission of transportation costs is for simplifying the analysis. We measure prices in terms of the commodity and the price of the commodity is unity. The assumption of zero transportation cost of the commodity implies price equality for the commodity in the global economy. We denote wage and interest rates by $w_{jq}(t)$ and $r_{jq}(t)$, respectively, of region q in country j. The interest rate is equalized throughout the global economy, *i. e.,* $r(t) = r_{jq}(t)$. Each country has a fixed and homogeneous population, denoted by N_j. This study neglects complicated issues related to international migration. There is no international migration but people in each country are free to choose their residential location. We assume that people work and reside in the same region. Each region has fixed homogenous land L_{jq} which is only for residential use. As amenity and land are immobile, the wage rates and land rents vary between regions. Let $\tilde{F}_{jq}(t)$ and $\hat{F}_{jq}(t)$ stand for the output levels of the capital good sector and service sector in region (j, q).

2.1. Behavior of the capital good sectors

We denote the two productive factors, capital by $\tilde{K}_{jq}(t)$ and labor by $\tilde{N}_{jq}(t)$, at each point in time t. The production functions are specified as

$$\tilde{F}_{jq}(t) = \tilde{A}_{jq} \tilde{K}_{jq}^{\tilde{\alpha}_{jq}}(t) \tilde{N}_{jq}^{\tilde{\beta}_{jq}}(t),\ \tilde{\alpha}_{jq}, \tilde{\beta}_{jq} > 0,\ \tilde{\alpha}_{jq} + \tilde{\beta}_{jq} = 1,\ j = 1, ..., J,\ q = 1, ..., Q_j. \quad (1)$$

The markets are competitive, thus labor and capital earn their marginal products. The sectors earn zero profits. The rate of interest and wage rates are determined by markets. The marginal conditions are given by

$$r(t)+\delta_{jq}=\frac{\tilde{\alpha}_{ji}\tilde{F}_{jq}(t)}{\tilde{K}_{jq}(t)},\ w_{jq}\left(t\right)=\frac{\tilde{\beta}_{jq}\tilde{F}_{jq}(t)}{\tilde{N}_{jq}(t)}, \tag{2}$$

where δ_{jq} is the depreciation rate of physical capital in region (*j*, *q*). It should be noted that our approach to firm location is oversimplified in the light of a rapidly increasing literature on identifying the factors that affect the location choice of firms (*e. g.*, Lee and Mansfield, 1996; Henisz, 2000; Busse and Hefeker, 2007; Almazan *et al.*, 2007; Johansson and Klaesson, 2011; De Beule and Duanmu, 2012; Colombo and Dawid, 2014). We may make our approach more comprehensive by taking account of other factors that affect location of production sectors.

2.2. Behavior of the service sectors

We denote the two productive factors, capital by $\hat{K}_{jq}(t)$ and labor by $\hat{N}_{jq}(t)$, at each point in time *t*. The production functions are specified as

$$\hat{F}_{jq}(t)=\hat{A}_{jq}\hat{K}_{jq}^{\hat{\alpha}_{jq}}\left(t\right)N_{jq}^{\hat{\beta}_{jq}}(t),\ \hat{\alpha}_{jq},\hat{\beta}_{jq}>0,\ \hat{\alpha}_{jq}+\hat{\beta}_{jq}=1,\ j=1,\ldots,J,\ q=1,\ldots.,Q_j. \tag{3}$$

We use $p_{jq}(t)$ to stand for region (*j*, *q*)'s service price. The marginal conditions are

$$r(t)+\delta_{jq}=\frac{\tilde{\alpha}_{ji}\tilde{F}_{jq}(t)}{\tilde{K}_{jq}(t)},\ w_{jq}(t)=\frac{\tilde{\beta}_{jq}\tilde{F}_{jq}(t)}{\tilde{N}_{jq}(t)}, \tag{4}$$

2.3. Behavior of consumers

In order to define incomes, it is necessary to determine land ownership structure. It can be seen that land properties may be distributed in multiple ways under various institutions. To simplify the model, we accept the assumption of absent landownership, which means that the income of land rent is spent outside the economic system. A possible reasoning for this is that as the land is owned by the government, people can rent the land in competitive market, and the government uses the income for military or other public purposes. Consumers make decisions on lot size, consumption levels of services and the commodity, as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993b, 2013). This approach makes it possible to solve many important (national) economic problems. Let $\bar{k}_{jq}(t)$ stand for the per household wealth in region (*j*, *q*). Consumer (*j*, *q*) obtains the current income $y_{jq}(t)$ from the interest payment and the wage payment. That is

$$y_{jq}(t)=r(t)\bar{k}_{jq}(t)+w_{jq}(t). \tag{5}$$

The total value of wealth that consumer (j, q) can sell to purchase goods and to save is equal to $\bar{k}_{jq}(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction barrier and cost. The disposable income $\hat{y}_{jq}(t)$ is the sum of the current come and the total value of wealth. That is

$$\hat{y}_{jq}(t) = y_{jq}(t) + \bar{k}_{jq}(t). \tag{6}$$

The disposable income is used for saving and consumption. It should be noted that the variable, $\bar{k}_{jq}(t)$, in the above equation is considered as a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we consider $\bar{k}_{jq}(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_{jq}(t)$ to distribute between consuming and saving. At each point in time, consumer (j, q) distributes the total available budget among the lot size, $l_{jq}(t)$ the saving, $s_{jq}(t)$ the consumption of the capital good, $c_{jq}(t)$ and consumption of services, $\hat{c}_{jq}(t)$. The budget constraint is given by

$$R_{jq}(t)\, l_{jq}(t) + c_{jq}(t) + p_{jq}(t)\, \hat{c}_{jq}(t) + s_{jq}(t) = \hat{y}_{jq}(t), \tag{7}$$

where $R_{jq}(t)$ is region (j, q)'s land rent. In our model, the consumer has four variables to decide. A consumer decides how much to consume housing (which is measured by lot size), capital goods and services, and how much to save. Equation (7) means that consumption and savings exhaust the consumers' disposable personal income.

We assume that the utility level $U_{jq}(t)$ that consumer (j, q) obtains is dependent on the consumption levels of the lot size, capital good, and services, and the saving. The utility level is represented by

$$U_{jq}(t) = \theta_{jq}(t)\, l_{jq}^{\eta_{j0}}(t)\, c_{jq}^{\xi_{j0}}\, \hat{c}_{jq}^{\gamma_{j0}}(t)\, s_{jq}^{\lambda_{j0}}(t), \;\; \eta_{j0}, \gamma_{j0}, \xi_{j0}, \lambda_{j0} > 0, \tag{8}$$

in which η_{j0}, ξ_{j0}, γ_{j0}, and λ_{j0} are the elasticities of utility of the consumer in country j with regard to the lot size, capital good, services, and savings in region (j, q). We call η_{j0}, ξ_{j0}, γ_{j0}, and λ_{j0} the propensities to consume lot size, capital good, and services, and to hold wealth (save), respectively. In (8), $\theta_{jq}(t)$ is called region (j, q)'s amenity level. Amenities are affected by infrastructures, regional cultures and climates (*e. g.*, Kanemoto, 1980; Diamond and Tolley, 1981; Blomquist, *et al.* 1988). Following Zhang (2016), we assume that amenity is affected by population. We specify θ_{jq} as follows

$$\eta_{j0}, \gamma_{j0}, \xi_{j0}, \lambda_{j0} > 0, \tag{9}$$

where $\bar{\theta}_{jq}(>0)$ and d_j are parameters and $N_{jq}(t)$ is region (j, q)'s population. We don't specify signs of d_j as the population may have either positive or negative effects on regional attractiveness. As Chen *et al.* (2013: 269) observe: «The presence of both positive and negative population externalities suggests that the steady state (or competitive) pattern may differ from an optimal pattern in which all the external benefits and costs of households' migration decisions are internalized». We will examine effects of changes in amenity parameters on not only steady state but also transitory process of the economic system.

Maximizing $U_{jq}(t)$ subject to the budget constraint yields

$$l_{jq}(t)R_{jq}(t)=\eta_j\hat{y}_{jq}(t),\ c_{jq}(t)=\xi_j\hat{y}_{jq}(t),\ p_{jq}(t)\hat{c}_{jq}(t)= \\ =\gamma_j\hat{y}_{jq}(t),\ s_{jq}(t)=\lambda_j\hat{y}_{jq}(t), \tag{10}$$

where

$$\eta_j\equiv\eta_{j0}\rho_j,\ \xi_j\equiv\xi_{j0}\rho_j,\ \gamma_j\equiv\gamma_{j0}\rho_j,\ \lambda_j\equiv\lambda_{j0}\rho_j,\ \rho_j\equiv\frac{1}{\eta_{j0}+\xi_{j0}+\gamma_{j0}+\lambda_{j0}}.$$

The saving behavior of the approach in this study is similar to these implied by the Keynesian consumption function and permanent income hypotheses, which are empirically more valid than the assumptions in the Solow model with a constant saving rate or the Ramsey model.

2.4. Wealth accumulation

According to the definition of $s_{jq}(t)$ the wealth accumulation of the representative person (j, q) is given by

$$\dot{\bar{k}}_{jq}(t)=s_{jq}(t)-\bar{k}_{jq}(t). \tag{11}$$

The equation simply implies that the change in wealth is the saving minus the missaving.

2.5. Equalization of utility levels between regions

As households are freely mobile between regions, the utility level of people in the same country should be equal, irrespective of in which region they live, *i. e.*

$$U_{jq}(t)=U_{jm}(t),\ q,m=1,\ldots,Q_j. \tag{12}$$

Possible costs for migration are omitted. Changing houses or moving to another region will cost. Taking account of such changes in the model makes it difficult to simulate the model. Wage equalization between regions is often used as the equilibrium mechanism of population mobility over space. This study assumes that households obtain the same level of utility in different regions as the equilibrium mechanism of population distribution between regions.

2.6. Capital and wealth

We use $K_{jq}(t)$ and $\bar{K}_{jq}(t)$ to denote the capital stocks employed by and the value of wealth owned by the population in region (j, q). We have

$$K_{jq}(t) = \tilde{K}_{jq}(t) + \hat{K}_{jq}(t), \; \bar{K}_{jq}(t) = \bar{k}_{jq}(t) N_{jq}(t). \tag{13}$$

We use $K_j(t)$ and $\bar{K}_j(t)$ to denote the capital stocks employed by and the value of wealth owned by country (j, q). We have

$$K_j(t) = \sum_q K_{jq}(t), \; \bar{K}_j(t) = \sum_q \bar{K}_{jq}(t). \tag{14}$$

The world capital stocks $K(t)$ employed by the production sectors is equal to the total wealth owned by the households of all the countries. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{K}_j(t). \tag{15}$$

2.7. Demand and supply for services

A region's supply of services is consumed by the region

$$\hat{c}_{jq}(t) N_j(t) = \hat{F}_{js}(t). \tag{16}$$

2.8. Full employment of the regional labor

The labor force is fully employed in each region

$$\tilde{N}_{jq}(t) + \hat{N}_{jq}(t) = N_j(t). \tag{17}$$

2.9. Full employment of the national labor force

The national labor force is fully used

$$\sum_{q=1}^{Q_j} N_{jq}(t) = N_j. \tag{18}$$

2.10. The regional land being fully used

Each region's land is fully occupied for residential use

$$l_{jq}(t) N_{jq}(t) = L_{jq}. \tag{19}$$

We thus built the model. The growth force is due to endogenous wealth accumulation with international and interregional capital and labor distribution in the global economy in which all the markets are perfectly competitive. The model is quite general in the sense that it includes, for instance, the Solow growth model, the Uzawa two-sector growth model, the Oniki-Uzawa trade model, Zhang's international growth model, and Zhang's interregional growth model as special cases.

3. Simulating the model

The dynamic system is complicated. For illustration, the rest of the study simulates the model. In the appendix, we show that the dynamics of the global economy can be expressed as $1+\sum_j Q_j$ differential equations (in which one is actually algebra equation as shown in the appendix). First, we introduce a variable $z_{11}(t)$

$$z_{11}(t) \equiv \frac{r(t)+\delta_{11}}{w_{11}(t)}.$$

3.1. Lemma

The motion of the global economy is given by the following $1+\sum_j Q_j$ differential equations with $z_{11}(t)$ and $\left(\bar{k}_j(t)\right)$ as variables

$$\begin{gathered}\dot{\bar{k}}_{jq}(t)=\Phi_{jq}\left(z_{11}(t), \bar{k}_{jq}(t)\right),\ j=1, \ldots, J,\ q=1, \ldots, Q_j, \\ \dot{z}_{11}(t)=\Phi_0\left(z_{11}(t), \left(\bar{k}_{jq}(t)\right)\right),\end{gathered} \tag{20}$$

where Φ_{jq} and Φ_0 are functions of $z_{11}(t)$ and $\left(\bar{k}_{jq}(t)\right)$ defined in the appendix. For any given positive values of $z_{11}(t)$ and $\left(\bar{k}_{jq}(t)\right)$ at any point in time, the other variables are uniquely determined by the following procedure: $r(t)$ by (A3) $\rightarrow w_{jq}(t)$ by (A4) $\rightarrow \hat{y}_{jq}(t)$ by (A9) $\rightarrow p_{jq}(t)$ by (A6) $\rightarrow N_{j1}(t)$ by (A12) $\rightarrow N_{jq}(t)$ by (A13) $\rightarrow R_{jq}(t)$ by (A13) $\rightarrow l_{jq}(t)=L_{jq}/N_{jq}(t) \rightarrow \hat{N}_{jq}(t)$ by (A14) $\rightarrow \tilde{N}_{jq}(t)$ by (A14) $\rightarrow \hat{K}_{jq}(t)$ and $\tilde{K}_{jq}(t)$ by (A1) $\rightarrow \tilde{F}_{jq}(t)$ by (1) $\rightarrow \hat{F}_{jq}(t)$ by (3) $\rightarrow c_{jq}(t)$, $\tilde{c}_{jq}(t)$, and $s_{jq}(t)$ by (10) $\rightarrow K_{jq}(t)$ by (10) $\rightarrow \bar{K}_{jq}(t)$ by (13) $\rightarrow K_j(t)$ by (14) $\rightarrow \bar{K}_j(t)$ by (14) $\rightarrow K(t)$ by (15) $\rightarrow$ $Y_{jq}(t)=\tilde{F}_{jq}(t)+p_{jq}(t)\hat{F}_{jq}(t) \rightarrow Y_j(t)=\sum_q Y_{jq}(t) \rightarrow Y(t)=\sum_j Y_j(t)$.

This result is important as it allows us to simulate the model with computer. Our dynamic equations are highly dimensional and nonlinear. Many studies identify strong nonlinearities in regional growth (see, *e. g.*, McMillen, 1996; Brueckner, 2000; Arbia and Paelinck, 2003; Goodman and Thibodeau, 2003; Kim and Bhattacharya, 2009; Bourassa *et al.*, 2010; Lee *et al.*, 2010; Azomahou *et al.*, 2011; Fotopoulos, 2012; Basile *et al.* 2014). Nevertheless, only a few regional dynamic models are based on microeconomic foundation. Our model is built on microeconomic foundation with international and regional characteristics. The lemma provides a computational procedure for following the motion of the economic system with any number of countries and any number of regions. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for a 3-country global economy, each country consisting of two regions. We specify the parameter values as follows

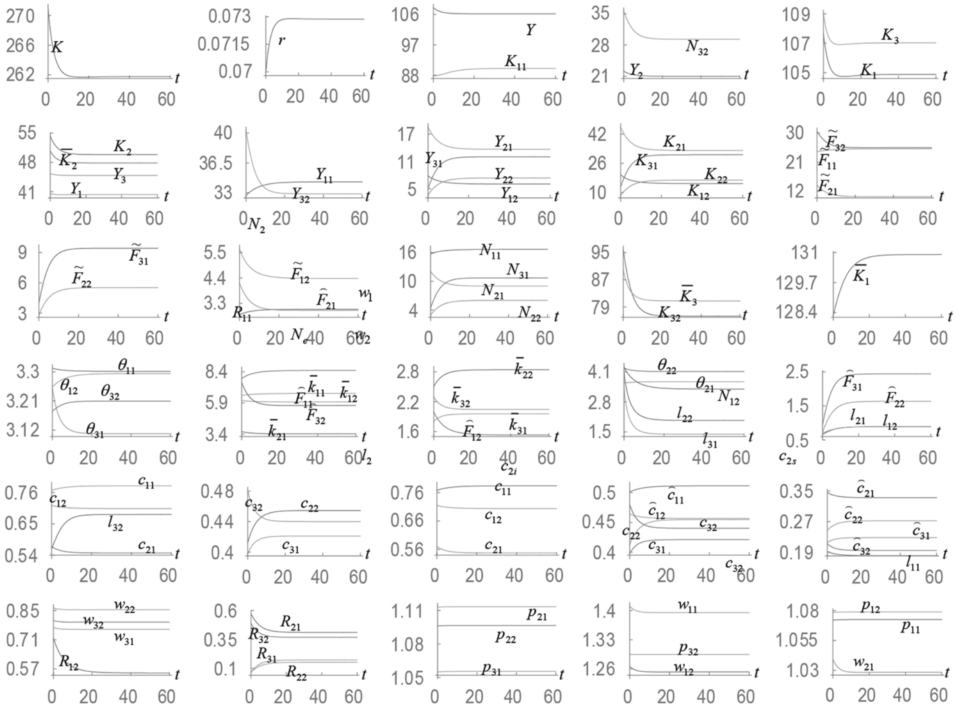
$$N_1=20,\ N_2=15,\ N_3=40,\ d_j=-0.05,\ \delta_{1q}=0.05,\ \delta_{2q}=0.06,\ \delta_{3q}=0.053,$$
$$\begin{pmatrix}\lambda_{10}\\ \lambda_{20}\\ \lambda_{30}\end{pmatrix}=\begin{pmatrix}0.85\\ 0.75\\ 0.65\end{pmatrix},\ \begin{pmatrix}\xi_{10}\\ \xi_{20}\\ \xi_{30}\end{pmatrix}=\begin{pmatrix}0.1\\ 0.12\\ 0.14\end{pmatrix},\ \begin{pmatrix}\eta_{10}\\ \eta_{20}\\ \eta_{30}\end{pmatrix}=\begin{pmatrix}0.07\\ 0.08\\ 0.08\end{pmatrix},\ \begin{pmatrix}\gamma_{10}\\ \gamma_{20}\\ \gamma_{30}\end{pmatrix}=\begin{pmatrix}0.07\\ 0.08\\ 0.08\end{pmatrix},$$
$$\begin{pmatrix}L_{11}\\ L_{12}\\ L_{21}\\ L_{22}\\ L_{31}\\ L_{32}\end{pmatrix}=\begin{pmatrix}3\\ 3\\ 8\\ 12\\ 15\\ 20\end{pmatrix},\ \begin{pmatrix}\tilde{A}_{11}\\ \tilde{A}_{12}\\ \tilde{A}_{21}\\ \tilde{A}_{22}\\ \tilde{A}_{31}\\ \tilde{A}_{32}\end{pmatrix}=\begin{pmatrix}1.2\\ 1.15\\ 1\\ 0.9\\ 0.8\\ 0.85\end{pmatrix},\ \begin{pmatrix}\hat{A}_{11}\\ \hat{A}_{12}\\ \hat{A}_{21}\\ \hat{A}_{22}\\ \hat{A}_{31}\\ \hat{A}_{32}\end{pmatrix}=\begin{pmatrix}1.1\\ 1.05\\ 0.9\\ 0.9\\ 0.75\\ 0.65\end{pmatrix},\ \begin{pmatrix}\hat{\alpha}_{11}\\ \hat{\alpha}_{12}\\ \hat{\alpha}_{21}\\ \hat{\alpha}_{22}\\ \hat{\alpha}_{31}\\ \hat{\alpha}_{32}\end{pmatrix}=\begin{pmatrix}0.32\\ 0.3\\ 0.32\\ 0.3\\ 0.32\\ 0.29\end{pmatrix},\ \begin{pmatrix}\tilde{\alpha}_{11}\\ \tilde{\alpha}_{12}\\ \tilde{\alpha}_{21}\\ \tilde{\alpha}_{22}\\ \tilde{\alpha}_{31}\\ \tilde{\alpha}_{32}\end{pmatrix}=\begin{pmatrix}0.33\\ 0.31\\ 0.33\\ 0.31\\ 0.33\\ 0.3\end{pmatrix},\ \begin{pmatrix}\theta_{11}\\ \theta_{12}\\ \theta_{21}\\ \theta_{22}\\ \theta_{31}\\ \theta_{32}\end{pmatrix}=\begin{pmatrix}3.8\\ 3.5\\ 4.1\\ 4.5\\ 3.5\\ 3.8\end{pmatrix},\tag{21}$$

Region (1, 1)'s levels of productivity of the two sectors are highest. We specify values of $\tilde{\alpha}_{jq}$ and $\hat{\alpha}_{jq}$ close to 0.3. With regard to the technological parameters, for illustration what are important in our international and interregional study are their relative values. The presumed productivity differences between the regions are not very large. It can be seen that the specified values of the land sizes, the preference parameters and the population will not affect our main concerns about interactions between the countries and regions. We specify the initial conditions as follows

$$z_1(0)=0.085,\ \bar{k}_{11}(0)=6.5,\ \bar{k}_{12}(0)=6.1,\ \bar{k}_{21}(0)=$$
$$=3.6,\ \bar{k}_{22}(0)=2.5,\ \bar{k}_{31}(0)=1.8,\ \bar{k}_{32}(0)=2.3.$$

The motion of the variables is plotted in Figure 1. The global output and wealth fall over time till they arrive at the equilibrium levels. Country 1's GDP changes scarcely. The falling in the global GDP is mainly caused by the fallings in country 2's and 3's GDPs. The rate of interest rises in association with falling wealth. The GDPs of regions (1, 1), (2, 2) and (3, 1) rise. The GDPs of regions (1, 2), (2, 1) and (3, 2) fall. The amenities in regions (1, 2), (2, 1) and (3, 2) rise. The amenities in regions (1, 1), (2, 2) and (3, 1) fall. In country 1, some people move from region (1, 2) to region (1, 1) in country 2, some people move from region (2, 1) to region (2, 2) and in country 3, some people move from region (3, 2) to region (3, 1). Hence, the lot sizes in regions (1, 1), (2, 2) and (3, 1) fall, and the lot sizes in regions (1, 2), (2, 1) and (3, 2) rise. Correspondingly the land rents in regions (1, 1), (2, 2) and (3, 1) fall, and the land rents in regions (1, 2), (2, 1) and (3, 2) rise. The wage rates and prices are scarcely changed. Figure 1 provides the motion of the other variables in the global economy. We see that there is no convergence in income and wealth between nations or between regions. The income and wealth gaps between nations or between regions are either enlarged or reduced. The persistent divergence is due to the differences in the amenity parameters, technology and resources.

Figure 1. The Motion of the Economic System



It is straightforward to confirm that all the variables become stationary in the long term. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (22)

$$Y = 105.9,\ K = 259.6,\ r = 0.074,\ Y_1 = 40.2,\ Y_2 = 20.95,\ Y_3 = 44.76,$$

$$\begin{pmatrix} K_1 \\ K_2 \\ K_3 \\ \bar{K}_1 \\ \bar{K}_2 \\ \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 104 \\ 49.5 \\ 106.2 \\ 131 \\ 47.8 \\ 80.7 \end{pmatrix}, \begin{pmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \\ Y_{31} \\ Y_{32} \end{pmatrix} = \begin{pmatrix} 34.2 \\ 6.06 \\ 13.6 \\ 7.34 \\ 11.9 \\ 32.8 \end{pmatrix}, \begin{pmatrix} N_{11} \\ N_{12} \\ N_{21} \\ N_{22} \\ N_{31} \\ N_{32} \end{pmatrix} = \begin{pmatrix} 16.64 \\ 3.36 \\ 8.99 \\ 6.01 \\ 10.68 \\ 29.32 \end{pmatrix}, \begin{pmatrix} K_{11} \\ K_{12} \\ K_{21} \\ K_{22} \\ K_{31} \\ K_{32} \end{pmatrix} = \begin{pmatrix} 89.22 \\ 14.76 \\ 32.84 \\ 16.62 \\ 30.39 \\ 75.76 \end{pmatrix}, \begin{pmatrix} \tilde{F}_{11} \\ \tilde{F}_{12} \\ \tilde{F}_{21} \\ \tilde{F}_{22} \\ \tilde{F}_{31} \\ \tilde{F}_{32} \end{pmatrix} = \begin{pmatrix} 25.04 \\ 4.37 \\ 10.33 \\ 5.52 \\ 9.37 \\ 25.45 \end{pmatrix},$$

$$\begin{pmatrix} \hat{F}_{11} \\ \hat{F}_{12} \\ \hat{F}_{21} \\ \hat{F}_{22} \\ \hat{F}_{31} \\ \hat{F}_{32} \end{pmatrix} = \begin{pmatrix} 8.51 \\ 1.54 \\ 2.99 \\ 1.64 \\ 2.43 \\ 5.69 \end{pmatrix}, \begin{pmatrix} \tilde{N}_{11} \\ \tilde{N}_{12} \\ \tilde{N}_{21} \\ \tilde{N}_{22} \\ \tilde{N}_{31} \\ \tilde{N}_{32} \end{pmatrix} = \begin{pmatrix} 12.24 \\ 2.45 \\ 6.85 \\ 4.54 \\ 8.41 \\ 22.81 \end{pmatrix}, \begin{pmatrix} \hat{N}_{11} \\ \hat{N}_{12} \\ \hat{N}_{21} \\ \hat{N}_{22} \\ \hat{N}_{31} \\ \hat{N}_{32} \end{pmatrix} = \begin{pmatrix} 4.4 \\ 0.92 \\ 2.14 \\ 1.48 \\ 2.27 \\ 6.51 \end{pmatrix}, \begin{pmatrix} \tilde{K}_{11} \\ \tilde{K}_{12} \\ \tilde{K}_{21} \\ \tilde{K}_{22} \\ \tilde{K}_{31} \\ \tilde{K}_{32} \end{pmatrix} = \begin{pmatrix} 64.84 \\ 10.1 \\ 24.73 \\ 12.39 \\ 23.69 \\ 58.3 \end{pmatrix}, \begin{pmatrix} \hat{K}_{11} \\ \hat{K}_{12} \\ \hat{K}_{21} \\ \hat{K}_{22} \\ \hat{K}_{31} \\ \hat{K}_{32} \end{pmatrix} = \begin{pmatrix} 24.38 \\ 4.16 \\ 8.1 \\ 4.23 \\ 6.69 \\ 17.46 \end{pmatrix},$$

$$\begin{pmatrix} p_{11} \\ p_{12} \\ p_{21} \\ p_{22} \\ p_{31} \\ p_{32} \end{pmatrix} = \begin{pmatrix} 1.07 \\ 1.08 \\ 1.1 \\ 1.11 \\ 1.06 \\ 1.3 \end{pmatrix}, \begin{pmatrix} w_{11} \\ w_{12} \\ w_{21} \\ w_{22} \\ w_{31} \\ w_{32} \end{pmatrix} = \begin{pmatrix} 1.39 \\ 1.25 \\ 1.03 \\ 0.85 \\ 0.76 \\ 0.79 \end{pmatrix}, \begin{pmatrix} R_{11} \\ R_{12} \\ R_{21} \\ R_{22} \\ R_{31} \\ R_{32} \end{pmatrix} = \begin{pmatrix} 3.04 \\ 0.55 \\ 0.41 \\ 0.15 \\ 0.17 \\ 0.37 \end{pmatrix}, \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{21} \\ \theta_{22} \\ \theta_{31} \\ \theta_{32} \end{pmatrix} = \begin{pmatrix} 3.3 \\ 3.29 \\ 3.67 \\ 4.11 \\ 3.11 \\ 3.21 \end{pmatrix}, \begin{pmatrix} \bar{k}_{11} \\ \bar{k}_{12} \\ \bar{k}_{21} \\ \bar{k}_{22} \\ \bar{k}_{31} \\ \bar{k}_{32} \end{pmatrix} = \begin{pmatrix} 6.66 \\ 5.99 \\ 3.42 \\ 2.84 \\ 1.95 \\ 2.04 \end{pmatrix},$$

$$\begin{pmatrix} l_{11} \\ l_{12} \\ l_{21} \\ l_{22} \\ l_{31} \\ l_{32} \end{pmatrix} = \begin{pmatrix} 0.18 \\ 0.89 \\ 0.89 \\ 2 \\ 1.4 \\ 0.68 \end{pmatrix}, \begin{pmatrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \\ c_{31} \\ c_{32} \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.7 \\ 0.55 \\ 0.46 \\ 0.42 \\ 0.44 \end{pmatrix}, \begin{pmatrix} \hat{c}_{11} \\ \hat{c}_{12} \\ \hat{c}_{21} \\ \hat{c}_{22} \\ \hat{c}_{31} \\ \hat{c}_{32} \end{pmatrix} = \begin{pmatrix} 0.51 \\ 0.46 \\ 0.33 \\ 0.27 \\ 0.23 \\ 0.19 \end{pmatrix}. \tag{22}$$

It should be noted that we also provide the labor and capital distributions between different sectors, while the dynamics of these variables are not provided in Figure 1 for convenience of representation. From (22) we see that in the long term country 3 has highest GDP among the countries and region (1, 1) has highest GDP among the regions. The wage rate, consumption levels of capital good and services in wealth in region (1, 1) are highest among the regions. The amenity and lot size of region (2, 2) are highest among the regions. The region's lot size and price of services are lowest and the land rent is highest. It is straightforward to calculate the six eigenvalues of the differential equations evaluated at the equilibrium point as follows

$$\{-0.218,\ -0.216,\ -0.170,\ -0.163,\ -0.003,\ -1.04\times 10^{-17}\}.$$

The six eigenvalues are real and negative. The equilibrium is locally stable. This result is important as it guarantees the validity of comparative dynamic analysis in the next section.

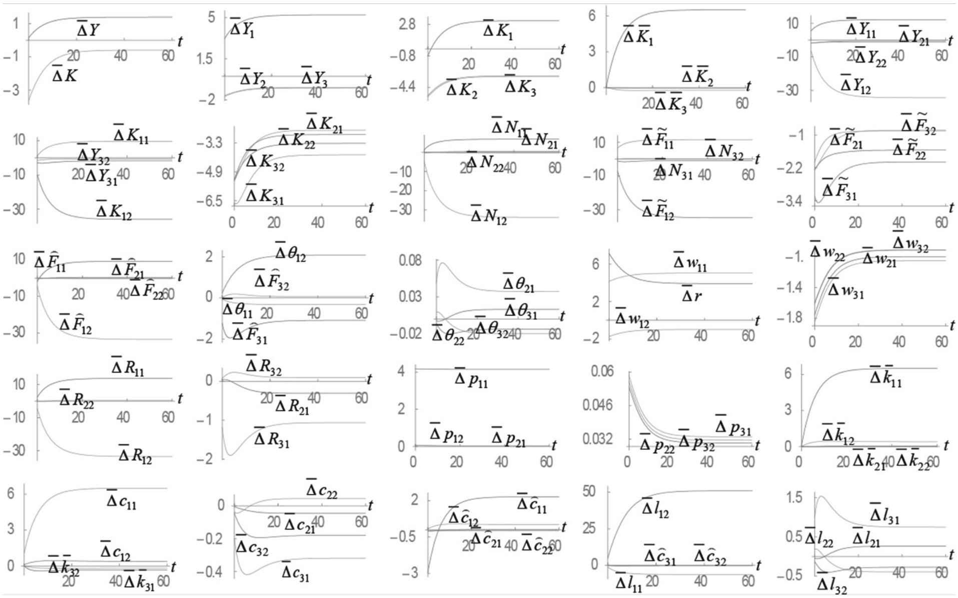
4. Comparative dynamic analysis

We simulated the motion of the global economy under (21). We now study how the economic system react to changes, for instance, in different countries' or region's preferences, amenity parameters, resources, and technologies. As the lemma gives a computational procedure to calibrate the motion of all the variables, it is straightforward to conduct comparative dynamic analysis. In the rest of this study we use $\bar{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$ in percentage due to changes in a parameter value.

4.1. The total factor productivity of region (1, 1)'s capital good sector being enhanced

We first study the effects of a technological improvement in region (1, 1)'s capital good sector. The technological progress is specified as follows: $\tilde{A}_{11}: 1.2 \Rightarrow 1.25$. The simulation results are plotted in Figure 2. A rise in the technology raises the global GDP but reduces the global physical wealth. As the region raises its technology, both the region's wage rate and global rate of interest are augmented. As the costs of capital inputs are globally enhanced, country 2's and country 3's GDPs are reduced in association with rising GDP in country 1. Country 1 employs more capital and accumulates more wealth, and the other two countries employ less capital and accumulate less wealth. Region (1, 1)'s GDP is enhanced, region (1, 2)'s GDP is lowered. The other regions' GDPs are slightly reduced. Some people migrate from region (1, 2) to region (1, 1) improving region (1, 2)'s amenity and deteriorating region (1, 1)'s amenity. Some people migrate from region (2, 1) to region (2, 2) and from region (3, 1) to region (3, 2). The service price in region (1, 1) is increased, and service prices in the other regions are slightly affected. The wealth levels per household in country 1 are increased, while the wealth levels per households in all the other regions are reduced. The lot sizes in regions (1, 1), (2, 2) and (3, 2) are reduced and the lot sizes in the other regions are increased. Correspondingly the land rents in regions (1, 1), (2, 2) and (3, 2) are enhanced and the land rents in the other regions are lowered. The technological change in region (1, 1) also causes economic structural changes in all the regions. Region (1, 1)'s wage rate is increased, while the wage rates in all the other regions are reduced. We see that wage disparities are caused by many factors, such as spatial differences in technology and amenity (see also, Glaeser and Maré, 2001; Duranton and Monastiriotis, 2002; Combes *et al.,* 2003; Rey and Janikas, 2005). From our simulation result, we see that the wage disparity is strongly affected by changes in technology. This also hints that if technological differences between regions are not large, the wage rates may tend to converge if the other factors weakly affect the differences. There are different studies on regional economic growth with endogenous knowledge (Florida *et al.*, 2008; Brunow and Hirte, 2009; Banerjee and Jarmuzek, 2010; Fleisher, *et al.*, 2010; Batabyal and Nijkamp, 2013). Although our study does not include endogenous technological changes, the literature of regional economic growth and knowledge should enable us to further generalize our modeling.

Figure 2. A Rise in the Productivity of Capital Good Sector (1, 1)



4.2. The total factor productivity of region (2, 1)'s service sector being enhanced

We now examine the effects of the following technological improvement in region (2, 1)'s service sector: $\hat{A}_{21}: 0.9 \Rightarrow 0.95$. The simulation results are plotted in Figure 3. The global GDP and wealth are augmented slightly. Country 2's GDP is increased and the other two countries' GDPs are scarcely affected. From Figure 3 we see that the productivity change in the country's service sector has little impact on the other countries' economic structures and growth. The capital stocks used by and wealth owned by country 2 are augmented, and the capital stocks used by and wealth owned by the other two countries are slightly reduced. Region (2, 1)'s GDP is increased and region (2, 2)'s GDP is lowered. Some of region (2, 2)'s labor force migrate to region (2, 1). The lot sizes and land rents in the two region are changed correspondingly. The amenity of region (2, 2) is improved and the amenity of region (2, 2) is deteriorated. The output levels of the two sectors in region (2, 1) are increased and the output levels of the two sectors in region (2, 2) are lowered. The price of services region (2, 1) falls and the per household consumption level of services in the region rises. The changes in the other variables are slight.

4.3. A rise in country 1's propensity to save

It is well known that the Keynesian economic theory predicts that a higher saving rate tends to reduce national income, while the neoclassical growth theory suggests

Figure 3. A Rise in the Productivity of the Service Sector in Region (2, 1)

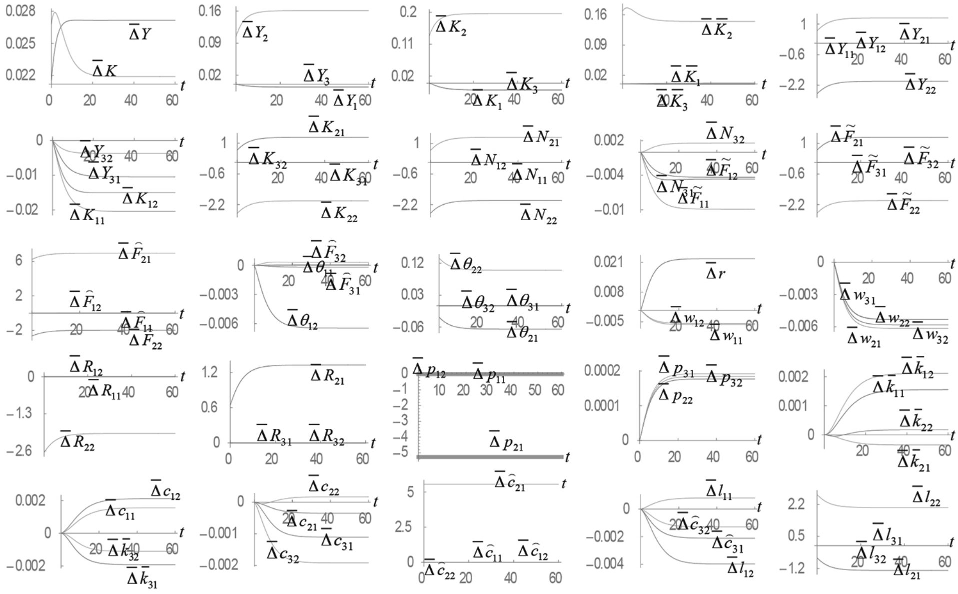
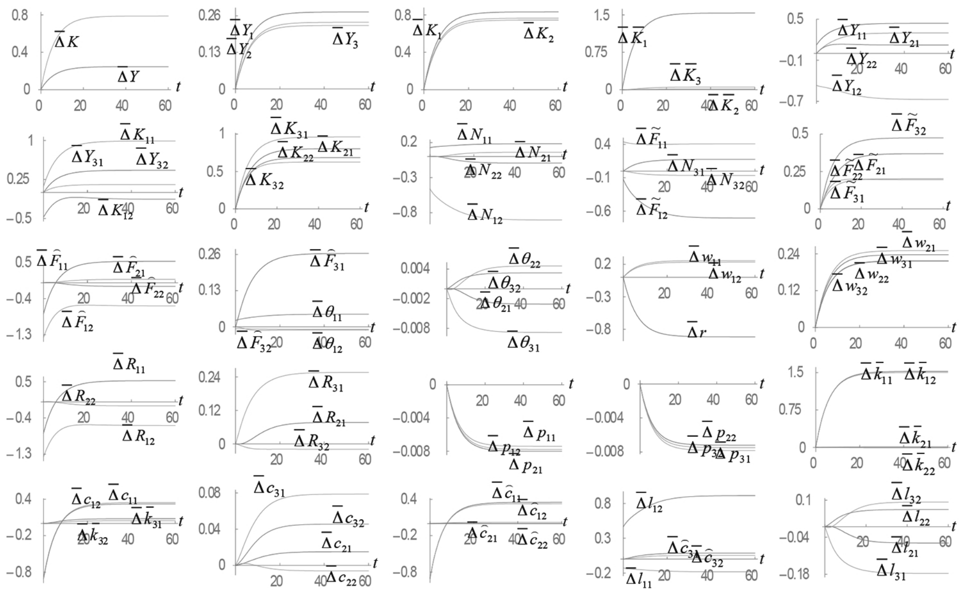


Figure 4. A Rise in Country 1's Propensity to Save



the opposite effect. Both trade theory and regional growth theory have not much to say on how a change in the propensity to save can affect global and regional economic growth and spatial agglomeration. As it describes interdependence among growth, economic structural change, and spatial agglomeration both for international and interregional economies within a comprehensive framework, our model can provide effects of a change in any country's propensity to save upon different aspects of the global economy. We now allow country 1's propensity to save to be changed as follows: $\lambda_{10}: 0.86 \Rightarrow 0.87$. The simulation results are plotted in Figure 4. The global output and wealth are increased. The rate of interest is lowered. The wage rates in all the regions are enhanced. Each region employs more capital inputs. All the national economies employ more capital stocks. Country 1's national wealth is enhanced, and the other two economies' national wealth levels are scarcely affected. The per household wealth in country 1 is increased and the per household wealth levels in the other two countries are slightly affected. In country 1 some people migrate from region (1, 2) to region (1, 1) in country 2 some people migrate from region (2, 2) to region (2, 1) and in country 3 some people migrate from region (3, 2) to region (3, 1). The lot sizes and land rents are changed correspondingly. It should be noted that in the long term the consumption levels of capital good are increased in all the regions, except in region (2, 2) where the consumption level is slightly reduced.

4.4. A rise in region (3, 2)'s amenity parameter

We now analyze the effects of the following rise in region (3, 2)'s amenity parameter: $\bar{\theta}_2: 3.8 \Rightarrow 4$. The simulation results are plotted in Figure 5. The change in the parameter has slight effects on the global GDP and reduces the global capital stocks. Region (3, 2) attracts more people and region (3, 1)'s population falls. The lot size falls in region (3, 2) and rises in region (3, 1). The amenity is improved in region (3, 2) and is deteriorated in region (3, 1). The land rent rises in region (3, 2) and falls in region (3, 1). Region (3, 2)'s GDP and output levels of the two sectors rise and region (3, 1)'s GDP and output levels of the two sectors fall. The other variables in the global economy are slightly affected. From Figure 5 we see that a change in the amenity parameter has a strong impact on the national economic structure and spatial agglomeration. It should be noted that our approach on regional housing markets can be related to hedonic price modelling (*e. g.*, Rosen, 1974; Helbich *et al.*, 2014). The approach is based in Lancaster's idea that it is a good's characteristics that creates utility. When we apply this idea to housing markets which are tied with environment and land, it implies that environment should have effects on housing prices (Dubin, 1992; Can and Megbolugbe, 1997; Sheppard, 1997; Malpezzi, 2003; McMillen, 2010; Ahlfeldt, 2011). Our model shows how the rent levels are closely related to different regional characteristics.

4.5. A rise in country 3's propensity to consume housing

We now study the effects of the following rise in country 3's propensity to consume housing: $\eta_{30}: 0.08 \Rightarrow 0.1$. The simulation results are plotted in Figure 6. As country 3

Figure 5. A Rise in a Region's Amenity Parameter

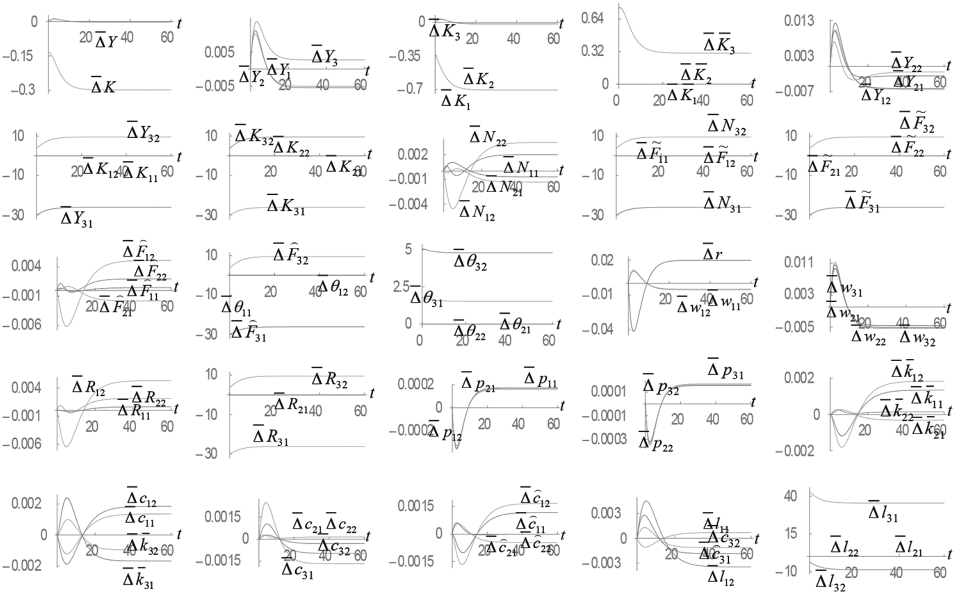
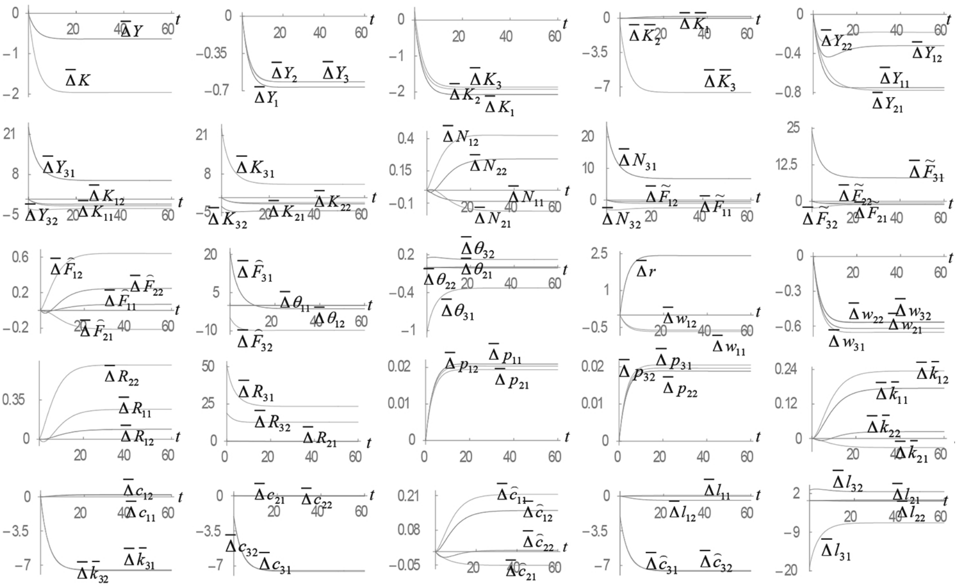


Figure 6. A Rise in Country 3's Propensity to Consume Housing

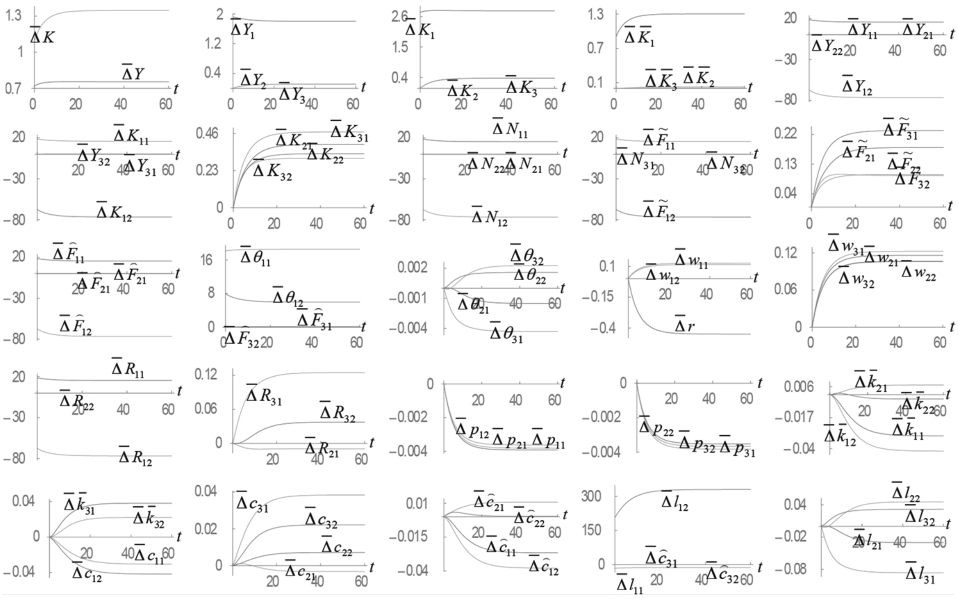


raises its propensity to consume housing, the global GDP, the GDP of any of the three countries, and global capital stocks are reduced. The wage rates of all the regions are reduced in tandem with rising in the rate of interest. All the national economies employ less capital stocks. Country 3 accumulates less wealth. The other two countries' national wealth levels are slightly affected. Region (3, 1)'s GDP is augmented and all the other regions' GDPs are reduced. The land rents in country 3 are increased. The lot size in region (3, 1) falls and the lot size in region (3, 2) rises. The change in the preference for housing affects the other variables as well as shown in Figure 6.

4.6. Country 1's population having a positive impact on the amenity

We assume that the population has negative impact on amenity. We now study what will happen in the global economy if country 1's population impact on amenity is changed as follows: $d_1: -0.05 \Rightarrow 0.01$. The simulation results are plotted in Figure 7. The change has a strong impact on country 1's spatial agglomeration. About 80 percent of region (1, 2)'s population migrate to region (1, 1) causing the population of region (1, 1) to increase about 20 percent. As more of country's labor force is reallocated to the most productive region, the national output is increased. The global GDP and global capital are increased. The rate of interest falls in tandem with rising global capital. All the region's wage rates are enhanced and the prices of services fall in all the regions. All the national economies employ more capital stocks and own more wealth. All the countries' GDPs are augmented. As region (1, 1) attracts more people, its amenity is enhanced and region (1, 1)'s amenity is deteriorated. The labor distributions and amenities in the other two countries are scarcely affected. It should

Figure 7. Country 1's Population Having a Positive Impact on Amenity



be noted that the per household wealth in regions (1, 1) and (1, 2) is merely affected, implying that the national wealth increase in caused by the interregional reallocation of the population.

5. Conclusions

This paper extended Uzawa's two-sector growth model to a global economy with any number of countries and each country with any number of regions. The model deals with dynamic interactions among international trade, national and global growth, interregional migration, wealth accumulation and regional amenities. The model is based on There is no international migration and interregional migration is free. The economy is built under assumptions of profit maximization, utility maximization, and perfect competition. We used the utility function proposed by Zhang (1993b) to determine the saving and consumption. The dynamics of the global economy is expressed with the $1+\sum_j Q_j$ differential equations. We simulated the model of the global economy with three countries and each country with two regions. We demonstrated the existence of equilibrium point and confirmed (local) stability of the equilibrium point. We also conducted comparative dynamic analysis with regard to the total factor productivity of regions' capital good sectors, the total factor productivities of the service sectors, the propensities to save, the amenity parameters, the propensities to consume housing, and the population effects on amenity. Our comparative analysis provides some important insights. Our model deals with complicated issues on the basis of some well-established economic theories. It is possible to further develop our model basing on traditional theories. It is straightforward to analyze behavior of the model with other forms of production or utility functions. Moreover, households in each country should be heterogeneous. Also issues related to tax competition between countries or regions have caused great attention in economic geography. The current model is deterministic. Random factors are important in making the morel more relevant for explaining reality.

Appendix: Proving the lemma

We now show a procedure to determine the dynamics of the system in differential equations. First, from equations (2) and (4) we obtain

$$z_{jq} \equiv \frac{r+\delta_{jq}}{w_{jq}} = \frac{a_{jq}\tilde{N}_{jq}}{\tilde{K}_{jq}} = \frac{b_{jq}\hat{N}_{jq}}{\hat{K}_{jq}}, \tag{A1}$$

where

$$a_{jq} \equiv \frac{\tilde{\alpha}_{jq}}{\tilde{\beta}_{jq}}, \; b_{jq} \equiv \frac{\hat{\alpha}_{js}}{\hat{\beta}_{js}}.$$

Insert $z_{jq}/a_{jq} \equiv \tilde{N}_{jq}/\tilde{K}_{jq}$ *in* $r+\delta_{jq}=\tilde{\alpha}_{jq}F_{jq}/K_{jq}$ from (2)

$$r\left(z_{jq}\right)=\frac{\tilde{\alpha}_{jq}\tilde{A}_{jq}}{a_{jq}^{\tilde{\beta}_{jq}}}z_{jq}^{\tilde{\beta}_{jq}}-\delta_{jq}. \tag{A2}$$

Especial we have

$$r\left(z_{11}\right)=\frac{\tilde{\alpha}_{11}\tilde{A}_{11}}{a_{11}^{\tilde{\beta}_{11}}}z_{11}^{\tilde{\beta}_{11}}-\delta_{11}. \tag{A3}$$

We can thus treat the rate of interest as a unique function of z_{11}. From (A2) we get

$$z_{jq}\left(z_{11}\right)=a_{jq}\left(\frac{r+\delta_{jq}}{\tilde{\alpha}_{jq}\tilde{A}_{ji}}\right)^{1/\tilde{\beta}_{ji}}. \tag{A4}$$

Hence we can treat all z_{jq} as a unique function of z_{11}. From (A1) and (A2), we have

$$w_{jq}\left(z_{11}\right)=\frac{r+\delta_{jq}}{z_{jq}}. \tag{A5}$$

From $z_{jq}=b_{jq}\hat{N}_{jq}/\hat{K}_{jq}$ and (1), we have

$$p_{jq}\left(z_{11}\right)=\frac{b_{jq}^{\hat{\beta}_{jq}}\left(r+\delta_{jq}\right)}{\hat{\alpha}_{jq}\hat{A}_{jq}z_{jq}^{\hat{\beta}_{jq}}}. \tag{A6}$$

From (1) and $p_{jq}\hat{c}_{jq}=\gamma_j\hat{y}_{jq}$ we have

$$\gamma_j\hat{y}_{jq}N_{jq}=p_{jq}\hat{F}_{jq}. \tag{A7}$$

Insert (4) in (A6)

$$\gamma_j\hat{y}_{jq}N_{jq}=\frac{w_{jq}\hat{N}_{jq}}{\hat{\beta}_{jq}}. \tag{A8}$$

By (3) we have

$$\hat{y}_{jq}\left(z_{11},\bar{k}_{jq}\right)=\left(1+r\right)\bar{k}_{jq}+w_{jq}. \tag{A9}$$

Substitute $l_{jq}=L_{jq}/N_{jq}$, $\theta_j=\bar{\theta}_jN_j^{d_j}$, and (7) into (6)

$$U_{jq}=\frac{\bar{\theta}_{jq}N_{jq}^{d-\eta_{j0}}L_{jq}^{\eta_{j0}}}{p_{jq}^{\gamma_{j0}}}\xi_j^{\xi_{j0}}\gamma_j^{\gamma_{j0}}\lambda_j^{\lambda_{j0}}\hat{y}_{jq}^{\xi_{j0}+\gamma_{j0}+\lambda_{j0}}. \tag{A10}$$

Apply $U_{jq} = U_{jm}$ to (A10)

$$N_{jq} = \Lambda_{jq} N_{j1}, \; q = 1, ..., Q_j, \tag{A11}$$

where

$$\Lambda_{jq}\left(z_{11}, \left(\bar{k}_{jq}\right)_j\right) \equiv \left(\frac{\bar{\theta}_{j1} L_{j1}^{\eta_{j0}} p_{jq}^{\gamma_{j0}}}{\bar{\theta}_{jq} L_{jq}^{\eta_{j0}} p_1^{\gamma_{j0}}}\right)^{\frac{1}{(d_j - \eta_{j0})}} \left(\frac{\hat{y}_{j1}}{\hat{y}_{jq}}\right)^{\frac{\xi_{j0} + \gamma_{j0} + \lambda_{j0}}{d_j - \eta_{j0}}},$$

where $\left(\bar{k}_{jq}\right)_j \equiv \left(\bar{k}_{j1}, ..., \bar{k}_{jQ_j}\right)$. Insert (A11) in (18)

$$N_{j1}\left(z_{11}, \left(\bar{k}_{jq}\right)_j\right) = \frac{N_j}{\sum_{q=1}^{Q_j} \Lambda_{jq}}, \; \Lambda_{j1} = 1. \tag{A12}$$

With (A11) and (A12) we determine the population distribution within country j as functions of z_{11} and $\left(\bar{k}_{jq}\right)_j$. By $l_{jq} R_{jq} = \eta \hat{y}_{jq}$ and $l_{jq} N_{jq} = L_{jq}$, we have

$$R_{jq}\left(z_{11}, \left(\bar{k}_{jq}\right)_j\right) = \frac{\eta_j \hat{y}_{jq} N_{jq}}{L_{jq}}. \tag{A13}$$

Insert (A8) in (A9)

$$\hat{N}_{jq}\left(z_{11}, \left(\bar{k}_q\right)_j\right) = \left(\frac{\left(1 + r\right)\bar{k}_{jq}}{w_{jq}} + 1\right) \gamma_j \hat{\beta}_{jq} N_{jq}. \tag{A14}$$

From $\tilde{N}_{jq} + \hat{N}_{jq} = N_{jq}$ and (A11), we have

$$\tilde{N}_{jq}\left(z_{11}, \left(\bar{k}_{jq}\right)_j\right) = N_{jq} - \hat{N}_{jq}. \tag{A15}$$

From equation (13)-(15), we have

$$\sum_{j=1}^{J} \sum_{q=1}^{Q_j} \left(\tilde{K}_{jq} + \hat{K}_{jq}\right) = \sum_{j=1}^{J} \sum_{q=1}^{Q_j} \bar{k}_{jq} N_{jq}. \tag{A16}$$

Insert (A1) in (A16)

$$\sum_{j=1}^{J} \sum_{q=1}^{Q_j} \left(\frac{a_{jq} \tilde{N}_{jq} + b_{jq} \hat{N}_{jq}}{z_{jq}}\right) = \sum_{j=1}^{J} \sum_{q=1}^{Q_j} \bar{k}_{jq} N_{jq}. \tag{A17}$$

Insert (A15) in (A17)

$$\sum_{j=1}^{J}\sum_{q=1}^{Q_j}\left(\frac{\left(b_{jq}-a_{jq}\right)\hat{N}_{jq}}{z_{jq}}\right)=\sum_{j=1}^{J}\sum_{q=1}^{Q_j}\left(\bar{k}_{jq}-\frac{a_{jq}}{z_{jq}}\right)N_{jq}. \tag{A18}$$

Insert (A14) in (A18)

$$\sum_{j=1}^{J}\sum_{q=1}^{Q_j}\left(\frac{\left(1+r\right)\bar{k}_{jq}}{w_{jq}}+1\right)\frac{\bar{a}_{jq}N_{jq}}{z_{jq}}=\sum_{j=1}^{J}\sum_{q=1}^{Q_j}\left(\bar{k}_{jq}-\frac{a_{jq}}{z_{jq}}\right)N_{jq}. \tag{A19}$$

where

$$\bar{a}_{jq}\equiv\left(b_{jq}-a_{jq}\right)\gamma_j\hat{\beta}_{jq}.$$

Insert (A10) in (A19)

$$\Phi\left(z_{11},\left(\bar{k}_{jq}\right)\right)\equiv\sum_{j=1}^{J}N_{j1}\sum_{q=1}^{Q_j}\left[\left(\frac{\left(1+r\right)\bar{k}_{jq}}{w_{jq}}+1\right)\frac{\bar{a}_{jq}}{z_{jq}}-\left(\bar{k}_{jq}-\frac{a_{jq}}{z_{jq}}\right)\right]\Lambda_{jq}=0. \tag{A20}$$

Substitute $s_{jq}=\lambda_j\hat{y}_{jq}$ and $\hat{y}_{jq}=\left(1+r\right)\bar{k}_{jq}+w_{jq}$ into equations (9)

$$\dot{\bar{k}}_{jq}=\Phi_{jq}\left(z_{1q},\bar{k}_{jq}\right)\equiv\left(1+r\right)\lambda_j\bar{k}_{jq}+\lambda_j w_{jq}-\bar{k}_{jq}. \tag{A21}$$

Taking derivates of equation (A20) with respect to *t* yields.

$$\dot{z}_{11}=-\left(\sum_{j=1}^{J}\sum_{q=1}^{Q_j}\frac{\partial\Phi}{\partial\bar{k}_{jq}}\dot{\bar{k}}_{jq}\right)\left(\frac{\partial\Phi}{\partial z_{11}}\right)^{-1}. \tag{A22}$$

Insert (A18) in (A19)

$$\dot{z}_{11}=\Phi_0\left(z_{1q},\bar{k}_{jq}\right)\equiv-\left(\sum_{j=1}^{J}\sum_{q=1}^{Q_j}\Phi_{jq}\frac{\partial\Phi}{\partial\bar{k}_{jq}}\right)\left(\frac{\partial\Phi}{\partial z_{11}}\right)^{-1}. \tag{A23}$$

Following the procedure in the lemma we describe the dynamics of the global economic system.

References

Ahlfeldt, G. (2011): «If Alonso Was Right: Modeling Accessibility and Explaining the Residential Land Gradient», *Journal of Regional Science,* 51, 318-338.

Almazan, A., DeMotta, A., Titman, S. (2007): «Firm Location and the Creation and Utilization of Human Capital», *Review of Economic Studies,* 74, 1305-1327.

Arbia, G., and Paelinck, J. (2003): «Spatial Econometric Modeling of Regional Convergence in Continuous Time», *International Regional Science Review,* 26, 342-362.
Azomahou, T., Ouardighi, J. E., Nguyen-Van, P., and Pham, T. (2011): «Testing Convergence of European Regions: A Semiparametric Approach», *Economic Modelling,* 28, 1202-1210.
Bairoch, P. (1993): *Economics and World History: Myths and Paradoxes*, Chicago: University of Chicago Press.
Banerjee, B., and Jarmuzek, M. (2010): «Economic Growth and Regional Disparities in the Slovak Republic», *Comparative Economic Studies,* 52, 279-403.
Bardhan, P. K. (1965): «Equilibrium Growth in the International Economy», *Quarterly Journal of Economics.*
Basile, R., Durbán, M., Mínguez, R., Montero, J. M., and Mur, J. (2014): «Modeling regional economic dynamics: Spatial dependence, spatial heterogeneity and nonlinearities», *Journal of Economic Dynamics & Control,* 48, 229-245.
Batabyal, A., and Nijkamp, P. (2013): «A Multi-Region Model of Economic Growth with Human Capital and Negative Externalities in Innovation», *Journal of Evolutionary Economics,* 23, 909-924.
Blomquist, G. C., Berger, M. C., and Hoehn, J. C. (1988): «New Estimates of Quality of Life in Urban Areas», *American Economic Review,* 78, 89-107.
Bond, E. W., Trask, K., and Wang, P. (2003): «Factor Accumulation and Trade: Dynamic Comparative Advantage with Endogenous Physical and Human Capital», *International Economic Review,* 44, 1041-1060.
Bourassa, S. C., Cantoni, E., and Hoesli, M. (2010): «Predicting House Prices with Spatial Dependence: A Comparison of Alternative Methods», *Journal of Real Estate Research,* 32, 139-159.
Brecher, R. A., Chen, Z. Q., and Choudhri, E. U. (2002): «Absolute and Comparative Advantage, Reconsidered: The Pattern of International Trade with Optimal Saving», *Review of International Economics,* 10, 645-656.
Brown, D., Fuguitt, G., Heaton, T., and Waseem, S. (1997): «Continuities in Size of Place Preferences in the United States, 1972-1992», *Rural Sociology,* 62, 408-428.
Brueckner, J. K. (2000): «Urban Sprawl: Diagnosis and Remedies», *International Regional Science Review,* 23, 160-171.
Brunow, S., and Hirte, G. (2009): «The Age Pattern of Human Capital and Regional Productivity: A Spatial Econometric Study on German Regions», *Papers in Regional Science,* 88, 799-823.
Busse, M., and Hefeker, C. (2007): «Political Risk, Institutions and Foreign Direct Investment», *European Journal of Political Economy,* 23, 397-415.
Can, A., and Megbolugbe, I. (1997): «Spatial Dependence and House Price Index Construction», *The Journal of Real Estate Finance and Economics,* 14, 203-222.
Charlot, S., Gaigné, C., Robert-Nicoud, F., Thisse, J.-F. (2006): «Agglomeration and Welfare: The Core-Periphery Model In the Light of Bentham, Kaldor, and Rawls», *Journal of Public Economics,* 90, 325-347.
Chen, Y., Irwin, E. G., and Jayaprakash, C. (2013): «Population Dispersion vs. Concentration in a Two-Region Migration Model with Endogenous Natural Amenities», *Journal of Regional Science,* 53, 256-273.
Clark, D., and Hunter, W. (1992): «The Impact of Economic Opportunity, Amenities and Fiscal Factors on Age-Specific Migration Rates», *Journal of Regional Science,* 32, 349-365.
Colombo, L., and Dawid, H. (2014): «Strategic Location Choice under Dynamic Oligopolistic Competition and Spillovers», *Journal of Economic Dynamics & Control,* 48, 288-307.
Combes, P. G., Duraton, G., and Gobillon, L. (2004): «Spatial Wage Disparities: Sorting Matters!», *CEPR Discussion Paper*, 4220.

De Beule, F., and Duanmu, J. L. (2012): «Locational Determinants of Internationalization: A Firm-level Analysis of Chinese and Indian Acquisitions», *European Management Journal,* 30, 264-277.

Deardorff, A. V., and Hanson, J. A. (1978): «Accumulation and a Long-Run Heckscher-Ohlin Theorem», *Economic Inquiry,* 16, 288-292.

Deller, S. C., Tsai, T. H., Marcoullier, D., and Donald, E. (2001): «The Role of Amenities and Quality of Life in Rural Economic Growth», *American Journal of Agricultural Economics,* 83, 352-365.

Diamond, D. B., and Tolley, G. S. (1981, eds.): *The Economics of Urban Amenities*, New York: Academic Press.

Dubin, R. (1992): «Spatial Autocorrelation and Neighborhood Quality», *Regional Science and Urban Economics,* 22, 433-452.

Duranton, G., and Monastiriotis, V. (2002): «Mind the Gaps: The Evolution of Regional Earnings Inequalities in the UK 1982-1997», *Journal of Regional Science,* 42, 219-256.

Findlay, R. (1984): «Growth and Development in Trade Models», in Jones, R. W., and Kenen, R. B. (eds.), *Handbook of International Economics*, Amsterdam: North-Holland.

Fleisher, B., Li, H., and Qiang, M. (2010): «Human Capital, Economic Growth, and Regional Inequality in China», *Journal of Development Economics,* 92, 215-231.

Florida, R., Mellander, C., and Stolarick, K. (2008): «Inside the Black Box of Regional Development - Human Capital, the Creative Class, and Tolerance», *Journal of Economic Geography,* 8, 615-649.

Forslid, R., and Ottaviano, G. I. P. (2003): «Trade and Location: Two Analytically Solvable Models», *Journal of Economic Geography,* 3, 229-240.

Fotopoulos, G. (2012): «Nonlinearities in Regional Economic Growth and Convergence: The Role of Eentrepreneurship in the European Union Regions», *Annals of Regional Science,* 48, 719-741.

Fujita, M., Krugman, P., and Venables, A. (1999): *The Spatial Economy*, Cambridge, MA: MIT Press.

Glaeser, E. L., and Maré, D. C. (2001): «Cities and Skills», *Journal of Labor Economics,* 19, 316-342.

Goodman, C., and Thibodeau, T. G. (2003): «Housing Market Segmentation and Hedonic Prediction Accuracy», *Journal of Housing Economics,* 12, 181-201.

Graves, P. (1979): «A Life-Cycle Empirical Analysis of Migration and Climate by Race», *Journal of Urban Economics,* 6, 135-147.

Henisz, W. J. (2000): «The Institutional Environment for Multinational Investment», *Journal of Law, Economics and Organization,* 6, 334-364.

Helbich, M., Brunauer, W., and Nijkamp, P. (2014): «Spatial Heterogeneity in Hedonic House Price Models: The Case of Austria», *Urban Studies,* 51, 390-411.

Hirschman, A. O. (1958): *The Strategy of Economic Development*, New Haven: Yale University Press.

Jensen, B. S., and Wong, K. Y. (eds.) (1998): *Dynamics, Economic Growth, and International Trade*, Ann Arbor: The University of Michigan Press.

Johansson, B., and Klaesson, J. (2011): «Agglomeration Dynamics of Business Services», *The Annals of Regional Science,* 47(2), 373-391.

Kanemoto, Y. (1980): *Theories of Urban Externalities*, Amsterdam: North-Holland.

Kemp, M. C. (1961): «Foreign Investment and National Advantage», *Economic Record,* 28, 56-62.

Kim, K. K., Marcouiller, D. W., and Deller, S. C. (2005): «Natural Amenities and Rural Development: Understanding Spatial and Distributional Attributes», *Growth and Change,* 36, 273-297.

Kim, S. W., and Bhattacharya, R. (2009): «Regional Housing Prices in the USA: An Empirical Investigation of Nonlinearity», *Journal of Real Estate Finance and Economics,* 38,443-460.

Krugman, P. (1991): «Increasing Returns and Economic Geography», *Journal of Political Economy,* 99, 483-499.
— (1993): «On the Number and Location of Cities», *European Economic Review,* 37, 293-298.
Kuznets, S. (1966): *Modern Economic Growth, Rate Structure, and Spread*, New Haven: Yale University Press.
Lee, L. F., Liu, X., and Lin, X. (2010): «Specification and Estimation of Social Interaction Models with Network Structures», *Economic Journal,* 13, 145-176.
Lee, J. Y., and Mansfield, E. (1996): «Intellectual Property Protection and US Foreign Direct Investment», *Review of Economics and Statistics,* 78, 181-186.
MacDougall, G. D. A. (1960): «The Benefits and Costs of Private Investment from Abroad: A Theoretical Approach», *Economic Record,* 27, 13-15.
Malpezzi, S. (2003): «Hedonic Pricing Models: A Selective and Applied Review», in O'Sullivan, T., and Gibb, K. (eds.), *Housing Economics and Public Policy*, Oxford: Blackwell, 67-89.
McGranahan, D. (1999): «Natural Amenities Drive Rural Population Change», *Agricultural Economic Report* No. 781. Washington, DC: Economic Research Service, U.S. Department of Agriculture.
McMillen, D. P. (1996): «One Hundred Fifty Years of Land Values in Chicago: A Nonparametric Approach», *Journal of Urban Economics,* 40, 100-124.
— (2010): «Issues in Spatial Data Analysis», *Journal of Regional Science,* 50, 119-141.
Myrdal, G. (1957): *Economic Theory and Underdeveloped Regions,* London: Duckworth.
Nishimura, K., and Shimomura, K. (2002): «Trade and Indeterminacy in a Dynamic General Equilibrium Model», *Journal of Economic Theory,* 105, 244-260.
Obstfeld, M., and Rogoff, K. (1998): *Foundations of International Macroeconomics*, Mass., Cambridge: MIT Press.
Oniki, H., and Uzawa, H. (1965): «Patterns of Trade and Investment in a Dynamic Model of International Trade», *Review of Economic Studies,* 32, 15-38.
Ono, Y., and Shibata, A. (2005): «Fiscal Spending, Relative-price Dynamics, and Welfare in a World Economy», *Review of International Economics,* 13, 216-36.
Ottaviano, G. I. P., Tabuchi, T., and Thisse, J.-F. (2002): «Agglomeration and Trade Revisited», *International Economic Review,* 43, 409-436.
Partridge, M. D., Rickman, D. S., Ali, K., and Olfert, M. R. (2008a): «The Geographic Diversity of U.S. Nonmetropolitan Growth Dynamics: A Geographically Weighted Regression Approach», *Land Economics,* 84, 241-266.
— (2008b): «Lost in Space: Population Dynamics in the American Hinterlands and Small Cities», *Journal of Economic Geography,* 8, 727-757.
Pflüger, M. P. (2004): «A Simple, Analytically Solvable Chamberlinian Agglomeration Model», *Regional Science and Urban Economics,* 34, 565-573.
Picard, P. M., and Tabuchi, T. (2010): «Self-organized Agglomerations and Transport Costs», *Economic Theory,* 42, 565-589.
Rey, S. J., and Janikas, M. V. (2005): «Regional Convergence, Inequality, and Space», *Journal of Economic Geography,* 5, 155-176.
Roback, J. (1982): «Wages, Rents, and the Quality of Life», *Journal of Political Economy,* 90, 1257-78.
Rosen, S. (1974): «Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition», *Journal of Political Economy,* 82, 34-55.
Sheppard, S. (1997): «Hedonic Analysis of Housing Markets», in Chesire, P., and Mills, E. (eds.), *Handbook of Regional and Urban Economics*, Amsterdam: Elsevier, 3, 1595-1635.
Shumway, J. M., and Otterstron, S. M. (2001): «Spatial Patterns of Migration and Income Change in the Mountain West: The Dominance of Service-Based, Amenity-Rich Counties», *Professional Geographer,* 53, 492-502.

Sorger, G. (2002): «On the Multi-Region Version of the Solow-Swan Model», *The Japanese Economic Review,* 54, 146-164.

Tabuchi, T. (2014): «Historical Trends of Agglomeration to the Capital Region and New Economic Geography», *Regional Science and Urban Economics,* 44, 50-59.

Uzawa, H. (1961): «On a Two-Sector Model of Economic Growth», *Review of Economic Studies,* 29, 47-70.

Wong, K. Y. (1995): *International Trade in Goods and Factor Mobility*, Mass., Cambridge: MIT Press.

Zhang, W. B. (1993a): «Wages, Service Prices and Rent - Urban Division of Labor and Amenities», *Seoul Journal of Economics,* 6, 97-113.

— (1993b): «Woman's Labor Participation and Economic Growth - Creativity, Knowledge Utilization and Family Preference», *Economics Letters,* 42, 105-110.

— (1995): «Leisure Time, Savings and Trade Patterns - A Two-Country Growth Model», *Economic Modelling,* 12, 425-434.

— (1996): *Economic Geography with Two Regions: Capital Accumulation and Economic Structure*. *Australian Economic Papers,* 35, 225-235.

— (2013): «Income and Wealth Distribution with Physical and Human Capital Accumulation: Extending the Uzawa-Lucas Model to a Heterogeneous Households Economy», *Latin American Journal of Economics,* 50 (2), 257-287.

— (2016): «Regional Disparities and Growth in an Extended Uzawa's Two-Sector Growth Model», *Journal of Studies in Dynamics and Change* (forthcoming).