Boletim de Educação Matemática
ISSN: 0103-636X
bolema@rc.unesp.br
Universidade Estadual Paulista Júlio de Mesquita Filho
Brasil

Asghary, Nasim; Shahvarani, Ahmad; Reza Medghalchi, Ali

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Boletim de Educação Matemática, vol. 27, núm. 47, diciembre, 2013, pp. 1007-1026
Universidade Estadual Paulista Júlio de Mesquita Filho
Rio Claro, Brasil

ISSN 0103-636X

Significant Process of Change for Elementary Teachers to Foster Functional Thinking

Sobre o Processo de Mudança de Professores das Séries Iniciais Relativo ao Desenvolvimento do Pensamento Funcional

Nasim Asghary*
Ahmad Shahvarani**
Ali Reza Medghalchi***

Abstract

The purpose of this study was to explore a professional development program that involved 15 teachers. Functional thinking was used as a centerpiece of the program for work with teachers of Grades 1-5 during 6 months of the study. We used the concern-based adaptation model (CBAM) as a methodology to track the process of change of teachers and to understand the trajectories through which teachers may progress. Two questions guided the investigations: 1. How does implementation of the professional development program focused on functional thinking impact teachers' concerns? 2. How did teachers' practice change due to the implementation of the innovation program? The result of the study showed effectiveness of process of change in teachers, both in stages of concerns and level of use of the innovation.

Keywords: Process of Changing. Concerns CBAM. Elementary Teachers. Functional Thinking. Arithmetics.

* Doctor in Science and Research Branch, Islamic Azad Univerity. Department of Mathematics, Central Tehran Branch, Islamic Azad University, Tehran, Iran. Adress: Simaye Iran Ave., Shahrake Ghods, Paiambare Azam Campus, Tehran, Iran. *E-mail*: Nas.Asghari@iauctb.ac.ir and nasimsana@yahoo.com.
** Doctor Southampton University, United Kingdom. Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran. *E-mail*: maths_ahmad@yahoo.com.
*** Doctor University of Sheffield, Sheffield, United Kingdom. Department of Mathematics and Computer Science, Kharazmi University, Tehran, Iran. *E-mail*: a_medghalchi@khu.ac.ir.

Resumo

Neste estudo discute-se o programa de desenvolvimento profissional que envolveu 15 professores de primeira a quinta série do sistema educacional iraniano. O pensamento funcional foi a peça central do trabalho cuja duração foi de 6 meses. Para acompanhar o processo de mudança dos professores e entender as trajetórias de desenvolvimento dos professores, utilizou-se como metodologia o chamado *concern-based adaptation model* (CBAM), cujo objetivo é avaliar a adaptação de indivíduos e grupos frente a determinadas demandas. Duas perguntas orientaram a investigação: 1. Como se dá a implementação de um programa de desenvolvimento profissional que pretende avaliar o impacto das preocupações dos professores, focando o pensamento funcional? 2. Quais alterações, resultaram do Programa? O estudo mostrou eficácia no processo de mudança de professores, tanto em termos das suas preocupações quanto relativas à utilização de inovações.

Palavras-chave: Processo de Mudança. Método CBAM. Professores das Séries Iniciais. Pensamento Funcional. Aritmética.

1 Introduction

In the past decade, the number of educators and researchers who believed that algebra should become part of the elementary education curriculum increased. "The work of NCTM Algebra Working Group (NCTM; MSEB, 1998) and the Early Algebra Group (KAPUT et al., 2008) shows the recent efforts in which researchers collaborate to find ways to integrate algebraic reasoning throughout Grades K-12" (JACOBS et al., 2007, p. 259). Many studies have been conducted on how students engage in algebraic thinking, but there is no systematic evidence concerning the effect of professional development programs on teachers' competencies. The lack of awareness concerning teachers' professional development in the area of functuional thinking and the need for engaging teachers in this area was emphasized. According to Blanton and Kaput (2005) "creating ability and robustness in teachers to develop children's algebraic reasoning requires a significant process of change in elementary teachers", because many elementary teachers see algebra as a set of rules and they need to "develop algebra eyes and ears" (p. 29).

The professional development program that we introduce in this paper was an innovation designed to support Grade 1-5 teachers' involvement for development of their functional thinking. This study was conducted in the Islamic Azad University, South Tehran Branch, in Iran. The study examined changes in teachers' thinking and practice in the result of their participation in the program. Two questions were investigated: 1.How does implementation of professional development program focused on functional thinking impact on teachers' concerns? 2. How did teachers' practice change as the result of implementation of innovation?

2 Literature

Principles and Standards for School Mathematics published by the National Council of Teachers of Mathematics (NCTM) (2000) state that instructional programs in all grades should enable students to understand patterns, relations, and functions. A number of different characterizations of algebra can be found in the mathematics education literature. One of the forms of algebraic reasoning is *functional thinking*. Blanton and Kaput (2005) proposed that generalized arithmetic and functional thinking offer rich entry points to the study of algebraic reasoning. Kaput (2008), in his analysis of algebra content, identified two core aspects of algebra: (A) Algebra as systematically symbolizing generalizations of regularities and constrains, (B) Algebra as syntactically guided reasoning and actions on generalizations expressed in conventional symbol systems, and stated that these two core aspects are embodied in three strands. Kaput (2008) identified one of these strands as "Algebra as the study of functions, relations, and joint variation" (p. 15). Therefore the construction and use of function is considered to be central to most mathematical investigations.

2.1 Functional thinking

Representational thinking along a particular line has been called functional thinking. Smith (2008) describes functional thinking as "representational thinking that focuses on the relationship between two (or more) varying quantities" (p. 143). Generalizing numerical patterns, for example, involves describing growth patterns or generalizations about sums of consecutive numbers. According to Carraher et al. (2000) "many of the difficulties that algebra and pre-algebra students experience are artifacts of their early mathematics instruction". He states that students' difficulties are accentuated and prolonged by the stark separation between arithmetic and algebra. He argued that this separation cannot be adequately handled by programs designed to ease the transition from arithmetic to algebra. Many of the notational tools used in arithmetic can be presented in new ways. Carraher et al. (2000) explores how this might be achieved with regard to the operations of addition, subtraction, multiplication, and division. Carraher et al. (2006) designed some proper activities and found that eight-and nine-year-old children not only can understand additive functions but also meaningfully use algebraic expression such as "$n \rightarrow n + 3$" and "$y = x + 3$". According to Chazzan (1996), *relation and transformation* is fundamental to the concept of function, a schema about how certain quantities relate, or are changed or transformed, to other quantities. The construction and use of function has been found to be notoriously difficult for most students at all levels of learning (CUOCO, 1995). From an epistemological perspective, Dubinsky and Harel (1992) suggest that when someone abstracts the notion of a function, there appears to be three main landmarks that occur, namely; functions as actions, function as processes, and functions as objects. Students see a function an action when they tend to it in terms of particular input and output. But as they chunk individual steps together into a coherent whole, the function becomes a process. Finally, when their focus changes from the calculations of outputs to the behavior of the function itself, they see it as an object.

2.2 Professional development of teachers

Effective professional development is considered to be the center of an educational reform. It is an area about which little is known. Policy makers, boards of education, legislators, funding agencies and taxpayers all want to know whether professional development makes a difference (GUSKEY, 1997). In his book, *Evaluating Professional Development*, Guskey (2000) states that professional development evaluation should focus on measuring its impacts on teachers in terms of change in the knowledge, skills, attitudes and beliefs of participating teachers in the program. In fact to become a good teacher, one should go through along process. One way to facilitate teachers' mathematical development is by deepening their mathematical understanding and changing their epistemological beliefs via professional development experiences (HILL; BALL, 2004; KAJANDER et al., 2006). During the study of the process of change in teachers, emphasis was given to the teachers' prejudices and beliefs. In teacher education programs, special attention has been given to the analysis of the beliefs that the teachers in training bring with them when they start out their professional career.

Kieran (1992) noted "there is scarcity of research emphasizing the role of the classroom teacher in algebra instruction" (p. 395). There also exists a need for more effective models of how teachers interpret the learning process (BALL, 1997). There are some definition for professional development that we are discussing here. Heideman, (1990) states that the professional development of teachers goes beyond a merely informative stage; it implies adaptation of the change with a view to changing teaching and learning activities, altering teacher attitudes and improving the academic results of students. The professional development of teachers is concerned with individual, professional and organizational needs. The professional development of teachers is the professional growth that teachers acquire as a result of their experience and also as a result of systematic analysis of his/her own practice (VILLEGAS-REIMERS, 2003).

2.3 Concerns-based adaptation model

The Concerns-Based Adaptation Model (CBAM) is a conceptual and methodological framework that measures, describes, explains, and predicts probable teacher behaviors in the change process resulting from implementation of curricular and instructional innovation. What CBAM actually does is provide a set of lenses through which one can view and understand the changing process. The CBAM is based on a number of assumptions about the change process:

1) Change is a process, not an event, it takes time to change.
2) Individuals must be the focus if changing is to be facilitated, and institutions will not change until their members change.
3) The changing process is an extremely personal experience and the way it is perceived by individuals has a huge influence in the outcome.
4) Individuals progress through various stages regarding their emotions and capabilities relating to the innovation.
5) The availability of a client-centered diagnostic/prescriptive model can enhance the individual's facilitation during staff development.
6) People responsible for the changing process must work in an adaptive and systematic way where progress needs to be constantly monitored (HALL; LOUCKS, 1978).

The three principal diagnostic dimensions of the CBAM are: *Stages of Concern (Soc)*, *Levels of Use (Lou)*, and *Innovation Configurations (Ic)*. These are tools of *CBAM* for measuring the change. The concept of *concern* has been described as follows: "Depending on individuals' personal make-up, knowledge, and experience, each person perceives and mentally contends with a given issue differently; thus there are different kinds of concerns" (HALL; HORD, 2001, p. 59). The *Soc*, based on assumptions 3 and 4 mentioned above, describes the progression of individual feelings, perceptions, and motivations that result from curricular and/or instructional change. The *Soc* reports an individual's concerns profiles at any point during an innovation (HALL; LOUCKS, 1978). CBAM describes adoption of innovation in terms of *levels of use*. It includes eight categories with corresponding behavioral indicators (HALL; HORD, 2001). The *Lou*, arising from assumptions 1 and 4, details eight distinct developmental levels in the use of an innovation. Observations and interviews are used to ascertain an individual's level of use of an innovation (HALL et al., 1975). The CBAM is a very well researched model which describes how people develop as they learn about an innovation and the stages of that process. Actually, the CBAM is a complex, multi-part system.

3 Materials and methods

Thirty teachers of grades 1-5 participated in the staff development program during the 2010-2011 school year extended to summer 2011. In this paper, we will report the concerns and behaviors of fifteen of those teachers. These teachers were themselves students in the master degree program in elementary education in the Islamic Azad University, South Tehran Branch, in Iran. They were all Iranian from Tehran State.

3.1 Procedure of staff development program

The structure of the staff development program was formed on the basis of four principles:

1. The concept of "algebraic thinking" as a way of thinking, especially focused on *functional thinking*;
2. The necessity of offering teachers *ongoing* and *over time* opportunities, based on the idea that the individual change is a *process not an event*;
3. Forming small *working groups* for active learning and promoting teachers' self-confidence and reducing mathematics anxiety, keeping in mind that teachers might be wonderful learners;
4. The application of *CBAM* as a methodology for describing, explaining, and predicting teachers' probable behaviors in the change process.

During the six-month instructional program, teachers attended two three-hour sessions a week including conferences and workshops. They were in touch with researchers online during the week. In some cases, additional lectures were organized. Functional thinking was categorized in six classes according to Blanton and Kaputs (2005) description of practices and some activities were chosen and designed in each category, as follows:

• *Category A*, Representing data as input-output, t-table: Participants modeled the input and output data using manipulatives, while trying to observe the rule that defines the relationship.

• *Category B*, Representing data graphically: This is a way for encoding information (graphically) that allows analysis of functional relationships.

• *Category C*, Representing data as an ordered pair.

• *Category D*, Finding functional relationships: This category includes instances in which it is to explore co-variation among quantities or recursive relationships and to develop a rule that describes those relationships.

• *Category E*, Predicting unknown states using known data: This category includes instances when one should make conjectures about what would happen for some unknown state based on analyzed data.

• *Category F*, Identifying and describing numerical and geometric patterns: This category involves identifying patterns in numbers that were generated geometrically; in addition, identifying patterns in the sequences of geometric shapes and in the sets of number sentences.

Initially, activities which concerned algebraic thinking were chosen so that various forms of algebraic thinking, especially functional thinking, were imployed in them (ASGHARI; SHAHVARANI; MEDGHALCHI, 2011). By the fourth week of the program, teachers began to get familiar with examples and activities that foster algebraic thinking. Activities were discussed in the sessions. Connection of each activity with aspects of algebraic thinking was recognized. Different activities that foster functional thinking explicitly were seperated. As the result of these sessions, examples and activities relating to functional thinking were sorted according to functional thinking categories and, therefore, teachers acquire a source of activities. They gradually gained the ability to recognize activities and started to gather activities by themselves from different sources.

Dreyfus (1991), in relation with how representations support learning, claims that the learning process proceeds through four states, namely, using one representation, using more than one representation in parallel, making links between parallel representations, and integrating representations and flexibly moving between them.

The instruction, focused on seeing *arithmetic as change* and function machines, were used (Figure 1) to generalize and formalize identified relations. Using function machine is a concrete approach for demonstrating relationship between different things, and it works as a means for introductory exploration of the concept of function.

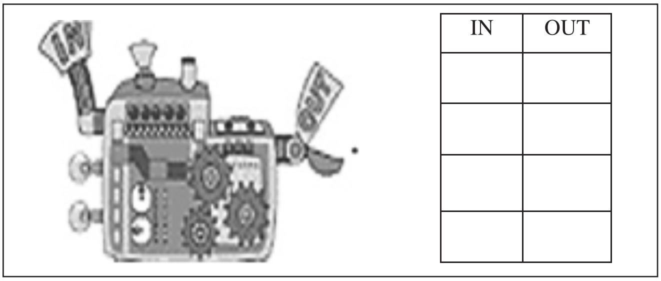


Figure 1 - Function machine and t-table to represent and record input-output values
Source: Developed by the authors

CBAM was used for managing and assessing the professional development program (HALL; HORD, 2001). To measure teachers' change processes, two instruments of CBAM were used, namely: Surveys of concerns (Soc) and Levels of use (Lou).

The first instrument, Soc, is a self-reporting instrument through a questionnaire (see Appendix) that allows users to rank their perceptions of concerns across seven categories according to three dimensions (Frame 1): self, task, and impact. The *self* dimension, with three stages (awareness, informational, and personal), reflects personal concerns. The *task* dimension refers to the concerns related to time management, for both planning and implementation; logistics, in terms of organization and delivery of the innovation; and content coverage, with regard to objects of development program. The impact dimension consists of three stages; the effect of the innovation on students, the efforts for collaborating with others, and refocusing for revision, which may lead to greater change (TUNKS; WELLER, 2009).

Dimension	Stage	Expression of Concern
Impact	6.Refocusing	Ideas that might work better
	5.Collaboration	Interaction of learners
	4.Consequence	The innovation had effect on learners
Task	3.Management	Spent time to prepare
Self	2.Personal	Affectted by the program
	1.Informational	Need to know more
	0.Awareness	Unconcerned

Frame 1 - Stages of concern
Source: Developed by the authors

The second instrument, Lou (see Frame 2), generates data through interviews and observations. The Lou describes adaptation of innovation. It includes 8 categories with corresponding behavioral indicators: level 0, non-use; level 1, orientation; level 2: preparation; level 3, mechanical; level 4, routine; level 5, refinement; level 6, integration; level 7, renewal (HALL; HORD, 2001). The data generated from observations and interviews were used to track the level of use of the innovation.

Levels of use	**Description of level**	**Behavioral indicators of level**
7	Renewal	The user is seeking more effective alternatives to the established use of the innovation
6	Integration	The user is making deliberate attempt to coordinate with others in using the innovation
5	Refinement	The user is making changes to previous established pattern to increase outcomes
4	Routine	The user is making few or no changes and has an established pattern of use
3	Mechanical	The user is making changes to better organize use of the innovation
2	Preparation	The user has definit plans to begin using the innovation
1	Orientation	The user is taking the initiative to learn more about innovation
0	Non-use	The user has no interest, is taking no action

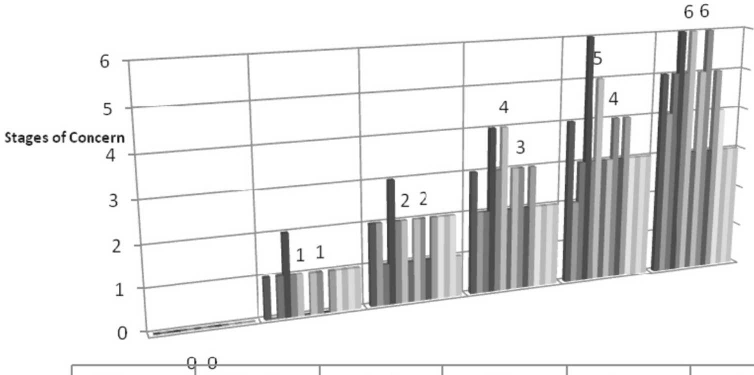
Frame 2 - Levels of use
Source: Developed by the authors

4 Results

Two questions were identified and data was collected by using the instruments of CBAM (Soc, Lou). Besides the informal evaluation, data generated from the Soc instrument were used for examining the first question and data generated from Lou instrument were used for examining the second question.

4.1 Question 1

The first research question was: How did implementation of the professional development program impact teachers' concerns? Teachers' concerns were measured using the Soc questionnaire and classroom observations check list for six months. The result of the measurements is shown in Figure 2 (table and graph). Figure 2 illustrates changes in concern stages (vertical axis) across time (horizontal axis). Teachers are denoted by T_is (i = 1, 2 …15). Levels of concern for each teacher were measured at the end of each month.



	February	March	April	May	June	July
T1	0	1	2	3	4	5
T2	0	0	2	2	2	4
T3	0	1	1	2	3	5
T4	0	2	3	4	6	6
T5	0	1	2	3	3	5
T6	0	1	2	4	5	6
T7	0	0	1	2	3	3
T8	0	1	2	3	3	5
T9	0	1	2	3	4	6
T10	0	0	1	2	3	3
T11	0	1	2	3	4	5
T12	0	1	2	2	3	4
T13	0	1	2	2	3	3
T14	0	1	2	2	3	3
T15	0	1	1	2	3	3

Figure 2 - Table and graph showing"Stages of concern for teachers"
Source: Developed by the authors

Figure 3 shows the frequency of teachers demonstrating each level of concern in two different times of the program: in the middle (April) and in the end of program (July).

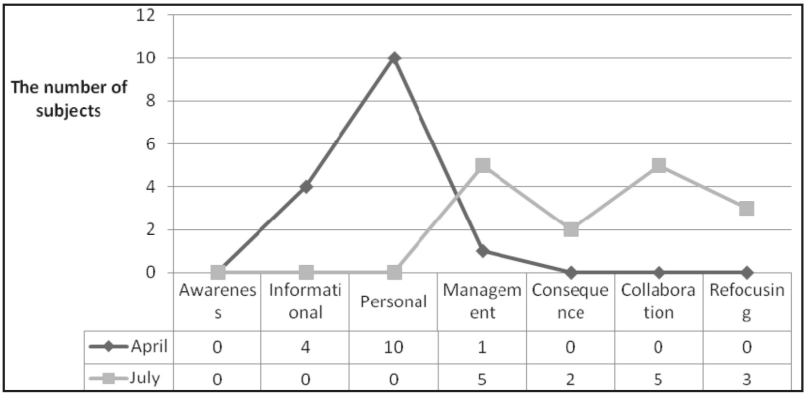


Figure 3 - Frequency of teachers in each stage in two months
Source: Developed by the authors

At the beginning of the program, since the algebra introduced to them was different from the algebra they were familiar to, teachers resisted to any change in their attitude and they were not hopeful about the usefulness of the program. But after passing one month they became interested in following the program. They began to be envolved in getting familiar with functional thinking. By April, ten of the teachers (T1, T2, T5, T6, T8, T9, T11, T12, T13, and T14) progressed into stage 2 (personal). Measurements of July showed more progress in all teachers. Some number of teachers progressed till stage 4, 5 and 6. Three groups of teachers were recognized: first group (A) included T7, T10, T13, T14, T15, progressed into stage 3 (Management), the second group (B) included T1, T2, T3, T5, T8, T11, T12, progressed into 4^{th} and 5^{th} stages (Consequence, collaboration) and a third group (C), that included T4, T6 and T9, progressed into stage 6.

As it was mentioned, at the beginning of the program, all teachers declared that the type of algebra introduced in the program was not the same algebra they had studied. Once we started to identify the concept of *function*, they were worried. This emotional stage was recognized as what we call *Mathematics anxiety*. They declared that this concept was difficult for them and believed it was even more difficult for students. They could not imagine how this concept could be taught to the students. To boost their confidence, some research papers were presented to them in order to show that students are capable of understanding the concept of function. According to the initial evaluations, teachers could not define the concept of function; also were unable to describe it using an example.

Two main priorities were set:

1. Teaching the concept of function and its various representations to the teachers;

2. Make teachers familiar with strategies for teaching function and fostering students' functional thinking.

At the beginning, definition of function and its various representation (set of ordered pair, formulating of the function, draw graphs, etc.) were discussed in the whole group. *Function machine* was used to teach function and to analyze situations as functions, in other words: functional approach. Index cards were other tool used. Teachers prepared one Index card for each lesson and categorized them systematically. Some of the teachers (T4, T6 and T9) began to prepare worksheets for each lesson. They prepared worksheets for teaching the *addition function* using function machine, and another worksheet for teaching *subtraction function* as reverse function of addition. Each of them performed as an *activity designer* during the program. Figure 4 illustrates T4's worksheet. At the end of the program, T4 designed an activity where two function machines were used and determined a formula for each one in order to teach combination of functions.

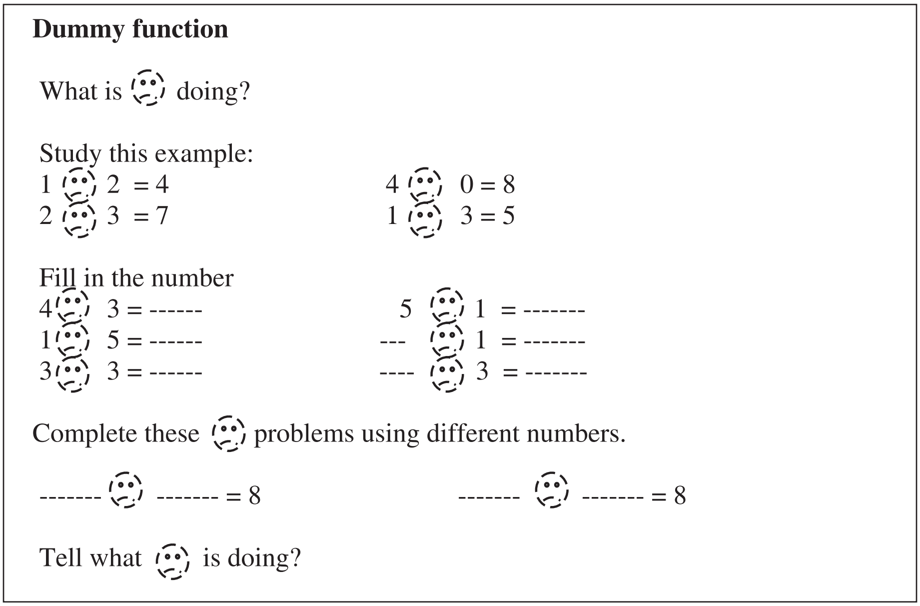


Figure 4 - The task that teacher T4 designed
Source: Developed by the authors

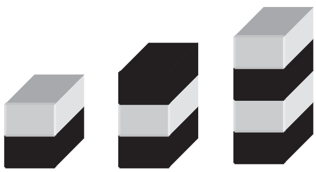
Teachers, with the help of instructors, categorized their activities in three groups:

1. Input value and function rule are known and the goal is determining the output;

2. Output value and function rule are known and the goal is determining input value;

3. Input and output values are known and the goal is to determine function rule.

At this stage, mathematical processes were introduced to teachers. One of these processes was *specializing and generalizing*. Teachers were required to bring some examples from daily lives into class and to discuss their specializing and generalizing in the group. All three groups of teachers showed active participation in this stage. In the next stage, introducing the numerical and geometrical patterns was the focus of the program; figure 5 is an example of the content of this stage (NCTM,2000).



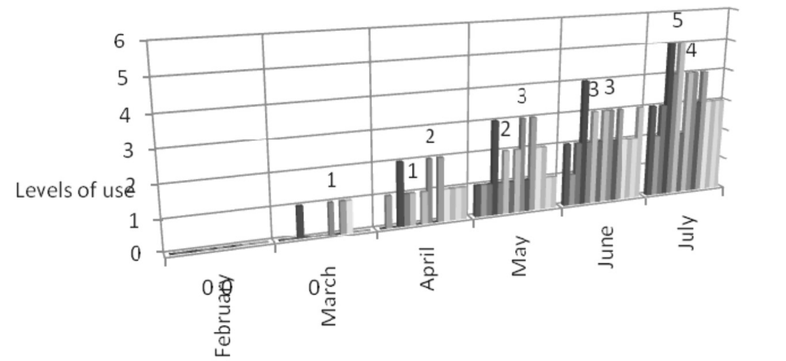
Number of cubes	Surface area in square units
1	6
2	10
3 14	
4 18	

Figure 5 - What is the surface area of each tower of cubes? As the towers get taller how does the surface area change?
Source: Developed by the authors

Teachers used t-table and noticed the number of cubes and surface area in square units, and tried to find a relationship between them. They described the pattern and tried to formulate the pattern. Group C of teachers often were successful in formulating the patterns, whereas group A and B had difficulty in formulating patterns. Teachers in Group A did not understand how we formulated the patterns even after representing the formula for the patterns. Gradually they began to find numerical and geometrical patterns using various resources, and began to communicate about these patterns in their small groups and then with their classmates.

4.2. Question 2

The results of the continous observations and monthly interviews were used to address the second question. Figure 6 illustrates table and graph relation to the level of use of the fifteen teachers during the six months of the program.



	February	March	April	May	June	July
T1	0	0	0	1	2	3
T2	0	0	1	1	1	2
T3	0	0	0	1	2	3
T4	0	1	2	3	4	5
T5	0	0	1	1	2	4
T6	0	0	1	2	3	5
T7	0	0	0	1	2	2
T8	0	0	1	2	3	4
T9	0	1	2	3	3	4
T10	0	0	0	1	2	3
T11	0	1	2	3	3	4
T12	0	1	1	2	2	3
T13	0	0	1	2	2	3
T14	0	0	1	1	2	3
T15	0	0	1	1	3	3

Figure 6 - Levels of use of teachers in 6 months
Source: Developed by the authors

The vertical axis shows level of use and the horizontal axis shows the timeline. By comparing Figure 2 and Figure 6, one realizes that there is a correlation between increasing teachers' Soc and teachers' Lou. T4, T6, T9, whose Soc moved from self to impact, had the highest Lou timeline.

5 Conclusion

This article explored a professional development program implemented with Functional thinking as the centerpiece of instruction. The ability to design activities was the specially focused in this program. Blanton and Kaput (2005) asserted that autonomy in task development is a key and a critical component in development of teachers, and this was also a goal of this study. Designing activities not only led to extend resources' domain but was also an *indicator of teachers' mastery* of subjects and processes defined as functional thinking. Teachers became proficient in finding, adopting, and designing the resources that led to developing algebraic reasoning. They come to construct links between the visual pattern and tables of values and to eventually integrate the representational systems, flexibily moving between them. As Dreyfus (1991) states, these linkage and integration of representations lead to abstraction of any mathematical concepts – in this case, function – and it is the sequential way of representation for supporting learning. Considering that resources in the field of functional thinking are limited, these types of activities could be used in professional development programs.

References

ASGHARI, N.; SHAHVARANI, A.; MEDGHALCHI, A. R. Preparing Elementary Teachers to Foster Functional Thinking. **Australian Journal of Basic and Applied Sciences**, Amman, Jordan, v. 5, n. 12, p. 485-489, 2011.

BALL, D. L. What do students know? Facing challenges of distance, context, and desire in trying to hear children. In: BIDDLE, B.; GOOD, T.; GOODSON, I. (Ed.). **International handbook of teachers and teaching**. 1997. p. 679-818. Vol. II.

BLANTON, M.; KAPUT, J. Developing elementary teachers' algebra eyes and ears. **Teaching Children Mathematics**, Reston, Va, n. 10, p. 70-77. 2003.

Blanton, M. L.; Kaput, J. J. Characterizing a Classroom Practice That Promotes Algebraic Reasoning. **Journal for Research in Mathematics Education**, Reston, Va., US, v. 36, n. 5, p. 412-446, Nov. 2005.

BLANTON, M. L.; KAPUT, J. J. Characterizing a classroom practice that promotes algebraic reasoning. **Journal for Research in Mathematics Education**, Reston, VA, v. 36, n. 5, p. 412-443, Jan. 2005.

CARRAHER, D. W.; SCHLIMANN, A. D.; BRIZUELA, B. M. Early Algebra, Early Arithmetic: treating operations as functions. In: NORTH AMERICAN CHAPTER OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION (PME-NA), 22nd, 2000, Tucson, AZ, **Proceedings…** Tucson, AZ: Ohio State University, 2000, p. 1-26. Available at: <http://earlyalgebra.terc.edu/our_papers/2003/Carraheretall_PMENA2000_PBS.pdf >. Accessed in: 18 Nov. 2013.

CARRAHER, D. W.; SCHLIEMANN, A. D.; BRIZUELA, B. M.; ERNEST, D. Arithmetic and algebra in early Mathematics Education, **Journal for research in Mathematics Education**, Reston, Va, v. 37, n. 2, p. 87-115, 2006.

CHAZAN, D. Algebra for All Students. **Journal of Mathematical Behavior**, Norwood, NJ, v. 15, n. 4, p. 455-477, 1996.

CUOCO, A. Computational media to support the learning and use of functions. In: DISESSA, A.; HOYLES, C.; NOSS, R. (Ed.). **Computers and exploratory learning**. Heidelberg: Springer, 1995. p. 79-108.

DREYFUS, T. Advanced mathematical thinking processes. In: TALL, D. (Ed.). **Advanced mathematical thinking**. Dordtrecht: KluwerAcademic Publishers, 1991. p. 25-41.

DUBINSKY, E.; HAREL, G. The nature of the process conception of function. In: DUBINSKY, E.; HAREL, G. (Ed.). **The concept of function–aspects of epistemology and pedagogy**. Washington, US: Mathematical Assn of Amer, 1992. p. 85-106.

GUSKEY, T. R. **Implementing Mastery Learning**, 2. ed. Belmont, CA: Wadsworth Publishing Company, 1997.

GUSKEY, T. R. **Evaluating Professional Development**. Thousand Oaks, CA: Corwin Press, 2000.

HALL, G.; HORD, S. **Implementing Change**: patterns, principles and potholes. Boston, MA: Allyn and Bacon, 2001.

HALL, G.; LOUCKS, S. Teacher concerns as a basis for facilitating and personalizing staff development. **Teachers College Record**, New York, v. 80, n. 1, p. 36-53, 1978.

HALL, G.; LOUCKS, S.; RUTHERFORD, W.; NEWLOVE, B. Levels of use of the innovation: A framework for analyzing innovation adoption. **Journal of Teacher Education**, Washington, US, v. 26, n. 1, p. 52-56, 1975. Doi: 10.1177/002248715026001l4.1975.

HEIDEMAN, C. Introduction to staff development. In: BURKE, P. et al. (Ed.). **Programming for staff development**. London: Falmer Press, 1990, p. 3-9.

HILL, H.; BALL, D. Learning mathematics for teaching: Results from California's mathematics professional development institutes. **Journal for Research in Mathematics Education**, Reston, Va., US, v. 35, n. 5, p. 330-351, 2004.

JACOBS, V. R.; FRANKE, M. L.; CARPENTER, T. P.; LEVI, L.; BATTEY, D. Professional Development Focused on Children's Algebraic Reasoning in Elementary School. **Journal for Research in Mathematics Education**, Reston, Va., US, v. 38, n. 3, p. 258-288, 2007.

KAJANDER, A.; KEENE, A. J.; SIDDO, R.; ZERPA, C. Effects of Professional Development on Intermediate Teachers' Knowledge and Beliefs Related to Mathematics. 2006. **Final Report of the PRISM-NWO Project** – Lakehead University, Ontario Ministry of Education, 2006.

KAPUT, J. J.; CARRAHER, D. W.; BLANTON, M. L. **Algebra in the Early Grades**. London: Taylor and Francis Group, 2008.

KIERAN, C. The learning and teaching of school algebra. In: GROUWS, D. (Ed.). **Handbook of Research on Mathematics Teaching and Learning**. New York: Macmillan Publishing Company, 1992. p. 390-419.

KIERAN, C. Algebraic Thinking in the Elementary Grades: What Is It? **The Mathematics Educator**, Athens, GA, v. 8, n. 1, p. 139-151, 2004.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM). **Principles and standards for school mathematics**. Reston, VA: NCTM, 2000.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM) and MATHEMATICS SCIENCES EDUCATION BOARD (MSEB). (Ed.). The nature and role of algebra in the K-14 curriculum. 1998. **Proceedings of a national symposium**. Washington, DC: National Academy Press, 2000.

SMITH, E. Representational thinking as a framework for introducing functions in the elementary curriculum. In: KAPUT, J. J. (Ed.). **Algebra in the Early Grades**. New York: Routledge Publishers, 2008. p. 133-160.

TUNKS, J.; WELLER. K. Changing practice, changing minds, from arithmetical to algebraic thinking: an application of the concerns-based adoption model (CBAM), **Educational Studies in Mathematics**, Dordrecht, v. 72, p. 161-183. 2009. DOI: 10.1007/s10649-009-9189-x.2009.

VILLEGAS-REIMERS, E. **Teacher Professional Development**: an international review of literature. Paris: UNESCO/International Institute for Educational Planning, 2003.

WARREN, E.; COOPERR, T. J. Theory and practice: developing an algebra syllabus for Years P-7. In: ANNUAL CONFERENCE OF THE INTERNATIONAL COMMISSION ON MATHEMATICS INSTRUCTION, 12th, 2001, Melbourne. **Proceedings…** Melbourne: CHICK, H.; STACEY, K.; VINCENT, J.; VINCENT, J. (Ed.), 2001. p. 641-648. Available at: <http://repository.unimelb.edu.au/10187/2812>. Accessed in: 18 Nov. 2013.

Submetido em Junho de 2013.
Aprovado em Outubro de 2013.

Appendix

Survey of Concerns

Name ______________________________

Read each statement and circle the number that best represents your perception about the functional thinking project. Use the rating scale provided below as your guide for representing your perception.

0	1 2	3 4	56 7
Irrelevant	Not true of me now	somewhat true of me now	Very true of me now

1. I am concerned about students' attitudes toward algebraic thinking.
   0 1 2 3 4 5 6 7
2. I now know of some other approaches that might work better in teaching functional thinking.
   0 1 2 3 4 5 6 7
3. I don't even know what this functional thinking project is about.
   0 1 2 3 4 5 6 7
4. I am concerned about not having enough time to organize myself each day.
   0 1 2 3 4 5 6 7
5. I would like to help other faculty in their incorporation of functional thinking into math teaching.
   0 1 2 3 4 5 6 7
6. I have a very limited knowledge of functional thinking.
   0 1 2 3 4 5 6 7
7. I would like to know the effect of the use of functional thinking in my math teaching on my professional status.
   0 1 2 3 4 5 6 7
8. I am concerned about conflict between my interests in functional thinking and my responsibilities to teach all math ideas.
   0 1 2 3 4 5 6 7
9. I am concerned about my revising my use of functional thinking.
   0 1 2 3 4 5 6 7
10. I would like to develop working relationships with both our faculty and outside faculty using functional thinking.
    0 1 2 3 4 5 6 7
11. I am concerned about how using algebraic thinking affects students.
    0 1 2 3 4 5 6 7
12. I am not concerned about functional thinking.
    0 1 2 3 4 5 6 7
13. I would like to know who will make the decisions in how we use algebraic thinking.
    0 1 2 3 4 5 6 7
14. I would like to discuss the possibility of using functional thinking.
    0 1 2 3 4 5 6 7
15. I would like to know what resources are available if we decide to use functional thinking.
    0 1 2 3 4 5 6 7
16. I am concerned about my inability to manage all that is required to include functional thinking in my mathematics teaching.
    0 1 2 3 4 5 6 7

17. I would like to know how my teaching and administration is supposed to change.
0 1 2 3 4 5 6 7
18. I would like to familiarize other departments or persons with the progress of incorporating functional thinking into my mathematics instruction.
0 1 2 3 4 5 6 7
19. I am concerned about evaluating my impact on students.
0 1 2 3 4 5 6 7
20. I would like to revise the instructional approach to algebraic thinking.
0 1 2 3 4 5 6 7
21. I am completely occupied with other things.
0 1 2 3 4 5 6 7
22. I would like to modify our use of algebraic thinking based on the experiences of the students.
0 1 2 3 4 5 6 7
23. Although I don't know about algebraic thinking, I am concerned about other things in the area.
0 1 2 3 4 5 6 7
24. I would like to excite my students about their part in learning about algebraic thinking.
0 1 2 3 4 5 6 7
25. I am concerned about my time spent working with nonacademic problems related to algebraic thinking.
0 1 2 3 4 5 6 7
26. I would like to know what the use of algebraic thinking will require in the immediate future.
0 1 2 3 4 5 6 7
27. I would like to coordinate my efforts with others to maximize the effect of using algebraic thinking in my mathematics teaching.
0 1 2 3 4 5 6 7
28. I would like to have more information on time and energy commitments required to introduce algebraic thinking into my mathematics teaching.
0 1 2 3 4 5 6 7
29. I would like to know what other teachers are doing with algebraic thinking.
0 1 2 3 4 5 6 7
30. At this time I am not interested in learning about algebraic thinking and how to introduce it in my mathematics teaching.
0 1 2 3 4 5 6 7
31. I would like to determine how to supplement, enhance, or replace the way I use algebraic thinking in my teaching.
0 1 2 3 4 5 6 7
32. I would like to use feedback from students to change the way I incorporate algebraic thinking into my mathematics teaching.
0 1 2 3 4 5 6 7
33. I would like to know how my role will change when I am incorporating algebraic thinking.
0 1 2 3 4 5 6 7
34. Coordination of tasks and people is taking too much of my time.
0 1 2 3 4 5 6 7
35. I would like to know how incorporating algebraic thinking is better than the way we teach mathematics now.
0 1 2 3 4 5 6 7