Mathematical modeling and simulation of hourly precipitation through rectangular pulses

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ABSTRACT. The recorded historical series of precipitation are usually available for short periods of time and with many failures. The use of mathematical modeling to simulate rainfall is a tool used to circumvent this problem and to simulate the operation of water systems in different scenarios. The present study applies mathematical modeling to the hourly pluviometric precipitation data simulation. A pluviographical data set from October 1980 to December 2007 was used in the study. Precipitation data sets were obtained through daily pluviometric digitalization from the Meteorologic Station of Epagri at Urussanga, in southern Santa Catarina, Brazil (latitude 28° 31' S and longitude 48° 19' W). To simulate the hourly rain series, the stochastic model was modified based on the Bartlett-Lewis rectangular pulses model with six parameters. Those parameters were fitted by minimizing a function related to the analytical expressions that define the average, variance, and autocorrelation coefficient at lag 1 and the probability of a dry period related to the estimated values from the observed data. Ten series were simulated for 100 years of data. Data analyses and results showed that fitting Bartlett-Lewis model parameters makes it possible to simulate hourly rainfall while preserving precipitation statistical properties at several temporal aggregation levels. In general, the probability of dry periods tended to be overestimated.

Keywords: hydrology, rainfall, simulation, probability, stochastic models.

RESUMO. Modelagem matemática e simulação da precipitação horária por meio de pulsos retangulares. As séries históricas de precipitação disponível, geralmente, são relativamente curtas e com muitas falhas nas observações. A utilização da modelagem matemática para simulação de chuvas é uma ferramenta utilizada para contornar esse problema, possibilitando a simulação do funcionamento de sistemas hidrológicos em diferentes cenários. Este trabalho teve como objetivo aplicar a modelagem matemática na simulação de série de dados de precipitação horária. Foi utilizada a série de dados pluviográficos do período de outubro de 1980 a dezembro de 2007. Os dados de precipitação foram obtidos pela digitalização dos pluviogramas diários da Estação Meteorológica da Epagri, Urussanga, Sul de Santa Catarina (latitude 28,31° S, longitude 48,19° W). Para a simulação das séries de chuva horária, o modelo estocástico adotado foi o modelo de pulsos retangulares de Bartlett-Lewis modificado com seis parâmetros. O ajuste dos parâmetros foi realizado tendo como base a minimização da função relacionada às expressões analíticas que definem a média, variância, e coeficiente de autocorrelação com retardo 1 e a probabilidade do período ser seco em relação aos valores estimados a partir dos dados observados. Foram simuladas dez séries com 100 anos de dados. A análise dos dados e os resultados nos levaram a concluir que o ajuste dos parâmetros do modelo de Bartlett-Lewis modificado possibilita a simulação de chuvas horária preservando as propriedades estatísticas da precipitação em vários níveis de agregação temporal. De forma geral, observou-se a tendência de superestimativa da probabilidade dos períodos serem secos.

Palavras-chave: hidrologia, chuvas, simulação, probabilidade, modelos estocásticos.

Introduction

Pluvial precipitation is a climate element that presents high spatial and temporal variability, and its excessive or rare occurrence usually causes damage to agricultural production as well as problems for people. To summarize the engineering attempts to solve that problem, rainfall values are used together with occurrence risks. This process is known as design rainfall. The rains are obtained through long term data sets; therefore, they can only be obtained where there is a long term daily pluvial precipitation recording.

In Brazil, it is relatively easy to obtain daily pluvial precipitation data sets, but for shorter

durations, the data sets are not available easily because of a lack of recording equipment, or the series are relatively short. Thus, mathematically modeling rainfall for simulation can be an important tool because it can generate a long series of correct data, which makes it possible to simulate the operation of hydrological systems and to estimate peak discharges related to historical rainfall. Rainfall precipitation modeling is not a simple task because of temporal and spatial variability. Paiva (2001) describes precipitation stochastic modeling as being very useful for studies involving hydrological phenomena, such as superficial drainage, soil infiltration and erosion.

There are a large number of references in which the precipitation stochastic models are discussed, such as Kelmann (1987), Waymire and Gupta (1981), and Foufola (1985). Clarke (2002) also presents a review of some stochastic methods that have been applied during the last 30 years. Precipitation modeling in daily intervals has been achieved by applying Markov series. Uggioni and Back (2005) showed that the first order Markov chain model in two states describes well the sequence of dry and rainy days south of Santa Catarina, and that the use of the exponential distribution allows daily pluvial precipitation values to be calculated while preserving the precipitation characteristics.

A significant development in precipitation modeling is presented by Rodriguez-Iturbe (1987). These models consider the precipitation to be formed by cells (precipitation basic units) whose distribution follows a definite stochastic process. Some processes have been already studied according to point process theory (COX; ISHAM, 1980, 1988). The Bartlett-Lewis process and Neyman-Scott process are two cell group point processes described by Rodriguez-Iturbe (1987) that gave rise to the Bartlett-Lewis rectangular pulse model and the Neyman-Scott rectangular pulse model. The difference between these models is relatively subtle, so it is unlikely that empirical analysis can be used to choose one of the models.

The Neyman-Scott and Bartlett-Lewis rectangular pulses models were applied to Denver's precipitation data set by Rodriguez-Iturbe (1987), who observed that these models are capable of preserving the precipitation statistical magnitude, including extreme values, for 1 to 24 hours. Nevertheless, they were not capable of maintaining dry periods for more than 1 hour at a time. Rodriguez-Iturbe (1988) modified the rectangular

pulses model by Bartlett-Lewis by permitting the cell duration exponential distribution parameter to vary from rain to rain according to a gamma distribution. With this modification, the model was also able to reproduce the dry periods proportionally in several time intervals. The modified model has six parameters (λ, α, υ, k, ϕ and μ_x) and is called the modified Bartlett-Lewis rectangular pulse model. Islam et al. (1990) studied the modified Bartlett-Lewis rectangular pulse model parameters dependence on the spatial and seasonal precipitation variation in the Arno River Basin in Italy and noted that the model parameters can be estimated at monthly intervals to reflect the seasonal precipitation variation. Entekhabi et al. (1989), Cowpertwait et al. (1996), Khaliq and Cunnane (1996) and Damé et al. (2007) all studied different ways to adjust the parameters and showed that these parameters can be estimated by different combinations statistics of historical series.

Back et al. (1999), using the Bartlett-Lewis model, generated series of hourly rain while maintaining rain structural characteristics. Therefore, the simulated extreme values were not verified and the simulation of shorter intervals was not studied. Damé et al. (2006) also used the modified Bartlett-Lewis rectangular pulse rain simulation model to verify whether or not the simulated rains preserve the historical statistical characteristics of the precipitation process by taking pluviographical data set from 1982 to 1998 in the city of Pelotas in the state of Rio Grande do Sul for their study of daily precipitation disaggregation and to estimate the intensity-duration-frequency curves, taking pluviographical data set from 1982 to 1998 in the city of Pelotas, in the state of Rio Grande do Sul, also used the modified Bartlett-Lewis rectangular pulse rain simulation model, verifying whether the simulated rains preserve the historical statistical characteristics of the precipitation process.

The objective of this study is to apply mathematical modeling to simulate hourly precipitation data series for Urussanga, Santa Catarina State, that are capable of producing long series of rainfall data without errors in the reporting.

Material and methods

This study used daily precipitation data sets from pluviograms of the Meteorological Station of Santa Catarina Agriculture-Cattle Raising Research and Rural Extension Company. The meteorological station is located at 28º 31' S latitude and 48º 19' W longitude and an altitude of 48.2 m. According to

the Köppen Climate Classification of this region, the climate is Cfa, which means Mesothermal with a wet and a hot summer and an annual total precipitation that varies from 1220 to 1660 mm; furthermore, the annual total number of days of rain varies between 98 and 150 days.

The pluviographical data series used was from October 1980 to December 2007. A Fuess-type pluviograph was used with daily intervals, a millimeter-graduated vertical scale that was subdivided decimally, and an hour-graduated horizontal scale that was subdivided at 10 minutes. These pluviograms were digitalized with GEDAC - *continuous data management* - (PEDROLLO, 1997) software and were stored in a databank after consistency analysis to compare the daily values to the values recorded in the pluviometer at the same meteorological station. Whenever rains were only recorded in the pluviometer, pluviographs data were recorded as information failures. The secondary files were generated with discretized data at 1-hour intervals.

The stochastic modified Bartlett-Lewis rectangular pulses model was used to simulate an hourly rain series with six parameters (λ, α, υ, k, ϕ and μ_x). The hourly data is recorded with a resolution of 0.1 mm, and whenever the observed precipitation is less than that value, the time interval is defined as dry. In order to consider seasonal variation of the precipitation values, model parameters are estimated for each month separately.

The study adopted the parameters adjustment procedures described by Entekhabi et al. (1989) and by Cowpertwait et al. (1996) that minimize the sum of squares of the differences between the observed values and the values calculated by the model. In this scheme, residues are normalized by their respective historical values. Therefore, considering $f_i \equiv f_i(\lambda, \alpha, \upsilon, k, \phi, \mu_x)$ as a function of the model and f_o as the sample value of a historical series of values, and if m functions are supposed, the parameters can be calculated by minimizing the following sum of squares:

$$S = \sum_{i=1}^{m} \left(1 - f_i / f_o\right)^2 \tag{1}$$

where:

f_i is the analytical function defined by the model;

f_o is the corresponding value estimated from observed data; and

m is the number of functions considered.

To minimize the total sum in equation (1), the Solver command from Excel was used (LAPPONI, 2005). This program uses an algorithm similar to the Newton method to find the minimal values. Functions representing the average over one hour, the variance, and the autocorrelation coefficient at lag 1 as well as the dry periods for intervals of 1, 6, 12 and 24h.

The average of the observed values for each duration interval was estimated by the function

$$\mu_k(h) = \sum_{i=1}^{n} \sum_{j=1}^{n_k^{(h)}} Y_{i,j,k}^{(h)} / \left\{ n_k^{(h)} n \right\} \tag{2}$$

where:

$\mu_k(h)$ is the observed average for h hour time interval (mm);

k is the calendar monthly index (k=1 for January, 2 for February etc.);

$Y_{i,j,k}^{(h)}$ is the total precipitation value of the jth interval of year i through month k;

$n_k^{(h)}$ is the total number of hourly time intervals in month k; and

n is the data year number.

The variance of the precipitation value will be estimated by the function:

$$\gamma_k(h) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n_k^{(h)}} \left\{ Y_{i,j,k}^{(h)} - \mu_k(h) \right\}^2}{n_k^{(h)} n} \tag{3}$$

where:

$\gamma(h)$ is the observed variance for an h-hour time interval (mm^{-2}).

The autocovariance at lag 1 is estimated by the function

$$\gamma_k(h,1) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n_k^{(h)-1}} \left\{ Y_{i,j,k}^{(h)} - \mu_k(h) \right\} \left\{ Y_{i,j+1,k}^{(h)} - \mu_k(h) \right\}}{(n_k^{(h)} - 1) n} \tag{4}$$

where:

$\gamma_k(h,1)$ is the autocovariance at lag 1 (mm^{-2}).

The autocorrelation coefficient at lag 1 is given by the relation between the autocovariance at lag 1 and the variance, that is

$$\rho(h,1) = \frac{\gamma_k(h,1)}{\gamma_k(h)} \tag{5}$$

where:

$\rho(h,1)$ is the autocorrelation coefficient at lag 1.

The dry interval ratio is estimated by the ratio between the number of h-hour dry intervals and the total number of h-hour intervals in month k as follows:

$$\phi_d(h) = n_d(h)/\ n_k^{(h)} \qquad (6)$$

where:

ϕ_d is the proportion of h-hour intervals without rain and n_d is the observed number of intervals.

The analytical expression of the Bartlett-Lewis rectangular pulse model that defines the average precipitation value in an h-hour interval is given by

$$E\left(Y_i^{\ h}\right) = \frac{\lambda\, h\, \upsilon\mu\ _x\mu_c}{\alpha\ -\ 1} \qquad (7)$$

where:

$E(Y^h)$ is the precipitation average in an h-hour interval (mm);

$\lambda, \upsilon, \mu_x, \alpha, \phi$, and K are the model parameters;

$$\mu_c\ = 1 + K/\phi \qquad (8)$$

where:

h is the time interval (h).

The precipitation variance is defined by the following expression:

$$\text{var}\left[Y_i^h\right] = 2A_1\left[(\alpha-3)h\upsilon^{2-\alpha} - \upsilon^{3-\alpha} + (\upsilon+h)^{3-\alpha}\right] \\ -2A_2\left[(\alpha-3)\phi h\upsilon^{2-\alpha} - \upsilon^{3-\alpha} + (\upsilon+\phi h)^{3-\alpha}\right] \qquad (9)$$

where:

Var[Y^h] are the precipitation variances in an h-hour interval (mm^2):

$$A_1 = \frac{\lambda\mu_c\upsilon^\alpha}{(\alpha-1)(\alpha-2)(\alpha-3)}\left[E\left(X^2\right) + \frac{\kappa\phi\mu_x^2}{\phi^2-1}\right] \qquad (10)$$

$$A_2 = \frac{\lambda\mu_c\kappa\mu_x^2\upsilon^\alpha}{\phi^2\left(\phi^2-1\right)(\alpha-1)(\alpha-2)(\alpha-3)} \qquad (11)$$

The depth of the cell X is assumed to be exponentially distributed; thus, $E(X^2) = 2\mu_x^2$.

The autocovariance at lag τ is defined by the expression

$$cov\left[Y_i^{\ h}, Y_{i+\tau}^{\ h}\right] = A_1\left\{ \begin{matrix}[\upsilon+(\tau+1)h]^{3-\alpha} \\ -2(\upsilon+\tau h)^{3-\alpha} + [\upsilon+(\tau-1)h]^{3-\alpha}\end{matrix}\right\} \\ -A_2\left\{\begin{matrix}[\upsilon+(\tau+1)\phi h]^{3-\alpha} \\ -2(\upsilon+\tau\phi h)^{3-\alpha} + [\upsilon+(\tau-1)\phi h]^{3-\alpha}\end{matrix}\right\} \qquad (12)$$

where:

$cov[Y_i, Y_{i+\tau}]$ is the autocovariance at lag τ (mm^2).

The probability that the h-hour depth period is dry is given by

$$\Pr\left(Y_i^h = 0\right) = \exp\left\{-\lambda h - \lambda\mu_T + \frac{\lambda\phi}{(\phi+\kappa)}B_1 + \frac{\lambda\kappa}{(\phi+\kappa)}B_2\right\} \qquad (13)$$

where:

Pr is the probability that the h-hour interval is dry and

$$\mu_T \approx \frac{\upsilon}{\phi(\alpha-1)}\left[\begin{matrix}1+\phi(\kappa+\phi)-\frac{1}{4}\phi \quad (\kappa+\phi)(\kappa+4\phi) \\ +\frac{1}{72}\phi(\phi+\kappa)\left(4\kappa^2+27\kappa\phi+72\phi^2\right)\end{matrix}\right] \qquad (14)$$

$$B_1 = \frac{\upsilon}{\phi(\alpha-1)}\left(1-\kappa-\phi+\frac{3}{2}\kappa\phi+\phi^2+\frac{1}{2}\kappa^2\right) \qquad (15)$$

$$B_2 = \frac{\upsilon^\alpha}{\phi(\alpha-1)}[\upsilon+(\kappa+\phi)h]^{1-\alpha} \\ \left(1-\kappa-\phi+\frac{3}{2}\kappa\phi+\phi^2+\frac{1}{2}\kappa^2\right) \qquad (16)$$

The rain is simulated according to the number of rainy cells associated with it as follows:

The time at which the rain starts follows a Poisson process with rate λ h^{-1}, that is, the periods between the beginnings of consecutive rains are independent arbitrary variables and exponentially distributed with parameter 1 λ^{-1}.

(2) Each rain has a number η associated with it that specifies its intensity. These numbers are independent arbitrary variables, with an average gamma of α/υ and variance $\alpha\ \upsilon^{-2}$.

(3) Each rain has one or more cells. The first cell starts at the time of the original rain and the subsequent cells begin according to a Poisson process with rate β ($\beta = \kappa\eta$) h^{-1}; after the time exponential distribution reaches the average 1 Υ^{-1} ($\Upsilon = \phi\eta$) h, no more cells start.

(4) Each cell is a rectangular pulse of rain, with exponentially distributed intensity at an average of μ_x mm h^{-1} and an exponentially distributed duration that is 1 η^{-1} h on average.

(5) The total precipitation is given by the sum of all cells and rains.

A computer program was created in the Delphi language to generate the rain series and calculate the statistics defined by equations 2 to 6.

Results and discussion

Table 1 shows the statistical values of a precipitation series in 1-hour intervals for the historical series (1) as well as the values calculated by the model (2) and the average (3) obtained from 10 series generated for January and July. Based on the observed statistical values, model parameters were adjusted (Table 2) to reflect the seasonal precipitation. There is significant seasonal variation in precipitation characteristics.

Table 1. Historical observed values (1), values calculated by the models (2), and those obtained from rain generated series (3) in 60-minute intervals in January and July.

Intervals	Statistic	January			July		
		1	2	3	1	2	3
1h	Average [1]	0.250	0.249	0.2409	0.104	0.103	0.0994
	Var[2]	2.357	2.764	2.5766	0.390	0.390	0.3634
	PD[3]	0.857	0.918	0.9240	0.908	0.939	0.9410
	Covar[4]	0.737	0.840	0.7346	0.314	0.258	0.2340
	Correl[5]	0.313	0.304	0.2848	0.620	0.662	0.6424
6h	Var[2]	26.100	27.554	24.9241	7.111	7.629	6.8892
	PD[3]	0.696	0.709	0.7245	0.813	0.862	0.8683
	Covar[4]	4.270	3.168	2.7643	3.655	3.064	2.6039
	Correl[5]	0.164	0.115	0.1105	0.514	0.402	0.3744
12h	Var[2]	63.420	61.445	55.5171	21.942	21.385	18.9780
	PD[3]	0.608	0.561	0.5799	0.760	0.784	0.7933
	Covar[4]	5.699	7.202	6.5747	7.926	6.666	5.2939
	Correl[5]	0.090	0.117	0.1186	0.361	0.312	0.2758
24h	Var[2]	132.014	137.294	125.1837	58.109	56.103	48.1082
	PD[3]	0.421	0.385	0.4036	0.661	0.648	0.6620
	Covar[4]	21.392	16.814	15.0933	11.588	13.180	9.8141
	Correl[5]	0.162	0.122	0.1216	0.199	0.235	0.2038

[1]Average - average precipitation in 1 hour interval (mm). [2]Var - precipitation variance in the interval (mm²). [3]PD - dry interval probability. [4]Covar - autocovariance at lag 1 (mm²). [5]Correl - autocorrelation coefficient at lag 1.

Table 2. Modified Bartlett-Lewis parameters model, adjusted for the hourly data from Urussanga, Santa Catarina State.

Month	λ(h^{-1})	υ (h)	μ_x (mm·h^{-1})	α	ϕ	κ
January	0.0265	1.4160	11.4170	5.4544	0.0150	0.0238
February	0.0269	1.1761	10.2543	3.7085	0.0150	0.0192
March	0.0222	2.2954	7.8594	5.4369	0.0150	0.0125
April	0.0208	0.6594	7.8282	3.1764	0.0287	0.0463
May	0.0126	0.5179	2.4253	2.3743	0.1006	1.1368
June	0.0158	0.7929	1.8443	2.4069	0.1690	0.8962
July	0.0132	2.6125	1.7266	4.0096	0.1332	0.7992
August	0.0070	1.7618	2.7623	4.1161	0.0397	0.4039
September	0.0185	1.9613	1.3390	4.3782	0.1852	1.8695
October	0.0215	0.2342	4.5803	3.2837	0.0385	0.6424
November	0.0258	0.4922	6.7017	3.2795	0.0397	0.1349
December	0.0221	0.2546	10.5493	2.3852	0.0176	0.0695

According to Vianello and Alves (2006), the phenomena related to atmospheric dynamics and geographical factors, such as topography, continentality and maritime, are the main determinants of the climatic characteristics of southern Brazil. Air masses that influence the climate in southern Brazil are the Mass Tropical Maritime, which originates from the anticyclone of the Atlantic (30ºS) is characterized as hot and humid and operates throughout the year; the Massa Polar Maritime, which originates in subpolar latitudes and is characterized as cold and wet and operates in the state throughout the year but is more active during the winter; the Mass Tropical Continental, which penetrates the west Santa Catarina and is characterized as hot and dry with more intense activity in summer; and the Mass Equatorial Continental, which is hot and wet and permeates from the northwest, especially in the summer. In general, the precipitation is well distributed throughout the year because of the mass of polar air mass from the Atlantic and Tropical Atlantic, which, as a result of its constancy, does not mean that there is a season (DUFLOTH et al., 2005).

The southern region of Brazil has diversified atmospheric dynamics, mainly because of the incursions of polar cold that favor the genesis of other weather systems, and the cold front is the most common (NIMER, 1989). This system operates throughout the year, even in summer, when it presents low intensity on the continent. This dynamic is precisely the latitudinal position of southern Brazil that produces the shock front discontinuities between the inter-tropical and polar systems (MONTEIRO, 2001). In the spring, cold fronts have less continental displacement compared to those in the winter. Even so, the spring has a slight increase in the frequency of this system compared to other seasons.

The atmospheric dynamics associated with the various systems that occur in southern Brazil can be modified when there are interference phenomena, such as El Niño, La Niña and air locks. In addition to changing the behavior of these meteorological parameters, the locks are conducive to increased rainfall in the spring of the year during winter when the next year will experience an El Niño affect (GRIMM; NATORI, 2006; GRIMM et al., 1998). Grimm (2003) noted that, in this region, El Niño increases the frequency of extreme events relative to normal years and that La Niña decreases the frequency of extreme events. According to Seluchi and Marengo (2000) and Marengo et al. (2002), the occurrence of low-level jets may also influence the occurrence of extreme rainfall in southern Brazil.

The hourly average precipitation varies from 0.274 mm·h^{-1} in February to 0.104 mm·h^{-1} in June (Figure 1). In the summer, the major precipitation variances are observed at several aggregation levels. The proportion of dry days presents low values in January and February and large values in June and July (Figure 2). Such behavior was reported by Back et al. (1999), who used the hourly precipitation data series from 1980 to 1996.

They attributed the seasonal differences to the differentiated activity of the air masses south of the country that influence the frequency and intensity of rains directly. Assis (1993) also claimed that the differentiated activity of the air masses is responsible for the differences presented in the pluviometric standard in the south and south-east of the country. The frontal systems that are characterized by long duration and medium and low intensity (TUCCI, 1993) occur throughout year. In the summer, convective rainfalls predominate and are characterized by a short duration and high intensity, which influence the intensity and variance of the rains observed in the period.
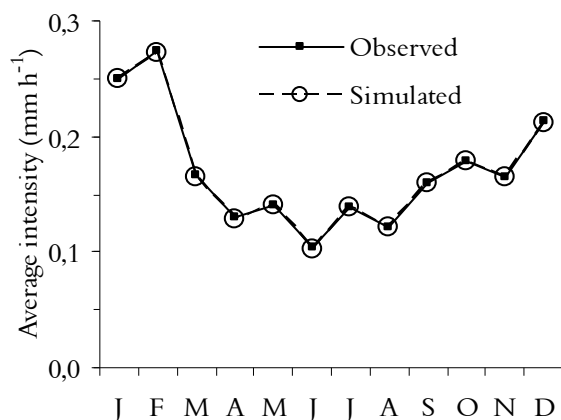
Figure 1. Average intensity values (mm h^{-1}) of the observed and simulated series.
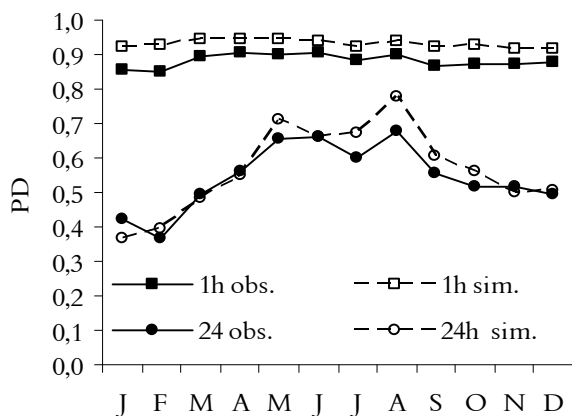

Figure 2. Dry interval probability (PD) for the observed (obs.) and simulated (sim.) series for group rains in 1 and 24h intervals.

The inverse of parameter λ determines the average interval between rains, and that interval varies from 37h in February to 143h in July. The intensity of rainy cells is given by the parameter μ_x, and they are more intense in summer. Figure 3 shows the average duration of rainy cells and the

average number of these cells. It was observed that, in January, the average number of cells is 2.585 for each rain, with an average duration of 0.318h. In July, there is an average of seven rainy cells with average durations of 0.868h.
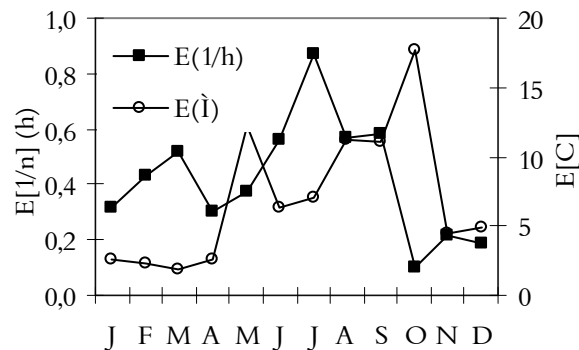

Figure 3. Rainy cell average duration values (E[1/η]) and average number of rainy cells (E[C]) in 1-hour intervals.

It is important to observe that the parameter values of the Bartlett-Lewis model can vary according to the statistical combination used in the model adjustment function, according to Damé et al. (2007). In the optimizing function, some parameters present major instability and vary according to the set of statistics applied. Damé et al. (2007) observed that the parameter λ was the most stable, while the parameter ν was the most unstable. Rodriguez-Iturbe (1987), Onof and Wheater (1994), and Khaliq and Cunnane (1996) also analyzed the sensitivity and stability of the parameters of the Bartlett-Lewis model using 5 different sets of statistics and verified that the parameter magnitudes varied considerably. The same authors used 16 moments to derive the six parameters of the model and suggested that different initial values for parameters α and ν must be used for optimization. The instability observed in parameter ν was understood by Damé et al. (2007) to be an indicator that the manner in which the parameters were estimated most affected rain cell duration. Onof and Wheater (1994) showed that, except for μ_x and λ, the parameters determined by two different sets of moments were very different, especially parameters α and ν. Therefore, both sets can maintain precipitation characteristics presenting that present errors that are 5% less than those of the historical values.

In general, parameters vary according to the physical processes that form the rains, that is, in summer, shorter duration rains predominate, and they have a smaller number of rainy cells of less duration but with a larger average intensity.

Monteiro (2001) claims that the various systems that are present in southern Brazil, provide southern Santa Catarina with accentuated climate dynamics and good rain distribution during the year, considering that all unstable systems produce rain. He presents a detailed description of the atmospheric systems that act in the Araranguá River Basin and emphasizes that the continental tropical air masses that occur in the southern region in a restricted way in summer cause short summers in southern Brazil. Moreover, he claims that the atmospheric system with the greatest climate influence on the region all year is the cold front. Rodrigues and Lopes (2008) identified an average of 3 to 3.5 cold fronts per month on the Santa Catarina state coast from 1990 to 1999. These cold fronts occur all year, but they act differently according to the season of the year. During the summer they act more on the Atlantic Ocean; during the fall they enter the continent; during the winter, because the continent is colder, the air masses that come from great latitudes become more important for the precipitation distribution in the south. Although the average number of monthly incursions is the same, cold fronts exhibit more continental action during the winter. According to Monteiro (2001), during the spring, cold fronts have less continental displacement than in winter, but during the spring the system presents a lighter frequency increase.

The convective rains depend on heat and wet and weak winds, and they occur frequently during the summer in southern Brazil; thus, they are responsible for the significant volume of precipitation in this period of the year. Monteiro (2001) states that the major precipitation in most localities around the Araranguá River Basin occurs during January and February. During this period, sudden downpours are common, and despite being very fast, perhaps lasting only 10 minutes, their precipitation volume can exceed that from a cold front that lasts more than 24 hours, which is common in winter. During the fall, the convective process is not very common, and the precipitation in the period is associated with the passage of cold fronts.

The seasonal parameter variation, especially that of λ, μ_x, α, and k, shows that the modified Bartlett-Lewis rectangular pulse model represents the precipitation characteristics well, that is, higher-intensity-shorter-duration rains dominate in the summer, and lower-intensity-longer-duration rains occur most often in the winter. There is also coherence regarding the number of rainy cells and their duration, which indicates the greater quantity

of cells and their longer duration during the winter but with less intensity. The biggest interval between rains also reflects the front-rain dominance.

The performance of the model can be evaluated by comparing the moments of the model to the characteristics of historical values. Table 1 presents the statistics of the simulated and observed data and shows that, in general, the model simulated hourly data series, while maintaining the hourly rain characteristics and those of several aggregation levels. The average values of hourly precipitation that were simulated for 10 series in 100 years during the month of August were 4.7% lower than the observed series (Figure 1). For the other months, differences were even less, which demonstrates the suitability of the model, though the parameter adjustment function of the model was restricted to equate the estimated average to the observed one.

Concerning the variances of the rains in different intervals, the most significant relationship between the simulated series variance and the observed series variance is 1.28, and it occurs in April for the 24-hour interval rains (Figure 4). This result indicates that the simulated series presented a variance that is 28% superior to the observed series variance. Results show that the simulated series presents values that indicate the probability that the 1-hour interval will be dry (PD1) are slightly superior to the observed values.
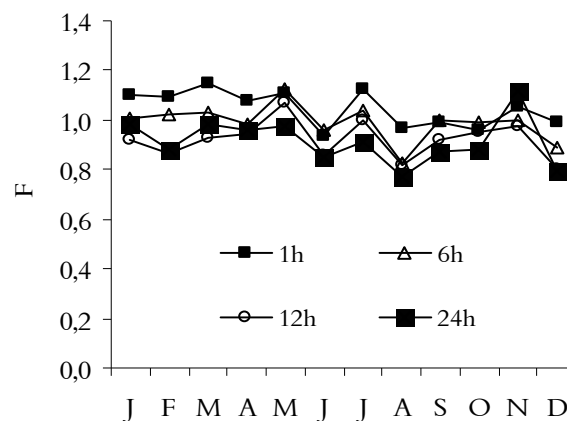

Figure 4. F-values for the simulated series variance in relation to the observed series variance at 1-hour intervals.

The biggest difference was observed in February (8.2%), and the smallest difference was in June (3.5%), with an annual average difference of 5.2%. Regarding the autocorrelation (Figure 5) for 1-hour interval rains, the differences between observed and simulated values vary from 10.8% greater in September to 1.4% lower in December, with an annual average of 2.4%. These results were reported by Gyasi-Agyei and Willgoose (1997), who observed

that the Bartlett-Lewis rectangular pulse model could not reproduce the autocorrelation as well as the probability of dry intervals, which are historical data characteristics in Queensland, Australia.
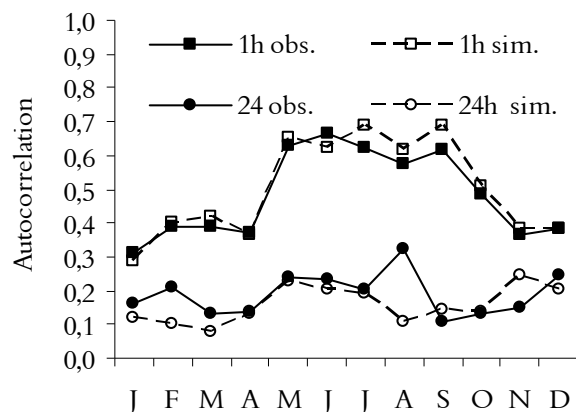

Figure 5. Autocorrelation values for the observed and simulated series and also the theoretical value of the model.

Figure 6 shows the annual total precipitation of the observed and simulated series, together with a 95% confidence interval, assuming that the annual total precipitation has a normal distribution. The simulated series presented 6% of the values out of the 95% confidence interval limits.
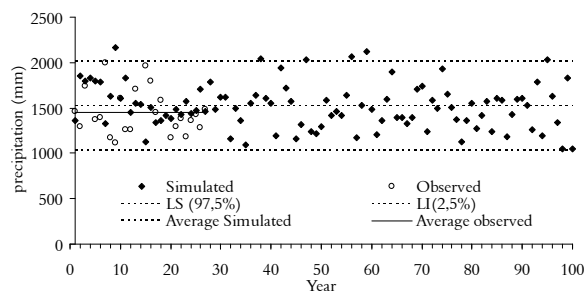

Figure 6. Annual total of the hourly series rain.

Based on average values, variance, and autocorrelation, the results are given by Rodriguez-Iturbe (1987) and Verhoest et al. (1997), who observed that the model preserves the first moments. The simulated data series maintains the precipitation characteristics and advances hydrological studies because the parameters can be adjusted after observing a data series for a relatively short period and a long data series can be generated without failures in data recordings.

Conclusion

Based on the results achieved in the study, the following remarks are important to mention: Parameter adjustment in the modified Bartlett-

Lewis model make the simulation of hourly rains possible while maintaining precipitation statistical properties in various temporal aggregation levels; the seasonal variation of atmospheric processes involved in the origin of the rain characteristics implies a seasonal variation of model parameters; the adjustment of the parameters for each month of the year shows the characteristics of the predominant rain in a coherent way; the statistical characteristics that are presented by the simulated series at various temporal aggregation levels are similar to the observed values; in general, there was a tendency to overestimate the probability that the periods will be dry and to underestimate the covariance for 24-hour intervals, mainly in the summer.

The annual total of simulated rain for all of the analyzed intervals of duration stays within the 95% confidence interval.

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