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Investigation of Eighth-Grade Students' Understanding of the Slope of the Linear Function

Investigando a Compreensão de Alunos do Oitavo Ano sobre a Inclinação de Funções Lineares

Osman Birgin*

Abstract

This study aimed to investigate eighth-grade students' difficulties and misconceptions and their performance of translation between the different representation modes related to the slope of linear functions. The participants were 115 Turkish eighth-grade students in a city in the eastern part of the Black Sea region of Turkey. Data was collected with an instrument consisting of seven written questions and a semi-structured interview protocol conducted with six students. Students' responses to questions were categorized and scored. Quantitative data was analyzed using the SPSS 17.0 statistical packet program with cross tables and one-way ANOVA. Qualitative data obtained from interviews was analyzed using descriptive analytical techniques. It was found that students' performance in articulating the slope of the linear function using its algebraic representation form was higher than their performance in using transformation between graphical and algebraic representation forms. It was also determined that some of them had difficulties and misunderstood linear function equations, graphs, and slopes and could not comprehend the connection between slope and the *x*- and *y*-intercepts.

Keywords: Mathematics Education. Eighth-grade Students. Linear Functions. Slope. Misunderstandings.

Resumo

Este estudo tem como objetivo investigar as dificuldades de alunos quando tratando de

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diferentes modos de representar a inclinação de funções lineares. Os participantes foram 115 estudantes de oitava série de uma cidade situada na parte leste da região do Mar Negro, na Turquia. Os dados foram coletados por meio de questionários escritos e complementados com um protocolo de entrevista semi-estruturada realizada com seis alunos. Os dados quantitativos foram analisados estatisticamente e para os qualitativos optou-se por uma análise descritiva. Verificou-se que o desempenho dos alunos em articular a inclinação da função linear usando sua forma de representação algébrica foi maior do que o desempenho nas questões relativas à articulação entre as formas de representação gráfica e algébrica. Outras dificuldades foram detectadas, como as relativas ao conceito de função linear e gráfico de funções; e a falta de entendimento sobre a conexão entre a inclinação e os pontos de interseção com os eixos.

Palavras-chave: Educação Matemática. Funções Lineares. Inclinação. Desempenho de Estudantes

1 Introduction

Linear functions are a complex, multifaceted idea whose power and richness permeate almost all areas of mathematics (LLOYD; WILSON, 1998). In school curricula, functions are taught at a fundamental level (line graphs, bar graphs, and circle graphs, etc.) until the secondary level, where this advanced topic is typically explored in depth (LEINHARDT; ZASLAVSKY; STEIN, 1990). In addition, it should be noted that the subject of functions is core for higher levels of both mathematics and other disciplines. Therefore, mathematics education reforms have called for changes in the ways that numerous topics, including the concept of functions, are taught. What students are expected to learn about functions and their graphs is not only single computational fluency, but also, as described in the Principles and Standards for School Mathematics (NCTM, 2000), the analyses of functions, both quantitative and qualitative, with conceptual understanding as a key goal for students' learning. Furthermore, NCTM (2000) explains that knowledge of functions is a central component of learning about algebra that should be emphasized throughout students' school years beginning in the elementary grades.

Because of their many real-world applications, linear functions are important for the role they play in supporting a conceptual understanding of more advanced mathematical topics such as the derivatives in calculus (DAVIS, 2007). They also represent linear combinations of quantities, situations of constant increase or decrease, lines as shortest distances, and approximations to curve (STUMP, 1996). An understanding of linear functions in one representation will not necessarily correspond to that in another representation, but the ability to translate among varied formats is necessary to effectively interpret problem situations (JANVIER, 1981; MOSCHKOVICH, 1996). Today, the students are expected to learn functions and their graphs with single procedural knowledge, and they are supposed to have a conceptual level of understanding and to carry out broader tasks such as quantitative and qualitative analyses.

Linear functions can be depicted using a variety of representational systems, of which the three most common are equations, tables, and graphs. Students need to be able to understand information presented in these different formats and to perform transitions among the various representations. Understanding the linear function concept is remarkably complex and involves many levels of abstraction (SHERIN, 2002). Therefore, linear functions are a complex domain where the development of interconnections of conceptual knowledge is essential for competence. Conceptual understanding of the domain of linear functions, including much more the procedural learning, involves understanding the connections between the graphical and algebraic representations (CHIU et al., 2001).

A main characteristic that makes a linear function interesting for research in mathematics education is that several representations are intertwined. Thus, previous research has investigated students' errors, conceptions, and misconceptions in linear functions and their graphs (ACUNA, 2007; BIRGIN, 2006; CHIU et al., 2001; DAVIS, 2007; HITT, 1998; GRAHAM; SHARP, 1999; KNUTH, 2000; LEINHARDT; ZASLAVSKY; STEIN, 1990; MEVARECH; KRAMARSKY, 1997; MOSCHKOVICH, 1996; 1999; NATHAN; KIM, 2007; STUMP, 1996; 2001; ZASLAVSKY; SELA; LERON, 2002). Many of these research studies have shown that students find linear functions quite difficult to understand, that they developed very limited conceptions about linear functions and their graphs, that they have difficulty translating among the different representations of linear functions, and that they do not appreciate the overall structure of the linear function concept. For example, Moschkovich (1996) determined that a number of eighth-grade students had difficulties regarding how to explain the functions of x-intercept, y-intercept, m and b, and the interrelations with one another in a linear equation given as $y = mx + b$. These difficulties may be primarily explained by the fact that the students cannot associate different forms of representations of functions and may have not learned mathematical concepts comprehensively. Knuth (2000) found that students who had mathematical backgrounds, ranging from first-year algebra to calculus, did not apply the Cartesian connection to translate a graphical representation to an algebraic one. Instead, students preferred to move in the opposite direction, from algebraic to graphical representations. In his study conducted with 135 10th-grade students, Acuna (2007) also found that although most students seemed to make a successful identification of the slope and the y-intercept using a construction easily recognizable by its graphic shape, this identification seemed to be different in other kinds of tasks, such as predicting or explaining.

2 The concept of slope

One of the defining attributes of linear functions is the concept of slope. The slope of linear functions represents the rate of change in one variable that results from a change in the other variable. Because linear function is represented geometrically as a line, the rate of change and the slope of the line are constant. The slope of linear functions may be represented in several forms. Slope is defined geometrically as a property of a line; given two points (x_1, y_1) and (x_2, y_2), the slope of the line is $(y_2 - y_1)/(x_2 - x_1)$, with slope as the ratio (rise/run). Horizontal lines have a slope of zero, and the slope of vertical lines is undefined. When functions are represented as a linear equation, slope is the m in the equation $y = mx + b$, called the slope-intercept form, and we can determine the slope of the line just by looking at the equation. However, when a linear equation is written such as $ax + by + c = 0$, some algebraic manipulation is required in order to determine the slope. Slope is also used to describe the steepness of physical objects such as the ratio of the change in y to the change in x of the line. Thus, the slope of a linear function is represented in geometric, algebraic, physical, trigonometric, functional, and ratio terms. However, traditional mathematics curricula tend to separate and isolate the learning of arithmetic, algebra, geometry, trigonometry, and functions; it is questionable whether students make connections among these various conceptions of slope and recognize them as representations of same concepts (STUMP, 1996).

The concept of slope has a life in both school mathematics and outside the classroom. Throughout the secondary mathematics curriculum, the concept of slope emerges in various forms. It appears algebraically in formulas and equations, geometrically in graphs, and trigonometrically as the tangent of an angle. In the study of calculus, slope appears as a limit. Real-world representations of slope exist in two different forms: physical situations, such as mountain roads, ski slopes, and wheelchair ramps; and functional situations, such as distance versus time or quantity versus cost (STUMP, 2001). In this study, we focused on slope as a measure of rate of change in terms of the linear function topic in the eighth-grade mathematics curriculum in Turkey.

Research has noted that students had difficulties and misconceptions associated with the concept of slope. Some studies have focused on students' understanding of slope within abstract contexts (LEINHARDT et al., 1990; ZASLAVSKY et al., 2002) and others on real-world contexts (NEMIROVSKY, 1997; STUMP, 2001). There are misconceptions associated with the calculation of slope, and when it is represented in decimal form, students have trouble considering slope as a ratio (BARR, 1980; STUMP, 1996; 2001). They also have difficulty interpreting linear functions and their graphs (BARR, 1980, MOSCHKOVICH, 1996; NATHAN; KIM, 2007; SCHOENFELD; SMITH; ARCAVI, 1993), connecting graphs to linear equations (Acuna, 2007; Birgin, 2006), and connecting graphs to the notion of rate of change (BELL; JANVIER, 1981; LEINHARDT et al., 1990; ORTON, 1984).

The concept of slope can pose challenges for students when they must translate between different representations (MOSCHKOVICH, 1996; SCHOENFELD et al., 1993; SMITH; ARCAVI; SCHOENFELD, 1989). There is confusion between the role of m and b when a function is written in the form $y = mx + b$. It may be difficult for students to consider the slope as ratio if m is an integer and whether slope is computed as "x over y" or "y over x". Moschkovich (1996) determined that one of the students' conceptions in this domain is that the x-intercept is relevant for equations of form $y = mx + b$, either in place of m or in place of b. One of them is that lines are moving to the right or to the left as a result of changing the b in an equation of this form. The slope of the line also depends on where the line originates, either from a given quadrant or from particular point. Another belief is that the slope and the y-intercept are not independent of one another. Stump (1996) investigated secondary teachers' mathematical understanding of the algebraic, geometric, trigonometric, and functional representations of slope and their abilities to translate among the various representations. She found that secondary mathematics teachers have a limited understanding of the concept of slope. Their mathematical understanding of slope is dominated by geometric representations, whereas algebraic, trigonometric, and functional representations are less understood. Kondratieva and Radu (2009) conducted a study with 499 students enrolled in a precalculus university course, and they found that the students more correctly recognized formulas for lines with a positive slope than for lines with a negative slope. They determined that the order of preference in recognizing the line formulas was horizontal, vertical, line with positive slope, and line with negative slope.

Stump (2001) found that the students demonstrated a better understanding of slope as a measure of rate of change than as a measure of steepness. The students in this investigation also exhibited a limited understanding of slope as a measure of steepness. They had trouble considering slope as a ratio. This study suggested a gap in students' understanding of slope as a measure of rate of change and implied that instruction should be focused on helping students form connections among rates involving time, rates involving other variables, and graphical representations of these relationships. In a case study, Zazlavsky et al. (2002) also pointed out that some 11th-grade students have experienced much confusion regarding the connections between the algebraic and geometric aspect of slope, scale, and angle.

This study focused on the concept of slope and its importance as a defining parameter of a linear function. Although many research studies have investigated secondary students' and mathematic teachers' knowledge of functions (ZAZLAVSKY et al., 2002; STUMP, 1996; 2001; KARATAŞ; GÜVEN, 2004), a search of literature reveals no evidence regarding eighth-grade students' knowledge of the concept of slope. The misconceptions and deficiencies of the students in linear functions also may lead to severe learning difficulties in the subjects of functions, limit, derivation, and integral through high school and the university years. For this reason, determination of students' understanding and competence in the connection between linear functions and their graphs seems to be important. So, this study aimed to investigate eighth-grade students' understanding and difficulties regarding the slope of linear functions. The anticipated benefits of this study include contribution to the literature on eighth-grade students' knowledge and misconceptions about the slope of a linear function. Especially, this study also reveals the negative impact of traditional mathematics teaching approaches to students' learning in the slope of a linear function. In this context, the following research questions were determined:

- What knowledge do students actually have of the slope of a linear function involving verbal, graphical, or algebraic representations? Are there any significant differences among students' competence in various representation modes of the slope of a linear function?
- What kind of difficulties and misconceptions do students have about the slope of a linear function?

3 Method

3.1 Participants

The participants in this study were 115 eighth-grade students from a public school in Trabzon, which is a city in the eastern part of the Black Sea region of Turkey. In Turkey, primary education, as a compulsory education, is intended for pupils aged 6 to 14 years. It is divided into two stages. The first stage consists of grades 1–5, and the second stage consists of grades 6–8. Formerly, middle schools were responsible for education in the second stage. In 1997, the duration of compulsory education was extended from five to eight years, and middle schools were integrated into the primary school system. Besides, eighth grades were seen as a particularly important grade in Turkey's context because Turkish students have some crucial national exams for their education and careers at this grade level. In addition, the slope of straight line concept under investigation in the study was taught in this grade level. Hence, eighth-grade students were selected as the study group. The students were between 14 and 15, and the 115 participants consisted of 62 males and 53 females.

3.2 The contexts of the study

The participants in this study were generally subjected to traditional mathematics instruction. In other words, mathematics lessons were taught in a teacher-centered manner. All content was provided in a face-to-face manner in which the teachers talked and the students listened for 70% of the class time. The instruction was combined with the use of lectures and skill practice. In this process, the concepts were lectured to the students according to the syllabus of the book, and the teacher wrote down the necessary parts from the lectures. While the teacher was writing notes on the board, he framed the important parts with a colorful chalk. During the process, the students sat in their seats silently and inactively and listened to the teacher. When the board was filled with some notes, the teacher gave the students a certain time to copy from the board to their notebooks. Meanwhile, the teacher discussed the problem and asked a student to go to the board to solve the problem. The teacher helped the student when he/she had any difficulty in solving the problem. So approximately 70%-75% of the lesson can be said to be composed of just the teacher's discourse. The teacher also asked during this process if they had any questions about the concept. Students were made aware of the procedures to be followed and rules to be obeyed during the lesson. They were given the opportunity to ask questions about points they did not understand, and short summaries were made available from the lecturer after every subject. After lecturing, the teacher asked the students to answer the questions at the end of the unit.

3.3 Instruments

An instrument, consisting of seven questions needing written answers, was used as a data collecting tool (see Appendix 1). Questions were related to analyzing and interpreting specific mathematical situations involving verbal, graphical, or algebraic representations of function. The questions of the instrument were primarily about understanding of the algebraic representation (UAR) of the slope (Q1a, Q1b, Q2a, Q2b), transferring from algebraic to graphical representation (TAGR) of the slope (Q3a, Q3b, Q4b, Q4d), and transferring from graphical to algebraic representation (TGAT) of the slope of linear function (Q5, Q6, Q7a, Q7b). That is, questions were related to translation from modes of one representation of slope to another so as to find the slope of linear functions given the algebraic representation form, to find the slope of line given the graph, and to sketch the graph of the line given the slope. The questions were prepared considering the gains mentioned in the eighth-grade mathematics curriculum in Turkey. For this reason, other types of representations were not covered. In order to ensure the content validity of the instrument, two mathematics teachers and three field experts were consulted.

Interview methods allowed investigating not only the methods used by the students during their problem-solving attempts, but also the reasoning behind those solutions. Therefore, in this study a semi-structured interview protocol was conducted with six students for in-depth investigation. These students were three females (S1, S4, S6) and three males (S2, S3, S5) and selected based on their performance on the test. One female (S1) and one male (S2) students' scores were below the average, one female (S4) and one male (S3) students' scores were close to average, and finally one male (S5) and one female (S6) students' scores were above average. Each interview session began with a careful explanation and assurance that the students were not being graded in any way. The questions used in the interviews were, for the most part, analogous to the problems on the instrument. All six participants also received the same problems.

3.4 Data collection and analyzes

The developed instrument was employed for 115 eighth-grade students. Students' answers to each item were coded into categories such as "*correct answer (CA),*" "*partly correct answer (PCA),*" "*incorrect answer (IA),*" or "*no answer (NA).*" The categories assigned to the answers denoted the following meanings: *correct answer*—the explanation, operation, or the solution strategy is scientifically true; *partly correct answer*—the explanations are true but not sufficient, the solution is true but partially flawed; *incorrect answer*—the explanations given as answers are unrelated, illogical, or flawed; *no answer*—the question is not answered. In order to provide coding reliability, the answers to the questions were coded by two different researchers independently, and afterwards these codes were compared and a consensus was attained on the categories. Cohen's kappa coefficient is a statistical measure of inter-rater agreement for qualitative (categorical) items (Cohen, 1960). Therefore, in this paper, we utilize the kappa coefficient to measure the agreement between coders, and the agreements between coders were found to range between 82% and 95% for each item. The categories were also scored from 0 to 2 as follows: 2 points for "*correct answer,*" 1 point for "*partial correct answer,*" and 0 point for "*incorrect answer*" and "*no answer.*" The SPSS 17.0 statistical software was used to analyze the quantitative data, using cross tabulations, one-way analysis of variance (ANOVA) for repeated measure and the Bonferroni post hoc technique.

In this study, the interviews were audio recorded, and the audiotapes together with the students' worksheets were transcribed. As recommended by Strauss and Corbin (1990), students' responses to the interview tasks were analyzed and coded around the research's problem. Each interview was outlined, summarized, and evaluated using categories such as the open-ended question in the instrument. In addition, a couple of examples of student mistakes were presented. Moreover, the extracts from the interviews and students' worksheets were given in English in this study. These were translated from Turkish.

4 Results

4.1 Students understanding of graphical and algebraic representations of the slope of linear functions

Descriptive statistics of the students' responses to questions related to the slope of the linear function and the representation mode of slope are given in Table 1 and Table 2. As shown in Table 1, students mostly answered correctly question Q1a (75.7%) on finding the slope of the $5x - y + 3 = 0$ (*M* = 1.59, *SD* = .75). Although question Q1b (related to finding the slope of the $2y - 3x = 0$) was similar to Q1a, only 59.1% of the students answered question Q1b correctly. Here, in finding the slope of the line equation given in the algebraic representation form as $ax + by + c = 0$, it can be noticed that many students perceived the coefficient of "x" as slope, as is the case in the form $y = mx + b$. This finding suggests that most students made mistakes in finding the slope of the line given in the algebraic form "$ax + by + c = 0$" and they lacked conceptual understanding.

Table 1 - Descriptive statistics related to question items

	Item	*n*	CA		PCA		ICA		NA		*M*	*SD*
			f	%	*f*	%	*f*	%	*f*	%		
Understanding of the Algebraic Representation (UAR)	Q1a	115	87	75.7	9	7.8	10	8.7	9	7.8	1.59	.75
	Q1b	115	68	59.1	12	10.4	28	24.3	7	6.1	1.28	.90
	Q2a	115	70	60.9	0	0.0	34	29.6	11	9.6	1.21	.98
	Q2b	115	67	58.3	0	0.0	35	30.4	13	11.3	1.16	.99
Transferring from Algebraic to Graphical Representation (TAGR)	Q3a	115	73	63.5	0	0.0	25	21.7	17	14.8	1.26	.96
	Q3b	115	66	57.4	19	16.5	21	18.3	9	7.8	1.31	.86
	Q4a	115	52	45.2	0	0.0	49	42.6	14	12.2	.90	.99
	Q4b	115	57	49.6	0	0.0	43	37.4	15	13.0	.99	1.01
Transferring from Graphical to Algebraic Representation (TGAR)	Q5	115	66	57.4	15	13.0	23	20.0	11	9.6	1.27	.89
	Q6	115	49	42.6	32	27.8	22	19.1	12	10.4	1.13	.84
	Q7a	115	50	43.5	0	0.0	42	36.5	23	20.0	.86	.99
	Q7b	115	63	54.8	28	24.3	17	14.8	7	6.1	1.33	.80

CA: Correct Answer, PCA: Partial Correct Answer, ICA: Incorrect Answer, NA: No Answer

On the other hand, the questions that were answered correctly by the least number of students were questions Q6 (42.6%), Q7a (43.5%), and Q4a (45.2%). Among these questions, question Q6 compares the slopes of the line graphs that pass through the origin differently, question Q7a shows the slope angle of a line that passed through two points other than the origin and whose graph was given, and question Q4a represents a line with a slope of $m = -1$ graphically. Based on these results, it can be claimed that students had difficulty in transferring between the graphical and algebraic representation forms in finding the slope of line.

As seen in Table 2, 63.5% of students gave correct answers, 23.3% gave incorrect answers, and 8.7% gave no response for questions requiring the understanding of the algebraic representation (UAR) of the slope of line (*M* = 1.31, *SD* = .85). On the other hand, 53.9% of students gave correct answers, 30% gave incorrect answers, and 12% gave no response for questions requiring transferring from algebraic to graphical representation (TAGR) of the slope of line (M = 1.12, SD = .88). For the questions requiring transferring from graphical to algebraic representation (TGAR) of the slope of line, 49.6% of students gave correct answers, 16.3% of them gave partially correct answers, 22.6% gave incorrect answers, and 11.5% left the questions blank (M = 1.15, SD = .82).

Table 2 - Descriptive statistics of the students' performance on the questions about the representation mode of slope

Performance on the Representation mode of slope	Items	CA	PCA	ICA	NA	*M*	*SD*
		%	%	%	%		
Understanding of the Algebraic Representation (UAR)	Q1a, Q1b, Q2a, Q2b	63.5	4.6	23.3	8.7	1.31	.85
Transferring from Algebraic to Graphical Representation (TAGR)	Q3a, Q3b, Q4b, Q4d	53.9	4.1	30.0	12.0	1.12	.88
Transferring from Graphical to Algebraic Representation (TGAR)	Q5, Q6, Q7a, Q7b	49.6	16.3	22.6	11.5	1.15	.82

CA: Correct Answer, PCA: Partial Correct Answer, ICA : Incorrect Answer, NA: No Answer

According to Table 2, it can be stated that the most successful point for the students was understanding of the algebraic representation of the slope of line (M = 1.31, SD = .85), whereas the least successful areas were transferring from algebraic to graphical representation (TAGR) and from graphical to algebraic representation (TGAR) of the slope of line. One-way ANOVA for repeated measures was used to compare the students' competence in representation modes of the slope of line. As seen in Table 3, results of ANOVA showed that there was a significant difference for students' competence in representation modes of slope of line [F (1, 114) = 234.21, $p < .001$]. Bonferroni post hoc analyses in Table 4 also showed that competence scores of the students in UAR (M = 1.31, SD = .85) were significantly higher than competence scores of the TAGR (*Mean differences* = .193, SE = .029, $p < .01$) and TGAR (*Mean differences* = .161, SE = .029, $p < .01$], but there was no significant difference between students' TAGR and TGAR competence scores ($p > .05$). These findings showed that eighth-grade students mostly succeeded in finding the slope of line given in algebraic representation form, whereas they could not demonstrate sufficient achievement in situations that required transferring between graphical and algebraic representations.

Table 3 - Results of One-way ANOVA

Source	Type III Sum of Squares	*df*	Mean Square	*F*	*p*
Between subjects	240.731	114	2.112		
Measure	2.456	2	1.228	32.271	.000*
Error	8.675	228	.008		
Total	251.862	344			

*$p < .01$

Table 4 - Pairwise comparisons

(I) Measure	(J) Measure	Mean Difference (I-J)	Std. Error	*p*
UAR	TAGR	.193	.029	.000*
UAR	TGAR	.161	.029	.000*
TAGR	TGAR	-.032	.018	.219

*$p < .01$

4.2 Students' difficulties and misconceptions of the slope of linear functions

In this section, the students' misconceptions and mistakes regarding the slope of linear functions are given in detail. Questions Q5, Q2b, and Q4b are related to articulating the slope of a line in forms of $y = a$, $x = b$; (where a and b are real numbers). As seen in Table 1, the percentages of CA categories related to these questions are 57.4% (Q5), 58.3% (Q2b), and 49.6% (Q4b), respectively. Thus, it can be said that some students have misconceptions and mistakes about the slope of a line that is parallel to the y-axis and x-axis in a Cartesian coordinate plane. For example, some students responses to question *Q5* were "*Slope of a line given in the form of x = -2 is zero. Because this line is parallel to y-axes.*" "*Slope of a line given in the form of y = mx + b is the coefficient of x. Therefore, slope of a line given in the form of x = -2 is -2. Because, coefficient of x is -2.*" Interview data related to question *Q5* also showed that two students (S1, S3) had serious confusion about the slope of a line. Furthermore, S3 had a misunderstanding about the angle concept as "*Tangent of an angle is not to be negative.*"

R : Researcher S: Student
R : What is the slope of line given in the form of x = - 2?
S1 : Zero
R : Why?

S1 : The slope of line parallel to y-axis is zero.
R : Why? Can you explain?
S1 : I don't know…This is a rule.
R : What do you remember about the slope of line?
S1 : It shows the aspect of the line and where the line is (first region (+), second region (-)).
…
R : What is the slope of line given in the form of x = - 2?
S3 : Its slope is zero. Because the slope is equal to tan (α), hence = 90^0, tan(90º) = 0 .
R : Is the slope of a line parallel to x-axis always zero?
S3 : I am not sure.
R : What is the slope of the line given form of y = - 2?
S3 : The slope of line parallel to x-axis is undefined.
R : Why?
S3 : The slope of line is equal to the tangent of angle between its graph and the horizontal axis. But, line d passes through the point (0, -2) and its algebraic form is y = - 2. Because of not being the negative tangent of the angle, its slope is undefined.

Question Q7 is related to examining the graphical representation of the data and exploring relationships between symbolic expressions and graphs of lines, paying particular attention to the meanings of intercept and slope. Graphical representation of this line was given and its angle of slope was asked in question Q7a, and its slope was asked in question Q7b. Only 43.5% of the students could correctly show the angle of slope of this line. As seen in Figures 1(a) and 1(b), most of the students who gave incorrect answers were found to show the angle with 45° as the angle of the slope. Of these students, 54.8% gave correct answers and 24.3% of them gave partially correct answers. Some of the students who calculated the slope of this line incorrectly were found to have calculated the slope as $m = 3/-3 = 1$ (Figure 1a) or as $m = tan(45^o) = 1$, and some of them were found to locate the points incorrectly in the slope formula as $m = (y_2 - y_1)/(x_2 - x_1)$. As seen in Figure 1(c), a student used the formula $y = mx$ for the algebraic equation of this line not passing through the origin. A number of students could calculate the slope of this line accurately, but could not state its algebraic representation. These findings show that students lack information in transforming a line whose graphical illustration was given half and half in algebraic form and in finding the slope of the line starting from the graph. The students also face challenges in relating *m*, *x*-intercept and *y*-intercept in an algebraic representation as $y = mx + b$.

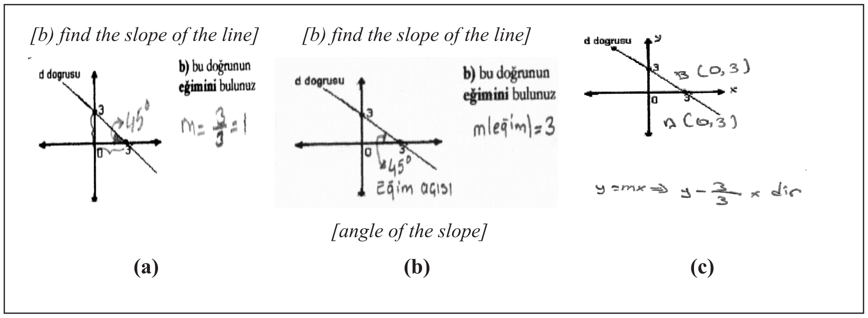

Figure 1 - Samples of the misunderstanding of students related to question Q7

Question Q6 compares the slopes of the lines across the origin differently and whose graph is given. As seen in Table 1, only 42.6% of students could correctly state the slope of lines. The percentage of the PCA, ICA, and NA categories are 27.8%, 19.1%, and 10.4%, respectively. It is seen in Figure 2(a) that few students misunderstood the angle of slope and the slope of linear functions such as "*when the measurement of the angle of slope increases, the slope of lines that pass through the origin is bigger.*" As seen in Figures 2(b) and 2(c), some students misunderstood about the slope of line such as "the *slope of line that passes through the origin is same and zero.*" Similarly, student S5 gave an explanation related to question Q6 as "*slope of these lines are zero, because these lines cross the origin,*" and student S2 who gave equal as the answer offered an explanation as "*all lines cross the origin.*"

For questions Q4a (a line with a slope is -1) and Q4b (a line with a slope is zero), students had to draw a line in a Cartesian coordinate plane. According to Table 1, the percentage of students who gave correct answers to questions Q4a and Q4b were 45.2% and 49.6% respectively. Some of them who gave incorrect answers to question Q4a, stated a line with a slope of -1 as the line passing through the point (-1, 0) as in Figure 3(a) or as the ratio between *y*- and *x*-intercept ($m = 3/-3 = -1$) as in Figure 3(b). Some students who gave incorrect answers to question Q4b were found to perceive slope as can be seen in Figure 3(c) and 3(d) as any line passing through the origin. These sketches show that some students made mistakes and lacked information regarding the slope of line.

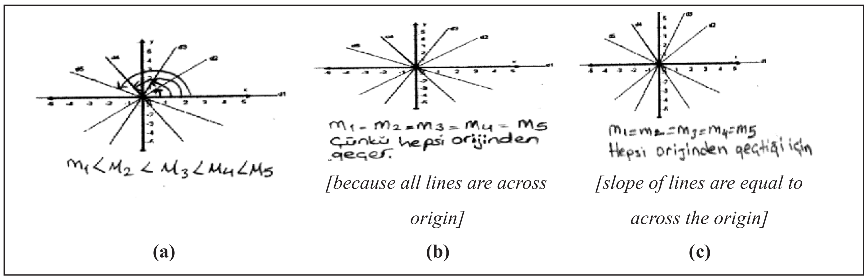

Figure 2 - Samples of the misunderstanding of students related to question Q6

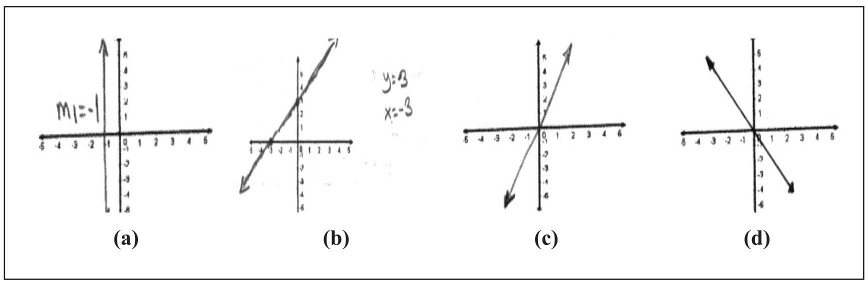

Figure 3 - Sample of the graphs of students related to Q4a and Q4b

Questions Q1a and Q1b were directed to the students so that they could find the slope of the line given in an algebraic equation as $ax + by = 0$ and $ax + by + c = 0$. As seen in Table 2, most of students (75.7%) did correctly state the accurate answer to question Q1a (find the slope for $5x - y + 3 = 0$), whereas 59.1% of students correctly stated the accurate answer to *Q1b* (find the slope for $2y - 3x = 0$). Most of the students giving incorrect answers to question Q1b were found to define the slope of the line $2y - 3x = 0$ as the slope of line of $2y - 3x = 0$ is - 3. For example, one student (S3) gave an explanation as *"2y - 3x = 0, 2y = 3x, the coefficient of x shows its slope, and it is 3"*. These findings showed that a number of students did not learn the concept and merely memorized the algebraic representation of $y = mx + b$.

5 Discussion and conclusion

Comprehension of the slope of a line is remarkably complex and involves many levels of abstraction (SHERIN, 2002). Therefore, future constructions, connections, and representations of the slope of linear function and their graphs rest on the development of conceptual knowledge (CHIU et al., 2001). The misconceptions and deficiencies of the eighth-grade students in understanding the slope of a line also may lead to severe learning difficulties in the subjects of functions, limit, derivation, and integral at the high school. For this reason, this study was conducted with eighth-grade students to determine their competencies, understandings, challenges, and mistakes regarding the slope of a linear function.

In this study, it was determined that most students (75.7%) accurately answered question Q1a (find the slope of $5x - y + 3 = 0$), whereas 59.1% of them accurately answered question Q1b (find the slope of $2y - 3x = 0$). Some of them answered question Q1b as: "*The slope of 2y - 3x = 0 was -3. Because the slope of a line that was given in algebraic form y = mx + b was coefficient of x.*" This result showed that nearly 40% of students had superficial knowledge and merely memorized the slope of a line that was given in the algebraic representation form $ax + by + c = 0$. On the other hand, it was found that some students had serious mistakes about the concept of the slope angle of a line (as seen in Figures 2-3) because of their deficient or incorrect information. In particular, some of the students made mistakes mostly in cases where the tangent of angle of inclination of a line is negative (Q2a, Q4, Q6, and Q7). These findings confirmed the findings of Kondratieva and Radu (2009). They found that students enrolled in a precalculus university course more correctly recognized formulas for lines with positive slope than for lines with negative slope. Moreover, this study revealed that some students had difficulties in ordering the points [(x_1, y_1) and (x_2, y_2)] for computing the slope of a line ($m = rise/run$) as seen in Figure 1(a). This finding is consistent with the findings of Bar (1980).

In this study, nearly 60% of the students could show the graphical representation of the slope of a line on the Cartesian coordinate plane by making use of the algebraic representation of a linear equation, whereas nearly 50% of the students had trouble in expressing the slope of a line whose graph had been given in the algebraic representation and in interpreting the graphics (Table 2). It was determined that the most successful points of understanding for the students were the algebraic representation of the slope of a line, whereas the least successful areas were transferring from algebraic to graphical representation and from graphical to algebraic representation of the slope of a line (Tables 3 and 4). This study also indicated that most students had difficulties with linear function equations, graphs, and slopes, and did not translate between different representation modes of slope a line. Furthermore, it was found that much confusion existed regarding the connection between the algebraic and geometric aspects of slope of a line. According to these findings, it can be said that placing more importance on applications could enhance procedural learning in the teaching process.

Similarly, the results of a sound body of research align with this study in finding that moving from a graph to an equation and vice versa are more difficult. In this context, many research studies indicated that students have difficulty interpreting linear functions graphs (BIRGIN, 2006; MOSCHKOVICH, 1996; 1999; NATHAN; KIM, 2007; PADILLA; MCKENZIE; SHAW, 1986; SCHOENFELD et al., 1993), connecting graphs to linear equations (KERSLAKE, 1981; MARKOVITS et al., 1983; ZASLAVSKY et al., 2002), and connecting graphs to the notion of slope (BELL; JANVIER, 1981; LEINHARDT et al., 1990). They had difficulty translating among the different representations of linear functions, and they do not appreciate the overall structure of the function concept. For example, Padilla et al. (1986) found that students in grades 7 through 12 were most successful with plotting points and determining the *x* and *y* coordinates of a point, while the most difficult skills appeared to be scaling axes and using a line of best fit. Kerslake (1981) found that while many 13- to 15-year-olds were able to read information from a graph or to plot given data, relatively few were able to understand the connection between an equation and a graph. Markovits et al. (1983) also found translation from graph to equations to be more difficult than the reverse when the functions were familiar. When the function was less familiar translations in both directions were found to be equally hard (cited in LEINHARDT et al., 1990).

Other studies (ACUNA, 2007; CARPENTER et al., 1981; STUMP, 1996; ZASLAVSKY et al., 2002) also confirmed these findings but with groups from different education levels. Acuna (2007) determined that most 10^{th}-grade students had difficulties in predicting or explaining about the graphical representation of the slope of linear function. Zaslavsky et al. (2002) pointed out that some 11th-grade students have experienced much confusion regarding the connections between the algebraic and geometric aspect of slope, scale, and angle. Carpenter et al. (1981) determined that when using a ruler and a sheet of paper with labeled axes, only 18% of 17-year-olds produced a correct graph corresponding to a linear equation. However, the reverse translation was even more difficult. Given a graph of a straight line with two intercept points indicated, only 5% of 17-year-olds could generate the equation. Stump (1996) also found that secondary mathematics teachers' conception of slope was restricted. They perceived slope mostly in geometric means and had a scarce understanding of algebraic, trigonometric, and functional representations. Therefore, some studies (NATHAN; KIM, 2007; RIDER, 2004) stated that students' understandings may be raised to the conceptual learning level by making reference to different types of representations. When using different types of representations, no single one should be preferred and each of these representations should be encouraged by demonstrating the relationships between them.

This study also indicated that most students, especially at poor and middle academic achievement levels, had mostly procedural learning rather than conceptual learning and demonstrated a range of mistakes and misunderstanding about linear function equations, graphs, and slope concepts. Some students were confused regarding algebraic and geometrics aspects of slope, and misunderstood about the tangent of angle and slope of linear functions. In this study, eighth-grade students' mistakes and difficulties can be explained with teacher-centered instruction such as describing the definition and the rule and making drills. The participants in this study were taught by traditional teaching methods that were still dominant in the Turkish education system. Similarly, Noss and Baki (1996) stated that mathematics teachers' priorities in Turkey were to follow the textbook; to spend the majority of their time lecturing to students using the blackboard; to stress algorithms, rules, definition, axioms, and to memorize formulas. Indeed, some studies (BAKI; ÖZTEKIN, 2003; BIRGIN; KUTLUCA, 2007) in Turkey also stressed that preferred teacher-centered approaches in the instruction of linear functions and graphs and technology were hardly used.

Based on the findings of this study, it can be stated that students' preconceptions and misconceptions about the slope of linear function affect their subsequent mathematics learning. Moreover, these deficient conceptions manifest great obstacles in future learning and understanding of mathematical concepts. For these reasons, it is important to remedy students' misconceptions and mistakes in time. In this context, Eisenberg (1992) emphasized that graph-to-equation translations receive little focus traditionally. Rider (2004) suggested that, in order to foster conceptual knowledge, linear functions should be taught with a method embracing multiple representations and providing linkages between these representations. Rizzuti (1991) showed that instruction that included multiple representations of functions allowed students to develop comprehensive and multifaceted conception of function. On the other hand, Moschkovich (1996) claimed that teachers can support group work by discussing and sharing meanings about the slope of linear function to attain students' conceptual learning, and to correct students' possible misunderstandings. In this regard, this study recommends the following curricular changes. Graph-to-equation translations and vice versa should be mostly emphasized in instruction and textbooks. The curriculum should also include spoken-language and real-world situations and specific activities such as representations to tables, graphs, and equations.

The results of this study indicated that many eighth-grade students had a limited understanding of the representation mode of the slope of a linear function. Their understanding about the slope was dominated by algebraic representations, while transformation between graphical and algebraic representations of slope was less understood. Therefore, students should be provided with the opportunity to view many graphs and their corresponding equations and to examine the relationship between graphical representations and algebraic parameters. In this context, many studies stated that computer technology could be used for translations between graphical and algebraic representations (BAKI, 2002; BIRGIN; KUTLUCA, 2006; LLOYD; WILSON, 1998; MOSCHKOVICH, 2004). Because many computer technologies, software programs, and graphical calculators open up the possibility of more visual constructions, and linking the graphical and algebraic representations of functions and graphs can be also generated quickly by the computer, freeing the student from the burden of calculating, plotting, and drawing. Under the dynamic environment, students can also change the slope and the *y*-intercept of the straight line by changing the values of *m* and *b*. Moreover, research conducted at the 10th grade level also showed that students exposed to computer-assisted instruction outperformed and learned the concepts and translations between a variety of the representation modes related to linear functions and its graphs better than via traditional instructions methods (BAKI; ÖZTEKIN, 2003; BIRGIN, KUTLUCA; GÜRBÜZ, 2008; CHIU et al., 2001; ISIKSAL; ASKAR, 2005; MOSCHKOVICH, 2004). Based on these results, this study recommended that some of the mathematics and geometry software programs such as Coypu, Drive, GeoCebra, or Sketchpad be used in order to enhance the students' conceptual learning and to overcome their learning difficulties and misconceptions about linear functions and graphs.

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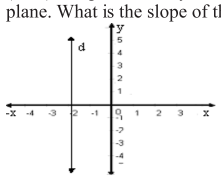
Appendix 1: Sample questions used in the instrument

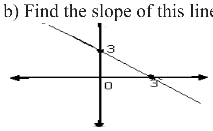
Q1.
a) Find the slope of the given linear functions,
$5x - y + 3 = 0$
b) Find the slope of the given linear function,
$2y - 3x = 0$

Q2.
a) Write the equation of a line in Cartesian coordinate plane whose slope is -1.
b) Write the equation of a line in Cartesian coordinate plane whose slope is zero.

Q5.
d is a straight line which crosses the point (-2, 0) and parallel the y-axis in Cartesian coordinate plane. What is the slope of the line?



Q7.
The graph of a straight line in the Cartesian coordinate plane is shown below.
a) Show the angle of slope of this straight line on the graph.
b) Find the slope of this line.



Q3.
d is a straight line which makes an angle of 45^0 with *x*-axis and passes through the origin in the Cartesian coordinate plane.
a) Sketch the graph of the straight line *d* in the Cartesian coordinate plane.
b) Find the slope of the straight line *d*?

Q4.
a) Sketch the graph of the straight line in Cartesian coordinate plane whose slope is -1.
b) Sketch the graph of the straight line in Cartesian coordinate plane whose slope is zero.

Q6.
The graphs of straight lines (d_1, d_2, d_3, d_4, d_5) in the Cartesian coordinate plane are shown below. Arrange the slope of the straight lines (m_1, m_2, m_3, m_4, m_5) in ascending order.