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Problemas matemáticos verbales de una operación, "cambiar" y "comparar": un estudio de los movimientos oculares

Andreas Moutsios-Rentzos[1]

y

Panagiotis J. Stamatis[2]

[1] Department of Mathematics, University of Athens, Athens, Greece
[2] Department of Sciences of Preschool Education and Educational Design (TEPAES), University of the Aegean, Rhodes, Greece

Grecia

Correspondencia: Andreas Moutsios-Rentzos. Filolaou 74, 11633, Athens, Greece. E-mail: moutsiosrent@math.uoa.gr

Resumen

Introducción. En este estudio, nos centramos en la relación entre pensamiento matemático y sus movimientos oculares no-mecánicos, identificados con el propósito de obtener una comprensió profundar de los procesos de razonamiento de los estudiantes y para investigar la viabilidad de incorporar la información de movimiento ocular en la instrucción cotidiana.

Método. Se trata de un estudio cuantitativo. Participaron 38 estudiantes de primer grado (6 años). Se les presentaron verbalmente presentó seis palabra en un problema aritmético: tres problemas de "cambio" y tres problemas de “comparación”. Los problemas fueron elegidos para estar dentro de las habilidades matemáticas de los estudiantes.

Resultados. Los resultados de este estudio parecen validar la técnica de recolección de datos no mecánico. Además, se encontraron diferencias en los movimientos colares hacia la derecha (lo que sugiere la actividad del hemisferio izquierdo) y la propagación movimientos oculares (que sugiere la actividad de ambos hemisferios) en línea con la literatura. Se encontró que los problemas más desafiantes estaban vinculado con una difusión más amplia de movimientos oculares y duranter más tiempo. Por otra parte, aunque los niños parecen ser más rápidos que las niñas en las tareas más fáciles, no se encontraron diferencias estadísticas en las tareas más difíciles.

Discusión. La técnica "más suave" utilizada era lo suficientemente sensible para estar de acuerdo con la literatura existente y al mismo tiempo, ayudar en la obtención de una comprensión más profunda de 'razonamiento acerca de los problemas comparer de “cambio” y de “comparación”.. Por ejemplo, las diferencias de género denunciados -en línea con la más amplia evidencia de la investigación de la educación y neuropsicología- sugieren indicacios cualitativos de diferentes procesos de pensamiento involucrados, que pueden estar relacionados con diferentes disposiciones de pensamiento relacionadas con el género. En consecuencia, se postuló que la técnica adoptada era está en línea con la evidencia de la investigación relevante, ofreciendo al mismo discernimiento en los complejos procesos involucrados, permitiendo así una mayor investigación que se llevó a cabo con el fin de investigar los beneficios instruccionales de la incorporación de tales técnicas "blandas" movimiento de movimieno ocular para identificar procesos de pensamiento en las prácticas de enseñanza cotidianas.

Palabras clave: problemas aritméticos orales, movimientos oculares, comunicación no verbal, razonamiento

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One-step ‘change’ and ‘compare’ word problems: Focusing on eye-movements

Abstract

Introduction. In this study, we focus on the relationship between the students’ mathematical thinking and their non-mechanically identified eye-movements with the purpose to gain deeper understanding about the students’ reasoning processes and to investigate the feasibility of incorporating eye-movement information in everyday pedagogy.

Method. This is a quantitative study. Thirty-eight (N=38) Grade 1 (6 years old) students were verbally presented with six word arithmetic problem: three problems of ‘change’ and three problems of ‘comparison’. The problems were chosen to be within the students’ mathematical abilities.

Results. The findings of this study appeared to validate the non-mechanical data collection technique. Furthermore, differentiations were found in the students’ rightwards eye-movements (suggesting the activity of the left hemisphere) and the eye-movements spread (suggesting the activity of both hemispheres) in line with the literature. The more challenging problems were found to be linked with a wider spread of eye-movements and to be more time consuming. Moreover, though boys appeared to be faster than the girls in the easier tasks, no statistical differences were found in the more challenging tasks.

Discussion. The adopted ‘softer’ technique was sensitive enough to be in accordance with the existing literature and at the same time helped in gaining deeper understanding in the students’ reasoning about ‘change’ and ‘compare’ problems. For example, the reported gender differences –in line with broader research evidence from education and neurophysiology– are hypothesised to indicate of qualitatively different thinking processes involved, which may be related to different gender-related thinking dispositions. Consequently, it is posited that the adopted technique was is in line with the relevant research evidence, offering at the same time insight in the complex processes involved, thus allowing for further research to be conducted in order to investigate the pedagogical benefits of the incorporation of such ‘softer ‘eye-movement identification techniques in everyday pedagogical practices.

Keywords: word arithmetic problems, eye-movements, non-verbal communication, reasoning

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Introduction

Mathematics educators have stressed the importance of both verbal and non-verbal communication in the students' thinking about mathematics (Cockburn, 2007; Morgan, 2006; Gorgorió, 1998; Tatsis & Moutsios-Rentzos, 2013). Regarding non-verbal communication, researchers have focussed on the students' eye-movements (Andrà et al, 2009), their gestures (Arzarello, Paola, Robutti, & Sabena, 2009), their facial expressions (Moutsios-Rentzos & Kalozoumi-Paizi, 2014) and their broader somatic experiences (Moutsios-Rentzos, Spyrou & Peteinara, 2014). Moreover, the students' reasoning with mathematical problems has been investigated from a plethora of perspectives and mathematical contents (including symbolic and word problems, the students' strategies and the interplay amongst the different representational systems involved; De Corte & Verschaffel, 1993; Geary, 1994; Mayer & Hegarty, 1996; Siegler & Shrager, 1984; Vergnaud, 1982).

Drawing upon neurophysiological evidence and upon the broader eye-movement research, left hemisphere activity has been found to be linked with logico-mathematical reasoning and problem-solving (Bear, Connors & Paradiso, 2007), as well as with language-related functions (including speech, reading, writing) and the processing of acoustic stimuli and abstract information (Gazzaniga, Ivry & Mangun, 2009). Though, Joseph (2011) argues that non-verbal communication is predominantly controlled by the right hemisphere, eye movements have been linked with both left and right hemisphere brain activity (Garrett, 2008; Gluck, Mercado & Myers, 2007; Smith & Kosslyn, 2007). Stamatis (2011) discusses the pedagogical usefulness of eye movement information in everyday practice, as the teachers may inform their synchronous assessment of their teaching and take appropriate action with respect, for example, task presentation and the allowed response time.

Following these, in a previous research project (Moutsios-Rentzos & Stamatis, 2013) we focussed on the eye-movements of forty Grade 3 students when they were verbally presented with simple arithmetic problems, considering the effect of the verbal information load of each problem and the type of the operation included in the problem (addition or subtraction) on the students' reasoning. Importantly, drawing upon a methodology proposed by Babad (2005), the collected data were based on *human identification abilities* (and *not* on an *eye-tracking device*), with the purpose to investigate the reliability and validity of such data and,

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thus, to allow the investigation of the pedagogical implementation of eye-movements information within everyday teaching along with other non-verbal communication pieces of information (including gestures, posture and facial expressions). Following the findings of our previous project, in this study we concentrate in one-step word mathematical ‘change’ and ‘compare’ problems (Riley, Greeno & Heller, 1983), addressing the fundamental question: What is the nature of the relationship between the non-mechanically identified eye movements of primary school 6-year old students and their thinking when they deal with arithmetic word problems?

Mathematical thinking in one-step addition and subtraction word problems

Simple addition and subtraction problems

The students’ thinking about simple arithmetic problems has been widely researched (Bebout, 1990; Carpenter, Moser & Romberg, 1982; Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Christou & Philippou, 1998; De Corte & Verschaffel, 1993; Fuson & Briars, 1990; Kamii, Lewis & Kirkland, 2001; Nesher, Greeno, & Riley, 1982; Siegler & Booth, 2004; Vergnaud, 1982). Moreover, researchers have stressed that the students’ ability to successfully cope with one-strep arithmetic word problems is affected by various factors, including the wording of the problems and the way information is presented, the students’ familiarity with mathematical language, their ability to execute an operation and their short-term memory (Geary, 1994; López, 2014; Reed, 1999; Riley, Greeno & Heller, 1983; Stern, 1993).

Word arithmetic problems have been categorised based on the semantic relationships amongst the quantities involved. Carpenter, Hiebert and Moser (1981) noted that the quantities involved in one-step word problems may be in dynamic or static relationships and they may be in ‘inclusion relationships’ or not. Greeno identified three categories of one-step world addition and subtraction problems: *change*, *combine* and *compare* (Riley, Greeno & Heller, 1983). Greeno’s categorisation roughly corresponds or is the basis of other schemes, whilst is the theoretical framework adopted by most researchers (Verschaffel, Greer & De Corte, 2007). Consequently, this scheme was adopted for the purposes of this study.

A ‘change’ problem refers to dynamic situations within which a transformation (*change*) is applied to the initial quantity (*start*), thus resulting to the final quantity (*result*). For example, a change problem with the result quantity being the unknown: ‘Nick has 4 pencils [‘start’; known]. Then George gave him 5 more pencils [‘change’; known]. How many pencils does Nick have now [‘result’; unknown]?’

1) A ‘compare’ problem describes the comparison of one quantity (*referent*) with another (*compared*), which results to a quantity representing their difference (*difference*). For example, a compare problem with the ‘difference’ quantity being the unknown: ‘Lea has 5 pens [‘referent’; known]. John has 3 pens [‘compared’; known]. How many more pens does Lea have [‘difference’; unknown]?’

2) A ‘combine’ problem refers to a static relationship between two quantities (*subset 1* and *subset 2*) that are combined in a set (*combine value*). For example, a combine problem with one of the combined quantities being the unknown: ‘Anna and Maria have 7 stickers altogether [‘combine value’; known]. Maria has 2 stickers [‘subset 2’; known]. How many stickers does Anna have [‘subset 3’; known]?’

Greeno’s basic categorisation scheme can be further elaborated into a 14-category scheme (Riley, Greeno & Heller, 1983) depending on which quantity is the unknown, the direction of change (increase or decrease) and the nature of the comparison (more or less). Broader schemes, such as the four-category scheme proposed by Carpenter and his colleagues (Carpenter, Hiebert & Moser, 1981; Carpenter & Moser, 1982) or Vergnaud’s (1982) six-category scheme roughly corresponds and expands Greeno’s.

This study

The arithmetic problems

In this study, we extend this line of research to investigate the students’ thinking about ‘change’ and ‘compare’ word problems, as indicated by their eye movements.

Six word problems were chosen including three word ‘change’ problems and three ‘comparison’ word problems (see Table 1). The three ‘change’ problems described an *increase* of the initial quantity with the unknown quantity being the ‘start’ (initial quantity), the ‘change’ (change quantity) or ‘result’ (final quantity). The three ‘comparison’ problems described a ‘more than’ relationship referring to the ‘compared’ (the initial quantity that is being compared), the ‘referent’ (the quantity with which the comparison is made) or the ‘difference’ (the difference between the ‘compared’ and the ‘referent’).

Table 1. **The arithmetic problems included in the study**

Abbreviated title	**Description**	**Unknown quantity**
ChangeRes	Peter has 4 apples. Anna gave him 2 more apples. How many apples does Peter have now?	Result
ChangeCha	Nick has 5 apples. Maria gave him some more apples. Now Nick has 7 apples. How many apples did Maria give to Nick?	Change
ChangeSta	Lea had some apples. Kostas gave her 2 more apples. Now Lea has 6 apples. How many apples did Lea have in the beginning?	Start
CompareCom	John has 4 apples. Lara has 2 apples more than John. How many apples does Lara have?	Compared
CompareDif	Fiona has 8 apples. Peter has 6 apples. How many apples does Fiona have more than Peter?	Difference
CompareRef	Alex has 8 apples. He has 2 more apples than Elena. How many apples does Elena have?	Referent

Considering that in this study we focus on Grade 1 primary school students (see *Participants*), all six problems described either an increase of ‘2’ or a ‘more than’ relationship of ‘2’ in order to be within the students’ mathematical abilities (as suggested by the curriculum). Furthermore, drawing upon our previous research findings (Moutsios-Rentzos & Stamatis, 2013), in this study the problems contained only necessary information to minimise the effect of information load to the students’ reasoning.

Consequently, in this study, *we concentrated in word ‘change’ and ‘comparison’ problems that describe an increase or a ‘more than’ relationship of ‘2, containing only ‘necessary information’.*

Expectations: problem type, response time, response correctness, eye-movements, gender

It has been suggested that ‘change’ problems, would be less challenging than ‘compare’ problems (Carpenter, Hiebert & Moser, 1981; Riley, 1981; Riley, Greeno & Heller, 1983). Furthermore, the ‘compare’ problems with the ‘compared’ being the unknown are expected to be dealt more successfully than those with the ‘referent’ being the unknown (Hiebert, 1981; Stern, 1993; Vergnaud, 1982). Moreover, synthesising the findings reported by

Riley, Greeno and Helle (1983) and by Dellarosa, Weimer and Kintsch (1985), the challenging 'change' problems may be more demanding than the challenging 'compare' problems.

Following these, we expected that:

I. The less demanding 'change' problems would be less challenging (in terms of correctness, time spent to answer and the students' eye movements) than the less demanding 'compare' problems. Thus, the 'change' problems with the unknown quantity being the 'change' (ChangeCha) and the 'result' (ChangeRes) would be the least challenging problems for the students.

II. The 'compare' problems with the unknown quantity being the 'difference' (CompareDif) and the 'compared' (CompareCom) would be less challenging than CompareRef.

III. The 'change' problem with the unknown quantity being the 'start' (ChangeSta) would be more challenging than the 'compare' problem with the unknown being the 'referent' (CompareRef).

Drawing upon our discussion in the previous section about the eye-mind hypothesis and the research findings linking brain activity with cognitive process, as well as the findings of our previous study (Moutsios-Rentzos & Stamatis, 2013), we expected that:

IV. The dominant eye-movements would be right or rightwards.

V. The most demanding problems would be linked with a wider eye-movement spread.

Moreover, based on the mixed results about gender differences in the children's ability to answer simple word problems with respect to response correctness, response time, as well as the chosen strategy (Desoete, 2009; Halpern, 2013; Royer, Tronsky, Chan, Jackson & Marchant III, 1999; Zheng, 2007), in this study we investigated the role of gender. For example, qualitatively different strategies were found with the boys being more likely to attempt to retrieve relevant information from memory and the girls to employ fingers or counters (Carr & Jessup, 1997). Royer et al (1999) suggested that boys would be faster than the girls in memory retrieval related tasks, which may result in the boys' higher performance. Nevertheless, girls have been found to outperform boys in computational arithmetic (Halpern, 2013). Furthermore, girls have been reported to employ a more cautious and time consuming approach

(Goldstein, Haldane & Mitchell, 1990), which may affect their performance. Fennema, Carpenter, Jacob, Frank and Levi (1998) suggested that boys and girls employ qualitatively different strategies, with the girls tending to employ more concrete strategies and the boys to use more abstract strategies. Furthermore, evidence from neurophysiological research suggest that gender differences exist in the humans' arithmetic reasoning with females appearing to employ broader areas of the brain (as shown in the electroencephalography; EEG) very early in the reasoning process (Skandries, Reik & Kunze, 1999). Following these mixed findings, we expected that:

VI. No gender differences would be found with respect to response correctness.
VII. Gender differences would be found with respect to the students' eye movements as indicators of different employed mental strategies.
VIII. Boys' response time would be shorter than the girls' response time.

Method

Participants

This study was conducted with thirty-eight 6-year old students (N=38; 18 girls and 20 boys) at May 2013 nearing the end of the school year. All the participants attended two classes of the Grade 1 of a primary school. The consent of the children's parents was obtained.

Both boys and girls were included in order to delineate whether *gender* would affect the identified relationships. Bearing in mind the fact that the participants' handedness has been linked with cerebral lateralisation (Haken, 2008), in this study we considered only *right-handed students* to facilitate our linking their eye-movements with specific hemisphere activity, such as linking the left hemisphere activity with right body activity and thus right eye-movements (Glannon, 2011). Furthermore, Smith, Jussim, and Eccles (1999) reported that the students' mathematics attainment as indicated by the teacher has been linked with the participant's performance in dealing with mathematical tasks. Hence, we chose to concentrate in students with *above average reported mathematical attainment*, as indicated by their teachers, in order to ensure that the under investigation situation will be mostly about students reasoning (since they possess the mathematical knowledge required), which would allow our identifying factors affecting their answers that go beyond mathematical attainment. Moreover, we concentrated in the students that *actually provided an answer* (correct or not) to the given problems, because we aimed to record the students' eye-movements linked with a specific answer, thus indicating their reasoning about it. Overall, we focus on 6 years old, right-handed, primary school students, with above average reported mathematical attainment who provided some answer to the given problems.

Procedure

Drawing upon theories and research findings (Ekman & Friesen, 2003. De Vito, 1988) which focus on issues related to eye-movements and, furthermore, on Babad (2005), the data collection focussed on the students' *reasoning* when listening, thinking about and answering each problem. Each of the six problems was uttered by the students' teacher in a clear steady voice, medium speed and good enunciation. The teacher was instructed to re-state the problem if asked by the student, as many times as the student would ask (except for when it was evident that the students could not answer the problem, in which case she would move on to the next problem and the data from that effort would not be included in the study). The students did not use any blocks when reasoning about the questions. Whilst the teacher was uttering the problems, a member of the research focussed on noting *eye-movement information* (referring to the direction of the participants' eye movements; right, left, up, down and their combinations) and another member recorded the participants' *reasoning time* (referring to the time that each participant spent to think about the answer until they uttered the answer). Both reasoning time and eye-movement information was noted in a log especially designed for the purposes of this study. The data collection process was completed in three consecutive days (lasting overall around 230 minutes) in a well-lit, quiet room to facilitate the eye-movement identification and the minimum of interruptions.

At this point, we wish to stress that this eye-movement data collection method is in contrast to using an admittedly more accurate eye-tracking device. This was chosen deliberately, because in this study we are interested predominantly to investigate the feasibility of incorporating eye-movement information in everyday pedagogy. Such information cannot be available during class through devices, but through each teacher's perception. We acknowledge of the importance of eye-tracking data in mathematics education, for example, in order to delineate the complex mental processes involved in mathematical reasoning. Nevertheless, we argue that 'softer' techniques, such as the one adopted in this study –less precise as they

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may be than eye-tracking devices– allow to investigate the pragmatics of utilising the eye-movement research findings in everyday pedagogical practices. Furthermore, the validation of such techniques would allow the incorporation of the identification of eye-movement techniques combined with the identification of other non-verbal information (such as facial expressions) that implicitly or explicitly are considered in everyday practices, would help in more validly identifying the students' thinking (Simonds & Cooper, 2011).

Data Analyses

The quantitative data analysis was conducted with SPSS 17 (SPSS, Inc., Chicago, IL). The collected data required the implementation of non-parametric statistical tests (Sheshkin, 2004). For the comparisons of two different groups the Mann-Whitney *U* test was used (ordinal data) or the Fisher's exact test (categorical data). For the comparison of nominal data of the same group in two or more occasions we employed, respectively, the McNemar exact change test (with Bonferroni corrections applied) and the Cochran's Q test. Finally, for the identification of change in two or more than two ordinal variables, we considered respectively the Wilcoxon's signed rank test or Friedman's ANOVA.

Results

Response time and response correctness

First, we investigated whether or not there were any statistically significant differences with respect the response time and the correctness of the responses. Focussing on the response time, it appears that questions ChangeRes and ChangeCha are the least time-consuming, followed by CompareCom, ChangeSta and CompareDif, whilst CompareRef is the most time-consuming for the students (see Table 2). Friedman's ANOVA revealed that there was statistically significant difference in the students' reasoning time with respect to the question answered ($\chi^2(5) = 53.2$, $p<0.001$). We followed these results with Wilcoxon signed rank tests in order to 'situate' the identified difference (with Bonferroni corrections applied). The results of the post hoc analyses revealed that ChangeRes and ChangeCha were found to be statistically significantly less time consuming than *all* the other questions (for ChangeSta respectively $T = 371.0$, $z = -1.173$, $p<0.001$ and $T = 328.0$, $z = -3.748$, $p<0.001$; for CompareCom respectively $T = 356.0$, $z = -4.014$, $p<0.001$ and $T = 276.5$, $z = -2.567$, $p=0.009$; for CompareDif respectively $T = 342.5$, $z = -4.185$, $p<0.001$ and $T = 280.0$, $z = -3.162$, $p=0.001$; for CompareRef respectively T = 318.0, z = -2.691, $p<0.001$ and T = 325.0, z = -3.798, $p<0.001$). Moreover, CompareRef was statistically significantly more time consuming than CompareCom (T = 62.5, z = -2.691, p=0.006).

Regarding the correctness of the students' responses, all the students answered ChangeRes correctly. For the rest of the questions, ChangeCha and CompareCom were the easiest for the students to answer correctly, followed by CompareRef, whilst ChangeSta and CompareDif appeared to be the most difficult for the students. Cochran's Q test confirmed that there was statistically significant difference in the students' response correctness linked with the question answered ($\chi^2(5) = 19.4$, p=0.001). We followed these results with McNemar's exact change tests in order to 'situate' the identified difference (with Bonferroni corrections applied). The results of the post hoc analyses revealed that the only statistically significantly differences were between the perfect score in ChangeRes and the lower scores in ChangeSta (p=0.008) and in CompareDif (p=0.016).

Furthermore, we investigated whether there were any gender-related differences. In general, boys *appeared* to outperform girls in speed and correctness (see Table 2). Nevertheless, the Mann-Whitney test and the Fisher's exact test suggested that the only statistically significant differences were in the boys' faster responses in ChangeRes and ChangeCha (see Table 2). Bearing in mind that those problems were the least challenging in terms of response time and correctness for the students, it appears that in the more trivial of the given questions the boys are faster, which may be related to a faster retrieval of the relevant information (see also Royer et al., 1999), the effect of which diminishes when the cognitive strain increases (the gender differences in the two most demanding questions appear to almost disappear or even to reverse).

***Table 2*. The participants' correct responses and response time.**

Question	**Whole**		**Boys**		**Girls**		**Boys vs Girls[5]**			
	Time[1,2]	**Corr[3,4]**	**Time**	**Corr**	**Time**	**Corr**	**Time[5]**			**Corr[6]**
							U	***P***	***r***	***P***
ChangeRes	4.1±4.3	28	2.9±4.4	15	5.5±4.0	13	38.0	0.004	-0.53	-[7]
	[1-18]	(100%)	[1-18]	100%	[1-14]	100%				
ChangeCha	7.4±11.5	26	2.9±2.1	15	12.6±15.3	11	39.5	0.006	-0.51	0.206
	[1-55]	(93%)	[1-8]	100%	[1-55]	85%				
ChangeSta	22.8±17.3	20	19.6±17.9	12	26.4±16.6	8	71.0	0.230	-0.23	0.410
	[1-61]	(71%)	[1-61]	80%	[4-51]	62%				
CompareCom	19.0±19.1	25	15.9±16.5	13	22.6±19.9	12	71.0	0.230	-0.23	1.000
	[1-60]	(89%)	[1-50]	87%	[2-60]	92%				
CompareDif	23.8±21.4	19	20.0±20.2	11	28.1±22.7	8	71.5	0.239	-0.23	0.689
	[1-18]	(68%)	[1-70]	73%	[3-76]	62%				
CompareRef	30.3±23.7	23	28.9±21.6	12	32.0±26.6	11	94.5	0.901	-0.03	1.000
	[1-85]	(82%)	[1-60]	80%	[2-85]	85%				

[1]: 'Time' refers to the time that the students spent in order to respond to the question, *excluding* the time that the teacher was uttering the question.
[2]: Time is described as 'Mean ± SD [Range]'.
[3]: 'Corr' refers to the number of correct response that the participants gave to a question.
[4]: 'Corr' is described as 'Frequency (Valid Percent)'.
[5]: Mann-Whitney *U* test.
[6]: Fisher's exact test.
[7]: Test was not computed due to no scoring difference.

The students' eye-movements

The students' eye-movements for each question are outlined in Table 3. Notice that in Table 3 the percentages refer to the percentage of the participants whose eyes moved towards a direction during their reasoning about a task (including the listening and the responding phase). Furthermore, in Figure 1, we diagrammatically present the students' most relevant to our study eye-movements (right, rightwards, up-right, left-right), as well as their eye-movements spread with respect to each task.

Table 3. **The participants' eye-movements for each problem.**

Problems	**Eye movements**												
Gender	**S-A**[1]	**U**	**D**	**L**	**R**	**U-D**	**L-R**	**U-R**	**U-L**	**D-R**	**D-L**	**Dom**[2]	**RW**[3]
ChangeRes	100%	0%	0%	0%	21%	11%	21%	36%	4%	21%	4%	U-R	26%
Boys	100%	0%	0%	0%	13%	13%	20%	27%	0%	13%	0%	U-R	18%
Girls	100%	0%	0%	0%	31%	7%	23%	46%	8%	31%	8%	U-R	36%
ChangeCha	100%	0%	4%	0%	18%	18%	32%	32%	7%	11%	0%	U-R/L-R	20%
Boys	100%	0%	0%	0%	13%	0%	20%	26%	0%	0%	0%	U-R	13%
Girls	100%	0%	8%	0%	23%	39%	46%	39%	15%	23%	0%	L-R	28%
ChangeSta	100%	4%	7%	4%	25%	36%	39%	57%	25%	29%	11%	U-R	37%
Boys	100%	7%	7%	7%	20%	40%	27%	53%	33%	20%	13%	U-R	31%
Girls	100%	0%	8%	0%	31%	31%	54%	62%	15%	39%	8%	U-R	44%
CompareCom	100%	4%	4%	7%	36%	21%	39%	46%	11%	18%	4%	U-R	33%
Boys	100%	7%	7%	0%	33%	20%	33%	47%	7%	7%	0%	U-R	29%
Girls	100%	0%	0%	15%	39%	23%	46%	46%	15%	31%	8%	U-R/L-R	39%
CompareDif	100%	0%	4%	0%	32%	32%	43%	39%	7%	18%	7%	L-R	30%
Boys	100%	0%	0%	0%	27%	33%	33%	40%	7%	13%	7%	U-R	27%
Girls	100%	0%	8%	0%	39%	31%	54%	39%	8%	23%	8%	L-R	33%
CompareRef	100%	4%	4%	0%	14%	29%	50%	50%	18%	36%	14%	U-R/L-R	33%
Boys	100%	0%	7%	0%	13%	20%	40%	67%	13%	40%	13%	U-R	40%
Girls	100%	8%	0%	0%	15%	39%	62%	31%	23%	31%	15%	L-R	26%

Note. The percentages refer to the percentage of the participants whose eyes moved towards a direction.
[1]: 'S-A': Straight-Ahead, 'U': Up, 'D': Down, 'L': Left, 'R': Right, 'U-D': Up-Down, 'L-R': Left-Right, 'U-R': Up-Right, 'D-R': Down-Right, 'D-L': Down-Left.
[2]: 'Dom' refers to the eye-movement direction most frequently identified considering all the participants for each phase of each problem (except for straight-ahead, which was noted for all students in the listening and response phase).
[3]: 'RW' refers to rightwards eye-movements including Right, Up-Right and Down-Right.

Up-right' (indicating intense thinking; Argyle, 1988) were the dominant (most frequently identified) eye-movements and in two question were as frequent as 'left-right'. Drawing upon the findings of our previous studies and the students' handedness, we concentrated our analysis in 'rightwards' eye-movements as indicators of their thinking about the question (since such eye-movement are linked with left-hemisphere activity). The students showed more eye-movements indicating the employment of the left-hemisphere in ChangeSta, followed by CompareCom and CompareRef. Linking the eye-movements with the difficulty of the tasks (as indicated by the students' response time and correctness), it was revealed that the least challenging tasks (ChangeRes and ChangeCha) attracted the fewer right and right wards eye-movements.

Nevertheless, the identified eye-movements in the more demanding tasks rendered the need to pursue these results with further analyses. For this purpose, we calculated the percentage of the eleven different eye movement types for each participant for each question (rather than the percentage of the whole sample looking towards a direction as we did in Table 3). The results of this analysis are outlined in Figure 1 and Table 4.

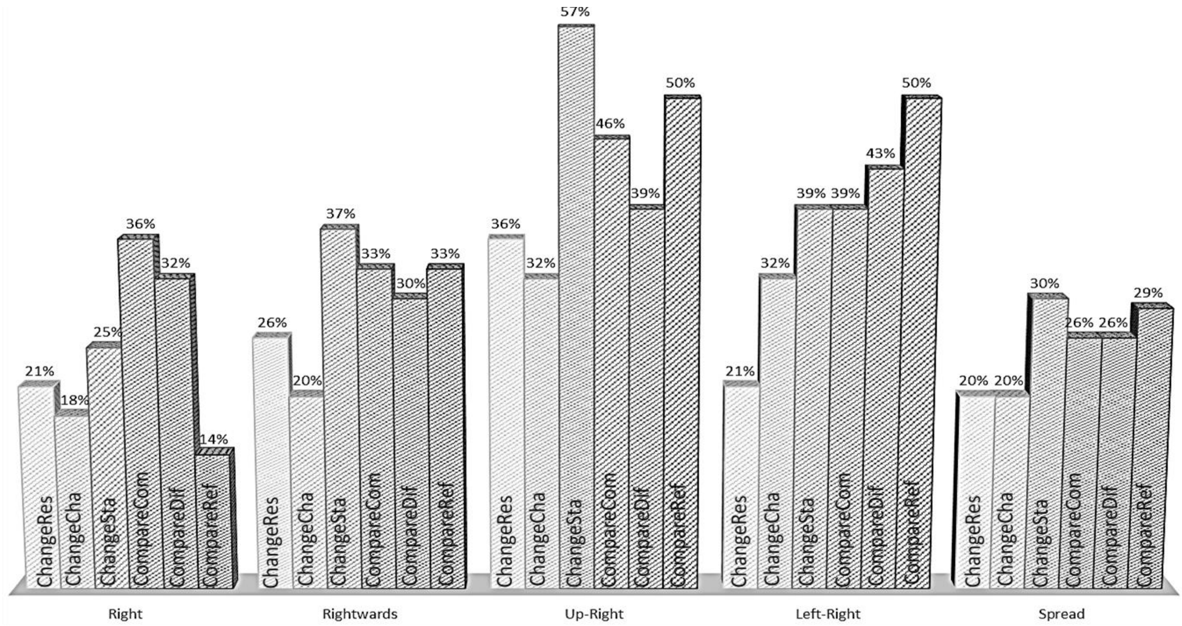

Figure 1. **Eye-movements during the students dealing with the problems**

Table 4. **Identifying the spread of eye-movements for each problem.**

	% of different eye-movements			**Boys vs Girls**		
Problem	**Whole**	**Boys**	**Girls**	***U***	***P***	***r***
ChangeRes	20%±13% [9%-55%]	19%±12% [9%-45%]	23%±14% [9%-55%]	73.0	0.234	-0.23
ChangeCha	20%±12% [9%-45%]	14%±8% [9%-36%]	26%±12% [9%-45%]	40.0	0.005	-0.52
ChangeSta	30%±15% [9%-64%]	30%±14% [9%-55%]	31%±17% [18%-64%]	96.5	0.976	-0.01
CompareCom	26%±14% [9%-55%]	24%±14% [9%-55%]	29%±13% [9%-55%]	76.5	0.322	-0.19
CompareDif	26%±13% [9%-55%]	24%±14% [9%-55%]	28%±11% [18%-55%]	72.0	0.232	-0.23
CompareRef	29%±16% [9%-64%]	28%±15% [9%-55%]	29%±17% [9%-64%]	96.5	0.981	-0.01

First, we noticed that the *least* demanding tasks were associated with a lower percentage of 'left-right' eye movements, suggesting the lower employment of both hemispheres. Moreover, the more demanding problems were linked with more 'up-right' and rightwards eye-movements. Subsequently, we focussed on the spread of the students' eye-movements during the reasoning phase for each type of arithmetic problem. It appeared that the expected to be less complex problems appeared to gather a smaller diversity in the students' eye movements (thus suggesting the smaller diversity in the areas of the brain engagement when coping with the task), which broadens as the task gets more demanding.

Considering the gender effect, the boys had the same dominant eye movement across all problems ('up-right'), whilst the girls had the same or 'left-right' (in ChangeCha, CompareDif and CompareRef). In the identified eye-movements spread, the boys appeared to have a narrower spread in their eye-movements when compared to girls. Nevertheless, statistical significant differences were found only in the 'ChangeCha' problem with the girls employing statistically significantly more eye-movement types than the boys. Notice also than in two of the most demanding problems (ChangeSta and CompareRef) the gender effect in the eye movement spread diminishes.

Discussion

Challenging' revisited

In this study, we focused on the eye-movements of Grade 1 students when they think about one-step 'change' and 'compare' word problems. Drawing upon existing research and our previous research project, three sets of expectations were formulated revolving around the level of challenge that each problem would pose to the students, as identified by the students' response time, response correctness and eye-movements (see *Expectations*). The findings of this study appear to reveal the complexity of what constitutes a 'challenging' or a 'demanding' task for the students. 'Challenging' appears to consist of different parameters, which seem to highlight related, yet qualitatively different aspects of the cognitive demands of a task concerning both the reasoning outcome and the reasoning process. Concerning eye-movements, we differentiated between two dimensions: rightwards eye-movements indicating left-hemisphere activity and eye-movements spread indicating both hemisphere activity.

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***Table 5.* Identifying the level of 'challenging' of each problem.**

	Reasoning outcome		**Reasoning process**	
Problem	**Time-consuming**	**Correctness**	**Rightwards eye-movements**	**Eye-movements spread**
ChangeRes	'low'	'low'	'low'	'low'
ChangeCha	'low'	'low'	'low'	'low'
ChangeSta	'medium'	'high'	'high'	'high'
CompareCom	'medium'	'medium'	'medium'	'medium'
CompareDif	'medium'	'high'	'medium'	'medium'
CompareRef	'high'	'medium'	'medium'	'high'

Note. 'high', 'medium', 'low' refer to the level of challenge a problem posed to the students as identified by the cut-point values that result in three equal groups of the collected data.

Consequently, a 'challenging' problem could be a result of a combination of some or all of the following: a) with respect to the reasoning outcome: time-consuming and/or difficult to answer it correctly, b) with respect to the reasoning process: requiring intense logico-mathematical thinking and/or requiring the combination of various mental activities. For each of those parameters, the given problems may be classified in three 'challenging' levels ('high', 'medium', 'low') as identified by the cut points that result in three equal groups of the collected data (see Table 5). In the following section, we argue that this bidimensional perspective of 'challenging' is crucial in gaining deeper understanding about the collected data.

Students' reasoning about one-step 'change' and 'compare' word problems

Following the aforementioned discussion, we argue that the findings of this study partially confirmed our expectations. First, considering the expectations deriving from the mathematics educations one-step problem literature, Expectation I was confirmed, since the expected to be less demanding 'change problems, ChangeRes and ChangeCha were clearly the least challenging, in all the considered aspects of challenging, while ChangeSta was overall more challenging than CompareDif and CompareCom. With respect to Expectation II, a mixed situation was revealed. CompareDif and CompareCom were indeed less time consuming and appearing to require less complex thinking than CompareRef. Nevertheless, no difference was identified amongst the three in the intensity of left-hemisphere activity, whilst CompareDif appeared to be harder for the students to answer correctly than CompareRef. Thus, Expectation II was partially confirmed in the sense that CompareRef appeared to require the orchestration of various mental activities for a longer period of time to be answered (usually correctly). Drawing upon the fact that this expectation derived from the related with one-step word problems literature, it is argued that the chosen methodology (not utilising an eye-tracking machine) appeared to be satisfactory validated. Thus, it is reasonable to investigate the employment of such techniques to inform everyday pedagogical practices.

Considering the expectations derived from the eye-mind hypothesis body of research, the identified links between cerebral activity and cognitive processes, as well as our previous research project, it appeared that both expectations (IV and V) were partially confirmed. First, Expectation IV was confirmed, as ‘Up-Right’ were the most frequently identified eye-movements in four of the problems and at par with ‘Left-Right’ in the two remaining problems. Nevertheless, it appears that the situation would be qualitatively different, had our focus been on the girls; in that case, ‘left-right’ eye-movements seemed to be dominant. Hence, it appears that there is a qualitatively different pattern in the reasoning process between boys and girls, with the girls appearing to employ more parts of the brain in their reasoning process. This is in line with the fact that the girls were found to have a broader eye-movements spread than the boys (albeit only in one case statistically significant). Furthermore, with respect to the remaining expectation deriving from this body of research and our redefinition of ‘challenging’, Expectation V could be re-stated as follows: the most challenging problems would be linked with a wider eye-movement spread (indicating a whole-brain activity linked with the reasoning about more complex tasks). In this sense, our expectation was confirmed. Consequently, it is posited that the adopted technique revealed information that is in line with the existing relevant research evidence, offering further insight in the complex processes involved.

Moreover, Expectations VI, VII and VIII that derived from research evidence about gender differences in the children’s ability to answer simple word problems were partially confirmed revealing a complex situation: in the least challenging problems the boys were indeed faster. We argued that this may be related with the faster retrieval of the relevant information that has been documented to be linked with the boys (Royer et al, 1999). Nevertheless, this effect appeared to diminish or reverse as the cognitive strain posed by more challenging problem increased. This is further supported by the eye-movement findings of this study, which revealed that in the more challenging tasks the gender differences in the identified eye-movements literally disappear. In specific, it was found that in the more challenging tasks the identified eye-movements indicated whole-brain activity. Moreover, the dominant eye-movements of the boys differ from the ones of the girls in the least demanding task, with the girls appearing to employ their whole brain even in the easy tasks, while the boys appearing to employ intense thinking followed, when necessary, by whole brain activity. It might be the result of a gender-related thinking disposition effect that lead the girls to prefer to employ their whole brain even in problems that it is not really necessary for the task to be accomplished. Hence, the girls' preference for more complex thinking might prevent them from quickly applying a simple algorithm, thus responding fast to the uttered problem. Importantly, it appears that this whole-brain activity implies the employment of mental processes that prevent the girls from providing a correct answer to the simpler problems. On the contrary, the whole-brain activity is rather efficient in the more complex tasks. It follows that further research should be conducted to delineate the identified reasoning process gender differences in the less challenging tasks. Overall, the findings of the inter-gender comparisons appear to be in line with evidence reported from the broader body of research and to help us in gaining deeper understanding in the complex phenomena involved in the students' reasoning about these seemingly simple arithmetic problems

Concluding remarks

In this study, we focused on the relationship between the students' mathematical thinking and their eye-movements as identified by a human being. This derived from our broader drive to investigate the feasibility of incorporating the findings of eye-movement research in everyday pedagogy. Six word arithmetic problems (three problems of 'change' and three problems of 'comparison') were verbally presented to thirty-eight 6 years old students. The findings of this study were multifaceted. First, they were in line with the existing literature, thus validating the chosen, 'softer' technique. Moreover, the eye-movement information helped in redefining what constitutes a challenging problem, differentiating the reasoning outcome (including response time and response correctness) from the reasoning process (eye-movement patters; dominant and spread). Importantly, this information derived from non-mechanical techniques which can be incorporated in the teachers' training to be employed in everyday pedagogies. By utilising this bidimensional, grounded in empirical evidence, conceptualisation of challenging, the teacher may gain deeper insight in the cognitive processes that are linked to a specific answer, thus more validly taking appropriate pedagogical actions.

Furthermore, the reported gender differences favouring the boys in terms of the response time were situated in the simpler tasks in which differences in eye-movements patterns were found. It is argued that –in line with broader research evidence from education and neurophysiology– these are indications of qualitatively different thinking processes involved, which may be related to different gender-related thinking dispositions. Nevertheless, the current research concentrated in only one wording variation for each of the identified word problem categories, which the research identifies as being one of the factors that may affect the students' strategies and/or overall performance (Verschaffel, Greer & De Corte, 2000). Thus, future projects may investigate the sensitivity and the educational relevance of the chosen method in arithmetic word problems of the same category, but with varied wording. Notwithstanding these limitations, it is posited that the adopted 'softer' technique was sensitive enough to be in accordance with the existing literature and at the same time helped in gaining deeper understanding in the students' reasoning about 'change' and 'compare' problems.

Considering the pedagogical implications, in order for the teachers to be more efficient in their everyday practices, they draw upon various sources of information including both verbal and non-verbal forms of communication in order, on the one hand, to gain deeper understanding in the students' cognitive and affective aspects of the learning process and, on the other, to obtain instant and constant feedback about their teaching. Within this framework, this study appears to support the argument that the eye-movement information can be valuable source of multifaceted information to facilitate the teachers' work (cf. Stamatis, 2013). Furthermore, non-verbal information, such as eye-movement and body-language, may constitute an alternative means of communication for the students that may face difficulties in expressing verbally their thinking (Pantazis & Stamatis, 2013). Consequently, further research should be conducted in order to facilitate the incorporation of such 'softer', thus feasible, eye-movement identification techniques in everyday pedagogical practices and to investigate their pedagogical benefits.

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