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Weakly continuous functions on mixed fuzzy topological spaces

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ABSTRACT. The notions of continuity was generalized in the fuzzy setting by Chang (1968). Later on Azad (1981) introduced some weaker form of fuzzy continuity like fuzzy almost continuity, fuzzy semi-continuity and fuzzy weak continuity. These are natural generalization of the corresponding weaker forms of continuity in topological spaces. Recently Arya and Singal (2001a and b) introduce another weaker form of fuzzy continuity, namely fuzzy subweakly continuity as a natural generalization of subweak continuity introduced by Rose (1984). In this paper we introduce fuzzy weak continuity in mixed fuzzy topological space.

Keywords: fuzzy weak continuity, fuzzy point, mixed fuzzy topological space, fuzzy subspace.

Funções contínuas fracas sobre espaços topológicos difusos misturados

RESUMO. As noções de continuidade foram generalizados no ambiente difuso de Chang (1968). Mais tarde, Azad (1981) apresentou formas mais fracas de continuidade difusa, como continuidade quase difusa, semi-continuidade difusa e continuidade difusa fraca. São generalizações naturais das formas correspondentes de continuidades mais fracas em espaços topológicos. Recentemente, Arya e Singal (2001a e b) apresentaram uma outra forma mais fraca de continuidade difusa, ou seja, continuidade sub-fraca difusa como uma generalização natural da continuidade sub-fraca de Rose (1984). Apresenta-se nesse trabalho a continuidade fraca difusa no espaço topológico difuso misto.

Palavras-chave: continuidade fraca difusa, ponto difuso, espaço topológico difuso misto, sub-espaço difuso.

Introduction

The notion of topological space has been generalized in many ways. The notion of bitopological space and mixed topological space has been introduced and investigated in the resent past. Bitopological spaces have recently been studied by Ganguly and Singha (1984), Tripathy and Sarma (2011b, 2012) and others. Mixed topology lies in the theory of strict topology of the spaces of continuous functions on locally compact spaces. The concept of mixed topology is very old. Mixed topology is a technique of mixing two topologies on a set to get a third topology on that set. The works on mixed topology is due to Cooper (1971), Buck (1952), Das and Baishya (1995), Tripathy and Ray (2012), Wiweger (1961) and many others.

In 1965 L. A. Zadeh introduced the concept of fuzzy sets. Since then the notion of fuzziness has been applied for the study in all the branches of science and technology. It has been applied for studying different classes of sequences of fuzzy numbers by Tripathy and Baruah (2010), Tripathy and Borgohain (2008, 2011), Tripathy and Dutta (2010), Tripathy and Sarma (2011a), Tripathy et al. (2012), and many workers on sequence spaces in the recent years. The notion of fuzziness has been applied in topology and the notion of fuzzy topological spaces has introduced and investigated by many researches on topological spaces. Different properties of fuzzy topological spaces have been investigated by Arya and Singal (2001a and b), Chang (1968), Das and Baishya (1995), Ganster et al. (2005), Ganguly and Singha (1984), Ghanim et al. (1984), Katsaras and Liu (1977), Petricevic (1998), Srivastava et al. (1981), Warren (1978), Wong (1974a and b) and many others. Recently mixed fuzzy topological spaces have been investigated from different aspect by Das and Baishya (1995) and others.

Preliminaries

Let X be a non-empty set and I, the unit interval [0, 1]. A fuzzy set A in X is characterized by a function $\mu_A : X \to I$, where μ_A is called the membership function of A. $\mu_A(x)$ represents the membership grade of x in A. The empty fuzzy set is defined by $\mu_\phi(t) = 0$ for all $t \in X$. Also X can be regarded as a fuzzy set in itself defined by $\mu_X(t) = 1$ for all $t \in X$. Further, an ordinary subset A of X can also be regarded as a fuzzy set in X if its membership function is taken as usual characteristic function of A that is $\mu_A(t) = 1$, for all $t \in X$ and $\mu_A(t) = 0$ for all $t \in X$- A. Two fuzzy sets A and B are said to be equal if $\mu_A = \mu_B$. A fuzzy set A is said to be contained in a fuzzy set B, written as $A \subseteq B$, if $\mu_A \le \mu_B$. Complement of a fuzzy set A in X is a fuzzy set A in X defined by $\mu_{A^c} = 1 - \mu_A$. We write $A^c = coA$. Union and intersection of a collection $\{A_i : i \in J\}$ of fuzzy sets in X, to be written as $\underset{i=1}{\cup} A_i$ and $\underset{i=1}{\cap} A_i$ respectively, are defined as follows:

$\mu_{\bigcup_{i\in I} A_i}(x) = \sup\{\mu_{A_i}(x) : i \in J\}$, for all $x \in X$ and

$\mu_{\bigcap_{i\in I} A_i}(x) = \inf\{\mu_{A_i}(x) : i \in J\}$ for all $x \in X$.

Definition 2.1. A fuzzy topology τ on X is a collection of fuzzy sets in X such that $\varnothing, X \in \tau$; if $A_i \in \tau$, $i \in J$ then $\underset{i=1}{\cup} A_i \in \tau$; if $A, B \in \tau$ then $A \cap B \in \tau$. The pair (X, τ) is called a fuzzy topological space (fts). Members of τ are called open fuzzy set and the complement of an open fuzzy sets is called a closed fuzzy set.

Definition 2.2. If (X, τ) is a fuzzy topological space, then the closure and interior of a fuzzy set A in X, denoted by *cl A* and *int A* respectively, are defined as *cl* $A = \bigcap$ { B: B is a closed fuzzy set in X and $A \subseteq B$}. The *int* $A = \bigcap$ {V : V is an open fuzzy in X and $V \subseteq A$}.

Clearly, cl A (respectively *int* A) is the smallest (respectively largest) closed(respectively open) fuzzy set in X containing (respectively contained in) A. If there are more than one topologies on X then the closure and interior of A with respect to a fuzzy topology τ on X will be denoted by τ-*cl A* and τ-*int A*.

Definition 2.3. A collection Ɓ of open fuzzy sets in a fts X is said to be an open base for X if every open fuzzy set in X is a union of members of Ɓ.

Definition 2.4. If A is a fuzzy set in X and B is a fuzzy set in Y then, $A \times B$ is a fuzzy set in $X \times Y$ defined by $\mu_{A \times B}(x,y) = \min\{\mu_A(x), \mu_B(y)\}$ for all $x \in X$ and for all $y \in Y$. Let f be a function from X into Y. Then, for each fuzzy set B in Y, the inverse image of B under f, written as $f^{-1}[B]$, is a fuzzy set in X defined by $\mu_{f^{-1}[B]}(x) = \mu_B(f(x))$ for all $x \in X$.

Definition 2.5. A fuzzy set A in a fuzzy topological space (X, τ) is called a neighborhood of a point $x \in X$ if and only if there exists $B \in \tau$ such that $B \subseteq A$ and $A(x) = B(x) > 0$.

Definition 2.6. A fuzzy point x_α is said to be quasi-coincident with A, denoted by $x_\alpha\, qA$, if and only if $\alpha + A(x) > 1$ or $\alpha > (A(x))^c$.

Definition 2.7. A fuzzy set A is said to be quasi-coincident with B and is denoted by AqB, if and only if there exists a $x \in X$ such that $A(x) + B(x) > 1$.

Remark 2.1. It is clear that if A and B are quasi-coincident at x both $A(x)$ and $B(x)$ are not zero at x and hence A and B intersect at x.

Definition 2.8. A fuzzy set A in a fts (X, τ) is called a quasi-neighbourhood of x_λ if and only if $A_1 \in \tau$ such that $A_1 \subseteq A$ and $x_\lambda\, qA_1$.

The family of all Q-neighbourhood of x_λ is called the system of Q-neighbourhood of x_λ. Intersection of two quasi-neighbourhood of x_λ is a quasi-neighbourhood. Let (X, τ_1) and (X, τ_2) be two fuzzy topological spaces and let $\tau_1(\tau_2)$ =

{$A \in I^X$: for every fuzzy set B in X with AqB, there exists a τ_2 –Q-neighbourhood A_α, such that $A_\alpha qB$ and τ_1-closure, $\overline{A_\alpha} \subseteq B$ }, Then $\tau_1(\tau_2)$ is a fuzzy topology on X and this is called *mixed fuzzy topology*, and the space $(X, \tau_1(\tau_2))$ is called *mixed fuzzy topological space*.

Main results

Theorem 3.1. If (X, τ_1, τ_2) and (Y, τ_1^*, τ_2^*) be two fuzzy bitopological spaces. If f : X $\rightarrow$Y is τ_1-τ_1^* and τ_2-τ_2^* continuous, then f is $\tau_1(\tau_2)$-$\tau_1^*(\tau_2^*)$ continuous.

Proof: Let A^* be any $\tau_1^*(\tau_2^*)$ open set in Y.

We show that $f^{-1}(A^*)$ is an open set in X.

Let A be any fuzzy set in X such that

$Aqf^{-1}(A^*) \Rightarrow A(x) + f^{-1}(A^*)(x) > 1$, for some $x \in X$

$\Rightarrow \lambda + f(x) > 1$ where $A(x) = \lambda$

$\Rightarrow (f(x))_\lambda qA^*$

Now $(f(x))_\lambda$ is a fuzzy set in Y and $(f(x))_\lambda\, q\, A^*$

Therefore by definition of mixed topology there exists τ_2^*-open set B^* such that τ_1^* – closure $\overline{B^*} \subseteq A^*$ and $(f(x))_\lambda qB^*$.

Since f is $\tau_1 - \tau_1^*$ continuous, so we have $f^{-1}\left(\overline{B^*}\right) \subseteq f^{-1}(A^*)$.

But $\overline{f^{-1}}(B^*) \subseteq f^{-1}\left(\overline{B^*}\right) \subseteq f^{-1}(A^*)$.

Since f is τ_2-τ_2^* continuous, so we have therefore for each fuzzy set A in X and τ_2^*-open set B^* in Y with $(f(x))_\lambda\, q\, B^*$ there exists open set B such that BqA and $f(B) \subseteq B^*$

$\Rightarrow B \subseteq f^{-1}(B^*) \subseteq f^{-1}(A^*)$

$\Rightarrow B \subseteq f^{-1}(A^*)$

$\Rightarrow f(B) \subseteq A^*$

Thus we have τ_2^*-open set B with BqA and so by definition of mixed topology, we have

τ_1 closure, $\overline{B} \subseteq \overline{f^{-1}(A^*)}$

Hence $f^1(A^\star)$ is $\tau_1(\tau_2)$-open, and so $f: X \rightarrow Y$ is continuous.

This completes the proof.

Theorem 3.2. Let (X, τ_1, τ_2) and (Y, τ_1^*, τ_2^*) be any two fuzzy bi-topological spaces and f: X→ Y be a mapping such that f is τ_1-τ_1^* and τ_2-τ_2^* weakly continuous. then f is $\tau_1(\tau_2)$-$\tau_1^*(\tau_2^*)$ weakly continuous.

Proof. Let B be any $\tau_1^*(\tau_2^*)$ fuzzy open set in Y. We show that $cl\ (f^1(B)) \subseteq f^{-1}(clB)$, in this case the closure is with respect to $\tau_1^*(\tau_2^*)$. We have $B \in \tau_1^*(\tau_2^*)$. Let BqV for any fuzzy set V in Y. Then there exist τ_2^*-open set U such that

UqV and τ_1^* -closure $cl\ (U) \subseteq B$ (1)

Again given that f is τ_2-τ_2^* weakly continuous and U is τ_2^*-open fuzzy set in Y so by definition of weakly continuity we have

$cl(f^{-1}(B)) \subseteq f^{-1}(clB)$ (2)

Also cl (U) is τ_1^*-closed fuzzy set in Y, so $co_Y(clU)$ is τ_1^*-fuzzy open set in Y. Hence f is $\tau_1 - \tau_1^*$ weakly continuous. By definition of weakly continuity we have

$cl\ f^{-1}(co_Y(cl(U)) \subseteq f^{-1}(co_Y(cl(U))$ (3)

Taking complement of both side of (3), we get

$f^{-1}(cl(U)) \subseteq int(f^{-1}(cl(U))$ (4)

Now from (1) we have $f^{-1}(cl(U)) \subseteq f^{-1}(B)$

Therefore using (2) we have

$cl(f^{-1}(U)) \subseteq f^{-1}(B)$ (5)

Further,

$int(f^{1}(cl(U)) \subseteq f^{-1}(cl(U))$ (6)

Therefore using (4) and (6) we have

$f^{-1}(cl(U) = int\ (f^{-1}(cl(U)))$ (7)

Again from (1) we have

$cl(U) \subseteq B$.

$\Rightarrow f^{-1}(cl(U)) \subseteq f^{-1}(B)$.

$\Rightarrow int(f^{-1}(cl(U)) \subseteq f^{-1}(B)$.

Taking closure of both side, we get

$$cl(f^{-1}(B) \supseteq cl(\text{int}(f^{-1}(cl(U)) = cl(f^{-1}(cl(U)$$
$$= f^{-1}(cl(U))$$

Hence $cl(f^1(B)) \subseteq f^1(clB) \Rightarrow f$ is $\tau_1(\tau_2)$ - $\tau_1^\star(\tau_2^\star)$ is continuous.

This completes the proof.

Theorem 3.3. Let (X, τ_1, τ_2) and (Y, $\tau_1^\star$, $\tau_2^\star$) be two fuzzy bi-topological spaces and $\tau_1^* \subset \tau_2^*$ and τ_1^* is fuzzy regular space. If f: X→Y is $\tau_1(\tau_2)$-$\tau_1^*(\tau_2^*)$ weakly continuous, then f is $\tau_2 - \tau_1^*$ weakly continuous.

Proof: Let B^* be any τ_1^* -fuzzy open set in Y.

We show that $cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*)$

By hypothesis we have $\tau_1^* \subset \tau_2^*$ and τ_1^* is fuzzy regular space, therefore

$\tau_1^* \subset \tau_1^*(\tau_2^*)$

$\Rightarrow$ B^* is $\tau_1^*(\tau_2^*)$ fuzzy open set in Y.

Since f: $X \rightarrow Y$ is $\tau_1(\tau_2)$-$\tau_1^*(\tau_2^*)$ weakly continuous, we have

$cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*)$ (8)

Also we know that $\tau_1(\tau_2) \subseteq \tau_2$, thus the result (8) is true for τ_2 -fuzzy topology also.

Therefore $cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*)$, closure being with respect to τ_2 topology and τ_1^*-fuzzy topology. Hence f is $\tau_2 - \tau_1^*$ weakly continuous.

This completes the proof.

Theorem 3.4. Let (X, τ_1, τ_2) and (Y, τ_1^*, τ_2^*) be two fuzzy bi-topological spaces, If f: X→Y is $\tau_1(\tau_2)$-$\tau_1^*(\tau_2^*)$ weakly continuous. Then f is $\tau_2 - \tau_1^*(\tau_2^*)$ weakly continuous.

Proof: Let B^* be any $\tau_1^*(\tau_2^*)$ open fuzzy set in Y.

We show that $cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*))$, the closure is being with respect to τ_2.

Let f: $X \to Y$ be $\tau_1(\tau_2)$ - $\tau_1^*(\tau_2^*)$ weakly continuous. Then $cl(f^{-1}(B^*) \subseteq f^{-1}(cl(B^*)$ closures are being with respect to $\tau_1(\tau_2)$ and $\tau_1^*(\tau_2^*)$ respectively.

We know that the mixed topology $\tau_1(\tau_2)$ is contained in τ_2.

Since $cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*))$, closure being with respect to $\tau_1(\tau_2)$.

$\Rightarrow$ $cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*))$, closure of the left hand side being with respect to τ_2.

$\Rightarrow$ f is $\tau_2 - \tau_1^*(\tau_2^*)$ weakly continuous.

This completes the proof.

Theorem 3.5. Let (X, τ_1, τ_2) and (Y, τ_1^*, τ_2^*) be two fuzzy bi-topological spaces. If f: X→Y is $\tau_1 - \tau_1^*$ weakly continuous and τ_1 is fuzzy regular, then f is $\tau_1(\tau_2) - \tau_1^*$ weakly continuous.

Proof: Let B^* be any τ_1^* fuzzy open set in Y.

Let f be $\tau_1 - \tau_1^*$ weakly continuous. Then by definition of weakly continuity, we have $cl(f^{-1}(B^*) \subseteq f^{-1}(cl(B^*)$ closure of left hand side is with respect to τ_1 and right hand side is with respect to τ_1^*.

Further, τ_1 is fuzzy regular and $\tau_1 \subset \tau_2$ and therefore $\tau_1 \subset \tau_1(\tau_2)$.

Thus closure of $f^{-1}(B^*)$ is with respect to τ_1 is also the closure of with respect to $\tau_1(\tau_2)$. Hence $cl(f^{-1}(B^*) \subseteq f^{-1}(cl(B^*)$, the closure of L.H.S. being with respect to $\tau_1(\tau_2)$. Hence f is $\tau_1(\tau_2) - \tau_1^*$ weakly continuous.

This completes the proof.

Theorem 3.6. Let (X, τ_1, τ_2) and (Y, τ_1^*, τ_2^*) be two fuzzy bi-topological spaces. If f: X→Y is $\tau_1(\tau_2) - \tau_1^*$ weakly continuous, then f is $\tau_2 - \tau_1^*$ continuous

Proof: Let f: $X \to Y$ be $\tau_1(\tau_2) - \tau_1^*$ weakly continuous. Thus we have for any τ_1^*-fuzzy open set B^* in Y, we have $cl(f^{-1}(B^*) \subseteq f^{-1}(cl(B^*)$, closure of the L.H.S. is being with respect to $\tau_1(\tau_2)$.

We know that $\tau_1(\tau_2) \subset \tau_2$

Thus, closure of $f^{-1}(B^*)$ with respect to $\tau_1(\tau_2)$ is same as the closure of $f^{-1}(B^*)$ with respect to τ_2.

Thus $cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*))$, the closure of left hand side is with respect to the fuzzy topology τ_2.

$\Rightarrow$ f is $\tau_2 - \tau_1^*$ weakly continuous.

This completes the proof.

Theorem 3.7. Let (X, τ_1, τ_2) and (Y, τ_1^*, τ_2^*) be two fuzzy bi-topological spaces. If f: X→Y is $\tau_1(\tau_2) - \tau_2^*$ weakly continuous, then f is $\tau_2 - \tau_2^*$ continuous.

Proof: Let B^* be τ_2^*-fuzzy open set in Y.

Let f: $X \to Y$ be $\tau_1(\tau_2) - \tau_2^*$ weakly continuous. Then we have $cl(f^{-1}(B^*) \subseteq f^{-1}(cl(B^*)$, the closure of left hand side being with respect to $\tau_1(\tau_2)$.

Further we have $\tau_1(\tau_2) \subset \tau_2$

Therefore closure of $f^{-1}(B^*)$ with respect to $\tau_1(\tau_2)$ is same as the closure of $f^{-1}(B^*)$ with respect to τ_2.

Hence $cl(f^{-1}(B^*)) \subseteq f^{-1}(cl(B^*))$, the closure of left hand side is being with respect to τ_2.

Thus f is $\tau_2 - \tau_2^*$ weakly continuous.

This completes the proof.

Conclusion

We have introduced fuzzy weak continuity in mixed fuzzy topological space and have investigated its different properties. The results of this article can be applied for futher investigations and applications in studying different properties weak continuity of functions in mixed fuzzy topological spaces.

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